CHAPTER 3

Publications based on this Chapter;


• *Effect of Viscous Dissipation on Dusty Fluid over a Stretching Sheet*, Communicated.
Chapter 3

Flow and heat transfer of a dusty fluid over a stretching sheet

3.1 Introduction

The flow and heat transfer of a viscous and incompressible fluid induced by a continuously moving or stretching surface in a ambient fluid is relevant to the field of chemical engineering processes. Many chemical engineering processes like metallurgical process, polymer extrusion process, drawing, annealing and tinning of copper wires, continuous stretching, rolling and manufacturing of plastic films and artificial fibres, materials manufactured by extrusion process and heat treated materials traveling between a feed roll and windup rolls or on conveyer belts, glass blowing, crystal growing, paper production. The behavior of boundary layer flow due to a moving flat surface immersed in an otherwise quiescent fluid was first studied by Sakiadis [52], who investigated it theoretically by both exact and approximate methods. Crane [18] presented a closed form exponential solution for the planar viscous flow of linear stretching case. Later, this problem has been extended to various aspects by considering non-Newtonian fluids, more general stretching
velocity, magnetohydrodynamic (MHD) effects, porous sheets, porous media and heat or mass transfer. Andreson et al. [5] extended the work of Crane [18] to non-Newtonian power law fluid over a linear stretching sheet. Chakrabarti et al. [11] have discussed the hydromagnetic flow and heat transfer over a stretching sheet.

Gebhart [25] was the first author who studied the problem taking into account the viscous dissipation. The MHD and viscous dissipation effects of the heat transfer analysis were studied many authors such as Mahmoud [43], Vajravelu et al. [67], Samad et al. [56] and Anjali Devi et al. [6]. Further, Grubka et al. [32] analyzed heat transfer study by considering the power-law variation of surface temperature. Cortell [17] studied the effects of viscous dissipation and work done by deformation on the MHD flow and heat transfer of a viscoelastic fluid over a stretching sheet. Abel [62] extended the work of [17] and studied the viscoelastic MHD flow and heat transfer of a fluid over a stretching sheet with viscous and Ohmic dissipation.

To study the two-phase flows, in which solid spherical particles are distributed in a fluid are of interest in a wide range of technical problems, such as flow through packed beds, sedimentation, environmental pollution, centrifugal separation of particles and blood rheology etc. The study of the boundary layer flow of fluid-particle suspension flow is important in determining the particle accumulation and impingement of the particle on the surface. In view of these applications, Chakrabarti [10] has analyzed the boundary layer flow of a dusty gas. Datta et al. [19] have investigated boundary layer flow of a dusty fluid over a semi-infinite flat plate. Recently many mathematicians like Evgeny et al. [23], XIE Ming-liang et al. [70], Palani et al. [47], Agranat [2] and Vajravelu et al.
[68] have obtained the analytical as well as numerical solutions of the study. Abdul Aziz [1] obtained the numerical solution for laminar thermal boundary over a flat plate with a convective surface boundary condition using the symbolic algebra software Maple.

In view of the above discussion, present chapter considers the study of two-dimensional steady state incompressible boundary layer flow of a dusty fluid over a stretching sheet. This chapter is divided into two cases, in the first case, we assumed that the flow exposed under the influence of uniform transverse magnetic field and thermal boundary condition depends on one of the different type of the heating process, namely prescribed surface temperature. In second case, we have neglected the magnetic field, taking the effect of suction and viscous dissipation under prescribed heat flux boundary condition. In above two cases the non-linear partial differential equations have been transformed by a similarity transformation into a system of non-linear ordinary differential equations. Highly non-linear momentum and heat transfer equations are solved numerically using RKF45 method. Further, we analyzed the effect of various physical parameters like fluid particle interaction parameter, magnetic parameter, suction parameter, Prandtl number and Eckert number on velocity profiles and temperature distribution.

3.2 Formulation and Solution of the Problem

Consider a steady two-dimensional laminar boundary layer flow of an electrically conducting viscous and incompressible dusty fluid over a stretching sheet. The sheet is coinciding with the plane \( y = 0 \), with the flow being confined to \( y > 0 \). The flow is generated by the action of two equal and opposite forces along the \( x-\) axis and \( y-\) axis being normal to the
flow. The sheet being stretched with the velocity $U_w(x)$ along the $x$-axis are shown in figure (3.1). The dust particles are assumed to be spherical in shape and uniform in size and number density of these is taken as a constant throughout the flow.

![Figure 3.1: Schematic of the two-dimensional stretching sheet problem.](image)

Under these assumptions, the governing two-dimensional boundary layer equations for momentum equations of dusty fluid in the usual notation are written as [68]:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3.2.1}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \frac{KN}{\rho} (u_p - u) - \frac{\sigma B_0^2}{\rho} u, \tag{3.2.2}
\]

\[
\frac{u_p \partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} = \frac{K}{m} (u - u_p), \tag{3.2.3}
\]

\[
\frac{u_p \partial v_p}{\partial x} + v_p \frac{\partial v_p}{\partial y} = \frac{K}{m} (v - v_p), \tag{3.2.4}
\]
where \((u, v)\) and \((u_p, v_p)\) are the velocity components of the fluid and dust particle phases along \(x\) and \(y\) directions respectively. \(\nu, \rho, \rho_p\) and \(N\) are the kinematic viscosity, density of the fluid, density of the particle phase, number density of the dust particles respectively, \(\sigma\) is the electrical conductivity of the fluid, \(B_0\) is the induced magnetic field, \(K = 6\pi a\mu\) is the stokes resistance (drag co-efficient), \(a\) is the spherical radius of dust particle, \(\mu\) is the viscosity of the fluid and \(m\) is the mass of the dust particle. In deriving these equations, the drag force is considered for the interaction between the fluid and particle phases.

The appropriate boundary conditions for the above described flow are given by

\[
u = u(x), \quad v = -v_0 \quad \text{at} \quad y = 0,
\]

\[
u \rightarrow 0, u_p \rightarrow 0, v_p \rightarrow v, \rho_p \rightarrow \rho k \quad \text{as} \quad y \rightarrow \infty,
\]

where \(u(x) = cx\) is a stretching sheet velocity, \(c > 0\) is the stretching rate, \(v_0\) is a constant and \(k\) is the density ratio.

To convert the governing equations into a set of similarity equations, introduce the following dimensionless coordinates in terms of similarity variable \(\eta\) and the similarity function \(f\) as

\[
u = cx f'(\eta), \quad v = -\sqrt{\nu c} f(\eta), \quad \eta = \sqrt{\frac{c}{\nu}} y,
\]

\[
u_p = cx F(\eta), \quad v_p = \sqrt{\nu c} G(\eta), \quad \rho_r = H(\eta),
\]

(3.2.7)
where prime denotes the differentiation with respect to $\eta$ and which are identically satisfies (3.2.1).

### 3.3 Heat Transfer Analysis

The governing boundary layer heat transport equations for dusty fluid in the presence of viscous dissipation for two dimensional flow are:

$$
\frac{u}{\partial x} + \frac{v}{\partial y} = \frac{k^*}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{N}{\rho \tau_T} (T_p - T) + \frac{N}{\rho c_p \tau_v} (u_p - u)^2 + \frac{\mu}{\rho c_p} \left( \frac{\partial u}{\partial y} \right)^2,
$$

(3.3.1)

$$
\frac{u_p}{\partial x} + \frac{v_p}{\partial y} = -\frac{c_p}{c_m \tau_T} (T_p - T),
$$

(3.3.2)

where $T$ and $T_p$ is the temperature of the fluid and that of the dust particles, $c_p$ and $c_m$ are the specific heat of fluid and dust particles. $\tau_T$ is the thermal equilibrium time which is the time required by the dust cloud to adjust its temperature to the fluid, $\tau_v$ is the relaxation time of the dust particle i.e., the time required by a dust particle to adjust its velocity relative to the fluid, $k^*$ is the thermal conductivity.

**CASE-3.1:**

In this case, the flow exposed under the influence of uniform transverse magnetic field and in the absence of suction and viscous dissipation. Substituting (3.2.7) into (3.2.2) to
(3.2.5), one can get the following non-linear ordinary differential equations,

\[ f''''(\eta) + f(\eta)f''(\eta) - [f'(\eta)]^2 + l^* \beta II(\eta)[F(\eta) - f'(\eta)] - M f'(\eta) = 0, \quad (3.3.3) \]

\[ G(\eta)F'(\eta) + [F(\eta)]^2 + \beta [F(\eta) - f'(\eta)] = 0, \quad (3.3.4) \]

\[ G(\eta)G'(\eta) + \beta [f(\eta) + G(\eta)] = 0, \quad (3.3.5) \]

\[ H(\eta)F(\eta) + H(\eta)G'(\eta) + G(\eta)H'(\eta) = 0, \quad (3.3.6) \]

where \( l^* = \frac{mN}{\rho} \) is the mass concentration of dust particles, \( \tau = \frac{m}{K} \) is the relaxation time of the particle phase, \( \beta = \frac{1}{\tau} \) is the fluid particle interaction parameter, \( \rho_r = \frac{\rho_f}{\rho_c} \) is the relative density and \( M = \frac{\sigma_h \kappa}{\rho_c} \) is magnetic parameter.

The boundary conditions defined as in (3.2.6) with the help of (3.2.7) will reduces to,

\[ f'(\eta) = 1, \quad f(\eta) = 0, \quad \text{at} \quad \eta = 0, \]

\[ f'(\eta) = 0, \quad F(\eta) = 0, \quad G(\eta) = -f(\eta), \quad H(\eta) = k \quad \text{as} \quad \eta \to \infty. \quad (3.3.7) \]

If \( \beta = 0 \), the analytical solution of (3.3.3) with boundary condition (3.3.7) can be written in the form:

\[ f(\eta) = 1 - e^{-\eta}. \quad (3.3.8) \]
In this case, to solve the temperature equations (3.3.1) and (3.3.2), we consider prescribed surface temperature boundary condition as,

\[ T = T_w = T_\infty + A \left( \frac{y}{l} \right)^2 \quad \text{at} \quad y = 0, \]

\[ T \to T_\infty, \quad T_p \to T_\infty \quad \text{as} \quad y \to \infty, \quad (3.3.9) \]

where \( T_w \) and \( T_\infty \) denote the temperature at the wall and at large distance from the wall respectively, \( A \) is a positive constant and \( l = \sqrt{\frac{c}{\varepsilon}} \) is a characteristic length.

We now define the non-dimensional fluid phase temperature \( \theta(\eta) \) and dust phase temperature \( \theta_p(\eta) \) as

\[ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \theta_p(\eta) = \frac{T_p - T_\infty}{T_w - T_\infty}, \quad (3.3.10) \]

where \( T - T_\infty = A \left( \frac{y}{l} \right)^2 \theta(\eta) \).

Using (3.3.10) into (3.3.1) to (3.3.2), we obtain the following non-linear ordinary differential equations

\[ \theta''(\eta) + Pr[f(\eta)\theta'(\eta) - 2f'(\eta)\theta(\eta)] + \frac{NP_T}{\rho c_T} [\theta_p(\eta) - \theta(\eta)] + \frac{NP_T Ec}{\rho \tau_v} [F(\eta) - f'(\eta)]^2 = 0, \quad (3.3.11) \]

\[ 2F(\eta)\theta_p(\eta) + G(\eta)\theta_p'(\eta) + \frac{c_p}{c_m \tau_T} [\theta_p(\eta) - \theta(\eta)] = 0, \quad (3.3.12) \]

where \( Pr = \frac{k_p}{c_p} \) is the Prandtl number, \( Ec = \frac{k\tau_p^{3/2}}{D_p \rho_{p'}^{1/2}} \) is the Eckert number.
The corresponding boundary conditions are

\[
\theta(\eta) = 1 \quad \text{at} \quad \eta = 0,
\]

\[
\theta(\eta) \to 0, \quad \theta_p(\eta) \to 0 \quad \text{as} \quad \eta \to \infty. \tag{3.3.13}
\]

CASE-3.2:

For this case, the heat transfer analysis is carried out taking into the effect of viscous dissipation and neglecting uniform magnetic field.

Under these assumptions, the corresponding momentum (3.3.3) and energy equations (3.3.11) will become

\[
f''''(\eta) + f(\eta)f''(\eta) - [f'(\eta)]^2 + \ell^* \beta H(\eta) [F(\eta) - f'(\eta)] = 0, \tag{3.3.14}
\]

\[
\theta''(\eta) + Pr [f(\eta)\theta'(\eta) - 2f'(\eta)\theta(\eta)] + \frac{NPr}{\rho c_p T} [\theta_p(\eta) - \theta(\eta)]
\]

\[
+ \frac{NPr Ec}{\rho \beta_v^2} [F(\eta) - f'(\eta)]^2 + Pr Ec f''''(\eta) = 0 = 0, \tag{3.3.15}
\]

Here we consider non-dimensional prescribed heat flux temperature boundary condition as follows

\[
-k^* \frac{\partial T}{\partial y} = q_w = D \left( \frac{x}{l} \right)^2 \quad \text{at} \quad y = 0,
\]

\[
T \to T_\infty, \quad T_p \to T_\infty \quad \text{as} \quad y \to \infty, \tag{3.3.16}
\]

where \( q_w \) is the rate of heat transfer, \( D \) is a positive constant, \( l = \sqrt{\frac{\beta}{c}} \) is a characteristic length and \( T_w - T_\infty = \frac{D}{k^*} \left( \frac{x}{l} \right)^2 \sqrt{\frac{\beta}{c}}. \)
The corresponding boundary conditions for velocity and temperature are

\[ f'(0) = 1, \quad f(0) = R, \quad f'(\infty) = 0, \quad F(\infty) = 0, \quad G(\infty) = -f(\infty), \]

\[ H(\infty) = k, \quad \theta'(0) = -1, \quad \theta(\infty) \to 0, \quad \theta_p(\infty) \to 0 \quad (3.3.17) \]

where \( R = \frac{\nu_0}{(\nu c)^{1/2}} \) is the suction parameter.

### 3.4 Results and discussion

A boundary layer flow and heat transfer of a dusty fluid problem for momentum, heat transfer over a stretching sheet is examined. In this chapter we studied the effect of magnetic field, suction parameter and viscous dissipation for two different heating process prescribed surface temperature and prescribed heat flux. To solve highly non-linear ordinary differential equations we adopted symbolic algebra software Maple, described by Aziz [1]. It is very efficient in using the well known Runge-Kutta-Fehlberg-fourth-fifth order method (RKF45 Method) to obtained the numerical solutions of a boundary value problem. The RKF45 algorithm in Maple has been well tested for its accuracy and robustness. In order to verify the validity and accuracy of the present analysis, results for the wall temperature gradient \( \theta'(0) \) are compared with those reported by Abel et al.[63] and Grubka et al.[32]. The Comparison in the above case is found to be in excellent agreement, as shown in Table 3.1. The results of thermal characteristics at the wall are examined for the values of \( \theta'(0) \) for case 3.1 and \( \theta(0) \) for case 3.2 are tabulated in Table
3.2 and Table 3.3. The effects of various physical parameters such as fluid particle interaction parameter $\beta$, suction parameter $R$, magnetic parameter $M$, Prandtl number $Pr$ and Eckert number $Ec$ on the velocity and temperature of both the fluid and dust phase are shown graphically.

The dimensionless velocity profiles for fluid phase $f'(\eta)$ and dust phase $F'(\eta)$ for different values of fluid-interaction parameter $\beta$ are shown in the figure 3.2(a) and 3.2(b) for both the cases respectively. It is found that, if $\beta$ increases the fluid phase velocity $f'(\eta)$ is decreases whereas in dust phase velocity $F'(\eta)$ increases. For large value of $\beta$ i.e., the mass of the dust particles is very small ultimately both the velocities of fluid and dust particle will be the same.

Figure 3.3(a) illustrate the effect of magnetic parameter $M$ on the velocity profiles when $\beta = 0.5$, $Pr = 1.0$ and $Ec = 2.0$. This figure shows that the velocity of both fluid phase and dust phase decreases with the increase of magnetic parameter. This is due to the fact that the presence of a magnetic field normal to the flow in an electrically conducting fluid produces a Lorentz force, which acts against the flow. Figure 3.3(b) shows that the effect of magnetic parameter $M$ on temperature profiles $\theta(\eta), \theta_p(\eta)$ versus ($\eta$). From these figures it reveals that the transverse magnetic field contributes to the thickening of thermal boundary layer. It is evident form these graphs that an applied transverse magnetic field produces a body force, called a Lorentz force, which opposes the motion. The resistance offered to the flow is responsible in enhancing the temperature.

The effect of Prandtl number $Pr$ on the temperature profiles versus $\eta$ for the case (3.1) and (3.2) are shown in the figures 3.4(a) and 3.4(b) respectively. By analyzing the
graph it reveal that the effect of increasing the $Pr$ decreases the temperature distribution in the flow region, and also it is evident that large values of Prandtl number results in thinning of thermal boundary layer. Throughout our thermal analysis the following values for different parameters, like $\tau_T = \tau_v = 0.5$ and $c_p = c_m = 0.2$, $\rho = 0.5$, $c = 1$ are used.

Figures 3.5(a) and 3.5(b) are plotted for the temperature profile, for different values of $Ec$ for both cases. One can observe from these figures that the effect of increasing values of Eckert number enhances the temperature at a point which is true for both the fluid phase as well as dust phase. It is observed that the effect of viscous dissipation is to amplify the temperature. Also it is observed that the fluid phase temperature is higher than the dust phase temperature.

Figure 3.6(a) shows the effect of suction parameter on velocity components of the fluid velocity $f'(\eta)$ and particle velocity $F(\eta)$. It is observed that boundary layer flow for fluid and dust phase velocity is decreases with the increase of suction parameter $R$. Both fluid and dust phase velocity profiles tending asymptotically to the horizontal axis, the non-dimensional velocities is observers maximum at the wall. It is fact that suction stabilizes the boundary layer growth. Figure 3.6(b) is plotted for temperature distribution for different values of $R$, when $Pr = 1.0$, $Ec = 2.0$ and $\beta = 0.5$. It is interesting to note that there is a significant enhancement of temperature on the wall when it is porous. The fluid and dust phase temperature profiles start to decreases monotonically from the very beginning, thus sucking the decelerated fluid and dust phase reduces the growth of the thermal boundary layer.
The graphs for temperature distributions from the sheet for different values of fluid-interaction parameter $\beta$ are plotted as in figures 3.7(a) and 3.7(b) for case (3.1) and case (3.2) respectively when $M = 0.5$, $Pr = 1.0$ and $Ec = 2.0$. It is noted that, the temperature of the fluid and dust phase increases with increase in the fluid-particle interaction parameter. It is also observed that the fluid phase temperature is higher than that of dust phase in all the graphs and also it indicates that the fluid particle temperature is parallel to that of dust particle.

### 3.5 Conclusions

The problem of two dimensional boundary layer flow and heat transfer of a dusty, viscous and incompressible fluid over a stretching sheet is investigated. The highly non-linear momentum and heat transfer equations are converted into coupled ordinary differential equations using similarity transformations. Numerical solutions of these equations are obtained using RKF-45 method. The effects of parameters like magnetic parameter $M$, fluid particle interaction $\beta$, suction parameter $R$, Prandtl number $Pr$ and Eckert number $Ec$ on velocity and temperature profiles are studied and discussed with the help of graphs. Some of the important conclusions of the present study are

- The thickness of the momentum boundary layer decreases with increase in the magnetic parameter in velocity profiles.

- Effect of magnetic parameter increases the temperature distributions in the flow region in both fluid and dust phase.
• Effect of suction parameter is to decrease the velocity and temperature profiles for both the fluid and dust phase.

• Fluid particle interaction parameter is decrease in fluid velocity and is increase in dust velocity.

• Fluid phase temperature is higher than the dust phase temperature.

• $Ec$ has significant effect on the boundary layer growth.

• The increase in Prandtl number decreases the thermal boundary layer thickness.

• The rate of heat transfer $\theta'(0)$ decreases with increasing the $Pr$, While it increases with increasing the $Ec$ and $\beta$.

• The prescribed heat flux boundary condition is better suited for effective cooling of the stretching sheet.

**Table 3.1:** Comparison results for the wall temperature gradient $-\theta'(0)$ in the case of $\beta = 0$ and $Ec = 0$.

<table>
<thead>
<tr>
<th>$Pr$</th>
<th>Abel et al.[63]</th>
<th>Grubka et al.[32]</th>
<th>Present Study $-\theta'(0)$</th>
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Table 3.2: Values of wall temperature gradient $\theta'(0)$ for different values of $M$, $\beta$, $Pr$ and $Ec$ for (case-3.1).

<table>
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<tr>
<th>$M$</th>
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<th>$Pr$</th>
<th>$Ec$</th>
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Table 3.3: Values of wall temperature gradient $\theta(0)$ for different values of $\beta$, $R$, $Pr$ and $Ec$ for (case-3.2).

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Figure 3.2(a): Effect of $\beta$ on velocity profiles (Case 3.1).

Figure 3.2(b): Effect of $\beta$ on velocity profiles (Case 3.2).
Figure 3.3(a): Effect of $M$ on velocity profiles (Case 3.1).

Figure 3.3(b): Effect of $M$ on temperature profiles (Case 3.1).
Figure 3.4(a): Effect of $Pr$ on temperature profiles (Case 3.1).

Figure 3.4(b): Effect of $Pr$ on temperature profiles (Case 3.2)
Figure 3.5(a): Effect of $Ec$ on temperature profiles (Case 3.1)

Figure 3.5(b): Effect of $Ec$ on temperature profiles (Case 3.2).
Figure 3.6(a): Effect of $R$ on the velocity profiles (Case 3.2).

Figure 3.6(b): Effect of $R$ on temperature profiles (Case 3.2).
Figure 3.7(a): Effect of $\beta$ on temperature profiles (Case 3.1).

Figure 3.7(b): Effect of $\beta$ on temperature profiles (Case 3.2).