CHAPTER 4

Publications based on this Chapter;

Chapter 4

Unsteady flow and heat transfer of fluid-particle suspension over an exponentially stretching sheet

4.1 Introduction

To date, an enormous amount of work has been done on the boundary layer flow and heat transfer with consideration of the stretching sheet problem. The engineering applications of the stretching sheet problem include polymer sheet extrusion from a dye, drawing, tinning and annealing of copper wires, glass fibre, paper production, cooling of a metallic plate in a cooling bath and so on. During the manufacture of these sheets, the melt issues from a slit and is subsequently stretched to achieve the desired thickness. The final product of desired characteristics are notably influenced by the rate of cooling in the process and the process of stretching. Sakiadis [54, 55] initiated the boundary layer behavior on a continuous solid surface moving at constant speed which is quite different from Blasious flow past a flat plate. Crane [13] was the first to obtain an elegant analytical solution to the boundary layer equations for the problem of steady two-dimensional flow due to a stretching surface in a quiescent incompressible fluid. The study of heat
transfer has become important industrially for determining the quality of final product which is greatly dependent on the rate of cooling. The heat transfer analysis for the flow over a linearly stretching sheet with the power law variation of surface temperature was investigated by Grubka and Bobba [32]. The problem of non-linear stretching sheet for different cases of fluid flow and heat transfer has been analyzed by different researchers. Among them Anjali and Thiyagarajan [3] have made an investigation on the study of non-linear stretching sheet for hydromagnetic flow and heat transfer with an assumption that the magnetic strength is non-linear.

The study of flow and heat transfer past an exponentially stretching sheet has gained tremendous interest among researchers due to its wider applications in technology. For example, in case of annealing and tinning of copper wires. The final product depends on the rate of heat transfer at the stretching continuous surface with exponential variations of stretching velocity and temperature distributions. Such studies have been carried out by Magyari and Keller [38] and have obtained the similarity solutions which describe the steady plane boundary layers on an exponentially stretching continuous surface with an exponential temperature distribution both analytically and numerically. The numerical solutions was obtained by Al-odat et al [2] for the effect of magnetic field in the thermal boundary layer on an exponentially stretching continuous surface with an exponential temperature distribution. Following them many researches like [10], [4], [8] investigated the numerical solutions for the boundary layer flow problem over an exponentially stretching sheet. Later, an analytical solution for the radiation effects on hydromagnetics Newtonian liquid flow due to an exponential stretching sheet was discussed by Kameswaran et al [35] and observed that the increase in values of magnetic parameter results in thickening of
the species boundary layer. Recently, the slip effects on MHD boundary layer flow over an exponentially stretching sheet with suction/blowing and thermal radiation was analyzed by Mukhopadhyay [62] and found that the temperature profile decreases with an increase in thermal slip parameter. On the other hand [46] and [16] have discussed the similarity solutions for flow over an exponentially stretching vertical sheet by considering the viscous dissipation effect.

All of the above-mentioned studies were carried out under a steady-state condition. However, in certain aspects, flow becomes time dependent due to a sudden stretching of the flat sheet or by a step change of the temperature or heat flux of the sheet and, consequently, it becomes necessary to consider unsteady flow conditions. Very recently Elbashbeshy et al [22] derived the similarity transformations for an unsteady exponentially stretching sheet which is different from the steady case and used Mathematica to solve the system of governing equations.

The above mentioned investigations deal with the flow and heat transfer only for fluids induced by stretching sheet. The fluid flow embedded with dust particles is encountered in a wide variety of engineering problems concerned with atmospheric fallout, dust collection, nuclear reactor cooling, powder technology, acoustics, sedimentation, performance of solid fuel rock nozzles, rain erosion, guided missiles and paint spraying etc. Saffman [52] initiated the work by formulating the governing equations for the flow of dusty fluid and has discussed the stability of the laminar flow of a dusty fluid in which the dust particles are uniformly distributed. Datta and Mishra [14] studied the boundary layer flow of a dusty fluid over a semi-infinite flat plate. An analysis of hydromagnetic flow of a dusty fluid over a stretching sheet with a view to throw adequate light on the effects
Chapter-4: Unsteady flow and heat transfer of a fluid-particle suspension over an... of fluid-particle interaction, particle loading, and suction on the flow characteristics was carried out by Vajravelu and Nayfeh [67]. In recent years Gireesha et al. [28] and [30] have discussed the stretching sheet flow problems for both steady and unsteady cases with non-uniform heat source/sink. Very recently, Gireesha and Pavithra [47] have made an investigation on an exponentially stretching sheet flow problem for a dusty fluid by considering the viscous dissipation and internal heat generation/absorption.

A quick review of literature shows that this problem is different from the above mentioned investigations since the exponential stretching sheet was not taken into consideration for unsteady dusty fluid. One of the important aspects in this study is the investigation of heat transfer processes. This is due to the fact that the rate of cooling influences a lot to the quality of the product with desired characteristics. Continuous surface heat transfer problems have many practical applications in industrial manufacturing processes. Such processes are hot rolling, wire drawing and glass fiber production. In modeling the boundary layer flow and heat transfer of these problems, the boundary conditions that are usually applied are either a specified surface temperature or a specified surface heat flux. Here two cases are studied for heat transfer analysis, namely, (i) variable exponential order surface temperature (VEST case) and (ii) variable exponential order heat flux at the sheet (VEHF case). Considering the importance of unsteady hydromagnetic flow and heat transfer due to an exponentially stretching sheet, a numerical solution is obtained for an unsteady two dimensional MHD heat transfer flow in an incompressible dusty fluid under the influence of thermal radiation, heat generation/absorption and viscous dissipation for VEST and VEHF case. The similarity transformations gives a system
of non-linear ordinary differential equations. The obtained equations are solved numerically using Runge-Kutta-Fehlberg 45 scheme with the help of Maple software. Graphical results for various values of the flow parameters are presented to gain thorough insight towards the physics of the problem.

4.2 Mathematical Formulation

Consider an unsteady two-dimensional laminar boundary layer flow and heat transfer of an incompressible viscous dusty fluid near a porous stretching sheet. It is assumed that the surface is stretched with exponential velocity \( U_w(x, t) = \frac{U_0}{(1-\alpha t)} e^{\pi/L} \) in quiescent fluid and the surface is maintained at a temperature \( T_w(x, t) = T_\infty + \frac{T_0}{(1-\alpha t)} e^{x/L} \). The \( x \)-axis is chosen along the sheet and \( y \)-axis normal to it. The flow is generated as a consequence of exponential stretching of the sheet, caused by simultaneous application of equal and opposite forces along \( x \)-axis keeping the origin fixed as in the figure - 4.1. A uniform magnetic field \( B \) is assumed to be applied in the \( y \)-direction and suction/injection \( S \) is normal to the sheet.

![Figure 4.1: Schematic representation of boundary layer flow.](image)
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Using the boundary layer approximations, the equations for mass and momentum for both phases are written in the usual notations as,

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \]  

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \frac{KN}{\rho} (u_p - u) - \frac{\sigma B^2}{\rho} u - \frac{\nu}{k^*} u, \]  

\[ \frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} = 0, \]  

\[ \frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} = \frac{K}{m} (u - u_p), \]

where \( x \) and \( y \) represents coordinate axes along the continuous surface in the direction of motion and perpendicular to it, respectively. \((u, v)\) and \((u_p, v_p)\) denotes the velocity components of the fluid and particle phase along the \( x \) and \( y \) directions respectively, \( \nu \) is the coefficient of viscosity of fluid, \( k^* \) is the permeability of the porous medium, \( \rho \) is the density of the fluid phase, \( K \) is the Stoke's resistance, \( N \) is the number density of dust particles, \( m \) is the mass of the dust particles, \( \tau_r = m/K \) is the relaxation time of particle phase and \( \sigma \) is the electrical conductivity.

In order to solve the governing boundary layer equations consider the following appropriate boundary conditions on velocity;

\[ u = U_0(x, t), \quad v = V_0(x, t) \quad \text{at} \quad y = 0, \]

\[ u \rightarrow 0, \quad u_p \rightarrow 0, \quad v_p \rightarrow v \quad \text{as} \quad y \rightarrow \infty, \]

where \( U_0(x, t) = \frac{U_0}{(1 - ai)} \) is the sheet velocity and \( V_0(x) = -S \sqrt{\frac{U_0v}{2L(1 - ai)}} \) is the suction/injection velocity, \( U_0 \) is reference velocity, \( L \) is the reference length and and if
$S > 0$ is a suction or if $S < 0$ is a injection parameter.

The governing equations (4.2.1) - (4.2.4) subject to the boundary conditions (4.2.5) can be expressed in a simpler form by introducing the following transformations:

\[
\begin{align*}
\eta &= \sqrt{\frac{U_0}{2\nu L(1-\alpha t)}} e^{\frac{\tau}{\tau_0}} y, \\
B &= \frac{B_0}{\sqrt{(1-\alpha t)}} e^{\frac{\tau}{\tau_0}}, \\
k^* &= \frac{2k_0}{\sqrt{(1-\alpha t)}} e^{-\frac{\tau}{\tau_0}},
\end{align*}
\]

where $B_0$ is the magnetic field flux density.

In view of these relations and on equating the co-efficients of $(\xi)^3$ on both sides, equations (4.2.2) and (4.2.4) become

\[
\begin{align*}
&f'''(\eta) + f'(\eta)f''(\eta) - 2f'(\eta)^2 + 2\beta [F'(\eta) - f'(\eta)] - A[2f'(\eta) + \eta f''(\eta)] - (M + K_p)f'(\eta) = 0, \\
&F(\eta)F''(\eta) - 2F'(\eta)^2 + 2\beta [f'(\eta) - F'(\eta)] - A[\eta F''(\eta) - 2F'(\eta)] = 0,
\end{align*}
\]

where prime denotes the differentiation with respect to $\eta$ and $l = \frac{mN}{\rho}$ is the mass concentration, $\beta = \frac{\nu_0}{\nu_0(1-\alpha t)}$ is the fluid-particle interaction parameter for velocity, $A = \frac{\alpha L}{\omega_0}$ is the unsteady parameter which measures the unsteadiness, $M = \frac{2\sigma B_0^2 L}{\rho l_0}$ is the magnetic parameter and $K_p = \frac{\nu L}{U_0 k_0}$ is the permeability parameter.
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With the help of similarity transformations (4.2.6) the boundary conditions (4.2.5) will become,

\[ f'(\eta) = 1, \quad f(\eta) = S \quad \text{at} \quad \eta = 0, \]
\[ f'(\eta) = 0, \quad F'(\eta) = 0, \quad F(\eta) = f(\eta) + \eta f'(\eta) - \eta F'(\eta) \quad \text{as} \quad \eta \to \infty. \]  (4.2.9)

The important physical parameter for the boundary layer flow is the skin-friction coefficient which is defined as in (2.2.12).

4.3 Heat Transfer Analysis

The unsteady boundary layer heat transport equations for both fluid and dust phases in the presence of thermal radiation with viscous dissipation and heat generation/absorption for two dimensional flows are given by:

\[
\rho C_p \left[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = k \frac{\partial^2 T}{\partial y^2} + \frac{N c_p}{\tau_T} (T_p - T) + \frac{N}{\tau_v} (u_p - u)^2
\]

\[ - \frac{\partial q_r}{\partial y} + \mu \left( \frac{\partial u}{\partial y} \right)^2 + Q(T - T_\infty), \]  (4.3.1)

\[
N c_m \left[ \frac{\partial T_p}{\partial t} + u_p \frac{\partial T_p}{\partial x} + v_p \frac{\partial T_p}{\partial y} \right] = - \frac{N c_p}{\tau_T} (T_p - T), \]  (4.3.2)

where \( T \) and \( T_p \) are the temperatures of the fluid and dust particle inside the boundary layer, \( c_p \) and \( c_m \) are the specific heat of fluid and dust particles, \( \tau_T \) is the thermal equilibrium time i.e., the time required by a dust cloud to adjust its temperature to that of fluid, \( k \) is the thermal conductivity, \( q_r \) is the radiative heat flux and \( Q \) represents the heat source when \( Q > 0 \) and the sink when \( Q < 0 \).
Using the Rosseland approximation [10] for radiation, radiative heat flux is simplified as

\[ q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}, \]  

(4.3.3)

where \( \sigma^* \) and \( k^* \) are the Stefan-Boltzmann constant and the mean absorption coefficient respectively. \( T^4 \) is expressed as a linear function of the temperature, and hence we get

\[ T^4 = 4T^3_{\infty} + 3T^3_{\infty}. \]

(4.3.4)

Using (4.3.3) and (4.3.4), equation (4.3.1) can be written as,

\[ \rho C_p \left[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = \left( k + \frac{16\sigma^*S_0^3}{3k^*} \right) \frac{\partial^2 T}{\partial y^2} + \frac{N_c p_p}{\tau_p} (T_p - T) + \frac{N}{\tau_v} (u_p - u)^2 \]

\[ + \mu \left( \frac{\partial u}{\partial y} \right)^2 + Q(T - T_{\infty}). \]

(4.3.5)

We solved the heat transfer phenomenon for following two types of heating process;

**Case-4.1: Variable exponential order surface temperature (VEST):**

For this heating process, employ the following boundary conditions;

\[ T = T_w(x, t) \text{ at } y = 0, \]

\[ T \longrightarrow T_{\infty}, \quad T_p \longrightarrow T_{\infty} \text{ as } y \longrightarrow \infty, \]

(4.3.6)

where \( T_w(x, t) = T_{\infty} + \frac{T_0}{(1-\alpha t)} e^{\frac{b}{2\kappa}} \) is the temperature distribution in the stretching surface, \( T_0 \) is a reference temperature and \( c_1 \) is a constant.

Using the similarity variable \( \eta \) and equation (2.3.4) into equations (4.3.2) and (4.3.5) and on equating the coefficients of \( (\xi \eta)^0 \) on both sides, one can arrive at the following
system of equations:

\[
\left(1 + \frac{4R}{3}\right)\theta''(\eta) + Pr [f(\eta)\theta'(\eta) - c_1 f'(\eta)\theta(\eta)] + 2\frac{N}{\rho} \beta_T Pr [\theta_p(\eta) - \theta(\eta)] + 2\lambda Pr \theta(\eta)
\]

\[+ 2\frac{N}{\rho} \beta Pr Ec [F'(\eta) - f'(\eta)]^2 - APR [\eta \theta'(\eta) + 4\theta(\eta)] + Pr Ec [f''(\eta)]^2 = 0, \quad (4.3.7)\]

\[c_1 F'(\eta)\theta_p(\eta) - F(\eta)\theta'_p(\eta) + 2\gamma \beta_T [\theta_p(\eta) - \theta(\eta)] + A[\eta \theta'_p(\eta) + 4\theta_p(\eta)] = 0, \quad (4.3.8)\]

where \( R = \frac{4\sigma^* T_0^3}{kk} \) is the radiation parameter, \( Pr = \frac{\nu c_p}{k} \) is the Prandtl number, \( Ec = \frac{U_k^2}{c_p T_0} \) is the Eckert number, \( \gamma = \frac{c_p}{c_v} \) is the the ratio of specific heat, \( \beta_T = \frac{L}{T_0} (1 - \alpha t) \) is the fluid-particle interaction parameters for heat transfer and \( \lambda = \frac{Q L^2}{\mu c_p Ec} (1 - \alpha t) \) is the dimensionless heat source/sink parameter.

Corresponding thermal boundary conditions become:

\[
\theta(\eta) = 1 \quad at \quad \eta = 0,
\]

\[
\theta(\eta) \rightarrow 0, \quad \theta_p(\eta) \rightarrow 0 \quad as \quad \eta \rightarrow \infty. \quad (4.3.9)
\]

Case-4.2: Variable exponential order heat flux (VEHF):

For this heating process, consider the boundary conditions as follows;

\[
\frac{\partial T}{\partial y} = -\frac{q_w(x,t)}{k} \quad at \quad y = 0,
\]

\[T \rightarrow T_{\infty}, \quad T_p \rightarrow T_{\infty} \quad as \quad y \rightarrow \infty, \quad (4.3.10)\]

where \( q_w(x,t) = \frac{T}{(1 - \alpha t)} e^{\frac{(\alpha + 1)t}{2}} \) and \( T_1 \) is reference temperature.

Again using the similarity variable \( \eta \) and equation (2.3.4) into equations (4.3.5) and (4.3.2) and by equating the co-efficients of \((\frac{x}{\xi})^0\) on both sides, we get the system of
equations as in equations (4.3.7) and (4.3.8) with Eckert number $E c = \frac{k u_0^2}{C_p R_h} \sqrt{\frac{\nu_R}{2\alpha}}$, which is different from the VEST case, and all other parameters are the same as in VEST case.

The corresponding thermal boundary conditions become:

$$
\theta'(\eta) = -1 \quad \text{at} \quad \eta = 0,
$$

$$
\theta(\eta) \to 0, \quad \theta_p(\eta) \to 0 \quad \text{as} \quad \eta \to \infty. \quad (4.3.11)
$$

The important physical parameter for the heat transfer coefficient is Nusselt number which is defined as in (2.3.15).

### 4.4 Numerical Solution

A numerical analysis is performed to investigate the structure of an unsteady boundary layer flow and heat transfer of a dusty fluid in a porous medium over an exponentially stretching sheet subject to suction/injection. Thermal radiation, viscous dissipation and internal heat generation/absorption effects are considered in the energy equation. Equations (4.2.7) and (4.2.8), (4.3.7) and (4.3.8) for both VEST and VEHF cases are the system of highly non-linear ordinary differential equations. These equations are solved numerically using RKF-45 method with the help of an algebraic software Maple. In this method, it is necessary to choose suitable finite values of $\eta \to \infty$ as $\eta = 5$.

Here we have given the comparison of our results of $f''(0)$ and $-\theta'(0)$ for both steady and unsteady cases with Elbashbeshy et al [22] as tabulated in table 4.1 and 4.3 respectively. From these tables, one can notice that there is a close agreement with these approaches and thus verify the accuracy of the method used. Further, we studied the effects of thermal radiation, viscous dissipation and heat generation/absorption for both
fluid and dust phases on velocity and temperature profiles and are depicted graphically and tabularly for several sets of values of the pertinent parameters.

4.5 Results and Discussion

By Runge-Kutta-Fehlberg 45 method the numerical results are obtained for a variety of physical parameters for both VEST and VEHF cases. The numerical calculations are presented in the form of non-dimensional velocity and temperature profiles for the purpose of discussing the result. The calculated results are presented in figures 4.2 - 4.18 to understand the effects of parameters on the flow and temperature field. In order to validate the method and to judge the accuracy of the present analysis, comparison of the skin friction coefficient and Nusselt numbers for steady and unsteady cases with available results of Elbashbeshy et al [22] are made (see tables 4.1 and 4.3) and found in excellent agreement. The results of the thermal characteristics at the wall are examined for the values of temperature gradient function \(-\theta'(0)\) in VEST case and the temperature function \[\theta(0)\] in VEHF case, which are tabulated in table 4.2.

- Figure 4.2 shows the plot of dimensionless velocity profiles \(f'(\eta)\) and \(F'(\eta)\) for different values of unsteady parameter \((A)\). From this, one can observe that the velocity for both fluid and dust phases decreases with increasing values of the unsteady parameter. It is interesting to note that the thickness of boundary layer increases with increasing values of \(A\).

- In figure 4.3, the velocity profiles are shown for different values of fluid-particle interaction parameter \((\beta)\). It is noticed from this figure that the velocity profile
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decreases with increasing values of $\beta$ for fluid phase and increases for dust phase in the boundary layer. The effect of increasing value of $\beta$ is to reduce the velocity $f'(\eta)$ and thereby increase the boundary layer thickness. Also it reveals that for large values of $\beta$ i.e., the relaxation time of the dust particles decreases, then the velocities of both fluid and dust particle will be the same.

- Figure 4.4 gives clearly the effect of magnetic parameter ($M$) on the velocity profile. Further, it explains that as magnetic field parameter increases, it decreases the velocity profiles. This is due to the fact that, the introduction of transverse magnetic field (normal to the flow direction) has a tendency to create a drag, known as Lorentz force which tends to resist the flow. By this we came to know that boundary layer thickness increases with increase in magnetic parameter.

- The influence of permeability parameter ($K_p$) on velocity profile is exhibited in figure 4.5. It is obvious that the presence of a porous medium causes higher restriction to the fluid flow which, in turn, slows its motion. Therefore, with increasing permeability parameter, the resistance to the fluid motion also increases. This causes the fluid velocity to decrease.

- In Figure 4.6, the velocity profiles are drawn for different values of the suction/injection parameter ($S$). It is observed that the velocity decreases significantly with increasing values of the suction parameter whereas it increases with the injection for both the fluid and dust phases so that the momentum boundary layers become thinner.

- Variation in temperature profiles for both fluid and dust phase due to the unsteady parameter ($A$) is visualized through figures 4.7(a) and 4.7(b) respectively. It is
observed that temperature of fluid and dust phase is found to decrease with increase in unsteady parameter. Physically it means that the temperature gradient at the surface increases as $A$ increases, which implies the increase of heat transfer rate $-\theta'(0)$ at the surface. This shows an important fact that the rate of cooling is much faster for higher values of unsteadiness parameter, where it may take a longer time in steady flows.

- The temperature profiles for different values of the fluid-particle interaction parameter ($\beta$) for both VEST and VEHF cases are presented graphically in figures 4.8(a) and 4.8(b) respectively. It shows that the temperature increases as $\beta$ increases for both fluid and dust phase, in both VEST and VEHF cases. Also one can observe that fluid phase temperature is higher and parallel to that of dust phase.

- Now we discussed the effects of magnetic parameter ($M$) on the temperature profiles $\theta(\eta)$ and $\theta_p(\eta)$ for both VEST and VEHF cases and are depicted as in figures 4.9(a) and 4.9(b) respectively. From these figures, we detect that the temperature profiles increase with increase in magnetic field parameter ($M$) and also it indicates that both the fluid and dust particle temperature are parallel to each other. This is true for both VEST and VEHF cases.

- Figures 4.10(a) and 4.10(b) depicts the temperature profiles $\theta(\eta)$ and $\theta_p(\eta)$ versus $\eta$ for different values of permeability parameter ($K_p$) respectively. As permeability parameter increases, there is rise in the temperature in the boundary layer which implies that Darcian body force improves the heat transfer rate. It can thus be inferred that an increase in permeability parameter decreases the boundary layer
thickness and consequently brings about an increase in the heat transfer rate.

- Figures 4.11(a) and 4.11(b) presents the effect of the suction/injection parameter \( S \) on the temperature profiles in both VEST and VEHF cases respectively. It reveals that the temperature decreases as the suction parameter increases which results in thinning of the thermal boundary layer thickness. However, by increasing the values of the injection parameter, the temperature increases with an increase in the thermal boundary layer thickness. Hence, suction can be used as a means for cooling the surface as it enhances the heat transfer coefficient much better than injection and thereby the thickness of the thermal boundary layer is reduced.

- Figures 4.12(a) and 4.12(b) portrays the temperature distributions \( \theta(\eta) \) and \( \theta_{\#}(\eta) \) for different values of heat source/sink parameter \( \lambda \) for both VEST and VEHF cases respectively. We infer from these figures that the thermal boundary generates the energy and this causes the temperature to increase with an increase in the heat source parameter, where as for the case of heat sink, the temperature decreases.

- Figures 4.13(a) and 4.13(b) represents the effect of radiation parameter \( R \) on temperature profiles for both VEST and VEHF cases respectively. Numerically increasing the radiation parameter enhances the heat transfer, physically increasing the radiation parameter produces a significant increase in the thickness of thermal boundary layer. In fact, the radiation parameter increases the fluid temperature. This is because as the radiation parameter increases, the mean Rosseland absorption co-efficient \( k^* \) decreases (for some thermal conductivity \( k \)). Hence the thermal radiation factor is better suitable for cooling process.
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- The profile in figures 4.14(a) and 4.14(b) exhibits the role of Prandtl number ($Pr$) on temperature profiles for both VEST and VEHF cases respectively. In heat transfer problems, the Prandtl number controls the relative thickness of the momentum and thermal boundary layers. When $Pr$ is small, it means that the heat diffuses very quickly when compared to the velocity (momentum). This means that for liquid metals the thickness of the thermal boundary layer is much bigger than that of velocity boundary layer. Hence Prandtl number can be used to increase the rate of cooling. Increase in the values of $Pr$ results in decrease of the temperature distribution and hence decrease in the boundary layer thickness.

- The effect of viscous dissipation in terms of Eckert number ($Ec$) on temperature profiles with $\eta$ is displayed graphically in figure 4.15(a) and 4.15(b) for both VEST and VEHF cases respectively. Viscous dissipation changes the temperature distribution by playing the role like an energy source, which affects heat transfer rates. Here the temperature increases with increase in the values of $Ec$, due to heat energy stored in the liquid by virtue of frictional heating. This is true in both the cases.

- Figures 4.16(a) and 4.16(b) shows the nature of skin-friction coefficient against suction parameter ($S$) for different values fluid-particle interaction parameter ($\beta$) and magnetic parameter ($M$) respectively. It reveals that skin-friction coefficient decreases with suction and magnetic parameter as well as for fluid-particle interaction parameter.

- The figures 4.17(a) and 4.17(b) displays the nature of heat transfer coefficient for both VEST and VEHF cases against radiation parameter ($R$) for different values
of Eckert number ($Ec$). Heat transfer increases for both radiation parameter and Eckert number. Rate of heat transfer increases with $\lambda$ but decreases with Prandtl number which is shown in figures 4.18(a) and 4.18(b).

4.6 Conclusion

The present discussion gives the solutions for an unsteady boundary layer flow and heat transfer of a dusty fluid over an exponential stretching surface with viscous dissipation and heat source/sink in presence of thermal radiation and suction. The efficient numerical method of Runge-Kutta-Fehlberg 45 scheme is used to solve the governing equations. The major findings from the present study can be summarized as follows;

- Fluid phase temperature is higher than that of dust phase.

- The effect of the fluid-particle interaction parameter is favorable for the dust phase velocity and unfavorable for fluid phase velocity.

- An increase in unsteady parameter is to decreases the momentum and thermal boundary layer thicknesses.

- The fluid will slow down as the magnetic parameter increases, so the effect of magnetic field becomes more significant.

- The momentum boundary layer becomes thinner for the effect of suction parameter.

- The effect of heat source/sink on temperature is quite opposite to that of suction parameter.
- Radiation should be kept minimum by regulating the temperature of the system for both the phases and in both VEST and VEHF cases.

- The temperature distribution in the flow region is increased by the effect of viscous dissipation of the fluid.

- Thermal boundary layer thickness decreases with increase in Prandtl number.

- The skin friction co-efficient decreases with increase in the unsteadiness, magnetic, suction and fluid-particle interaction parameters.

- The Nusselt number increases with increase in magnetic field, thermal radiation, heat generation/absorption and fluid-particle interaction parameter, while it decreases with increasing values of Prandtl number, number density, unsteadiness and suction parameter.

- It is found that under some limiting cases, our results coincide with the results of Elbashbeshy et al [22] for both the steady and unsteady case.

Table 4.1: Comparison of the results of skin friction co-efficient \([f''(0)]\) and wall temperature gradient \([-\theta'(0)]\) for an unsteady case.

<table>
<thead>
<tr>
<th></th>
<th>Elbashbeshy et al [22]</th>
<th>Present study</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f''(0))</td>
<td>-1.41266</td>
<td>-1.41263</td>
</tr>
<tr>
<td>(-\theta'(0))</td>
<td>0.48752</td>
<td>0.48755</td>
</tr>
</tbody>
</table>
Table 4.2: Values of skin friction co-efficient \( f''(0) \), rate of heat transfer \(-\theta'(0)\) (for VEST Case) and wall temperature \( \theta(0) \) (for VEHF Case).

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( A )</th>
<th>( M )</th>
<th>( S )</th>
<th>( \lambda )</th>
<th>( R )</th>
<th>( N )</th>
<th>( Pr )</th>
<th>( Ec )</th>
<th>( f''(0) )</th>
<th>( -\theta'(0) )</th>
<th>( \theta(0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>-1.7832</td>
<td>0.6820</td>
<td>1.3745</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
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Table 4.3: Comparison of the results for the dimensionless wall temperature gradient $[-\theta'(0)]$ [VEST case] by varying the $Pr$ with $\beta = M = A = R = \lambda = N = S = 0$.

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Figure 4.2: Effect of unsteady parameter ($A$) on velocity profiles.
Figure 4.3: Effect of fluid-particle interaction parameter ($\beta$) on velocity profiles.

Figure 4.4: Effect of Magnetic parameter ($M$) on velocity profiles.
Chapter 4: Unsteady flow and heat transfer of a fluid-particle suspension over an

1. On the dust phase and fluid phase with permeability parameter $K_p = 0.1, 1.0, 2.0$

Figure 4.5: Effect of permeability parameter ($K_p$) on velocity profiles.

Figure 4.6: Effect of Suction parameter ($S$) on velocity profiles.
Figure - 4.7(a): Effect of $A$ on temperature profile (VEST).

Figure - 4.7(b): Effect of $A$ on temperature profile (VEHF).
Chapter 4: Unsteady flow and heat transfer of a fluid-particle suspension over an...
Figure - 4.9(a): Effect of $M$ on temperature profile (VEST).

Figure - 4.9(b): Effect of $M$ on temperature profile (VEHF).
Figure - 4.10(a): Effect of $K_p$ on temperature profile (VEST).

Figure - 4.10(b): Effect of $K_p$ on temperature profile (VEST).
Figure - 4.11(a): Effect of $S$ on temperature profile (VEST).

Figure - 4.11(b): Effect of $S$ on temperature profile (VEHF).
Figure - 4.12(a): Effect of $\lambda$ on temperature profile (VEST).

Figure - 4.12(b): Effect of $\lambda$ on temperature profile (VEHF).
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Figure 4.13(a): Effect of $R$ on temperature profile (VEST).

Figure 4.13(b): Effect of $R$ on temperature profile (VEHF).
Figure - 4.14(a): Effect of $Pr$ on temperature profile (VEST).

Figure - 4.14(b): Effect of $Pr$ on temperature profile (VEHF).
Figure - 4.15(a): Effect of $Ec$ on temperature profile (VEST).

Figure - 4.15(b): Effect of $Ec$ on temperature profile (VEHF).
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Figure - 4.16(a): Effect of $M$ on skin friction coefficient against $S$.

Figure - 4.16(b): Effect of $\beta$ on skin friction coefficient against $S$. 
Figure - 4.17(a): Effect of $R$ and $Ec$ on heat transfer (VEST).

Figure - 4.17(b): Effect of $R$ and $Ec$ on heat transfer (VEHF).
Figure - 4.18(a): Effect of $\lambda$ and $Pr$ on heat transfer (VEST).

Figure - 4.18(b): Effect of $\lambda$ and $Pr$ on heat transfer (VEHF).