CHAPTER 6

Publications based on this Chapter;

• *Effect of radiation on hydromagnetic flow and heat transfer of a Dusty fluid between two parallel plates*, COMMUNICATED
Chapter 6

Effect of Radiation on Hydromagnetic Flow and Heat Transfer of a Dusty fluid between Two Parallel plates

6.1 Introduction

The fluid flow between two parallel plates in the presence of transversely applied magnetic field is a classical problem that has encountered in a variety of applications in many devices such as magnetohydrodynamic (MHD) power generators, MHD pumps, aerodynamics heating, electrostatic precipitation, purification of molten metals from non-metallic inclusions and fluid droplets-sprays. Also it has encountered in many material processing applications such as extrusion, metal forming, continuous casting, wire and glass fiber drawing, etc. The different physical effects of Hartmann flow on a Newtonian fluid with heat transfer have been studied by many authors like Alpher [5], Attia et al. [9]. The results obtained in these papers are important for the design of the duct wall
and the cooling arrangements. Bhargava et al. [19] investigated the effect of Hall currents on the MHD flow and heat transfer of a second order fluid between two parallel porous plates. Bodosa et al. [20] discussed the MHD Coutte flow with heat transfer between two horizontal plates in the presence of a uniform transverse magnetic field.

The study of the flow of dusty fluid has attracted the attention of many authors like Prasad et al. [79], Mitra et al. [74], Saxena et al. [102] and Debnath et al. [32] due to its important applications in the fields of fluidization, combustion, use of dust in gas cooling systems, centrifugal separation of matter from fluid, petroleum industry, polymer technology and electrostatic precipitation. Ganguly et al. [43] have determined the oscillatory motion of dusty viscous incompressible fluid between two parallel plates. Datta et al. [27] obtained the solution for an unsteady flow and heat transfer of a dusty viscous incompressible fluid in a channel. Heat transfer in unsteady MHD oscillatory flow was analyzed by Anjali Devi et al. [8]. Ghosh et al. [44] obtained the solution of hydromagnetic flow of a dusty visco-elastic fluid between two infinite parallel plates.

Attia [57] has obtained the numerical solution of an unsteady MHD Coutte flow and heat transfer of dusty fluid between parallel plates by varying some variable physical properties. Ganesh et al. [42] studied an unsteady Magnetohydrodynamic stokes flow of viscous fluid between two parallel porous plates. A note on an unsteady flow of viscous fluid due to an oscillating plane wall was analyzed by Erdogan [38]. Gireesha et al. [48] investigated the flow of an unsteady dusty fluid through rectangular channel under varying pulsatile pressure gradient, in frenet frame field system. Sreeharireddy et al. [104] studied the MHD flow of a dusty viscous conducting liquid between two parallel plates. Ajadi
[108] obtained an analytical solutions of unsteady oscillatory particulate visco-elastic fluid between two walls.

The effect of thermal radiation is important in many engineering applications such as in advanced types of power plants for nuclear rockets, high-speed flights, re-entry vehicles and processes involving high temperatures. Elsayed et al. [37] investigated simultaneous convection and radiation in flow between two parallel plates. Mebine [70] discussed the effect of radiation on MHD Couette flow with heat transfer between two parallel plates. Fahad et al. [40] studied the combined effects of forced convection and surface radiation between two parallel plates.

In view of the above discussions, present chapter is concentrated to investigate the effect of radiation on flow and heat transfer of a dusty fluid between two parallel plates. Here, the flow analysis is obtained for four different cases of boundary conditions. In all the four cases, the fluid is flowing between two infinite parallel plates maintained at constant different temperatures and the fluid is acted upon by a pulsatile pressure gradient and an external uniform magnetic field is applied perpendicular to the plates. The flow and temperature distributions of both fluid and dust phase particles are governed by the coupled set of the momentum and energy equations. The radiation effect in energy equation is taken into consideration. The governing coupled partial differential equations are solved numerically using finite difference method with the help of Matlab software. Further the effect of the magnetic field, number density on velocity and Prandtl number, Eckert number and Radiation parameter on temperature profiles for both the fluid and dust particles are discussed.
6.2 Equations of Motion

The governing equations of motion for fluid and dust are (2.2.2) and (2.2.4) respectively, and energy for two phases are given by

For fluid phase:
\[
\rho \left\{ \frac{\partial E}{\partial t} + (\vec{u} \cdot \nabla) E \right\} = Q + (\vec{v} - \vec{u}) \cdot F + k \nabla \cdot (\nabla T) - \frac{\partial q_r}{\partial y}, \quad \text{(Energy)} (6.2.1)
\]

For dust phase:
\[
N \left\{ \frac{\partial E_p}{\partial t} + (\vec{u} \cdot \nabla) E_p \right\} = -Q, \quad \text{(Energy)} \quad (6.2.2)
\]

where \( E = c_p T, \ E_p = c_m T_p, \ Q = N c_p (T_p - T) / \tau_T \) is the thermal interaction between fluid and dust particle phase, \( F = N (\vec{v} - \vec{u}) / \tau_v \) is the velocity interaction force between the fluid and dust particle phase, \( \tau_v = m/6 \pi a \mu = m/k \) is the velocity relaxation time of the dust particles, \( \tau_T = m c_p / 4 \pi a k \) is the thermal relaxation time of the dust particles, \( k \nabla \cdot (\nabla T) \) is the rate of heat added to the fluid by conduction in unit volume, \( \vec{u}, \rho, p, v, T, c_p \) & \( k \) are respectively, the velocity vector, density, pressure, kinematic viscosity, temperature, specific heat and thermal conductivity of the fluid, \( \vec{v}, N, T_p, c_m \) & \( m \) are respectively, the velocity vector, number density, temperature, specific heat and mass concentration of dust particles, \( k \) the Stoke’s resistance co-efficient (for spherical particles of radius \( a \) is \( 6 \pi a \mu \)), \( t \) is the time and \( \vec{J} \) and \( \vec{B} \) given by Maxwell’s equations and Ohm’s law, namely,
\[
\nabla \times \vec{H} = 4 \pi \vec{J}, \quad \nabla \times \vec{B} = 0, \quad \nabla \times \vec{E} = 0, \quad \vec{J} = \sigma [\vec{E} + \vec{u} \times \vec{B}].
\]

Here \( \vec{H} \) - magnetic field, \( \vec{J} \) - current density, \( \vec{B} \) - magnetic Flux, \( \vec{E} \) - electric field and \( \sigma \) - the electrical conductivity of the fluid.
It is assumed that the effect of induced magnetic fields produced by the motion of the electrically conducting gas is negligible and no external electric field is applied. With those assumptions the magnetic field $\mathbf{J} \times \mathbf{B}$ of the body force in (5.2.2) reduces simply to $-\sigma B_0^2 \mathbf{\hat{u}}$, where $B_0$ - the intensity of the imposed transverse magnetic field.

Using the Rosseland approximation for radiation [22], radiative heat flux $q_r$ is simplified as

$$q_r = -\frac{4\sigma^* \partial T^4}{3k_1 \partial y},$$

(6.2.3)

where $\sigma^*$ is the Stefan-Boltzmann constant and $k_1$ is the mean absorption coefficient. Assuming that the differences in the temperature within the flow such that the term $T^4$ can be expressed as linear combination of the temperature, we expand $T^4$ in a Taylor's series about $T_\infty$ as follows

$$T^4 = T^4_\infty + 4T^3_\infty(T - T_\infty) + 6T^2_\infty(T - T_\infty)^2 + \cdots$$

(6.2.4)

and neglecting higher order terms beyond the first degree in $(T - T_\infty)$, we get

$$T^4 \approx -3T^4_\infty + 4T^3_\infty T.$$  

(6.2.5)

Substituting equation (6.2.5) in the equation (6.2.3), we obtain

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T^3_\infty}{3k_1} \frac{\partial^2 T}{\partial y^2}.$$  

(6.2.6)
6.3 Formulation of the Problem

Consider an unsteady laminar flow and heat transfer of an incompressible, viscous dusty fluid between two parallel plates. It is assumed that, the two plates are electrically non-conducting and kept at constant temperatures $T_0$ for the lower plate and $T_1$ for the upper plate with $T_1 > T_0$. The flow is due to the influence of the pulsatile pressure gradient and uniform magnetic field are applied in positive $x$ and $y$-directions respectively. A uniform magnetic field of strength $B_0$ is imposed normal to the plate. Both the fluid and the dust particle clouds are suppose to be static at the beginning. The dust particles are assumed to be spherical in shape and uniform in size and number density of the dust particles is taken as a constant throughout the flow. Under these assumptions the geometry of the flow configuration is shown as in the figure-1.

![Figure 6.1: Geometry of the flow configuration.](image)
For the above described flow the velocities of both fluid and dust particles are given by

\[ \overline{u} = u(y, t)i, \quad \overline{v} = v(y, t)i. \]

### 6.4 Solution of the Problem

The governing equations from (2.2.2), (2.2.4), (6.2.1) and (6.2.2) can be decomposed as,

**For fluid phase:**

\[
\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} + \frac{k N}{\rho} (v - u) - \frac{\sigma B_0^2}{\rho} u,
\]

\[
\rho c_p \frac{\partial T}{\partial t} = \left( k + \frac{16 \sigma T^3}{3 k_1} \right) \frac{\partial^2 T}{\partial y^2} + \frac{N c_p}{\tau_T} (T_p - T) + \frac{k N}{m} (v - u)^2,
\]

**For dust phase:**

\[
\frac{\partial v}{\partial t} = \frac{k}{m} (u - v),
\]

\[
\frac{c_m}{\tau_T} \frac{\partial T_p}{\partial t} = -\frac{c_p}{\tau_T} (T_p - T),
\]

where \( u(y, t) \) and \( v(y, t) \) denote the velocity of the fluid and of the dust phase respectively.

The initial and boundary conditions on the velocity fields are taken as

**Case (i):**

\[
\text{Initial conditions : } u = 0, \ v = 0 \quad \text{at} \quad t \leq 0 \quad \text{for all} \ y,
\]

\[
\text{Boundary conditions : } u = U_i \quad \text{at} \quad y = 0 \quad \text{for} \ t > 0,
\]

\[
\quad u = V_i \quad \text{at} \quad y = h.
\]
Here the lower and upper plates start moving with uniform velocities $U_i$ and $V_i$ respectively. **Case (ii):**

**Initial conditions:** $u = 0, \ v = 0 \ at \ t \leq 0 \ for \ all \ y,$

**Boundary conditions:** $u = U_i e^{-\lambda t} \ at \ y = 0 \ for \ t > 0,$ \hspace{1cm} (6.4.6)

$u = V_i e^{-\lambda t} \ at \ y = h.$

where $\lambda$ is constant.

**Case (iii):**

**Initial conditions:** $u = 0, \ v = 0 \ at \ t \leq 0 \ for \ all \ y,$

**Boundary conditions:** $u = U_i \cos (\lambda t) \ at \ y = 0 \ for \ t > 0,$ \hspace{1cm} (6.4.7)

$u = V_i \cos (\lambda t) \ at \ y = h.$

**Case (iv):**

**Initial conditions:** $u = 0, \ v = 0 \ at \ t \leq 0 \ for \ all \ y,$

**Boundary conditions:** $u = U_i + \alpha \cos (\beta t) \ at \ y = 0 \ for \ t > 0,$ \hspace{1cm} (6.4.8)

$u = V_i + \alpha \cos (\beta t) \ at \ y = h.$

The common initial and boundary conditions of the temperature fields for the above four cases are

**Initial conditions:** $T = T_p = T_0 \ at \ t \leq 0,$

**Boundary conditions:** $T = T_p = T_0 \ at \ y = 0, \ t > 0,$ \hspace{1cm} (6.4.9)

$T = T_p = T_1 \ at \ y = h.$
It is assumed that the pulsatile pressure gradient is influenced on the flow and it is of the form,

\[
- \frac{1}{\rho} \frac{\partial p}{\partial x} = A[1 + \epsilon e^{i\sigma t}],
\]

(6.4.10)

where \( \epsilon \) is a small quantity, \( A \) and \( \sigma \) are constants.

Let us consider the following non-dimensional flow variables as

\[
\bar{u} = \frac{u}{\nu}, \quad \bar{v} = \frac{v}{\nu}, \quad \bar{t} = \frac{vt}{h^2}, \quad \bar{x} = \frac{x}{h}, \quad \bar{y} = \frac{y}{h},
\]

\[
\bar{p} = \frac{\rho h^2}{\nu^2}, \quad \theta = \frac{T - T_0}{T_1 - T_0}, \quad \theta_p = \frac{T_p - T_0}{T_1 - T_0},
\]

(6.4.11)

where \( \theta \) and \( \theta_p \) are the dimensionless fluid and dust phase temperatures.

Using (6.4.11) in the equations (6.4.1)-(6.4.4), then one can get the following non-dimensionalized forms of the equations (on dropping the bars) as follows

\[
\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \frac{f}{\tau_v} (v - u) - M^2 u,
\]

(6.4.12)

\[
\frac{\partial v}{\partial t} = \frac{1}{\tau_v} (u - v),
\]

(6.4.13)

\[
\frac{\partial \theta}{\partial t} = \frac{(1 + R)}{Pr} \frac{\partial^2 \theta}{\partial y^2} + \frac{l_1}{\tau_T} (\theta_p - \theta) + \frac{l_1 N}{\tau_v} Ec (v - u)^2,
\]

(6.4.14)

\[
\frac{\partial \theta_p}{\partial t} = l_2 (\theta - \theta_p),
\]

(6.4.15)

where \( f = \frac{N m}{\rho} \) the mass concenration of dust particles, \( \tau_v = \frac{\nu}{kh^2} \) relaxation time of particles, \( M = B_0 h \sqrt{\frac{\nu}{\mu}} \) Hartmann number, \( l_1 = \frac{h^2}{\mu} \), \( l_2 = \frac{h^2}{c_m \tau_T} \), \( Pr = \frac{\nu c_p}{\kappa} \) Prandtl number, \( Ec = \frac{h^2}{k c_p (T_1 - T_0)} \) Eckert number and \( R = \frac{16 \sigma^2 T_0^3}{3 \kappa \epsilon k} \) radiation parameter.

The non-dimensional form of initial and boundary conditions on the velocity fields are
For Case (i)

Initial conditions: \( u = 0, \ v = 0, \ \text{for} \ t \leq 0, \)

Boundary conditions: \( u = U_2 \) at \( y = 0, \) \hfill (6.4.16)
\[
\begin{align*}
\frac{u}{u} &= V_2 \quad \text{at} \quad y = 1.
\end{align*}
\]

where \( U_2 = \frac{b}{\nu} U_1 \) and \( V_2 = \frac{b}{\nu} V_1. \)

For Case (ii)

Initial conditions: \( u = 0, \ v = 0, \ \text{for} \ t \leq 0, \)

Boundary conditions: \( u = U_2 e^{-\omega t} \) at \( y = 0, \) \hfill (6.4.17)
\[
\begin{align*}
\frac{u}{u} &= V_2 e^{-\omega t} \quad \text{at} \quad y = 1.
\end{align*}
\]

where \( \omega = \frac{\lambda h^2}{\nu}. \)

For Case (iii)

Initial conditions: \( u = 0, \ v = 0, \ \text{for} \ t \leq 0, \)

Boundary conditions: \( u = U_2 \cos (\omega t) \) at \( y = 0, \) \hfill (6.4.18)
\[
\begin{align*}
\frac{u}{u} &= V_2 \cos (\omega t) \quad \text{at} \quad y = 1.
\end{align*}
\]

For Case (iv)

Initial conditions: \( u = 0, \ v = 0, \ \text{for} \ t \leq 0, \)

Boundary conditions: \( u = U_2 + a \cos (bt) \) at \( y = 0, \) \hfill (6.4.19)
\[
\begin{align*}
\frac{u}{u} &= V_2 + a \cos (bt) \quad \text{at} \quad y = 1.
\end{align*}
\]

where \( a = \frac{a}{\nu} \) and \( b = \frac{b h^2}{\nu}. \)
In the same way, the dimensionless form of initial and boundary conditions on the temperature fields are

**Initial conditions**: \( \theta = \theta_p = 0 \) for \( t \leq 0 \),

**Boundary conditions**: \( \theta = \theta_p = 0 \) at \( y = 0 \),

\( \theta = \theta_p = 1 \) at \( y = 1 \).  

(6.4.20)

The non-dimensional form of pressure gradient as given by

\[
\frac{\partial p}{\partial x} = A \left[ 1 + \epsilon e^{\gamma t} \right],
\]

(6.4.21)

where \( \gamma = \frac{ab^2}{\nu} \).

### 6.5 Numerical Solution

The system of partial differential equations from (6.4.12) to (6.4.15) with the initial and boundary conditions (6.4.16) to (6.4.19) are solved numerically using finite difference technique with the help of MATLAB PDE solver. In the absence of pressure gradient, the equations (6.4.12) and (6.4.13) with the initial and boundary conditions (6.4.16) are solved analytically by Mitra and Bhattacharyya [74]. The accuracy of this method is verified by comparing our obtained numerical solutions with the analytical solution of [74] for all four cases, which are given in the table 1. From this table, we can observe that a very good agreement between the results. Further table 2 shows the variations of temperature of the fluid phase for different values of the parameters like Prandtl number, Eckert number, number density and Radiation parameter at \( y = 0.2 \). Analyzing this table
reveals that increase values of the Prandtl number and number density will decrease the
temperature of the fluid phase $\theta(y, t)$. The table 2 also shows that temperature between
the plates increases gradually with increase of Eckert number and radiation parameter in
all the cases. **Table-1:** Comparison of analytical and numerical solutions of fluid velocity
$u(y, t)$ for all the cases.

<table>
<thead>
<tr>
<th></th>
<th>Analytical Solution [32]</th>
<th>Case (i) $\lambda = 0$</th>
<th>Case (ii) $\lambda = 0$</th>
<th>Case (iii) $\alpha = 0$</th>
<th>Case (iv) $\alpha = 0$</th>
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### 6.6 Results and Discussion

Consider an unsteady flow and heat transfer of a dusty fluid between two non-conducting
parallel plates with the effect of radiation. Velocity profiles are obtained for four different
types of boundary conditions. The governing coupled partial differential equations from
(6.4.12) to (6.4.15) are solved numerically by applying finite difference method using
Matlab software. Results show that the fields were influenced appreciably by the effects
of the flow parameters like Hartmann number $M$, Number density of the dust particles
$N$, Prandtl number $Pr$, Eckert number $Ec$ and radiation parameter $R$. The variation of
velocity and temperature profiles for the fluid and dust particles for all the four cases are
plotted in graphs 6.2 to 6.33, and they are parabolic nature. Further one can observe
that if the dust is very fine, relaxation time of dust particle decreases and ultimately as
\( \tau_v \to 0 \), the velocities of fluid and dust particles will be the same. Throughout the flow analysis we have consider \( U_1 = 1, V_1 = 2, \omega = 1, \epsilon = 0.2, m = 1, N = 10, \tau_v = 1, \tau_T = 0.5, c_p = c_m = 0.2 \) and \( t = 0.2 \).

The dimensionless velocity profiles for fluid and dust phase for different values of magnetic parameter \( M \) (Hartmann number) are illustrated in the figures 6.2 to 6.9 when \( N = 0.5 \). It is evident from the figures that, velocity decreases with the increase of magnetic parameter \( M \). This is due to the fact that the presence of a magnetic field normal to the flow in an electrically conducting fluid produces a Lorentz force, which acts against the flow. Also one can notice that the drag on the lower plate and total volume flow in between the plates decreases as magnetic field increases.

The velocity profiles of both fluid and dust phase against the vertical distance \( y \) for different values of Number density \( N \) are shown in the figures 6.10-6.17 when \( M = 0.2, Pr = 0.72, Ec = 1.0 \) and \( R = 2.0 \). Here one can note that the decreasing of velocity while the number density of the dust particles is increasing. This result is very relevant to the traffic problem when we consider the vehicles as dust particles. From the figures 6.18 – 6.21, one can observe that the temperature of fluid and dust particles decreases with the increase of \( N \).

Figures 6.22 – 6.25 are obtained by plotting the temperature distributions for both fluid and dust phase against variable \( y \) for different values of Prandtl number \( Pr \) for all the four cases when \( N = 0.5, M = 0.2, Ec = 1.0 \) and \( R = 2.0 \). Form these graphs, it is clear that the temperature distribution between the plates decreases gradually with the increase of \( Pr \). Also, the increase of Prandtl number means that the slow rate of thermal
diffusion.

Figures 6.26 – 6.29 depict the effect of Eckert number on temperature distribution $\theta(y, t)$ and $\theta_p(y, t)$ versus $y$ for the case (i)-(iv) respectively. By analysing these graphs, it reveals that the effect of increasing values of $Ec$ increases the temperature distributions in flow region in all the cases when $N = 0.5$, $M = 0.2$, $Pr = 0.72$ and $R = 2.0$. This is due to the fact that heat energy is stored in the liquid due to the frictional heating.

In cases (i)-(iv), figures 6.30 – 6.33 are plotted to demonstrate the influence of thermal radiation parameter $R$ on temperature profiles of both fluid and dust phase when $N = 0.5$, $M = 0.2$, $Pr = 0.72$ and $Ec = 2.0$. It is noted that the temperature between the plates increases with increase in the value of the thermal radiation parameter. This is due to the fact that the divergence of the radiative heat flux $\partial q_r / \partial y$ increases as the Rosseland radiative absorptivity $k_1$ decreases (see expression for $R$) which in turn increases the rate of radiative heat transfer to the fluid, which causes the fluid temperature to increase. In view of this fact, the effect of radiation becomes more significant as $R \to \infty$ and the radiation effect can be neglected when $R = 0$. 
Figure 6.2(a): Effect of Magnetic field on the velocity profiles for fluid phase (case (i)).

Figure 6.3(b): Effect of Magnetic field on the velocity profiles for dust phase (case (i)).
Figure 6.4(a): Effect of Magnetic field on the velocity profiles for fluid phase (case (ii)).

Figure 6.5(b): Effect of Magnetic field on the velocity profiles for dust phase (case (ii)).
Figure 6.6(a): Effect of Magnetic field on the velocity profiles for fluid phase (case (iii)).

Figure 6.7(b): Effect of Magnetic field on the velocity profiles for dust phase (case (iii)).
Figure 6.8(a): Effect of Magnetic field on the velocity profiles for fluid phase (case (iv)).

Figure 6.9(b): Effect of Magnetic field on the velocity profiles for dust phase (case (iv)).
Figure 6.10(a): Effect of Number density on the velocity profiles for fluid phase (case (i)).

Figure 6.11(b): Effect of Number density on the velocity profiles for dust phase (case (i)).
Figure 6.12(a): Effect of Number density on the velocity for fluid phase (case (ii)).

Figure 6.13(b): Effect of Number density on the velocity for dust phase (case (ii)).
Figure 6.14(a): Effect of Number density on the velocity for fluid phase (case (iii)).

Figure 6.15(b): Effect of Number density on the velocity for dust phase (case (iii)).
Figure 6.16(a): Effect of Number density on the velocity for fluid phase (case (iv)).

Figure 6.17(b): Effect of Number density on the velocity for dust phase (case (iv)).
Figure 6.18: Effect of Number density on the temperature for case (i).

Figure 6.19: Effect of Number density on the temperature for case (ii).
Figure 6.20: Effect of Number density on the temperature profiles for case (iii).

Figure 6.21: Effect of Number density on the temperature profiles for case (iv).
Figure 6.22: Effect of Prandtl number on the temperature profiles for case (i).

Figure 6.23: Effect of Prandtl number on the temperature profiles for case (ii).
Figure 6.24: Effect of Prandtl number on the temperature profiles for case (iii).

Figure 6.25: Effect of Prandtl number on the temperature profiles for case (iv).
Figure 6.26: Effect of Eckert number on the temperature profiles for case (i).

Figure 6.27: Effect of Eckert number on the temperature profiles for case (ii).
Chapter 6: Effect of Radiation on Hydromagnetic Flow and Heat Transfer

1.0 - 0.4
fluid phase
dust phase

Figure 6.28: Effect of Eckert number on the temperature profiles for case (iii).

Figure 6.29: Effect of Eckert number on the temperature profiles for case (iv).
Figure 6.30: Effect of Radiation on the temperature profiles for case (i).

Figure 6.31: Effect of Radiation on the temperature profiles for case (ii).
Figure 6.32: Effect of Radiation on the temperature profiles for case (iii).

Figure 6.33: Effect of Radiation on the temperature profiles for case (iv).
Table 2: The variation of temperature of the fluid phase for different values of the parameters $Pr$, $Ec$, $N$ and $R$ at $y = 0.2$.

<table>
<thead>
<tr>
<th>Pr</th>
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<th>R</th>
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<th>Case(ii)</th>
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6.7 Conclusions

In this paper, a mathematical analysis has been carried out on momentum and heat transfer characteristics in an incompressible, viscous, unsteady dusty fluid between two parallel plates with the effect of radiation. Here the flow analysis is studied on four different types of boundary conditions. In all the four cases, the effect of pulsatile pressure gradient is considered. The governing equations for this investigation were non-dimensionalized and solved numerically using finite difference method with the help of MATLAB PDE solver. Table 1 shows the accuracy of this numerical solution was validated for the all cases by a comparison with the analytical solutions reported by [74]. From this table, one can
seen that a very good agreement between the existing results. Further graphical results
of velocity profiles for both fluid and dust phase are presented and discussed to show
the effects of the Hartmann number and the number density. In addition, the numeri­
cal solutions of energy equations for case (i) to case (iv) were performed graphically to
show the influence of the Prandtl number, Eckert number and radiation parameter on the
temperature profiles. Some of the important conclusions of the present study are

- Velocities of both fluid and dust phase decreases for the effects of increasing the
  strength of the magnetic field and number density.

- The increasing values of the Prandtl number reduces temperature distributions of
  both fluid and dust phase.

- The temperature distribution between plates increases gradually with the increase
  of Eckert number $Ec$.

- The effect of thermal radiation parameter increases the temperature profiles of both
  fluid and dust phases in all the four cases.

- Fluid phase temperature is higher than the dust phase temperature.

- In the absence of pressure gradient, our numerical results coincides with the ana­
  lytical solutions obtained by [74].