CHAPTER 2

Publications based on this Chapter;


Chapter 2

Unsteady Flow of a Conducting Dusty Fluid Between Two Parallel Plates

2.1 Introduction

The influence of dust particles on viscous flows has great importance in petroleum industry and in the purification of crude oil. Other important applications of dust particles in boundary layer, include soil erosion by natural winds and dust entrainment in a cloud during nuclear explosion. Also such flows occur in a wide range of areas of technical importance like fluidization, flow in rocket tubes, combustion, paint spraying and more recently blood flows in capillaries.

Considerable work has already been done on models of dusty fluid flow. P.G.Saffman [77] formulated the basic equations for the flow of dusty fluid. Regarding the plate problems, Lokenath Debnath et al [44], Liu [43], Michael et.al., [46], [47] have studied the flow produced by the motion of an infinite plane in a steady fluid occupying the semi-infinite space above it. Later, M.C.Baral [11] has discussed the plane parallel flow of conducting
dusty gas. To investigate the kinematical properties of fluid flows in the field of fluid mechanics some researchers like Kanwal [41], Truesdell [89], Indrasena [38], Purushotham et al [58], Bagewadi et al [7,8,9] have applied differential geometry techniques. Further, recently the authors [8,9] have studied two-dimensional dusty fluid flow in Frenet frame field system. The present chapter considers the flow of a conducting viscous incompressible fluid with embedded nonconducting identical spherical particles between two infinite parallel plates under the influence of constant magnetic field. Initially the fluid and dust particles are assumed to be at rest. Applying Laplace transform technique, the velocity fields for fluid and dust particles have been obtained. Finally the changes in the velocity profiles at of fluid and dust particles are shown graphically.

2.2 Equations of Motion

The equations of motion of unsteady viscous incompressible fluid with uniform distribution of dust particles are given by

For fluid phase

\[ \nabla \cdot \mathbf{u} = 0, \quad (\text{Continuity}) \tag{2.2.1} \]

\[ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} \]

\[ + \frac{k N}{\rho} (\mathbf{v} - \mathbf{u}) + \frac{1}{\rho} (\mathbf{J} \times \mathbf{B}) \quad (\text{Linear Momentum}) \tag{2.2.2} \]

For dust phase

\[ \nabla \cdot \mathbf{v} = 0, \quad (\text{Continuity}) \tag{2.2.3} \]

\[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = \frac{k}{m} (\mathbf{u} - \mathbf{v}) \quad (\text{Linear Momentum}) \tag{2.2.4} \]
We have following nomenclature:

\( \vec{u} \) — velocity of the fluid phase, \( \vec{v} \) — velocity of the dust phase, \( \rho \) — density of the gas, \( p \) — pressure of the fluid, \( N \) — number density of dust particles, \( \nu \) — kinematic viscosity, \( k = 6\pi a \mu \) — Stoke's resistance (drag coefficient), \( a \) — spherical radius of dust particle, \( m \) — mass of the dust particle, \( \mu \) — the co-efficient of viscosity of fluid particles, \( t \) — time and \( \vec{J} \) and \( \vec{B} \) given by Maxwell’s equations and Ohm’s law, namely,

\[
\nabla \times \vec{H} = 4\pi \vec{J}, \quad \nabla \times \vec{B} = 0, \quad \nabla \times \vec{E} = 0, \quad \vec{J} = \sigma(\vec{E} + \vec{u} \times \vec{B})
\]

Here \( \vec{H} \) — magnetic field, \( \vec{J} \) — current density, \( \vec{B} \) — magnetic Flux, \( \vec{E} \) — electric field and \( \sigma \) — the electrical conductivity of the fluid.

It is assumed that the effect of induced magnetic fields produced by the motion of the electrically conducting gas is negligible and no external electric field is applied. With those assumptions the magnetic field \( \vec{J} \times \vec{B} \) of the body force in (2.2.2) reduces simply to \(-\sigma B_0^2 \vec{u}\), where \( B_0 \)—the intensity of the imposed transverse magnetic field.

### 2.3 Formulation of the Problem

The present discussion considers a viscous incompressible, dusty fluid bounded by two infinite flat plates in which one plate is moving with the constant speed \( u_0 \). Both the fluid and the dust particle clouds are supposed to be static at the beginning. The dust particles are assumed to be spherical in shape and uniform in size. The number density of the particles is taken as a constant throughout the flow. It is assumed that the dust particles are electrically nonconducting and neutral. The motion of the dusty fluid is due
to magnetic field of constant strength $B_0$. Under these assumptions the flow will be a parallel flow in which the streamlines are along the tangential direction as in figure 2.1.

\[ b = h \]

\[ b = 0 \]

Figure 2.1: Schematic diagram of dusty fluid flow.

For the above described flow the velocities of fluid and dust are of the form

\[ \vec{u} = u_b \hat{b}, \quad \vec{v} = v_b \hat{b} \]  

(2.3.1)

i.e., $u_s = u_n = 0$ and $v_s = v_n = 0$, where $(u_s, u_n, u_b)$ and $(v_s, v_n, v_b)$ denote the velocity components of fluid and dust respectively.

Since the flow is in between two moving plates, we can assume the velocity of both fluid and dust particles do not vary along tangential direction. Suppose the fluid extends to infinity in the principal normal direction, then the velocities of both may be neglected in this direction.
2.4 Solution of the Problem

By virtue of system of equations (1.3.1) the intrinsic decomposition of equations (2.2.2) and (2.2.4) give the following forms:

\[
\frac{\partial u_s}{\partial t} = \nu \left[ \frac{\partial^2 u_s}{\partial b^2} - C_r u_s \right] + \frac{kN}{\rho} (v_s - u_s) - D u_s \tag{2.4.1}
\]

\[
2u_s^2 k_s = \nu \left[ 2\sigma_b^2 \frac{\partial u_s}{\partial b} - u_s k_s^2 \right] \tag{2.4.2}
\]

\[
0 = \nu \left[ u_s k_s \tau_s - 2k^b \frac{\partial u_s}{\partial b} \right] \tag{2.4.3}
\]

\[
\frac{\partial v_s}{\partial t} = \frac{k}{m} (u_s - v_s) \tag{2.4.4}
\]

\[
2v_s^2 k_s = 0 \tag{2.4.5}
\]

where \( D = \frac{\sigma B^2}{\rho} \) and \( C_r = (\sigma_b^2 + k_n^2 + k_b^2 + \sigma_b^2) \) is called curvature number [9].

From equation (2.4.5) we see that \( v_s^2 k_s = 0 \) which implies either \( v_s = 0 \) or \( k_s = 0 \). The choice \( v_s = 0 \) is impossible, since if it happens then \( u_s = 0 \), which shows that the flow doesn’t exist. Hence \( k_s = 0 \), it suggests that the curvature of the streamline along tangential direction is zero. Thus no radial flow exists.

**Case-2.1:** Suppose the flow is due to the movement of upper plate in addition to the uniform magnetic field. Hence consider the boundary and initial conditions as

\[
\begin{align*}
\text{Initial condition:} & \quad \text{at} \ t = 0; \ u_s = 0, v_s = 0 \\
\text{Boundary condition:} & \quad \text{for} \ t > 0; \ u_s = 0, \ at \ b = 0 \text{ and } u_s = u_0 \text{ at } b = h
\end{align*}
\tag{2.4.6}
\]

We define Laplace transformations of \( u_s \) and \( v_s \) as

\[
U = \int_0^\infty e^{-xt}u_s dt \quad \text{and} \quad V = \int_0^\infty e^{-xt}v_s dt \tag{2.4.7}
\]
Chapter 2: Unsteady Flow of a Conducting Dusty Fluid Between Two Parallel Plates

Applying the Laplace transform to equations (2.4.1), (2.4.4) and to boundary conditions (2.4.6), then by using initial conditions one obtains

\[ xU = \nu \left[ \frac{\partial^2 U}{\partial b^2} - C_r U \right] + \frac{l}{\tau} (V - U) - DU \]  \hspace{1cm} (2.4.8)

\[ xV = \frac{1}{\tau} (U - V) \] \hspace{1cm} (2.4.9)

\[ U = 0, \text{ at } b = 0 \text{ and } U = \frac{u_0}{x} \text{ at } b = h \] \hspace{1cm} (2.4.10)

where \( l = \frac{mN}{\rho} \) and \( \tau = \frac{m}{k} \). Equation (2.4.9) implies

\[ V = \frac{U}{1 + x \tau} \] \hspace{1cm} (2.4.11)

Eliminating \( V \) from (2.4.8) and (2.4.11) we obtain the following equation

\[ \frac{d^2 U}{db^2} - Q^2 U = 0 \] \hspace{1cm} (2.4.12)

where \( Q^2 = \left( C_r + \frac{2}{\nu} + \frac{D}{\nu} + \frac{x l}{\nu (1 + x \tau)} \right) \).

The velocities of fluid and dust particle are obtained by solving the equation (2.4.12) subjected to the boundary conditions (6.3.5) as follows

\[ U = \frac{u_0}{x} \left\{ \frac{\sinh(Qb)}{\sinh(Qh)} \right\} \, . \]

Using \( U \) in (2.4.11) we obtain \( V \) as

\[ V = \frac{u_0}{x(1 + x \tau)} \left\{ \frac{\sinh(Qb)}{\sinh(Qh)} \right\} \, . \]
Chapter 2: Unsteady Flow of a Conducting Dusty Fluid Between Two Parallel Plates

By taking inverse Laplace transform to $U$ and $V$, one can obtain

$$u_s = u_0 \frac{\sinh(yb)}{\sinh(gh)} + \frac{2u_0\pi\nu}{h} \sum_{r=0}^{\infty} r(-1)^{r+1} \sin \left( \frac{r\pi}{h} \right)$$

$$\times \frac{e^{x_1t}(1 + x_1\tau)^2}{x_1 [(1 + x_1\tau)^2 + l]} + \frac{e^{x_2t}(1 + x_2\tau)^2}{x_2 [(1 + x_2\tau)^2 + l]}$$

$$v_s = u_0 \frac{\sinh(yb)}{\sinh(gh)} - u_0e^{-\tau}$$

$$+ \frac{2u_0\pi\nu}{h} \sum_{r=0}^{\infty} r(-1)^{r+1} \sin \left( \frac{r\pi}{h} \right)$$

$$\times \frac{e^{x_1t}(1 + x_1\tau)}{x_1 [(1 + x_1\tau)^2 + l]} + \frac{e^{x_2t}(1 + x_2\tau)}{x_2 [(1 + x_2\tau)^2 + l]}$$

Shearing Stress/Skin Friction:

The shearing stress at the lower plate and upper plate is given by

$$D_0 = \frac{\mu u_0 y}{\sinh(gh)}$$

$$+ \frac{2\mu u_0\pi^2\nu}{h^2} \sum_{r=0}^{\infty} r^2(-1)^{r+1} \left[ \frac{e^{x_1t}(1 + x_1\tau)^2}{x_1 [(1 + x_1\tau)^2 + l]} + \frac{e^{x_2t}(1 + x_2\tau)^2}{x_2 [(1 + x_2\tau)^2 + l]} \right]$$
Chapter 2: Unsteady Flow of a Conducting Dusty Fluid Between Two Parallel Plates

\[ D_h = \mu u_0 y \left\{ \frac{\cosh(yh)}{\sinh(yh)} \right\} \]

\[ + \frac{2\mu u_0 \pi^2 \nu}{h^2} \sum_{r=0}^{\infty} r^2 (-1)^{2r+1} \left\{ \frac{e^{x_1t}(1 + x_1\tau)^2}{x_1 [(1 + x_1\tau)^2 + l]} + \frac{e^{x_2t}(1 + x_2\tau)^2}{x_2 [(1 + x_2\tau)^2 + l]} \right\} \]

Case-2.2: Assume that the flow is due to the movement of both lower and upper plates with different uniform speed in addition to the applied magnetic field. Hence for this case the boundary and initial conditions can be taken as

\[ \begin{align*}
\text{Initial condition; } & at \ t = 0; \ u_s = 0, v_s = 0 \\
\text{Boundary condition; } & for \ t > 0; \ u_s = u_0, \ at \ b = 0 \ and \ u_s = v_0 \ at \ b = h
\end{align*} \]

(2.4.13)

where \( u_0 \) and \( v_0 \) are some constants.

Now using Laplace transform technique as in the case 2.1 one can get the solution for both fluid and dust phase velocity as

\[ u_s = \frac{v_0 \sinh(yb) - u_0 \sinh(y(b - h))}{\sinh(yh)} \]

\[ + \frac{2\pi \nu}{h} \sum_{r=0}^{\infty} r(-1)^r \left[ u_0 \sin \left( \frac{r\pi}{h} (b - h) \right) - v_0 \sin \left( \frac{r\pi}{h} b \right) \right] \]

\[ \times \left\{ \frac{e^{x_1t}(1 + x_1\tau)^2}{x_1 [(1 + x_1\tau)^2 + l]} + \frac{e^{x_2t}(1 + x_2\tau)^2}{x_2 [(1 + x_2\tau)^2 + l]} \right\} \]
Chapter 2: Unsteady Flow of a Conducting Dusty Fluid Between Two Parallel Plates

\[ \begin{align*}
\dot{v}_s &= \frac{v_0 \sinh(yb) - u_0 \sinh(y(b - h))}{\sinh(yh)} + (u_0 - v_0) e^{-\frac{r}{\nu}} \\
&+ \frac{2\pi\nu}{h} \sum_{r=0}^{\infty} r(-1)^r \left[ u_0 \sin \left( \frac{r\pi}{h} (b - h) \right) - v_0 \sin \left( \frac{r\pi}{h} b \right) \right] \\
&\times \left[ \frac{e^{x_1 t}(1 + x_1 \tau)}{x_1 [(1 + x_1 \tau)^2 + l]} + \frac{e^{x_2 t}(1 + x_2 \tau)}{x_2 [(1 + x_2 \tau)^2 + l]} \right]
\end{align*} \]

Shearing Stress/Skin Friction:

The shearing stress at the lower plate and upper plate is given by

\[ D_0 = \frac{\mu y}{\sinh(yh)} [v_0 - u_0 \cosh(yh)] \]

\[+ \frac{2\mu \pi^2 \nu}{h^2} \sum_{r=0}^{\infty} r^2 (u_0 - v_0) \left[ \frac{e^{x_1 t}(1 + x_1 \tau)}{x_1 [(1 + x_1 \tau)^2 + l]} + \frac{e^{x_2 t}(1 + x_2 \tau)}{x_2 [(1 + x_2 \tau)^2 + l]} \right] \]

\[ D_h = \frac{\mu y}{\sinh(yh)} [v_0 \cosh(yh) - u_0] \]

\[+ \frac{2\mu \pi \nu}{h} \sum_{r=0}^{\infty} r^2 \left( (-1)^r u_0 - v_0 \frac{r\pi}{h} \right) \left[ \frac{e^{x_1 t}(1 + x_1 \tau)}{x_1 [(1 + x_1 \tau)^2 + l]} + \frac{e^{x_2 t}(1 + x_2 \tau)}{x_2 [(1 + x_2 \tau)^2 + l]} \right] \]
where

\[ x_1 = -\frac{1}{2\tau} \left( 1 + l + D\tau + \nu C_r\tau + \nu \tau \frac{r^2\pi^2}{h^2} \right) \]

\[ + \frac{1}{2\tau} \sqrt{\left( 1 + l + D\tau + \nu C_r\tau + \nu \tau \frac{r^2\pi^2}{h^2} \right)^2 - 4\tau \left( C_r\nu + D + \nu \tau \frac{r^2\pi^2}{h^2} \right)} \]

\[ x_2 = -\frac{1}{2\tau} \left( 1 + l + D\tau + \nu C_r\tau + \nu \tau \frac{r^2\pi^2}{h^2} \right) \]

\[ - \frac{1}{2\tau} \sqrt{\left( 1 + l + D\tau + \nu C_r\tau + \nu \tau \frac{r^2\pi^2}{h^2} \right)^2 - 4\tau \left( C_r\nu + D + \nu \tau \frac{r^2\pi^2}{h^2} \right)} \]

\[ y = \frac{C_r\nu + D}{\nu} \]

### 2.5 Conclusion

The velocity profiles for the fluid and dust particles are drawn as in figures 2.1 and 2.2, which are parabolic. According Frenet approximation of a curve in the osculating plane, the path of the curve near origin is parabolic. Hence the results obtained here are analogous to the above [10]. It is concluded that velocity of fluid particles is parallel to velocity of dust particles. Also it is evident from the graphs that, as we increase the strength of the magnetic field, it has an appreciable effect on the velocities of fluid and dust particles. Further one can observe that if the magnetic field is zero then results are in agreement with the plane Couette flow. Also one can find that the drag on the lower plate and the total volume flow in between the plates decreases as magnetic field increases.

If \( B_0 = 0 \), and the plates are vibrating then the results analogous to [65]. Also if we
consider the fluid in absence of pressure gradient and in the presence of magnetic field in [28] the results satisfied with this paper.

Figure-2.2: Variation of fluid and dust phase velocity (case-2.1).
Figure-2.3: Variation of fluid and dust phase velocity (case-2.2).