CHAPTER 4

Publications based on this Chapter;

Chapter 4

Unsteady flow of a conducting dusty fluid between two circular cylinders

4.1 Introduction

The influence of dust particles on viscous flows has great importance in petroleum industry and in the purification of crude oil. Other important applications of dust particles in boundary layer, include soil erosion by natural winds and dust entrainment in a cloud during nuclear explosion. Also such flows occur in a wide range of areas of technical importance like fluidization, flow in rocket tubes, combustion, paint spraying and more recently blood flows in capillaries. P.G. Saffman [96] formulated the equations for dusty fluid flow and has studied the laminar flow of a dusty gas. Michael and Miller [71] investigated the motion of dusty gas with uniform distribution of the dust particles occupied in a cylinder and between two rotating cylinders. Samba Siva Rao [94] has obtained unsteady flow of a dusty viscous liquid through circular cylinder. E. Amos [7] studied magnetic effect on pulsatile flow in a constricted axis symmetric tube. A.J. Chamkha [4] has attained unsteady hydromagnetic flow and heat transfer from a non-isothermal stretching sheet.
immersed in a porous medium. Datta and Dalai [27] have deliberated the solutions for pulsatile flow and heat transfer of a dusty fluid through an infinitely long pipe. Liu [68] has studied flow induced by an oscillating infinite flat plate in a dusty gas. Indrasena [61] has obtained the solution of steady rotating hydrodynamic-flows. Girishwar Nath [51] has studied the dusty viscous fluid flow between rotating coaxial cylinders. Yang Lei and Bakhtier Farouk [117] investigated three-dimensional mixed convection flows in a horizontal annulus with a heated rotating inner circular cylinder. Colette Calmelet-Elulu and Philip Crooke [24] have inspected unsteady conducting dusty gas flow through a circular pipe in the presence of an applied and induced magnetic field. The authors Bagewadi and Gireesha [13], [14] have studied two-dimensional dusty fluid flow in Frenet frame field system and recently the authors [48], [50] have obtained the solutions for the flow of an unsteady dusty fluid under varying time dependent pressure gradients through different regions like parallel plates, rectangular channel and open rectangular channel.

The present chapter includes the study of unsteady flow of a conducting dusty fluid between two rotating concentric circular cylinders. Here the flow is due to the influence of time dependent pressure gradient and differential rotations of the cylinders. The fluid and dust particles are assumed to be at rest initially. The analytical expressions are obtained for velocities of fluid and dust particles. Further, the skin friction at the boundaries are calculated and graphical representation of the velocity profile are given for different values of flow parameters like Hartmann number and time.
4.2 Formulation of the Problem and Solution

Consider a flow of viscous incompressible, conducting dusty fluid between two rotating circular cylinders. The inner cylinder is of unit radius and outer cylinder is of radius \( b \). The flow is due to the influence of time dependent pressure gradient and differential rotations of the cylinders. It is assumed that the inner and outer cylinders rotate with the different angular velocities. Both the fluid and the dust particle clouds are supposed to be static at the beginning. The dust particles are assumed to be spherical in shape and uniform in size. The number density of the dust particles is taken as a constant throughout the flow. As figure-4.1 shows, the axis of the channel is along \( z \)-axis, radial distance and \( \theta \) be the angle. The velocity components of both fluid and dust particles are respectively given by:

\[
\begin{align*}
    u_r &= 0; \quad u_\theta = 0; \quad u_z = u_z(r, t), \\
    v_r &= 0; \quad v_\theta = 0; \quad v_z = v_z(r, t).
\end{align*}
\]  

(4.2.1)

Figure 4.1: Schematic diagram of the flow.
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The intrinsic decomposition of equations (2.2.2) and (2.2.4) in cylindrical polar coordinates give the following forms:

\[-\frac{1}{\rho} \frac{\partial p}{\partial r} = 0, \quad (4.2.2)\]

\[\frac{\partial u_z}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left[ \frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r} \frac{\partial u_z}{\partial r} \right] + \frac{kN}{\rho} (v_z - u_z) - \frac{\sigma B_0^2}{\rho} u_z, \quad (4.2.3)\]

\[\frac{\partial v_z}{\partial t} = \frac{k}{\mu} (u_z - v_z). \quad (4.2.4)\]

Let us introduce the following non-dimensional quantities:

\[R = \frac{r}{a}, \quad \bar{z} = \frac{z}{a}, \quad \bar{p} = \frac{pa^2}{\rho \nu^2}, \quad T = \frac{a \nu}{T}, \quad \bar{u} = \frac{u a^2}{\nu}, \quad \bar{v} = \frac{v a^2}{\nu}, \quad (4.2.5)\]

\[\beta = \frac{l}{\gamma} = \frac{N a^2}{\rho \nu}, \quad l = \frac{Nm}{\rho}, \quad \gamma = \frac{\nu m}{ka^2}. \]

Transform the equations (4.2.2) to (4.2.4) to the non-dimensional forms as,

\[-\frac{\nu^2}{a^3} \frac{\partial p}{\partial R} = 0, \quad (4.2.6)\]

\[\frac{\partial u}{\partial T} = -\frac{\partial p}{\partial \bar{z}} + \left[ \frac{\partial^2 u}{\partial R^2} + \frac{1}{R} \frac{\partial u}{\partial R} \right] + \beta(v - u) - M^2 u, \quad (4.2.7)\]

\[\gamma \frac{\partial v}{\partial T} = (u - v), \quad (4.2.8)\]

where \(M = B_0 a \sqrt{(\sigma/\mu)}\) is the Hartmann number.
Since we have assumed that the time dependent pressure gradient to be impressed on the system for \( t > 0 \), so we can write

\[
\frac{-1}{\rho} \frac{\partial p}{\partial z} = c + d e^{i\alpha t},
\]

where \( c, d \) and \( \alpha \) are reals.

Eliminating \( v \) from (4.2.7) and (4.2.8) and then substituting the expression for pressure gradient, one can get

\[
\gamma \frac{\partial^2 u}{\partial T^2} + (l + 1 + M^2 \gamma) \frac{\partial u}{\partial T} - \gamma \frac{\partial}{\partial T} \left[ \frac{\partial^2 u}{\partial R^2} + \frac{1}{R} \frac{\partial u}{\partial R} \right] = c + d e^{i\alpha t} + \left[ \frac{\partial^2 u}{\partial R^2} + \frac{1}{R} \frac{\partial u}{\partial R} \right] - M^2 u. \tag{4.2.9}
\]

Let the solution of the equation (4.2.9) in the form [112], [65],

\[
u = U(R) + V(R, T), \tag{4.2.11}
\]

where \( U \) is the steady part and \( V \) is the unsteady part of the fluid velocity.

Separating the equation (4.2.9) into a steady part and unsteady part as

\[
\frac{\partial^2 U}{\partial R^2} + \frac{1}{R} \frac{\partial U}{\partial R} - M^2 U = -c, \tag{4.2.12}
\]

\[
\gamma \frac{\partial^2 V}{\partial T^2} + (l + 1 + M^2 \gamma) \frac{\partial V}{\partial T} - \gamma \frac{\partial}{\partial T} \left[ \frac{\partial^2 V}{\partial R^2} + \frac{1}{R} \frac{\partial V}{\partial R} \right] = d e^{i\alpha t} + \left[ \frac{\partial^2 V}{\partial R^2} + \frac{1}{R} \frac{\partial V}{\partial R} \right] - M^2 V. \tag{4.2.13}
\]
Case-4.1: (Periodic Motion) For the present case consider the boundary conditions as,

\[ u = u_1 \sin(\alpha T), \quad \text{at} \quad R = 1, \]
\[ u = u_2 \sin(\alpha T), \quad \text{at} \quad R = b, \]

where \( u_1 \) and \( u_2 \) are uniform angular velocities.

Since \( u = U(R) + V(R, T) \) one can see that the boundary conditions becomes

\[ U = 0 \quad \text{and} \quad V = u_1 \sin(\alpha t) \quad \text{at} \quad R = 1, \]
\[ U = 0 \quad \text{and} \quad V = u_2 \sin(\alpha t) \quad \text{at} \quad R = b. \] (4.2.15)

Now, by solving equation (4.2.12) using the boundary conditions (4.2.15) one can get

\[ U = \frac{c}{M^2} \left( \frac{T_1 J_0(MR) + T_2 K_0(MR)}{T_0} - 1 \right), \] (4.2.16)

where \( J_0 \) and \( K_0 \) are Bessel's function of first and second kind respectively of order zero.

Assume that the solution of the equation (4.2.13) is in the form

\[ V = g(R)e^{\alpha T}, \] (4.2.17)

where \( g(R) \) is an unknown function to be determine.

Using equation (4.2.17) in (4.2.13) one can obtain

\[ \frac{\partial^2 g}{\partial R^2} + \frac{1}{R} \frac{\partial g}{\partial R} - \lambda_2^2 g = -\lambda_2, \] (4.2.18)

where \( \lambda_1 = \frac{(M^2 - \gamma^2) + \imath \alpha(1+\gamma)}{(1+\imath \alpha \gamma)} \) and \( \lambda_2 = \frac{d}{(1+\imath \alpha \gamma)} \).
Using the boundary conditions (4.2.15) one can obtain the solution of (4.2.18) as,

\[
g(R) = \frac{\lambda_2}{\lambda_1^2} \left[ \frac{Q_1 J_0(\lambda_1 R) + Q_3 K_0(\lambda_1 R)}{Q_0} - 1 \right] \\
+ \frac{\sin(\alpha T)}{e^{i\alpha t}} \left[ \frac{Q_2 J_0(\lambda_1 R) + Q_4 K_0(\lambda_1 R)}{Q_0} \right]. \tag{4.2.19}
\]

Using this in (4.2.17) we get

\[
V = \frac{\lambda_2 e^{i \alpha T}}{\lambda_1^2} \left[ \frac{Q_1 J_0(\lambda_1 R) + Q_3 K_0(\lambda_1 R)}{Q_0} - 1 \right] \\
+ \sin(\alpha T) \left[ \frac{Q_2 J_0(\lambda_1 R) + Q_4 K_0(\lambda_1 R)}{Q_0} \right]. \tag{4.2.20}
\]

Now, using equations (4.2.20) and (4.2.16) in (4.2.11) we obtain the fluid velocity \(u\) as

\[
u = \frac{c}{M^2} \left[ \frac{T_1 J_0(M R) + T_2 K_0(M R)}{T_0} - 1 \right] \\
+ \frac{\lambda_2 e^{i \alpha T}}{\lambda_1^2} \left[ \frac{Q_1 J_0(\lambda_1 R) + Q_3 K_0(\lambda_1 R)}{Q_0} - 1 \right] \\
+ \sin(\alpha T) \left[ \frac{Q_2 J_0(\lambda_1 R) + Q_4 K_0(\lambda_1 R)}{Q_0} \right].
\]

Also, the dust phase velocity is obtained from equation (4.2.17) as,

\[
u = \frac{c}{M^2} \left[ \frac{T_1 J_0(M R) + T_2 K_0(M R)}{T_0} - 1 \right] \\
+ \frac{\lambda_2 e^{i \alpha T}}{\lambda_1^2(1 + i \alpha \gamma)} \left[ \frac{Q_1 J_0(\lambda_1 R) + Q_3 K_0(\lambda_1 R)}{Q_0} - 1 \right] \\
+ \frac{1}{1 + \alpha^2 \gamma^2} [\sin \alpha T - \alpha \gamma \cos \alpha T] \left[ \frac{Q_2 J_0(\lambda_1 R) + Q_4 K_0(\lambda_1 R)}{Q_0} \right] + Ae^{-\frac{1}{2}T}.
\]
Shearing Stress (Skin Friction):

The shear stress at the boundary $R = 1$ and $R = b$ are respectively given by

$$D_1 = \frac{\mu c}{M} \left[ \frac{T_1 J_0(M) + T_2 K_0(M)}{T_0} \right] + \frac{\mu \lambda_2 e^{i\alpha T}}{\lambda_1} \left[ \frac{Q_1 J_0(\lambda_1) + Q_2 K_0(\lambda_1)}{Q_0} \right]$$

$$+ \mu \lambda_1 \sin(\alpha T) \left[ \frac{Q_2 J_0(\lambda_1) + Q_4 K_0(\lambda_1)}{Q_0} \right],$$

$$D_0 = \frac{\mu c}{M} \left[ \frac{T_1 J_0(Mb) + T_2 K_0(Mb)}{T_0} \right] + \frac{\mu \lambda_2 e^{i\alpha T}}{\lambda_1} \left[ \frac{Q_1 J_0(\lambda_1 b) + Q_3 K_0(\lambda_1 b)}{Q_0} \right]$$

$$+ \mu \lambda_1 \sin(\alpha T) \left[ \frac{Q_2 J_0(\lambda_1 b) + Q_4 K_0(\lambda_1 b)}{Q_0} \right].$$

Case-4.2: (Impulsive Motion) Consider the boundary conditions for impulsive motion as,

$$u = u_1 \delta(T), \; \text{at} \; R = 1,$$

$$u = u_2 \delta(T), \; \text{at} \; R = b,$$

where $\delta(T)$ is the Dirac delta function.

Using these boundary conditions one can see that the solution for velocities of fluid and dust phases are obtained as,

$$u = \frac{c}{M^2} \left[ \frac{T_1 J_0(MR) + T_2 K_0(MR)}{T_0} - 1 \right]$$
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\[ + \frac{\lambda_2 e^{i\alpha T}}{\lambda_1^2} \left[ \frac{Q_1 J_0(\lambda_1 R) + Q_3 K_0(\lambda_1 R)}{Q_0} - 1 \right] \]

\[ + \delta(T) \left[ \frac{Q_2 J_0(\lambda_1 R) + Q_4 K_0(\lambda_1 R)}{Q_0} \right], \]

\[ v = \frac{c}{M^2} \left[ \frac{T_1 J_0(MR) + T_2 K_0(MR)}{T_0} - 1 \right] \]

\[ + \frac{\lambda_2 e^{i\alpha T}}{(1 + i\alpha \gamma)\lambda_1^2} \left[ \frac{Q_1 J_0(\lambda_1 R) + Q_3 K_0(\lambda_1 R)}{Q_0} - 1 \right] \]

\[ + \frac{e^{\frac{\pi}{4} T}}{\gamma} \left[ \frac{Q_2 J_0(\lambda_1 R) + Q_4 K_0(\lambda_1 R)}{Q_0} \right] + A_1 e^{-\frac{\pi}{4} T}. \]

Shearing Stress (Skin Friction):

The shear stress i.e., the skin friction at \( R = 1 \) and \( R = b \) are, respectively given by

\[ D_1 = \frac{\mu c}{M} \left[ \frac{T_1 J_0'(M) + T_2 K_0'(M)}{T_0} \right] + \frac{\mu \lambda_2 e^{i\alpha T}}{\lambda_1} \left[ \frac{Q_1 J_0'(\lambda_1) + Q_3 K_0'(\lambda_1)}{Q_0} \right] \]

\[ \quad + \mu \delta(T) \lambda_1 \left[ \frac{Q_2 J_0'(\lambda_1) + Q_4 K_0'(\lambda_1)}{Q_0} \right], \]

\[ D_b = \frac{\mu c}{M} \left[ \frac{T_1 J_0'(Mb) + T_2 K_0'(Mb)}{T_0} \right] + \frac{\mu \lambda_2 \mu}{\lambda_1} \left[ \frac{Q_1 J_0'(\lambda_1 b) + Q_3 K_0'(\lambda_1 b)}{Q_0} \right] \]

\[ \quad + \mu \delta(T) \lambda_1 \left[ \frac{Q_2 J_0'(\lambda_1 b) + Q_4 K_0'(\lambda_1 b)}{Q_0} \right]. \]
Case-4.3: (Transition Motion)

For the transition motion, we consider the boundary conditions as,

\[ u = u_1 H(T)e^{\alpha T}, \quad \text{at} \quad R = 1, \]
\[ u = u_2 H(T)e^{\alpha T}, \quad \text{at} \quad R = b, \]

where \( H(T) \) is the Heaviside’s unit step function.

Using these boundary conditions the solution for velocities of fluid and dust phase can be written as

\[
\begin{align*}
\frac{u}{M^2} &= \frac{c}{M^2} \left[ \frac{T_1 J_0(MR) + T_2 K_0(MR)}{T_0} - 1 \right] \\
&\quad + \frac{\lambda_2 e^{i\alpha T}}{\lambda_1^2} \left[ \frac{Q_1 J_0(\lambda_1 R) + Q_3 K_0(\lambda_1 R)}{Q_0} - 1 \right] \\
&\quad + H(T)e^{\alpha T} \left[ \frac{Q_2 J_0(\lambda_1 R) + Q_4 K_0(\lambda_1 R)}{Q_0} \right],
\end{align*}
\]

and

\[
\begin{align*}
\frac{v}{M^2} &= \frac{c}{M^2} \left[ \frac{T_1 J_0(MR) + T_2 K_0(MR)}{T_0} - 1 \right] \\
&\quad + \frac{\lambda_2 e^{i\alpha T}}{\lambda_1^2(1 + i\alpha\gamma)} \left[ \frac{Q_1 J_0(\lambda_1 R) + Q_3 K_0(\lambda_1 R)}{Q_0} - 1 \right] \\
&\quad + \frac{e^{-\frac{1}{2}T} \left[ e^{(1+\alpha)T} - 1 \right]}{\left(1 + \alpha\gamma\right)} H(T) \left[ \frac{Q_2 J_0(\lambda_1 R) + Q_4 K_0(\lambda_1 R)}{Q_0} \right] + A_2 e^{-\frac{1}{2}T}.
\end{align*}
\]
Shearing Stress (Skin Friction):

The skin friction at $R = 1$ and $R = b$ are respectively given by

$$D_1 = \frac{\mu c}{M} \left[ \frac{T_1 J_0(M) + T_2 K_0(M)}{T_0} \right]$$

$$+ \frac{\mu \lambda_2 e^{\alpha T}}{\lambda_1} \left[ \frac{Q_1 J_0(\lambda_1) + Q_3 K_0(\lambda_1)}{Q_0} \right]$$

$$+ \mu \lambda_1 H(T)e^{\alpha T} \left[ \frac{Q_2 J_0(\lambda_1) + Q_4 K_0(\lambda_1)}{Q_0} \right].$$

$$D_b = \frac{\mu c}{M} \left[ \frac{T_1 J_0(Mb) + T_2 K_0(Mb)}{T_0} \right]$$

$$+ \frac{\mu \lambda_2 e^{\alpha T}}{\lambda_1} \left[ \frac{Q_1 J_0(\lambda_1 b) + Q_3 K_0(\lambda_1 b)}{Q_0} \right]$$

$$+ \mu \lambda_1 H(T)e^{\alpha T} \left[ \frac{Q_2 J_0(\lambda_1 b) + Q_4 K_0(\lambda_1 b)}{Q_0} \right].$$

Case-4.4: (Motion for a Finite Time)

For this case, we consider the boundary conditions as,

$$u = u_1[H(T) - H(T - t)], \quad at \quad R = 1,$$

$$u = u_2[H(T) - H(T - t)], \quad at \quad R = b,$$

where $H(T)$ is the Heaviside’s unit step function.
Using these boundary conditions we found the solution for velocities of fluid and dust phase as,

\[
u = \frac{c}{M^2} \left[ \frac{T_1 J_0 (MR) + T_2 K_0 (MR)}{T_0} - 1 \right] + \frac{\lambda_2 e^{i\alpha T}}{\lambda_1^2} \left[ \frac{Q_1 J_0 (\lambda_1 R) + Q_3 K_0 (\lambda_1 R)}{Q_0} - 1 \right] + \left[ H(T) - H(T - t) \right] \left[ \frac{Q_2 J_0 (\lambda_1 R) + Q_4 K_0 (\lambda_1 R)}{Q_0} \right],
\]

and

\[
v = \frac{c}{M^2} \left[ \frac{T_1 J_0 (MR) + T_2 K_0 (MR)}{T_0} - 1 \right] + \frac{\lambda_2 e^{i\alpha T}}{\lambda_1^2 (1 + i\alpha \gamma)} \left[ \frac{Q_1 J_0 (\lambda_1 R) + Q_3 K_0 (\lambda_1 R)}{Q_0} - 1 \right] + \frac{e^{-\frac{\gamma T}{1+\alpha\gamma}} - 1}{(1 + \alpha\gamma)} \left[ H(T) - H(T - t) \right] \left[ \frac{Q_2 J_0 (\lambda_1 R) + Q_4 K_0 (\lambda_1 R)}{Q_0} \right] + A_3 e^{-\frac{\gamma T}{1+\alpha\gamma}}.
\]

Shearing Stress (Skin Friction):

The shear stress i.e., the skin friction at the boundary \( R = 1 \) and \( R = b \) are respectively given by

\[
D_1 = \frac{\mu c}{M} \left[ \frac{T_1 J_0 (M) + T_2 K_0 (M)}{T_0} \right] + \frac{\mu \lambda_2 e^{i\alpha T}}{\lambda_1} \left[ \frac{Q_1 J_0 (\lambda_1) + Q_3 K_0 (\lambda_1)}{Q_0} \right] + \mu \lambda_1 \left[ H(T) - H(T - t) \right] \left[ \frac{Q_2 J_0 (\lambda_1) + Q_4 K_0 (\lambda_1)}{Q_0} \right],
\]
\[ D_b = \frac{\mu c}{M} \left[ T_1 J_0(Mb) + T_2 K_0'(Mb) \right] + \frac{\mu \lambda_2 e^{i\alpha T}}{\lambda_1} \left[ \frac{Q_1 J_0(\lambda_1 b) + Q_3 K_0'(\lambda_1 b)}{Q_0} \right] \]

\[ + \mu \lambda_1 [H(T) - H(T - t)] \left[ \frac{Q_2 J_0(\lambda_1 b) + Q_4 K_0'(\lambda_1 b)}{Q_0} \right]. \]

where

\[ A = \frac{a\gamma}{1 + \alpha^2 \gamma^2} \left[ \frac{Q_2 J_0(\lambda_1 R) + Q_4 K_0(\lambda_1 R)}{Q_0} \right] - \frac{c}{M^2} \left[ \frac{T_1 J_0(MR) + T_2 K_0(MR)}{T_0} \right] - 1 \]

\[ - \frac{\lambda_2(1 - i\alpha \gamma)}{\lambda^2_2(1 + \alpha^2 \gamma^2)} \left[ \frac{Q_1 J_0(\lambda_1 R) + Q_3 K_0(\lambda_1 R)}{Q_0} \right] - 1, \]

\[ A_1 = - \frac{1}{\gamma} \left[ \frac{Q_2 J_0(\lambda_1 R) + Q_4 K_0(\lambda_1 R)}{Q_0} \right] - \frac{\lambda_2}{\lambda^2_2(1 + i\alpha \gamma)} \left[ \frac{Q_1 J_0(\lambda_1 R) + Q_3 K_0(\lambda_1 R)}{Q_0} \right] - 1 \]

\[ - \frac{c}{M^2} \left[ \frac{T_1 J_0(MR) + T_2 K_0(MR)}{T_0} \right] - 1, \]

\[ A_2 = - \frac{\lambda_2}{\lambda^2_2(1 + i\alpha \gamma)} \left[ \frac{Q_1 J_0(\lambda_1 R) + Q_3 K_0(\lambda_1 R)}{Q_0} \right] - 1 \]

\[ - \frac{c}{M^2} \left[ \frac{T_1 J_0(MR) + T_2 K_0(MR)}{T_0} \right] - 1, \]

\[ A_3 = - \frac{\lambda_2}{\lambda^2_2(1 + i\alpha \gamma)} \left[ \frac{Q_1 J_0(\lambda_1 R) + Q_3 K_0(\lambda_1 R)}{Q_0} \right] - 1 \]

\[ - \frac{c}{M^2} \left[ \frac{T_1 J_0(MR) + T_2 K_0(MR)}{T_0} \right] - 1, \]
\[ T_0 = J_0(M)K_0(Mb) - J_0(Mb)K_0(M), \quad T_1 = K_0(Mb) - K_0(M), \]

\[ T_2 = J_0(M) - J_0(Mb), \quad Q_0 = J_0(\lambda_1)K_0(\lambda_1 b) - J_0(\lambda_1 b)K_0(\lambda_1), \]

\[ Q_1 = K_0(\lambda_1 b) - K_0(\lambda_1), \quad Q_2 = u_1K_0(\lambda_1 b) - u_2K_0(\lambda_1), \]

\[ Q_3 = J_0(\lambda_1) - J_0(\lambda_1 b), \quad Q_4 = u_2J_0(\lambda_1) - u_1J_0(\lambda_1 b). \]

### 4.3 Conclusion

Here we have studied the laminar flow of a conducting dusty fluid between two circular cylinders. The four different cases are discussed based on the time dependent pressure gradient. Variable separable and Eigen expansion methods are employed to solve the governing equations. The graphs for velocity profiles are shown in figure 4.2 to 4.18 for different values of parameters like Hartmann number \((M)\) and Time \((T)\) which shows that they are parabolic in nature. From figures 4.2 to 4.10 one can observe that the appreciable effect of Hartmann number on the flow of both fluid and dust phases i.e., the magnetic field has retarding influence. Also, it is evident from the graphs 4.11 to 4.18 that as time increases the velocity of both the phases decreases, which is desirable in physical situations. Further we can see that if \(\gamma \to 0\) i.e., if the dust is very fine then the velocities of both fluid and dust particles will be the same.
Figure 4.2: Variation of fluid phase velocity with $R$ for Case-1.

Figure 4.3: Variation of dust phase velocity with $R$ for Case-1.
Figure 4.5: Variation of fluid phase velocity with $R$ for Case-2.

Figure 4.6: Variation of dust phase velocity with $R$ for Case-2.
Figure 4.7: Variation of fluid phase velocity with $R$ for Case-3.

Figure 4.8: Variation of dust phase velocity with $R$ for Case-3.
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Figure 4.9: Variation of fluid and dust velocity with $R$ for Case-4.

Figure 4.10: Variation of dust phase velocity with $R$ for Case-4.
Figure 4.11: Variation of fluid phase velocity with $R$ for Case-1.

Figure 4.12: Variation of dust phase velocity with $R$ for Case-1.
Figure 4.13: Variation of fluid phase velocity with $R$ for Case-2.

Figure 4.14: Variation of dust phase velocity with $R$ for Case-2.
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Figure 4.15: Variation of fluid phase velocity with $R$ for Case-3.

Figure 4.16: Variation of dust phase velocity with $R$ for Case-3.
Figure 4.17: Variation of fluid phase velocity with $R$ for Case-4.

Figure 4.18: Variation of dust phase velocity with $R$ for Case-4.