COLLAPSE OF CONICAL CAVITIES IN ALUMINIUM METAL UNDER SHOCK WAVE IMPACT

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Summary—Under shock wave impact the conical free surface of the metal collapses and a metallic jet with a velocity exceeding 10 km/s is squirted out of the metallic target. Variation in the jet tip velocity when the cavity half angle ranges from 60° down to 7.5° has been studied over three different shock pressures. The jet velocity has been found to increase with the decrease in the angle of the conical free surface and as the angle approaches zero the jet tip velocity attains a value near to the limit set by hydrodynamic theory for non-compressible fluids. The effect of aerodynamic drag on this type of jet has been studied and it has been found that the jet formed by the collapse of a small angle conical cavity quickly slows down while travelling in air, whereas the jet coming out of large angle cavity suffers a small retardation. Theoretical explanation and experimental evidence in support of this fact are also presented.

NOTATION

\( a, b \) material constants
\( A \) projected area
\( C_D \) drag coefficient
\( F_d \) force due to air drag
\( K(a) \) constant dependent upon the cavity angle
\( M \) mach number
\( P \) shock pressure
\( r_0 \) original radius of any infinitesimal small element of the conical free surface
\( r \) radius of collapsed material or jet radius
\( U_f \) free surface velocity
\( U_s \) velocity with which the material flows behind the shock front
\( U_p \) shock velocity in the metal
\( V_j \) jet tip velocity
\( \theta \) collapse angle
\( \alpha \) cavity half angle
\( \rho_m \) density of metal
\( \rho \) density of air

INTRODUCTION

A large amount of experimental and theoretical work has been published on hollow charge jet formation, stretching and penetration processes, since their introduction in World War II. A complete mathematical theory on jet formation from lined cavities, and its penetration into thick metallic targets, was presented by Birkhoff et al. [1], on the basis of the steady state hydrodynamic flow of non-compressible fluids. A somewhat modified theory of non-uniform collapse was put forward by Pugh et al. [2], giving account of the velocity gradient along the length of a jet, along with experimental evidence [3]. Metallic jets are also produced by collapse of conical [4,5], hemispherical [6,7] and wedge shaped [8,9] free surfaces of the metal under shock wave impact. A jet produced in this type of collapse is sometimes given the name “microjet” as it contains a very small mass and has a comparatively large velocity. A large amount of work has also been done on the jet produced by the collision of two explosive propelled metallic plates at some angles [10–12]. The hydrodynamic flow of non-compressible fluids predicts the jet formation for all angles of conical cavities [1] and the jet velocity increases with decrease in the cavity angle. Walsh et al. [10] derived the limiting condition for the jet formation in the high velocity collision of two explosive propelled metallic plates. They derived a critical collision angle
below which no jet formation takes place. Simonov [8] also calculated the critical angle at which the flow changes over from wave formation to jet formation. Changeover angles calculated by him for aluminium, copper and steel were much smaller than those calculated by Walsh et al. [10]. Yadav et al. [5] working on the collapse of conical metallic free surfaces, calculated the critical angle of conical cavities below which the jet formed is incoherent, with poor penetrating capability. They further concluded that for a small cavity angle, though the jet breaks up into particles, the jet tip still moves with the theoretically predicted velocity.

In the present studies the microjet squirted out of the aluminium targets with conical cavities has been studied for cavity half angles ranging from 60 to 7.5°. It has been observed that the jet formation takes place for all the angles studied and that the jet resulting from the collapse of a larger angle cavity is fine tipped and coherent as compared to that resulting from a small angle (less than 15°) cavity, which is slightly scattered. Moreover, the jet coming out of a small angle cavity travels with the theoretically predicted velocity only for a very brief time (few microseconds) before it quickly slows down, whereas the microjet from a large angle cavity suffers a small retardation. Variation in the jet tip velocity with the cavity half angle has been studied in three configurations of shock wave experiments, namely, plane shock by contact explosive, plane shock by flyer plate impact and spherical converging shock by contact explosive.

THEORY

The motion of the metal occurring as a result of the incidence of a plane shock wave on a conical cavity has been studied theoretically and experimentally by Yadav et al. [5]. As the shock wave pressure is too high, the yield strength of the metal is neglected, thus assuming that it behaves like a perfect fluid [13]. Consider a cylindrical metallic target with small conical cavity of half angle $\alpha$ at one of its plane faces, as shown in Fig. 1. The shock wave is induced in the target from the opposite plane face so that it travels parallel to the axis of the conical cavity. As shock sweeps the slant height of the cavity the metal fall symmetrically on the axis of the cavity and a high speed metallic jet results, which travels in a direction bisecting the cavity angle. An expression for jet velocity can be written [5] as

$$V_j = \frac{r_0}{r} U_f \sin (\theta_0 - \alpha) + U_f \cos (\theta_0 - \alpha). \quad (1)$$

Fig. 1. Schematic diagram of the collapse process.
Where $U_f$ is the free surface velocity, the direction of which is determined by the collapse angle $\theta_0$. Symbol $r_o$ is the initial radius of any small annular element along the slant height and $r$ is the radius of the jet formed. Convergence factor $r_o/r$ is responsible for the high velocity of the jet formed. The continuous flow of mass into the cavity during the process of collapse, fills and flattens the apex of the cavity, or in other words, the diameter of jet increases from tip to tail side. Thus the portions of the cavity having larger values of $r_o$ also have large values of $r$ and therefore one can assign an average value to $r_o/r$ in the expression for jet velocity.

Collapse velocity $U_t$ in Eqn (1) can be written as

$$U_t^2 = U_p^2 [2(1 + \sin \alpha)]$$

which for a plane face ($\alpha = 90^\circ$) leads to the famous doubling rule [14]

$$U_t = 2U_p.$$  

(3)

Since the collapse angle $\theta_0$ is also dependent on cavity angle $\alpha$, the simplified form of Eqn (1) can be written as

$$V_j = K(\alpha)U_p$$

(4)

where $K(\alpha)$ is the factor which depends upon cavity angle and $U_p$ is the velocity with which the mass flows behind the shock front. Jet velocity is thus dependent on cavity angle and particle velocity; increasing with a decrease in cavity angle and also increasing with an increase in particle velocity. Two metals will produce microjets with equal velocities, provided the induced particle velocity behind the shock front in the two metals is the same. Thus it is the inertia which determines the velocity not the strength of material, which further confirms the assumption that the strength of material is neglected in comparison with shock pressure.

Experimental determination of the jet velocity also serves as a pressure sensor. For a known value of $K(\alpha)$, $U_p$ can be calculated from Eqn (4). The equation of state for the metal is

$$U_s = a + bU_p$$

(5)

which determines the shock velocity in the metal. Constants $a$ and $b$ are experimentally measured [15] for aluminium and their values are $5.692 \pm 0.21$ mm/µs and $1.102 \pm 0.16$, respectively. Making use of jump conditions,

$$P = \rho_0U_sU_p$$

(6)

one can calculate the shock pressure. $\rho_0$ in Eqn (6) is the density of non-compressed material, which for the aluminium used is 2.712.

**IMPACT EXPERIMENTS**

To work at different shock wave pressures, three experimental set-ups shown in Figs 2, 3 and 4 were used for the present studies. In the first experimental set up shown in Fig. 2, the plane shock wave was induced in the aluminium target by detonating a 76.2 mm diameter and 50 mm thick RDX/TNT (60:40) pad, in contact with the plane face of the metal target. In order to transmit a plane shock into the metal, high explosive charge was initiated through a plane wave shaper. A conical cavity of depth 10 mm was drilled in the opposite plane face of the metal target. The distance of the apex of the cavity from explosive metal interface was fixed at 10 mm for one set of trials. In the second configuration shown in Fig. 3 the chemical energy of the explosive was stored as the kinetic energy of the flyer plate, which on impact with the target plate generated a plane shock much more intense compared with that of the first geometry. An air gap of 5 mm was kept in between the explosive and flyer plate, to generate a planar impact. Flyer plate diameter and thickness were 60 and 3 mm, respectively, with flight distance kept at 30 mm. Diameter and thickness of the target were 76.2 and 10 mm, respectively, with the depth of the cavity kept at 7 mm. In the third experimental set up shown in Fig. 4, a spherical RDX/TNT (60:40) pad of...
diameter, 76.2 mm, and thickness, 50 mm, was detonated in contact with a metal target of diameter, 76.2 mm, and thickness, 20 mm, with radius of curvature, 30 mm. A wave shaping device with radius of curvature, 80 mm, was used to initiate the high explosive simultaneously. The cavity depth was kept at 10 mm, fixed for this set of trials. This technique made use of spherically converging shock waves. The cross-sectional area of mass flow decreases and in accordance with the law of conservation of mass and non-compressibility of the fluids, the velocity of the fluid increases, and hence the jet velocity. A metallic collar of 25–50 mm in width was kept at the explosive metal interface in all the experiments to block the gaseous products of detonation from camera viewing area.

The flight of the jet in the air was recorded on a Beckman Whitley Model 189-A framing camera with framing capability up to $1.2 \times 10^6$ frames/s. In the present studies, the camera was run at $2.4 \times 10^5$ frames/s with interframe timing of 4.16 $\mu$s. Microjet flight in air is
\( V_j \) in the above equations is the jet tip velocity in mm/\( \mu s \) and \( \alpha \) is the cavity half angle in degrees. Hydrodynamic theory of non-compressible fluids predicts the maximum jet velocity for a cavity half angle approaching zero to be twice the shock velocity in metal [1], which in the present case works out to be 14.76 mm/\( \mu s \) for the experimental set up of Fig. 2. From Eqn (8) the limiting value of jet tip velocity for a cavity half angle approaching zero is 12.64 mm/\( \mu s \). This difference is attributed to the aerodynamic drag, and in support of this fact we have also conducted some trials. Figure 9 shows the microjet flight in a glass tube, evacuated by a two stage rotary pump. The experimental set up was that of Fig. 2, with a cavity half angle of 37.5° and a glass tube vacuum sealed on the cavity portibn. The jet tip velocity calculated from this record is 8.41 mm/\( \mu s \), against 7.96 mm/\( \mu s \) calculated for the jet flight in air, with the same cavity angle. In another trial also, the jet tip velocity, measured in vacuum for cavity half angle of 30° is 9.94 mm/\( \mu s \), against 8.9 mm/\( \mu s \) in air. Though no attempt has been made to study all the cavity angles in vacuum, from the observed difference it is sufficient to conclude that the effect of air drag cannot be neglected in the case of hypersonic flow.

**CONCLUSION**

From the experimental observations it is concluded that the jet velocity depends only on the angle of the conical free surface and the mass flow velocity behind the shock front. Air drag causes the jet velocity to drop considerably, especially for the jet coming out of small angle conical cavities. If the air resistance is accounted for, the limiting value of jet tip velocity for a cavity angle approaching zero may be equal to twice the shock velocity in metal, which indicates that the collapse process of the conical free surface can be treated in terms of the hydrodynamic theory of non-compressible fluids.

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**REFERENCES**

Fig. 8. Variation in jet tip velocity with cavity half angle for three different configurations of shock wave experiments.

Fig. 9. Microjet from aluminium target of cavity half angle of 37.5° in a plane geometry. Free flight of jet in a vacuum tube.
Fig. 7. Variation in jet tip velocity with time for four different cavity half angles.

Table 3. Measured jet tip velocity with different cavity half angle

<table>
<thead>
<tr>
<th>Sr. number</th>
<th>Cavity half angle (°)</th>
<th>Measured jet tip velocity $V_j$ (mm/µs)</th>
<th>Shock pressure at apex of cavity $P$ (kbar)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Plane geometry (contact explosive)</td>
<td>Plane geometry (flyer plate impact)</td>
<td>Plane geometry (contact explosive)</td>
</tr>
<tr>
<td>1. 7.5</td>
<td>12.33*</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>2. 15.0</td>
<td>10.90</td>
<td>12.24*</td>
<td>13.15*</td>
</tr>
<tr>
<td>3. 22.5</td>
<td>9.38</td>
<td>11.47</td>
<td>11.64</td>
</tr>
<tr>
<td>4. 30.0</td>
<td>8.80</td>
<td>10.40</td>
<td>10.59</td>
</tr>
<tr>
<td>5. 37.5</td>
<td>7.96</td>
<td>9.81</td>
<td>9.74</td>
</tr>
<tr>
<td>6. 45.0</td>
<td>7.13</td>
<td>9.42</td>
<td>8.85</td>
</tr>
<tr>
<td>7. 60.0</td>
<td>5.95</td>
<td>8.19</td>
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</table>

*Jet velocity measured for first 4 µs of jet travel.

jet from the small angle cavity has a large jet tip velocity and hence the aerodynamic drag suffered by this type of jet is far greater than that suffered by the jet coming out of a large angle cavity. The jet tip velocities, $V_j$, for different cavity half angles are given in Table 3. The asterisked entries are the values of the jet tip velocity for the first 4 µs of jet travel as the jet from small angle quickly slows down. The rest of the entries are taken as the average values. From the measured jet tip velocities, for a cavity half angle of 30° and from known value of $K(a)$ (Table 1), the values of the shock pressure at the apex of the cavity are calculated, and are given in Table 3. As shown in Fig. 8 the jet tip velocity increases linearly with the decrease in angle $a$, and the straight line equations fitted by the method of least squares for all three geometries are given below:

$$V_j = 12.64 - 0.119a$$  \hspace{1cm} (8)

plane geometry (contact explosive)

$$V_j = 13.37 - 0.09a$$  \hspace{1cm} (9)

plane geometry (flyer plate impact)

$$V_j = 14.93 - 0.137a$$  \hspace{1cm} (10)

spherical/geometry (contact explosive).
Collapse of conical cavities

Metallic jet

Luminous envelope

Fig. 6. Microjet from aluminium target of cavity half angle of 45° in spherical geometry. Free flight of jet in air.

Table 2. Jet tip velocity measured at different timings

<table>
<thead>
<tr>
<th>Time (µs)</th>
<th>7.5° (spherical geometry)</th>
<th>15° (flying plate)</th>
<th>30° (spherical geometry)</th>
<th>45° (spherical geometry)</th>
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<tr>
<td>0.00</td>
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<td>4.16</td>
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<td>20.84</td>
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<td>10.44</td>
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</tr>
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<td>29.12</td>
<td>8.60</td>
<td>—</td>
<td>10.30</td>
<td>8.00</td>
</tr>
<tr>
<td>33.28</td>
<td>7.97</td>
<td>—</td>
<td>—</td>
<td>8.00</td>
</tr>
</tbody>
</table>

The body and speed of flow, \( A \) is the projected area, \( \rho \) is the density of air, and \( V_j \) is the jet velocity. For the hypersonic flow in the range of Mach 20–Mach 40, not only the jet tip area but approximately the whole of the jet area acts as a projected area. The drag coefficient \( C_D \) also takes some constant value in this range of material flow. As a microjet from a small angle cavity is scattered and has a large velocity gradient along its length, it presents a large surface area as compared to a jet from large angle cavity. Moreover, the
Coming back to Fig. 1, the component of shock wave sweeping the slant height of the cavity is $U_i \sec \alpha$, which decreases as the cavity half angle decreases. During the collapse process the shock pressure is released by the mass flow at the free inner surface of the cavity which gives rise to a rarefaction wave. The rarefaction wave thus generated tries to overtake the shock front with a speed equal to the sound velocity in compressed material plus the component of mass flow velocity along the slant height of the cavity. For a small cavity angle the component $U_i \sec \alpha$ is also small and the rarefaction wave quickly overtakes the shock front, thus reducing the collapse velocity. This gives rise to a non-uniform collapse and velocity gradient along the length of the jet formed. The component $U_i \sec \alpha$ has a large value for a large cavity angle and by the time the rarefaction wave overtakes the shock front most of the collapse process is completed and hence the jet formed is fine tipped with good penetrating capability. Moreover, it is practically difficult to fabricate the small angle conical cavities with pointed apex as some of the material is left at the apex while machining the target. This material moves out of the cavity with free surface velocity prior to the jet formation and forms the front part of the jet. The overtaking of slow moving mass by the fast moving jet, formed later by collapse of slant free surface of the cavity, causes scatter in the jet.

When a metallic jet travels in air, there is aerodynamic drag which opposes its motion. The basic equation of aerodynamic drag can be written as:

$$F_d = \frac{1}{2}C_D \rho A V_i^2$$  \hspace{1cm} (7)$$

where $F_d$ is the drag force, $C_D$ is the drag coefficient, which is dependent on the shape of
self luminous due to associated air shock and the aim was to measure jet tip velocity so no artificial light source was used. In some of the trials where the jet was fired in a vacuum, an argon flash bomb was used as background light source. Experimental determination of free surface velocity was also required in order to find out the value of \( K(a) \). Five different high explosive compositions (Table 1) were detonated in contact with the aluminium disc of thickness, 10 mm, and diameter 100 mm. The free surface velocity in each trial was recorded on Beckman Whitley Model 770 streak camera. The same five explosives were also used in plane geometry to study the effect of particle velocity on the jet velocity, keeping the angle of conical cavity fixed.

**RESULTS AND DISCUSSIONS**

Figures 5 and 6 show the framing camera record of the microjets from aluminium in spherical geometry with cavity half angles of 7.5 and 45°, respectively. The record shows the envelope of luminous trials with the jet hidden somewhere inside the luminous cloud. Jet is slightly visible as a shadowgraph in Fig. 6. At very high speeds \( M > 20 \) of the jet, the shape of the air shock associated with the jet resembles that of the jet, with small spread towards the tail end. Air behind the shock front heats up to very high temperature at which the process of energy transfer in the form of radiation takes place. Dissociation, ionization and recombination [16] of oxygen and nitrogen takes place across the shock front, which gives strong luminosity to the surroundings of the jet. White et al. [17] has taken the spectrogram of the luminosity associated with metallic pallets flying in air with velocities 4–6 km/s and identified the bands of metallic oxides apart from the ionized oxygen and nitrogen. To confirm that the source of luminosity is air, we have fired the microjet in a vacuum tube with the result that no luminosity has been observed.

The position of the jet at different times is written under each frame, taking camera opening time as the zero time. Jet tip velocity is the velocity with which the luminous front advances vertically downwards. By comparing the two records one can easily distinguish the coherent and fine tipped microjet from cavity half angle of 45° as compared to that from cavity half angle of 7.5°. A jet formed by the collapse of small angle cavity is non-coherent and scattered, which quickly slows down while travelling in air, whereas the microjet for large angle cavity suffers small retardation. The variation in jet tip velocity with time for four different cavity half angles of 7.5, 15, 30 and 45° is given in Table 2. Jet tip velocity in the case of the 7.5° cavity half angle, drops from 12.59 to 9 m\( \text{m/s} \) in a time span of 16.64 \( \mu \text{s} \). Whereas in the case of the 45° cavity half angle it drops from 8.61 to 8.07 m\( \text{m/s} \) over the same time span. Zero time in Table 2 is taken arbitrarily when the jet first appears in the camera record. Jet tip velocity versus time of flight is plotted in Fig. 7 and it has been observed that the retardation suffered by the jet decreases as the cavity angle increases and becomes negligibly small for a cavity angle exceeding 30°.
15. TBRL Report No. 95/74.
PARTICULATION OF METALLIC JETS UNDER HIGH STRAIN RATE

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Summary—High-speed metallic jets are squeezed out of the conical cavities in a metallic block when loaded with shock waves. The particulation studies on these jets, which elongate under a strain rate of the order of $10^4$-$10^5$ s$^{-1}$, have been carried out theoretically as well as experimentally. The synchro-streak technique (SST) has been used to measure the unparticulated jet distance (UPJD). The time of particulation calculated from the UPJD has been found to be equal to the total of the collapse time and the stretching time. It has been found that as the angle of conical cavity increases, the jet diameter and the particulation time increase, whereas the jet tip velocity and existing strain rate decrease. From the experimentally measured jet diameter the time of particulation has also been calculated by the Hirsch approximation. This time period is in close agreement with the particulation time calculated by the authors. From the measured jet diameter and jet length, the mass of the jet and the thickness of the block material squeezed into the jet have been estimated. A comparative study on aluminium and copper jets has also been conducted and it has been observed that for the given explosive metal system the jets produced by aluminium are solid and coherent as compared to those produced by copper.

NOTATION

- \( d_i \): Initial diameter of the jet
- \( d_b \): Base diameter of the elongated jet
- \( L_s \): Length of the elongated jet
- \( L_c \): Length of the jet formed in the collapse time
- \( l \): Slant height of the conical cavity
- \( M \): Mass of the jet
- \( r \): Radius of the base of the conical cavity
- \( s \): Thickness of the block material squeezed into the jet
- \( t_c \): Collapse time
- \( t_p \): Particulation time
- \( t_s \): Stretching time
- \( U_c \): Collapse velocity
- \( U_m \): Material velocity given by the release of hydrostatic pressure
- \( U_p \): Particle velocity
- \( V \): Jet tip velocity
- \( V_t \): Jet tail velocity
- \( V_p \): Plastic velocity
- \( X_p \): Unparticulated jet distance (UPJD)
- \( \theta \): Collapse angle
- \( \alpha \): Cavity half angle
- \( \sigma_y \): Metal dynamic yield stress
- \( \Delta \sigma \): Difference between isothermal and adiabatic stress versus strain characteristics of the metal
- \( \rho \): Density of the block material
- \( \eta \): Strain rate

INTRODUCTION

The ductility of some metals can increase by an order of magnitude when the strain rate increases from $10^{-2}$ to $10^{+5}$ s$^{-1}$. Shaped charge jets provide a unique opportunity to test the behaviour of metals undergoing elongation of a few hundred to a thousand per cent. Experimental observations of this order of elongation are difficult to see in other geometries. The extent of elongation depends upon the strain rate, initial dimensions and the nature
of the material. The cross-sectional area of the material undergoing elongation decreases until necking regions develop at some weak points along the stretched length. On further elongation the material breaks at these necking regions—the onset of particulation.

Shaped charge jet break-up is one method of studying materials under high strain rate. Another method is by explosively induced cavity collapse [1] which is the method adopted by the authors. Under the impact of shock waves the conical cavities in the metal collapse and a metal protrusion resembling the original cavity is formed. By increasing the shock pressure the metal flows in a fluid jet followed by the metallic block. It is a simple test technique which simulates high strain and high strain rate in shaped charge jet formation. The jet velocity, diameter and the strain rate can be varied inexpensively by changing either the cavity shape, quantity of the explosive or the nature of the material.

The break-up of a material under high strain rate has been described by Hirsch [2,3]. He calculated the break-up time of an explosive loaded metallic cylinder and shaped charge jets using a very simple formula

$$ t_p = \frac{d}{V_{pl}} $$

where \(d\) is the initial diameter of the jet before elongation starts and \(V_{pl}\) is the characteristic velocity of the material. In shaped charge jets this velocity is identified as the velocity increment between successive fragments of a particulated jet. In the explosive loaded cylinder it is called the plastic velocity and is given by the formula

$$ V_{pl} = \sqrt{\sigma_p / \rho} $$

where \(\sigma_p\) is the metal dynamic yield stress and \(\rho\) is the specific density of the material. The ambiguity in the relation (1) lies in the measurements of \(V_{pl}\) at high strain rate. To account for the dependency of \(V_{pl}\) on the strain rate, Eqn (2) was modified in Ref. [4] as

$$ V_{pl} = \sqrt{(d\sigma_m / \rho)} $$

where \(d\sigma_m\) is the difference between isothermal and adiabatic stress versus strain characteristics of the metals, at the point where the adiabatic stress becomes maximum. The factor \(d\sigma_m\) is weakly dependent on strain rate because the adiabatic and isothermal characteristics are influenced by the strain rate in a similar way. The value of \(V_{pl}\) has been calculated by many authors [5,6] and for aluminium it is between 110 and 250 m s\(^{-1}\). Beside its simplicity, Eqn (1) predicts the break-up time of elongating materials with a reasonable accuracy.

The particulation time of the shaped charge jets was found experimentally by Held [7,8]. He used the synchro-streak technique (SST) and flash radiography to record the jet break-up. The average particulation time was found to be equal to the length of the jet divided by the velocity gradient between the two ends of the jet. Held also calculated the individual particulation time of the different segments of the jet and found that the particulation time increased from the tip end to the tail end of the shaped charge jet.

In a previous paper [9] we have studied the variation in the jet tip velocity with the angle of the conical cavity in different geometries of explosive metal interactions. The jet tip velocity in plane geometry was found to vary from 5.4 mm μs\(^{-1}\) to 10.8 mm μs\(^{-1}\) as the cavity half angle changed from 60° to 15°. This order of velocity creates the strain rate of the order of 10\(^4\)-10\(^5\) s\(^{-1}\) across the jet length. By dividing the whole process of the jet formation into three parts, namely, the collapse process, stretching and particulation, we have empirically calculated the collapse and stretching times of the jet coming out of the aluminium blocks with different cavity angles. The addition of both times gives the particulation time; the time elapsed from when the shock front hit the apex of the conical cavity until the onset of particulation. The particulation time of the jets formed by the collapse of conical cavities of different angles are verified experimentally by using SST.

An attempt has been made to measure the jet diameter just after the collapse process is
over. The jet diameter divided by the plastic velocity of 145 m s\(^{-1}\) \cite{6} gives the break-up time \cite{2} from when elongation starts. It is assumed that elongation starts only after the collapse process is over and the tip of the jet is formed by the apex portion of the cavity. The particulation timings calculated by the two approaches are in close agreement with the time of particulation found experimentally. By measuring the diameter and the length of the elongated jets the mass of the jet and the thickness of the aluminium block squeezed into the jets have also been estimated.

**BREAK-UP TIME CALCULATIONS**

Figure 1 shows the process of jet formation by the interaction of shock waves with a V-shaped cavity machined at a plane face of a cylindrical metallic block. Another plane face of the block was placed in contact with the high explosive. As the shock pressure is released at the free inner surface of the cavity, the material collapses towards the axis of the cavity. Taking into consideration the conservation of mass and the non-compressibility of the material, the collapse velocity of the material increases as it approaches the cavity axis. For simplicity in calculations an average value is given for the collapse velocity \cite{10}

\[ U^2 = U^2_2 (1 + \sin \alpha). \]  

(4)

The velocity vector \( U_f \) makes an angle \( \theta \) with the original position of the cavity surface and is given by

\[ \theta = \pi/4 + \alpha/2, \]  

(5)

where \( \alpha \) is the cavity half angle. These two relations have been written by assuming that the particle velocity, \( U_p \), in the direction of shock wave and the velocity, \( U_m \), given to the material by the release of hydro-static pressure are equal. The collapse process is over when the last particle at the free inner surface of the cavity reaches the axis of the cavity and forms the tail of the jet. The collapse time is the time taken by the shock front to travel from the apex to the base of the cavity plus the time taken by the material at the base to reach the axis of the cavity. By simple trigonometry and by making use of (4) and (5), the collapse time can be calculated for a given cavity half angle \( \alpha \). The particle velocity \( U_p \) measured experimentally for the given explosive metal system is 1.5 mm \( \mu \) s\(^{-1}\).

It has been found in experimental trials that the metallic sample follows the metallic jet and is completely driven through the hole created by the jet in thin target plates. The rear

![Fig. 1. Schematic diagram of the jet formation by explosive-metal interaction.](image-url)
end of the jet remains attached to the moving metal block while the jet tip advances at a rate determined by the cavity angle. The variation in jet tip velocity with the angle of the cavity was found experimentally [9] and a linear relation fitted by the method of least squares was obtained

\[ V_j = 12.64 - 0.12a, \]

where \( V_j \) is the jet tip velocity in mm \( \mu s^{-1} \) and \( a \) is the cavity half angle in degrees. The tail velocity of the jet \( V_0 \) is taken to be equal to the free surface velocity of the metallic block which is twice [11] the particle velocity, i.e. 3 mm \( \mu s^{-1} \). The velocity gradient between the jet's two ends means that it elongates at a strain rate given by

\[ \dot{\eta} = L_0(V_j - V_0), \]

where \( L_0 \) is the length of jet formed in the collapse time and can be written as

\[ L_0 = V_j t_c. \]

An assumption has been made that the jet tip velocity remains constant throughout the time \( t_c \); this is true for a large angle cavity, an appreciable error [9] can be introduced for small angle cavities. The extent of elongation, and hence the break-up time, depends upon the quality of the material, strain rate and the initial diameter of the jet formed. On stretching, the diameter of the jet decreases uniformly until some necking regions develop along the jet length. On further elongation the material fails at these weak points and the jet becomes particulated. Particulation starts from the tip portion and progressively travels towards the tail end. In other words, the portions of jet moving faster are particulated earlier. The time of particulation measured from the arrival of the shock front at the apex of cavity can be written as the sum of the collapse time and the stretching time

\[ t_p = t_c + t_s = t_c + L_0(V_j - V_0), \]

\[ t_p = t_c + t_s(V_j - V_0). \]

The second factor in Eqn (10) is simply the inverse of the strain rate. With an increase in the angle of the conical cavity both the factors on the right-hand side of Eqn (10) increase, hence the particulation time also increases. Hirsch's formula (1) also predicts a longer particulation time for large angle cavities, as the diameter of the jet formed increases in accordance with the angle of the cavity.

The stand-off distance from the apex of the cavity where the particulation starts has been measured experimentally. This is also known as the unparticulated jet distance (UPJD) [12] and is related to the particulation time by

\[ X_p = t_p V_j. \]

EXPERIMENTAL PROCEDURES

The diagnostic techniques available for jet particulation studies are flash radiography and SST. Although the details of SST and its advantages and disadvantages compared to flash radiography are given elsewhere [13,14], a brief review will be given here.

The synchro-streak technique resembles a sports field photofinish. The speed of the event to be recorded is matched with the recording speed on the film, i.e. the writing rate of the streak camera. The camera slit is set perpendicular to the predetermined path of the event to be recorded. The streak camera record thus gives a time history of the event in one direction, at a point where the event crosses the camera slit. By using SST the same jet
Particulation of metallic jets under high strain rate

DETONATORS

Fig. 2. Experimental set-up for recording the jet by bi-synchro streak technique (Bi-SST).

The experimental set-up is shown in Fig. 2. A cylindrical metallic block of 76.2 mm diameter and 30 mm thickness was placed in contact with the RDX/TNT (60:40) pad of 76.2 mm diameter and 50 mm thickness. High explosive was detonated through a plane wave shaper using an EBW detonator. A conical cavity of depth 20 mm was drilled in the opposite plane face of the metallic block. The cavity half angle was changed from 15° to 60° to produce metallic jets of different velocities and, therefore, different strain rates. A metallic collar of 25–50 mm width was kept at the explosive metal interface in order to keep the detonation gases away from the camera viewing area. The plane of observation was kept sufficiently away from the explosive metal assembly in order to avoid the explosion debris obstructing the camera optical path.

Most of the trials were conducted using SST in which the metallic jets have been recorded at two stand-off distances. The streak camera slit was set perpendicular to the jet axis at one point and with the help of a pair of mirrors, M1 and M2, the plane of observation of the second point was rotated through 90° to bring the image on the same split. Two argon filled PVC pipes of 100 mm diameter and 1400 mm length were used to provide back lighting in order to record the jet as a shadowgraph. To induce the shock waves in argon, a high explosive pad of 76.2 mm diameter and 50 mm thickness was placed in contact with the rear end of each pipe. The front ends of the pipes were covered with a translucent paper to provide the diffused light. Argon flash bombs (AFB) were covered with black PVC tape of 12.7 mm width to facilitate the jet diameter measurements. The relative delay between the two AFBs was introduced depending upon the jet velocity and the distance between the two recording points. In some of the trials the jet was recorded at three stand-off distances using four mirrors. The adjustment of the mirrors in different planes and focusing the three observation points on one slit was difficult in practical use. We recorded the jet at a number of stand-off distances in a single trial without using any mirror. In this set-up multislits were made on the argon flash bomb and the camera was kept in the open view mode. The jet axis was rotated with reference to the camera optical axis to ensure that no overlapping occurred between the individual sweeps of the slits. The length of each slit was kept just sufficient to record the cross-sectional view of the metallic jet. A typical streak camera record of the metallic jet from aluminium block of cavity half angle of 30° recorded at four stand-off distances is shown in Fig. 5.

The jet break-up was recorded on a Beckman & Whitley model 770 Streak camera. The writing rate of the camera was adjusted to be between 0.83–1.67 mm μs⁻¹ depending upon the jet velocity and the magnification of the test event on the film plane. One typical SST record of the metallic jet from aluminium block of cavity half angle of 22.5° is shown in...
Fig. 4. The record does not give the length of the jet but gives the time of passage. The length of the jet is equal to the time of passage multiplied by the jet velocity.

RESULTS AND DISCUSSION

The type of explosive metal interaction under discussion is also known as explosively induced cavity collapse. The velocity, diameter and the length of the metal jet are dependent upon the cavity dimensions, nature of the material, and the nature and the quantity of the explosive detonated. In this study most of the work was with an aluminium metallic block placed in contact with RDX/TNT (60:40) explosive. Only the cavity dimensions have been changed to produce metallic jets of different velocity, length and diameter. The amount of material going into the jet is determined by the area of conical free surface interacting with the shock waves. As the shock wave sweeps the conical cavity from apex to the base region, the free surface interacting with the shock waves also increases, at a rate determined by the angle of the cavity. Assuming that there is no appreciable shock attenuation from the apex to the base of the cavity, the amount of collapsing material and hence the diameter of jet should increase from tip to tail end. The gradient in jet diameter from tip to tail end is dependent on the cavity angle. In other words the jets produced resemble the inversion of the original cavity, which is elongated from the apex.

The synchro-streak records of the jets produced by the shock-loaded aluminium blocks with different cavity angles are shown in Fig. 3. The writing rate of the camera was set at 1.67 mm µs⁻¹ for the jets of Fig. 3(a)–(d) and at 1.25 mm µs⁻¹ for Fig. 3(e) and (f). The records give an impression that as the cavity half angle increases from 15° to 60° the shape of the jet changes from a thin cylindrical rod to a right circular cone. In SST recording the tip and the tail portions of the jet are recorded at two different times. When the tip of the jet was being recorded, the diameter of the base portion of the jet was different to what had actually been recorded. The decrease in the jet diameter due to elongation and the arrival of different portions of the jet in front of the camera slit, at different times, makes the jet look like a thin cylindrical rod, not a right circular cone as would be expected.

<table>
<thead>
<tr>
<th>Cavity half angle (degree)</th>
<th>Stand off distance (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>a 80</td>
</tr>
<tr>
<td>22.5</td>
<td>b 80</td>
</tr>
<tr>
<td>30</td>
<td>c 100</td>
</tr>
<tr>
<td>37.5</td>
<td>d 120</td>
</tr>
<tr>
<td>45</td>
<td>e 150</td>
</tr>
<tr>
<td>60</td>
<td>f 250</td>
</tr>
</tbody>
</table>

Fig. 3. Synchro-streak records of the aluminium jets from different cavity angles.
For larger cavity angles the gradient in the jet diameter is greater and because of low strain rate it cannot be compensated for by the elongation, hence the conical shaped jets are recorded. A 12.7 mm wide black patch comes into most of the dynamic records for the measurements of jet diameter. By assuming the density of the jet material to be equal to the block material the mass of the jet has been calculated by the formula

\[ M_j = \frac{1}{12\pi} (d_e)^3 L_e \cdot \rho, \]  

(12)

where \( L_e \) is the length of the elongated jet measured from the record, \( d_e \) is the diameter of the elongated jet measured at the base portion and \( \rho \) is the density of the block material \((=2.712)\). The mass in the jet can also be calculated by taking the original length of the jet \( L_0 \) \((= V_j t_j)\) formed in the collapse time, but in that case the correction has to be applied in the jet diameter. In the records of Fig. 3 we are measuring the diameter of the elongated jet which is related to the jet diameter of the original jet length

\[ d_e/d_e = \sqrt{(L_e/L_0)} \]  

(13)

which means that the shrinkage in area is the square root of the axial stretching. Mass calculated by both the approaches will be the same. Mass in the jet divided by the area of conical free surface of the original cavity gives the thickness \( s \) of the block material squeezed into the jet. The required relation is

\[ s = \frac{M_j}{\pi r l}, \]  

(14)

where \( r \) is the radius of the base of conical cavity and \( l \) is the slant height of the conical cavity. Experimentally measured jet parameters such as the jet tip velocity, jet diameter, mass in the jet and the thickness of the material participated in the jet formation are given in Table 1. The thickness of the material going into the jet increases with the angle of the cavity because of the shock wave impact on the conical free surface. This was grazing for a small angle and becomes oblique and near normal for larger cavity angles.

The conical shaped metallic jets once formed, elongate because of the velocity gradient between the two ends. The extent of elongation depends upon the strain rate, diameter of the jet and the quality of the material. For a given explosive-metal system the jet tip velocity increases linearly with the decrease in the angle of the conical cavity. By changing the angle of the conical cavity alone the strain rate changes because the tail velocity of the jet in all the cases is equal to the free surface velocity of the block. Due to continuous stretching, the diameter of the jet decreases uniformly initially, until necking regions develop along the jet length. On further elongation the material fails at these necking regions and becomes particulated. The number and sizes of the particles are governed by the wavelength

<table>
<thead>
<tr>
<th>Cavity half angle, x (degree)</th>
<th>Experimental measured</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Original jet length, ( L(= t_j V_j) ) (mm)</td>
</tr>
<tr>
<td>15</td>
<td>69</td>
</tr>
<tr>
<td>22.5</td>
<td>86</td>
</tr>
<tr>
<td>30</td>
<td>104</td>
</tr>
<tr>
<td>37.5</td>
<td>126</td>
</tr>
<tr>
<td>45</td>
<td>155</td>
</tr>
<tr>
<td>60</td>
<td>250</td>
</tr>
</tbody>
</table>
Stand off distance (mm)

80
130
180
230
280

Time
Writing rate = 1.67 mm / μsec

Fig. 4. Stretching and particulation of the aluminium jet from cavity half angle of 22.5°.

Stand off distance (mm)

140
180
220
260

Time
Writing rate = 1.67 mm / μsec

Fig. 5. Multipoint recording of the metallic jet in synchro-streak mode.

[15] of any small disturbance originating during the collapse process. The SST record of the jet stretching and particulation is shown in Fig. 4. The jet from the aluminium block with a cavity half angle of 22.5° has been recorded at stand-off distances of 80, 130, 180, 230 and 280 mm from the apex of the cavity. The jet tip velocity in the present case is 9.9 mm μs⁻¹ and the jet is stretching under the initial strain rate of 8 × 10⁶ s⁻¹. The jet can be seen particulated at 230 mm against 210 mm calculated from Eqns (10) and (11). Due to aerodynamic drag, erosion and ablation [16,17] occurs at the jet tip which can be seen as a thick opaque cloud engulfing the tip portion of the jet. A weakly shaded portion surrounding the jet and diverging towards the tail end is the air shock, generated by the jet passage.

It is rather difficult to locate the exact stand-off distance at which the jet break-up occurs. It requires repetition of trials at different stand-off distances. However this procedure can be made easy by using a multislit streak recording. One typical record of multislit recording of the aluminium jet at four stand-off distances is shown in Fig. 5. The recording points are selected in the vicinity of the roughly estimated particulation stand-off. Although the
Particulation of metallic jets under high strain rate

Stand-off distance (mm) vs. Synchro streak record

Time ———

Writing rate = 0.83 mm / μsec

FIG. 6. Synchro-streak record of the aluminium jet from cavity half angle of 60° at different stand-off distances.

FIG. 7. Enlarged view of neck formation and jet break-up.

stand-off distances at which the particulation starts can be found accurately using this technique, a 20-30 mm variation in UPJD has still been observed from trial to trial.

For a metallic jet of a perfect right circular cone shape the particulation should start from the very beginning. Jet break-up at very initial stages has actually been observed in the experimental trials with large angle conical cavities because the jet formed by the large angle cavities is fine tipped, as can be seen from Fig. 3. The SST record of the jet from an aluminium block with a cavity half angle of 60° is shown in Fig. 6. The jet having a tip velocity of 5.4 mm μs⁻¹ and stretching under the strain rate of the order of 10⁴ s⁻¹ has been recorded at the stand-off distances of 250, 400, 600, 800 and 1000 mm from the cavity apex. The writing rate of the camera was 0.83 mm μs⁻¹, except for the recording at the stand-off distance of 250 mm which was made at 1.25 mm μs⁻¹. The jet tip can be seen to be particulated at the very initial stages. A major portion of the jet starts particulating only after the stand-off distance of 800 mm. A magnified view of the formation of the necking regions and the jet break-up is shown in Fig. 7. The average particulation times of the aluminium jets produced by the collapse of conical cavities of angles 15, 22.5, 30, 37.5, 45 and 60° are given in Table 2. Figure 8 gives the result in graphical form. The unparticulated jet distance (UPJD), the stand-off distance at which the major portion of the jet starts particulating, has been measured experimentally. This distance divided by the jet tip velocity gives an average time of particulation. There is an excellent agreement between the time of particulation calculated with the help of relation (10) and that found experimentally.

The Hirsch break-up model of a uniformly elongating material can also be applied to
Table 2. Comparison of particular times calculated from Eqn (10) and found experimentally

<table>
<thead>
<tr>
<th>Cavity half angle, ( \alpha ) (degree)</th>
<th>Jet tip velocity, ( V_j ) (mm ( \mu )s(^{-1}))</th>
<th>Strain rate, ( \eta ) (s(^{-1}) ( \times ) 10(^4))</th>
<th>Collapse time, ( t_c ) (( \mu )s)</th>
<th>Stretching time, ( t_s ) (( \mu )s)</th>
<th>Particulation time, ( t_p = t_c + t_s ) (( \mu )s)</th>
<th>Unparticulated jet distance, ( X_p ) (mm)</th>
<th>Particulation time, ( t_p = X_p/V_j ) (( \mu )s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>10.8</td>
<td>11</td>
<td>6.4</td>
<td>8.8</td>
<td>15.2</td>
<td>160 ± 20</td>
<td>14.8 ± 1.8</td>
</tr>
<tr>
<td>22.5</td>
<td>9.9</td>
<td>8</td>
<td>8.7</td>
<td>12.5</td>
<td>21.2</td>
<td>200 ± 20</td>
<td>20.2 ± 2</td>
</tr>
<tr>
<td>30</td>
<td>9</td>
<td>5.8</td>
<td>11.9</td>
<td>17.3</td>
<td>29.2</td>
<td>270 ± 20</td>
<td>30 ± 2.2</td>
</tr>
<tr>
<td>37.5</td>
<td>8.1</td>
<td>4</td>
<td>15.6</td>
<td>24.7</td>
<td>40.3</td>
<td>330 ± 25</td>
<td>40.7 ± 3.1</td>
</tr>
<tr>
<td>45</td>
<td>7.2</td>
<td>2.7</td>
<td>21.6</td>
<td>36.9</td>
<td>58.5</td>
<td>430 ± 30</td>
<td>59.7 ± 4.2</td>
</tr>
<tr>
<td>60</td>
<td>5.4</td>
<td>1</td>
<td>46.2</td>
<td>104.2</td>
<td>150.4</td>
<td>800 ± 30</td>
<td>148 ± 5.5</td>
</tr>
</tbody>
</table>

X - EMPIRICAL POINTS
I - EXPERIMENTAL POINTS
\( \cdot \) - HIRSCH APPROXIMATED POINTS

Fig. 8. Results of break-up studies in aluminium block, summarized in graphical form.

determine the particulation time of the jet. Since the gradient in the jet diameter for small angle cavities is very small and the jet tips are also not too sharp, so these jets can be treated as equivalent to the elongating cylindrical rods. The jet diameter measured just after the passage of the tip portion of the aluminium jets is given in Table 3. Hirsch break-up time under pure elongation can be calculated with the help of Eqn (1). The plastic velocity \( V_{pl} \) has been measured experimentally [6] to be equal to 145 m s\(^{-1}\). To calculate the particulation time of the jet the collapse time has to be added to the Hirsch break-up time and is given in Table 3. Reasonably good agreement is observed.

In this study, although the major work has been undertaken with aluminium, some trials were also conducted with copper as the block material. The SST record of a copper jet from a cavity half angle of 45° recorded at stand-off distances of 150, 300 and 500 mm is shown in Fig. 9. The jet with a tip velocity of 4.2 mm \( \mu \)s\(^{-1}\) was recorded at a writing rate of 0.83 mm \( \mu \)s\(^{-1}\). By comparing this record with the record of Fig. 4, one can see that the
Particulation of metallic jets under high strain rate

Table 3. Hirsch break-up time measurements

<table>
<thead>
<tr>
<th>Cavity half angle, α (degree)</th>
<th>Jet diameter at the leading part, ( d_0 ) (mm)</th>
<th>Stretching time, ( t_s = \frac{d_0}{V_{ps}} ) (µs)</th>
<th>Collapse time, ( t_c ) (µs)</th>
<th>Particulation time, ( t_c + t_s ) (µs)</th>
<th>Particulation time calculated from Eqn. (10), ( t_p ) (µs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>1.3</td>
<td>8.97</td>
<td>6.4</td>
<td>15.37</td>
<td>15.2</td>
</tr>
<tr>
<td>22.5</td>
<td>2</td>
<td>13.79</td>
<td>8.7</td>
<td>22.49</td>
<td>21.2</td>
</tr>
<tr>
<td>30</td>
<td>2.4</td>
<td>16.55</td>
<td>11.9</td>
<td>28.45</td>
<td>29.2</td>
</tr>
</tbody>
</table>

aluminium jet can be elongated as a single rod (Figs 4 and 6) until neck formation and jet break-up occurs, whereas the copper jet on elongation (Fig. 9) breaks into metal fragments without any neck formation. Two reasons may be given for the difference observed in elongation of the two materials. One is the temperature rise in the collapse process may be sufficient to melt aluminium but not enough for copper and the second is for the given cavity angle the stretching rate in the formed aluminium jet is almost twice that of the copper jet from a similar explosive metal assembly. Further studies on this type of explosive-metal interaction, with an emphasis on the suitability of different metals for block materials, are continuing.

CONCLUSIONS

The main conclusions drawn from this study are:

(i) the time of particulation measured from the arrival of the shock front at the apex of cavity to the onset of particulation is the sum total of the collapse time and the stretching time;
(ii) the particulation time of the jets from small angle cavities calculated by Hirsch relation (1) agree with the author’s particulation time;
(iii) the shape of the conical cavity is retained in the jet formed;
(iv) the material mass in the jet and the thickness of the block material squeezed into the jet increase with the increase in the angle of the conical cavity;
(v) the fact that the aluminium jet is elongated as a single rod reveals that the temperature rise in the jet formation is above the melting point of the aluminium.
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REFERENCES

Expansion of Metallic Cylinders under Explosive Loading

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Terminal Ballistics Research Laboratory, Chandigarh-160 020

ABSTRACT

The behaviour of expanding metallic cylinders under explosive loading was studied. Using ultra high speed photography, the expansion characteristics of aluminium and copper metallic cylinders have been evaluated with different c/m ratio, and by changing the nature of high explosive. The results obtained are comparable to those predicted by the Gurney's energy and momentum balance equations. A cylinder test has been established for comparative evaluation of the ability of high explosives to accelerate the metal. The relative energies delivered to the metal by octol, TNT, PEK-1, baratol and composition B are calculated. The results are in close agreement with those calculated by Kury et al.

1. INTRODUCTION

The motion of the metals driven by detonating explosives is of great importance for designing suitable explosive metal systems, for both armament and civil applications. The ability of an explosive to impart energy to the metal has been referred to as 'brisance'. Many complex computer codes, based on different equations of the state for detonation products, have been worked out to predict the motion of the metal driven under different geometrical conditions, such as plane, spherical and cylindrical configuration. However, the most simple and practical method for estimating the velocity imparted to metal by an explosive is the Gurney Method, based on energy and momentum balances. This method was first devised by RW Gurney in 1943 to compute the velocity of fragments from artillery shells. Despite a number of simplifying assumptions, the calculations done by this method often show an excellent correlation with the experimental data, and hence it remains most widely used method especially for calculating the final velocity imparted to the metal elements. In earlier studies at TBRL suitable experimental methods were successfully developed for projecting metallic plates at extremely high velocities with the help of explosives and also for monitoring the motion of these plates. The present study mainly deals with the loading of metallic tubes with different explosives and recording their motion using ultra high speed cameras. Considerable data has been generated, both by varying the wall thickness of the metallic tubes and by changing the explosive composition. A good comparison has been established between the experimental values and those predicted by Gurney's equations.

These results have been used in development of explosive-driven high current generators based on the principle of magnetic flux compression by expanding metallic cylinders. Further, since the experimental configuration used in this study was similar to that used by various authors in 'standard copper cylinder test' for comparative evaluation of the ability of explosives to accelerate metals, the above results could successfully be used for precise relative ordering of a number of explosive compositions for their applications in explosive metal systems.

2. THEORY

The hydrodynamic theory of detonation combined with the assumption that the detonation products follow the constant equation of state \( P = \rho^\gamma \) gives the
following set of relations among various parameters:

\[ P = pD^2 (\gamma + 1) \]  
\[ U = D' (\gamma + 1) \]  
\[ C = \gamma D' (\gamma + 1) \]  
\[ Q = D' Q' (\gamma + 1) \]

where \( P \) is the detonation pressure at C-J plane, \( U \) the velocity with which the material flows behind the detonation front, \( C \) is the sound velocity, \( D \) the detonation velocity and \( Q \) is the total chemical energy liberated during detonation.

Gurney assumed that out of the total chemical energy released, specific energy \( E \) (kcal/g), having a characteristic value for each explosive, is converted from chemical energy in the initial state to kinetic energy in the final state. This value of Gurney energy and heat of detonation are experimentally measured and are available for most of the explosives. It has been observed for most of the explosives that the ratio \( E/Q \) lies between 0.61 and 0.76. In the absence of suitable data for calculation of \( E \), one can assume \( E = 0.7 Q \) (5)

This kinetic energy is partitioned between the metal and the detonation products. Gurney coupled these assumptions with the law of conservation of energy and momentum, and derived simple relations for estimating the final velocity of the moving particle in different geometrical configurations such as plane, spherical and cylindrical systems. Gurney's relation for velocity \( V \) of an exploding metallic cylinder is

\[ V = \sqrt{2E} \left( m/c + 1/2 \right) \]  

where \( V \) is the terminal velocity imparted to the metal cylinder, \( m \) is the metal mass per unit length, \( c \) is the explosive mass per unit length and \( E \) is the kinetic energy per unit explosive mass also called the Gurney energy.

The quantity \( 2E \) which occurs in the above expression has the unit of velocity and is known as the Gurney characteristic velocity for a given explosive. It is clear from Eqn (6) that for a given explosive, the final metal velocity is a function of only charge-to-metal mass ratio.

The final velocity imparted to the metal is a sensitive function of the gas constant \( \gamma \) in the early stages of expansion and comparatively less sensitive to \( \gamma \) in the later stages of expansion. When the solid explosive is converted into a gas after the passage of detonation wave, the gases have initially the same density as that of the original explosive. The behaviour of the detonation gases is similar to that of a liquid under high pressure. A small initial increase in volume is accompanied by a large decrease in pressure, which is consistent with a value of \( \gamma > 3 \). Hence, a large acceleration is expected in the early stages of the expansion. After the volume of gaseous products has increased a little, the behaviour of the detonation gases will be nearer to that of the normal gas with a value of \( \gamma = 1.4 \), which suggests a small acceleration in the later stage of expansion.

3. DIRECTION OF METAL PROJECTION

When the detonation wave encounters the metal in a normal angle of incidence, the metal moves in a direction normal to the surface, i.e., in the direction of the detonation wave. However, this is not true when the metal is driven by "grazing detonation" which propagates parallel to the metal surface. The direction of the metal driven by the grazing detonation is such that it bisects the angle between the normal to the original and the deflected metal wall. If \( \theta \) is the angle of deflection of the cylinder as detonation wave progresses along the charge axis, the angle of metal projection \( \theta/2 \) is given by

\[ \theta/2 = \sin^{-1} \left( \sqrt{2E} \right) \]  

With the streak camera slit set perpendicular to the rest plane of the charge axis, an apparent velocity \( V_a \) measured is given by the relation

\[ V_a = D \tan \theta \]  

Eliminating \( D \) from the above two equations, the relation between the apparent velocity measured \( V_a \) and the actual velocity \( V \) can be written as

\[ V/V_a = \cos \theta/\cos (\theta/2) \]

The apparent velocity measured is only 4 to 7 per cent higher than the actual velocity.

4. EXPERIMENTAL TECHNIQUES

It is clear from the foregoing considerations that the experimental measure of the velocity of residual expansion of a metallic cylinder loaded with the explosive, along with the knowledge of the detonation velocity of an explosive enables to compute the motion of the cylinder, both in magnitude and the direction. The ultra high speed camera was found to be very useful tool for most accurate monitoring of the radial motion of an exploding cylinder. The complete layout is shown
in Fig. 1. A precision machined aluminium or copper cylinder of internal diameter 52 mm, length 200 mm and having wall thickness of 1 to 5 mm filled with high explosive, was placed horizontally on a wooden stand in front of the objective lens of the streak camera. The slit of the camera was adjusted perpendicular to the cylinder axis and was set at a distance of 120 mm from the initiating surface of the explosive. From the theoretical as well as the experimental considerations, it was essential to initiate explosive with a plane detonation wave generator, as shown in Fig. 1. An argon flash bomb was used as a source of back lighting so as to record the motion of the cylinder as a shadowgraph. Both the explosively loaded cylinder and argon flash bomb were fired by means of electronic detonators and their timings were synchronised through the delay generators incorporated in the control systems of the camera. A typical streak camera photographic record
taken on Beckman and Whitley Model 770 (nanoseconds camera) at a writing rate of 1.67 mm/s is shown in Fig. 2. The experiment was repeated several times by varying the wall thickness of the aluminium and copper cylinders and for different explosive compositions. In separate experiments, the detonation velocities of the explosives used were also measured.

Up to this stage it was assumed that either the velocity of detonation of the explosive used was known or it was measured from separate set of experiments. The knowledge of velocity of detonation was essential to compute the angle of deflection of the metallic walls. Hence, in the absence of such knowledge, it may not be advisable to use streak camera as it would not give sufficient information for monitoring the motion. However, in such a situation the answer for replacing streak camera by a suitable ultra high speed framing camera was found, which recorded motion in space coordinates, thereby making it possible to compute the angle of deflection without measuring the velocity of detonation. The experimental set-up was similar to that described above and a typical set of framing pictures taken by a Beckman and Whitley Model 189 (a framing camera at a framing rate of 2,40,000 frames) is shown in Fig. 3. In this experiment the explosive was initiated from both the ends simultaneously.

5. RESULTS AND DISCUSSION

In Fig. 4 the experimental radius-time curves for aluminium cylinder loaded with composition B are plotted. Wall thickness of the metal cylinder was the only variable with internal diameter of the tube fixed at 52 mm. It is clear from the curves that for a large \( \text{d/m} \) ratio the acceleration phase is quickly over and the cylinder expands with more or less a constant velocity with small accelerations whereas, for a small \( \text{d/m} \) ratio
the tube takes few microseconds to attain the terminal velocity. In the early stages of expansion, streak camera records slightly high velocity due to spray ejected from the metal surface on the arrival of the shock wave. Due to these reasons, the starting experimental points lie above the radius-time history. The theory suggests that the expansion of about two radii are required to approach the hydrodynamic terminal state. However, cylinders tend to burst after some finite expansion and the motion may be perturbed before the terminal state is obtained. The expansion velocity was measured at \( R/R_q = 1.7 \) and the results are given in Table 1.

Experimental values of the expansion velocity are slightly less than that estimated from Gurney relation (Eqn (6)). Possible sources of this discrepancy may lie in the effect of the compressibility of the metal or because the true terminal state has not been reached at \( R/R_q = 1.7 \). Expansion velocity with different \( \Delta R \) ratio for different explosives are plotted in Fig. 5. The values of Gurney’s characteristic velocity\(^1\) taken for octol, composition B and TNT are 2.8, 2.71 and 2.37 respectively. Most of the experimental points in the study lie slightly below the Gurney’s curves, though the difference is within 10 per cent. The results of expanding copper cylinder with different \( \Delta R \) ratio and with different explosives are given in Table 2.

For relative evaluation of the ability of explosives to impart energy to the metal a copper cylinder of internal diameter 52 mm and external diameter of 60

Table 1. Expansion characteristics of aluminium cylinders

<table>
<thead>
<tr>
<th>( \Delta R )</th>
<th>Wall thickness ((\text{mm}))</th>
<th>Deflection (\theta^\circ)</th>
<th>Apparent (V_e) ((\text{mm/m}))</th>
<th>Corrected (V) ((\text{mm/m}))</th>
<th>Velocity (Gurney)'s relation ((\text{mm/m}))</th>
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Inner diameter of cylinder = 52 mm

mm was exploded with five different high explosives, such as octol, composition B, TNT, PEK-1, and baratol. Radius-time history for different explosives is plotted.
Figure 5. Gurney’s curves vs experimental data.

Table 2. Expansion characteristics of copper cylinders

<table>
<thead>
<tr>
<th>cm</th>
<th>Wall thickness (mm)</th>
<th>Deflection angle (°)</th>
<th>Apparent velocity V_a (mm/μs)</th>
<th>Corrected velocity V (mm/μs)</th>
<th>Gurney’s relation V_r (mm/μs)</th>
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<td>20.26</td>
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</tbody>
</table>

Inner diameter of cylinder = 52 mm

Figure 6. Radial expansion of 4 mm thick explosive loaded Cu tube.

in Fig. 6 and the expansion velocity measured at $R/R_0 = 1.7$ for all five explosives is given in Table 3. Relative energy delivered to the metal by an explosive was found by squaring the cylinder wall velocity at $R/R_0 = 1.7$ for that explosive and comparing it to that of composition B for the same expansion. Relative energy values for octol, composition B and TNT are in close agreement with that calculated by Kury et al. The values of Gurney’s characteristic velocity for PEK-1 and baratol are not given, but from the experimental data the calculated velocity values for PEK-1 and baratol are 2.4 and 1.8, respectively.

Table 3. Relative grading of different explosives on the basis of cylinder test

<table>
<thead>
<tr>
<th>Explosive</th>
<th>Rate of expansion at $R/R_0 = 1.7$</th>
<th>Relative energy delivered</th>
<th>Relative energy values by Kury et al.</th>
</tr>
</thead>
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<td>1.0</td>
</tr>
<tr>
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<td>1.86</td>
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<td>1.15</td>
</tr>
<tr>
<td>Composition B (RDX/TNT (70:30))</td>
<td>1.44</td>
<td>0.72</td>
<td>0.74</td>
</tr>
<tr>
<td>TNT</td>
<td>1.48</td>
<td>0.76</td>
<td>-</td>
</tr>
<tr>
<td>PEK-1</td>
<td>1.23</td>
<td>0.52</td>
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</tr>
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</table>

Wall thickness of copper cylinder = 4 mm
Figure 7. Expansion of 4 mm thick (ID/OD = 52/60 mm) Al and Cu cylinders under TNT loading.

Figure 2 shows the streak camera records of the expanding aluminium and copper cylinders of inner diameter 52 mm and wall thickness 4 mm. Radius-time history of each record is plotted in Fig. 7; the broken part of the curves indicate the fracture of the tube. Aluminium being light and brittle is accelerated at a faster rate and fractures quickly (16 μs) whereas, copper being heavier and ductile can be expanded slowly before it fractures (27 μs). Therefore, copper is preferred to aluminium for the cylinder test.

6. CONCLUSION

A large amount of data on the radial expansion of copper and aluminium has been generated for various explosive metal systems. The present data has been successfully used in the design and development of explosive-driven current generator based on the principle of magnetic flux compression by expanding metal cylinders. The cylinder test has been established for relative evaluation of the ability of various explosive compositions to impart energy to the metals. With the help of this test five high explosives have been graded; but a novel technique has been established for future studies on various plastic-bonded explosives.

ACKNOWLEDGEMENTS

The authors express their sincere thanks to Shri M Balakrishnan, Director, TBRL, for his permission to publish the present work. The authors also acknowledge the experimental and computational assistance by Shri Balwinder Singh.

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ELONGATION OF ALUMINIUM & COPPER METALS UNDER STRAIN RATE OF $10^4$ - $10^5$ S$^{-1}$

MANJIT SINGH & M.S. BOLA
Terminal Ballistics Research Laboratory.
Sector - 30, Chandigarh - 160020
INDIA.

ABSTRACT
The present paper is a study on the dynamic elongation and particulation of metal jets under high strain rate. The jets were produced by Shock Induced Cavity Collapse in metallic samples of aluminium and copper. Because of the high velocity gradient between their tip and tail ends the jets were allowed to elongate till the onset of particulation. The process of elongation and breakup was recorded by Synchro Streak Technique. Instability parameter associated with the jets of different diameter and strain rate was calculated. Elongation of 600 to 1000 percent before breakup occurs, has been measured in copper and aluminium jets. Interparticle velocity difference and the particle length have been measured and the effect of strain rate on these two parameters has been evaluated.

INTRODUCTION
Under extremely high strain rates, the ductility of the materials increases by an order of magnitude. High strain rates of the order of $10^4$ - $10^5$ $S^{-1}$, are realised only in certain explosive metal interactions or in impact phenomena. Two or three types of explosive metal systems have been discussed in the literature which can subject the materials to this order of strain rate. First is the shaped charge mechanism (Birkhoff et al., 1948, Pugh et al., 1962, Eichclberger & Pugh, 1955) in which a thin metallic liner is axisymmetrically collapsed by detonating explosives, to form a high speed metal jet. Computer simulations have shown that a strain rate up to $10^5$ $S^{-1}$ (Miller et al., 1984) is achievable in this form of material motion.

Second explosive metal system which can deform the metal at strain rate up to $10^5$ $S^{-1}$ (Mogilevskii et al., 1994) is by explosive loading of tubular specimens. A metal cylinder filled with high explosive is detonated from one end. The wall thickness of the cylinder decreases as it expands and ultimately it breaks (Bola et al., 1992). This system is famous as the 'Copper Cylinder Test' (Kury et al., 1965) which is utilized in finding out the 'Brisance' and 'Equation of State' of different high explosive compositions. Out of the two systems described, a large amount of efforts both theoretical as well as experimental, have been devoted to shaped charge mechanisms in which the materials are elongated in the form of jets. This is because of its increased application in the development of armament stores.

The metal jet is treated as a slender rod which stretches because of the velocity difference between the two ends. Under ideal conditions the jet length would increase indefinitely with run, however after some finite stretching a series of disturbances, also called necking regions and jet breakup occurs. The breakup process or particulation starting from the tip portion, progresses...
towards the tail end with further elongation. Neglecting the forces due to aerodynamic drag, different segments of the particulated jet move with a constant velocity and size.

The present paper is an experimental study on the metal jets produced by Shock Induced Cavity Collapse. Conical cavities in aluminium and copper metal plates were axisymmetrically collapsed by a plane shock wave, to form metal jets. Cavity dimensions were changed to produce jets of varying velocities, length and strain rate. Based on the theoretical relations of shock induced surface collapse by Yadav (1982), together with some experimental observations, the initial length of the aluminium jets have been calculated after the collapse process is over. From the measured jet diameter, the instability parameter $\psi$ (Walsh, 1984) has been calculated for these jets, which is a measure of the instability associated with the elongating jets. Particulated metal jets were then recorded by the Synchro Streak Technique (Held, 1984) to calculate the elongated jet length for estimation of the strain. Individual particle velocity and size have been measured and their dependence on the jet diameter and strain rate have been established. Possible factors have also been identified which affect the jet elongation and breakup process.

COLLAPSE PROCESS AND JET FORMATION

Consider a cylindrical metal target, "Fig. 1.," with conical cavity of half angle $\alpha$ at one of its plane faces. The opposite plane face is loaded with plane shock waves by detonating high explosive, so that the direction of shock propagation is along the axis of the cavity. Any material particle at the slant conical surface is subjected to two velocities, the particle velocity $U_p$, in the direction of shock propagation and velocity $U_m$, due to release of shock pressure at the free surface, in a direction perpendicular to the slant surface. The material collapses under the effect of these two velocities with a velocity $U_f$, towards the axis of the cavity which has been written (Yadav, 1982) as

$$U_f^2 = U_p^2\left[2\left(1 + \sin \alpha\right)\right]$$  \hspace{1cm} (1)

The Collapse angle ($\psi$) is a function of cavity angle and can be written as

$$\psi = \frac{\pi}{4} + \frac{\alpha}{2}$$  \hspace{1cm} (2)

Material collapses to form a jet with a velocity $V_j$ given by the expression

$$V_j = \frac{r_0}{r} \left[ U_f \sin (\psi - \alpha) + U_f \cos (\psi - \alpha) \right]$$  \hspace{1cm} (3)

where $r_0$ is the initial radius of any small collapsing element and $r$ is the radius of the jet formed by that particular element. From the experimental observations (Singh & Bola, 1993) it has been observed that the jet diameter increases towards the tail end depending upon the cavity angle, so the ratio $r_0/r$ can be assigned a constant value during the collapse process. The jet tip velocity, is simply a function of cavity half angle and the particle velocity.

$$V_j = K(\alpha) U_p$$  \hspace{1cm} (4)

where $K(\alpha)$ is a constant depending upon the cavity angle and $U_p$ is the particle velocity behind the shock front.

As the shock wave sweeps the conical cavity from apex to base, the shock pressure is attenuated, with the result that the slant surface collapses with a velocity decreasing from the apex to the base end of the cavity causing the velocity gradient in the formed jet. Let $l_0$ be the original slant surface length of the cavity, which becomes $l$, the jet length after collapse process is over. The length of the jet $l$, can be estimated from the known parameters of shock propagation in metals. Collapse time $t_c$ is being defined here as the time interval between the arrival of shock wave at the cavity apex and the last particle at the cavity base collapses to reach the cavity axis, to form the tail of the jet. From Fig. (1) it can be written as

$$t_c = \frac{d}{U_f} + \frac{d \tan \alpha}{U_f \sin(\psi(90-\alpha)/2)}$$  \hspace{1cm} (5)

where $d$ is the cavity depth, which is fixed at 25 mm for the present studies. The length of the jet can be written as

$$270$$
where $V_j$ is the jet tip velocity which has been measured for different angles (Singh et al, 1991). Due to large aerodynamic drag, erosion and ablation of the material takes place at the jet tip (Costello & Backofen, 1983), thus making the experimental measurements difficult at the jet tip. For the calculation of the jet length from "Eq. (6)", $V_j$ is taken to be the velocity of the first clearly visible particle of the jet after the tip. Because of the finite elongation in the jet, while the collapse process is still continuing, the length of the formed jet $l$, will be considerably more than the original length $l_o$ of the cavity slant surface. This jet will further elongate under initial strain rate given by the relation

$$\eta = (V_j - V_0) / l$$  \hspace{1cm} (7)

where $V_j$ is the jet tip velocity and $V_0$ is the tail velocity of the jet which has been taken equal to free surface velocity of the metal target. The jet will elongate uniformly initially, until it necks down on a number of locations and breaks into discrete particles. The elongated length of the jet has been taken equal to the cumulative length of the jet particles added together.

$$l_n = \sum_{i=0}^{n} X_j$$  \hspace{1cm} (8)

where $X_j$ is the length of individual particle. Engineering strain is then calculated by dividing the change in jet length to the original slant length (Zemow & Chapyak, 1993) of the cavity.

$$\xi = (l_n - l_o) / l_o$$  \hspace{1cm} (9)

By dividing the whole cavity into annular rings of 1mm thickness, the whole collapse process has been evaluated on the computer. Collapse velocity has been calculated from "Eq (1)" by making use of equation of state

$$U_s = a + b U_p$$  \hspace{1cm} (10)

and shock jump condition

$$P = \rho_s U_s U_p$$  \hspace{1cm} (11)

of the metal. Exponent decay $\alpha$ in the shock pressure, as the shock travels from apex to the base of the conical cavity, has been given by the equation

$$P = P_0 \exp(-\alpha x/x_0)$$  \hspace{1cm} (12)

$P_0$ in "Eq. (12)" is the shock pressure transmitted into the metal at explosive metal interface, which has been taken equal to 330 kbar and 460 kbar, respectively for aluminium and copper metal, $\alpha$ is attenuation coefficient, $x$ is the distance measured from apex towards the base of the cavity and $x_0$ is the thickness of explosive detonated in contact with the metal. The attenuation constant has been taken from previous experiments at TDRL to be 0.91 and 0.78 for aluminium and copper, respectively.

**EXPERIMENTAL PROCEDURES**

The diagnostic techniques, commonly used for jet formation and particulation studies are Flash Radiography and Synchro-Streak Technique (SST). Although the details of SST and its advantages and disadvantages in comparison to Flash Radiography are given elsewhere (Held, 1984), a brief review will be given here.

The Synchro-Streak Technique resembles a sports field photofinish. The speed of the event to be recorded is matched with the writing rate of the streak camera. The camera slit is set perpendicular to the predetermined path of the event to be recorded. The streak camera record thus gives the time history of the event in one direction, at a point where the event crosses the camera slit. By using Bi-SST, the same jet can be recorded at two stand-off distances.

**Figure 2:** Synchro-Streak Technique for Recording the Particulated Metal Jets.

The experimental set up is shown in "Fig. 2." A cylindrical metallic block of 150mm diameter and 30mm thickness was placed in contact with the RDX / TNT (60:40) pad of 150mm diameter and 50-100 mm thickness. The high explosive was detonated through a plane wave shaper (PWS) using an EBW detonator. A conical cavity of depth 25mm was machined in one plane face of the metal target, opposite to that of explosive...
face. The cavity half angle was changed from 15° to 60° to produce metallic jets of different velocities, and therefore of different strain rates. A metallic collar of 25-50mm in width was kept at the explosive metal interface in order to keep detonation gases away from the camera viewing area. The plane of observation was kept sufficiently away from explosive metal assembly in order to avoid any explosive debris obstructing the camera optical path.

Most of the trials were conducted using Bi-SST in which the metallic jets were recorded at two stand-off distances. The streak camera slit was set perpendicular to the jet axis at one point and with the help of a pair of mirrors, the plane of observation of the second point was rotated through 90° to bring the image on the same slit. Two argon filled PVC pipes of 100mm diameter and 1.4m length were used to provide back lighting in order to record the jet as a shadowgraph. To induced the shock wave in argon, a high explosive pad of 76.2mm diameter and 50mm thickness was placed in contact with the rear end of each pipe. The front ends of the PVC pipes were covered with a translucent paper, to provide diffused light. The Argon Flash Bombs (AFB) were covered with black PVC tape of 12.7mm in width to facilitate the jet diameter measurements. The relative delay between the two AFB's was introduced depending upon the jet velocity and the distance between the two recording points.

The metal jets were recorded on a Beckman & Whitley Model 770 Streak Camera. The writing rate of the camera was adjusted to be between 0.83-1.67mm/μs depending upon the jet velocity and the magnification of the test event on the film plane. For the studies on the velocity and size of the individual metal jet particles and their break-up time, the Synchro-Streak Technique was used. Beckman & Whitley Model 189-A Rotating Mirror Framing Camera capable of recording 25 frames of the test event on 35mm film was used for the measurement of the tip velocity.

STRETCHING & PARTICULATION OF JETS

Metal jets undergo plastic stretching because of the velocity gradient between the two ends. After some finite stretching, the jet breaks into a number of particles with no further elongation in individual segments or particles. Walsh (1984) found out that the particulation process is dependent upon the perturbation structure and single dimensionless flow parameter ϕ, which is a measure of the instability associated with the elongating jets. This parameter was named the instability parameter and was written as

\[ \phi = \left( \frac{\sigma}{\rho} \right) / \eta R^2 \]  

Instability growth rate is a function of jet radius R, density \( \rho \), yield strength \( \sigma \) and strain rate \( \eta \). Low values of the instability parameter favours large elongation, which means that early breakup in jets will occur if the material has a higher yield strength and/or if the stretching rate is low. Since \( \eta \) decreases as \( I^3 \) and \( R_2 \) decreases as \( I^4 \), the parameter \( \phi \) increases at the rate of the cube of the jet length. It is apparent that the jets very rapidly become more unstable as they stretch.

Let \( I \) be the length of the jet at some early stage of flight, then its subsequent behaviour depends upon the instability parameter \( \phi_0 \). The jet will particulate when \( \phi \) attains some critical value \( \phi_c \). Then the length of the elongated jet can be written as

\[ l_0 = \phi_0^{1/3} (1 + \phi_0^{1/3}) \]  

The elongated jet length depends not only upon the initial length because of the \( I^3 \) dependence of the associated flow parameter \( \phi \). To increase the jet length, the yield strength of the material should be smaller and/or the diameter of the jet should be larger. (1984) gives the general formula for the jet length before break-up occurs as

\[ l_b = \frac{1}{\phi_0^{1/3}} \left[ C \phi_0^{1/3} \Pi \right] \]  

where \( C \) is a constant and \( \Pi \) is a general perturbation function combining the effects of random variation in jet surface roughness, non_uniform velocity gradient, and non_uniform yield strength.

Experimental Data

"Figure 3" shows an aluminium jet formed by the collapse of conical cavity of half angle 22.5°. The jet, with a tip velocity of 9.9 km/s and tail velocity of 3 km/s, is stretching under a strain rate of 7.3x10^5 S. This is an intermediate stage of jet flight after the collapse process is over. A 12.7 mm wide black patch is used as a fiducial for the measurement of the jet diameter. The streak record does not give directly the length of the jet, but it gives the time of passage of the jet across the camera slit. The jet length can be calculated by multiplying the distance between the recording points. For particulated jets the individual particle lengths are added together to calculate the total elongated jet length. "Figure 4." shows the same jet as shown in "Fig. 3" but is now particulated at a stand-off distance of 300mm from cavity apex.
Different parameters of the aluminium and copper jets calculated from “Eqns 1. to 12.” are given in Table 1 and 2. Column 4 in Table 1 gives the total elongated length of the jets measured experimentally. Engineering strain of the order of 8-10 for aluminium and 5-6 for copper has been observed for these jets. Two reasons may explain the difference in elongation. The yield strength of aluminium is approximately one half to that of copper and the strain rate (Table 2) in aluminium is more than that in copper jets from same cavity angle. From the measurement of jet stretching rate and diameter, the instability parameter $\phi$ has been calculated for a number of jets formed by the collapse of different angled cavities. As the diameter of the jet increases from tip to tail end, only the jet tail diameter has been taken for calculation purposes. Though the yield strength of the material changes remarkably during the collapse process and jet stretching, only the static values of 50 MPa and 100 MPa for aluminium and copper has been taken. It is to be noted here that the values of instability parameter $\phi$ are some intermediate values when sufficient jet elongation has already been taken place. The initial value of $\phi$, must be smaller than that given in Table 2. As the jet stretches the value of $\phi$ increases very rapidly until the onset of particulation. There are several factors which support such a high dynamic ductility: the elevated jet temperature and its effect upon the yield strength, deformation induced porosity in the necking regions and dynamic recrystallization in the metal during collapse process.

**TABLE 1. MEASURED / CALCULATED JET LENGTHS AT DIFFERENT STAGES OF JET FLIGHT**

<table>
<thead>
<tr>
<th>Cavity</th>
<th>Original Strain</th>
<th>Half</th>
<th>Material Length</th>
<th>Jet Slant at Length</th>
<th>Length Formation</th>
<th>$\alpha$ (deg)</th>
<th>$l_1$ (mm)</th>
<th>$l_2$ (mm)</th>
<th>$\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALUMINIUM</td>
<td></td>
<td>22.5</td>
<td>27.1</td>
<td>94</td>
<td>290</td>
<td>9.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td>28.9</td>
<td>106.6</td>
<td>317.2</td>
<td>9.98</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>45</td>
<td>35.4</td>
<td>178.4</td>
<td>341.2</td>
<td>8.64</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>60</td>
<td>50</td>
<td>267.5</td>
<td>489</td>
<td>8.78</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>COPPER</td>
<td></td>
<td>22.5</td>
<td>27.1</td>
<td>64.6</td>
<td>177</td>
<td>5.53</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td>28.9</td>
<td>93.4</td>
<td>200</td>
<td>5.92</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>45</td>
<td>35.4</td>
<td>134.7</td>
<td>250</td>
<td>6.06</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 2. INSTABILITY PARAMETER FOR JETS OF DIFFERENT STRAIN RATES.**

<table>
<thead>
<tr>
<th>Cavity</th>
<th>Initial Breakup Jet Diameter</th>
<th>Instability Parameter</th>
<th>Angle Rate</th>
<th>Rate Initial Breakup</th>
<th>$\alpha$</th>
<th>$\eta_i$</th>
<th>$\eta_b$</th>
<th>$\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ALUMINIUM</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22.5</td>
<td>7.3x10$^4$</td>
<td>2.4x10$^4$</td>
<td>3.85</td>
<td>0.934</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>5.4x10$^4$</td>
<td>1.4x10$^4$</td>
<td>4.73</td>
<td>1.13</td>
<td>41.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>2.7x10$^4$</td>
<td>1x10$^4$</td>
<td>7.1</td>
<td>3.8</td>
<td>2.008</td>
<td>51.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>COPPER</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22.5</td>
<td>6.4x10$^4$</td>
<td>2.05x10$^4$</td>
<td>1.9</td>
<td>29.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>3.8x10$^4$</td>
<td>1.26x10$^4$</td>
<td>3.2</td>
<td>27.64</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>1.7x10$^4$</td>
<td>8x10$^4$</td>
<td>4.25</td>
<td>38.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Keeping in view the exponential velocity gradient (Singh et al., 1995) along jet length and increasing jet diameter towards tail end, the $\phi$ values may vary at different locations within the jet.

273
A number of experimental trials were conducted with aluminium and copper metal jets stretching under different strain rates. Size and velocity of individual segments/particles were measured. Figure 6 shows the distribution of velocity and length of all the jet particles of the aluminium jet formed by the collapse of the cavity half angles 60° and 45°, where $\sigma_0$ and $\sigma$ are the stresses existing in the uncurved and curved portion of the jet, R is the current radius of the jet and x is the distance along the jet length. Because of the decrease in the jet cross-sectional area at the neck, stress enhancement takes place. In the later stage of stretching, plastic deformation takes place only in the necking regions, whereas elastic conditions exist in between the two necking regions. Figure 5 shows an aluminium jet during the process of neck formation. If the total force $\pi R_0^2 \sigma$ on a cross-section through the neck is less than the force $\pi R_0^2 \sigma_a$ for the nearby uncurred rod, then necking down will occur and the perturbation is unstable. Walsh (1984) & Chou (1977) derived independently the critical condition for stability of the stretching jets by equating the two forces

\[
\pi R_0^2 \sigma = \pi R_x^2 \sigma_a
\]

(17)

They further derived the least stable wavelength $\lambda_0$ as

\[
\lambda_0 / R_0 = \pi / \sqrt{2}
\]

(18)

Figure 5: Enlarged View of Neck Formation in a Stretching Jet.

EFFECT OF STRAIN RATE ON PARTICLE SIZE

Numerical simulations performed by Chou and Carbone (1977) shows that small initial disturbances in either the rod radius, flow stress or strain rate, all lead to instabilities that eventually cause necking and fragmentation in the jet. Irrespective of the initial structure of perturbation wavelengths, a certain least stable wave length $\lambda_0$ causes the actual necking and fragmentation of the rod like jet. Assuming a sinusoidal perturbation of wavelength $\lambda_0$ in the rod radius, there is a stress distribution in the stretching jets. The Bridgeman formula for stress distribution in a cross section through the neck is

\[
\sigma = \sigma_a \left(1 + \frac{1}{2} \cdot \frac{\sigma_0}{\sigma_a} \cdot \frac{R}{R_0} \cdot \frac{dR}{dx} \right)
\]

(16)

where $R_0 = \frac{R}{2} \left(\frac{d^2 R}{dx^2}\right)$

Here $\sigma_a$ and $\sigma$ are the stresses existing in the uncurved and curved portion of the jet, R is the current radius of the jet and x is the distance along the jet length. Because of the decrease in the jet cross-sectional area at the neck, stress enhancement takes place. In the later stage of stretching, plastic deformation takes place only in the necking regions, whereas elastic conditions exist in between the two necking regions. Figure 5 shows an aluminium jet during the process of neck formation. If the total force $\pi R_0^2 \sigma$ on a cross-section through the neck is less than the force $\pi R_0^2 \sigma_a$ for the nearby uncurred rod, then necking down will occur and the perturbation is unstable. Walsh (1984) & Chou (1977) derived independently the critical condition for stability of the stretching jets by equating the two forces

\[
\pi R_0^2 \sigma = \pi R_x^2 \sigma_a
\]

(17)

They further derived the least stable wavelength $\lambda_0$ as

\[
\lambda_0 / R_0 = \pi / \sqrt{2}
\]

(18)

where $\eta_0$ is the initial strain rate. With the increase in the jet strain rate, the least stable wave length decreases as well as the particle size.

Shorter wavelength initial disturbances will diminish in size and larger ones will grow. This treatment however, assumed the least stable wavelength or the particle size to be independent of strain rate, which was later contradicted by HEMP (Wilkins, 1962, Giroux, 1971) code calculations. Haugsted (1983) also showed this wavelength to be dependent upon the jet stretching rate by the formula

\[
\lambda_0 = a \left(\frac{\sigma_0 / \eta_0}{\sigma_a}ight)^{\eta_0 / 3}
\]

(19)

Experimental Data

A number of experimental trials were conducted with aluminium and copper metal jets stretching under different strain rates. Size and velocity of individual segments/particles were measured. "Figure 6" shows the distribution of velocity and length of all the jet particles of the aluminium jet formed by the collapse of the cavity half angles 60° and 45°.

Figure 6: Particle Size and Velocity Distribution in Aluminium Jet.
The segment velocity decreases exponentially from tip to tail end. The segment length distribution is random with the general trend of increasing segment length towards the tail end. The increase in segment or particle length towards the tail end is more prominent for the jets formed by large angled conical cavities. This may be due to the large gradient in jet diameter from tip to tail end, formed by these cavity. "Figure 7." shows a copper jet recorded by the Synchro-Streak Technique at three stand-off distances. The jet has been particulated at a break-up strain rate of \( 1.26 \times 10^5 \) s\(^{-1}\) with an average particle length of 9.6 mm. The experimental data of particle length measurements with strain rate is plotted in "Fig. 8." as \( V_{pl} \), the Hirsch (1981) breakup parameter. Hirsch parameter \( V_{pl} \), which is a measure of the metal quality, is used in a very simple way to calculate the breakup time of elongating jets and expanding metal cylinder under explosive loading. Its value is taken roughly to be 100 m/s for copper shaped charge jets which is on the lower side of the present value of 130 m/s for copper jets. Experimental data shows a weak dependence of \( \Delta V \) on the strain rate.

CONCLUSIONS
- Dynamic ductility up to 1000 percent has been achieved in the jets formed by shock induced cavity collapse.
- Ductility of the metal jet depends upon the yield strength, density, jet radius and strain rate.
- Particle length decreases with the increase in the strain rate.
- Interparticle velocity difference is weakly dependent on the strain rate.
- Particle length is 3-5 times the particle diameter.

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Dynamic tensile deformation and fracture of metal cylinders at high strain rate

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An experimental study is presented on the radial expansion of metal cylinders of internal diameter 52mm and wall thickness 1-6mm, internally loaded with high explosives. A rotating mirror streak camera monitors the distance time (x-t) history of the cylinder wall, expanding under strain rate of $10^4$-10^5 s^{-1}, till it is ruptured. Aluminium cylinder is found to rupture at strain ($\varepsilon$) of 75-150 %. For a fixed wall thickness, the rupture strain is found to increase with the strain rate. However, when the wall thickness is changed, a maxima is observed in the graph showing the rupture strain and strain rate relationship. Probably, aluminium follows the Ivanov rupture criteria. In deformation of copper cylinder the cracks initiation at strain of $\varepsilon$=30-50 % is followed by rupture at very high strain up to 300 %. Recovered fragments show wall thinning by 50-60% and also exhibit that the shear fracture dominates the radial fractures.

1. INTRODUCTION

The one-dimensional geometry of the radially expanding ring [1] is perhaps the simplest in considering the fundamental aspects of the fracture and fragmentation process. A ductile metal ring subjected to an outward radial impulse will decelerate due to circumferential tensile force. The magnitude of this force will vary due to the work hardening in the metal and geometric softening caused by the thinning of the ring. Initially hardening dominates but eventually a maximum in the tensile force is achieved at which the flow changes from stable to unstable. The unstable flow is manifested by the onset of plastic necking and fragmentation. In a ductile metal, fracture proceeds through the multiple nucleation and growth [2] of necking regions. Concepts, which are found to govern the fracture and fragmentation in the expanding ring are generalized for the cylindrical and spherical shells.

Researchers have used different techniques of driving the metal such as gun technique [3], electromagnetic force [4] and explosives [5,6]. Under a small impulse, by electromagnetic loading, cylinder wall will attain terminal velocity and then decelerate. The retardation history of the cylinder wall gives the stress-strain [4] behaviour of the metal. When the explosives are used as the driving force, the impulse generated is large enough to rupture the cylinder in the accelerating phase. Present studies are on the tensile deformation of metal cylinder when loaded internally with high explosives. Detonation of high explosives gives an outward impulse to the cylinder wall that expands under strain rate of $10^4$-10^5 s^{-1}, till the rupture occurs. The rupture strains of aluminium and copper cylinders have been found out under different strain rates.
2. FRACTURE HYPOTHESES

Taylor [7] proposed a model to predict the radius associated with fracture based upon the assumptions that the fractures are radial. Compressive hoop stresses exist over the inner portion of the tube wall and are governed predominately by explosive pressure. The radial fracture cannot propagate into this compressive zone. Taylor concluded that the fractures would initiate at the outer surface of the tube wall where hoop stresses are tensile and penetrate up to the depth of \( \sigma T/P \) which defines the boundary between the tensile and compressive hoop stresses. Here \( \sigma, T \) and \( P \) are tensile strength, wall thickness and internal explosive pressure, respectively. Since \( P \) is a function of \( r/r_0 \), ratio of instantaneous to initial internal tube radius, the fracture radius is defined as the radius associated with \( P = \sigma \). This theory is essentially correct and is well supported by experimental observations. However, observations of fracture mode do not support, in general, the assumption of radial fracture.

Hoggat and Recht [8] further developed this model for a cylinder. This model is based on the early appearance of small radial cracks in the tensile hoop stress region near the outer surface of the tube. With in the compressive hoop stress zone the tube expansion is accomplished by extrusion, which activates shear planes, rotated approximately 45° from the radial direction. When the component of stress normal to the shear directions changes from compressive to tensile, the preferentially weakened material in the shear zone, fractures. However, the material in the shear zone has been thermally softened by the heat of plastic deformation, resulting in the fracture along shear planes and the appearance of shear lips rather than the radial fractures.

2.1. Compressive Radial Stress in the Cylinder Wall

Let \( P \) is internal explosive pressure acting on the wall of internal radius \( r \). Then equation of motion for cylinder wall can be written as [8]

\[
P(2\pi r) = \rho \pi (R_o^2 - r^2) \frac{d^2 a}{dt^2}
\]

where \( r_0 \) is initial internal radius of the tube, \( R_o \) is initial external radius of the tube, and \( \rho \) is density of the tube material. The pressure acting to produce the radial acceleration, \( \frac{d^2 a}{dt^2} \), at any arbitrary radius \( 'a' \) within the wall, is defined by the equation of motion for the portion of the wall external to this radius, which is written in the same form as Equation (1) i.e.

\[
p_a(2\pi a) = \rho \pi (R^2 - a^2) \frac{d^2 a}{dt^2}
\]

From Equations (1) and (2) one gets

\[
p_a = \left( \frac{r}{a} \right) \left[ \frac{R^2 - a^2}{R_o^2 - r_o^2} \right] P
\]

Neglecting elastic strains and considering axial strain, \( \varepsilon_a \), to be zero, equations for \( 'a' \) and \( R \) are written

\[
a = \left( r^2 + a_o^2 - r_o^2 \right)^{1/2} \quad \text{and} \quad R = \left( r^2 + R_o^2 - r_o^2 \right)^{1/2}
\]
Above equations are written on the basis of conservation of mass of the expanding ring and also assuming that no appreciable change in density of the ring material occurs during expansion. Substituting Eqn. (4) into Eqn. (3) provides the required expression for the radial pressure in the wall at any radius 'a' as a function of the internal explosive pressure \( P \)

\[
P_a = \frac{P_r}{\left( r^2 + a^2 - r_0^2 \right)^{1/2}} \left( \frac{R_o^2 - a^2}{R_o^2 - r_0^2} \right)^{1/2}
\]

The internal explosive pressure can be represented by an isentropic expansion

\[
P = P_0 \left( \frac{r}{r_0} \right)^{-2\gamma}
\]

where \( P_0 \) is the effective detonation pressure acting on the wall when \( r = r_0 \) and \( \gamma \) is the expansion exponent. The gas expansion exponent \( \gamma \) actually varies during expansion, typically approaching a value of five during the early stages, and decreases as the expansion proceeds. A constant value of about \( \gamma = 3 \) [9] is generally taken for the calculations purposes. During expansion the radial pressure decreases and when it becomes equal to the tensile strength of the metal the cylinder fractures.

3. EXPERIMENTAL PROCEDURES

Rotating mirror framing and streak cameras were used to record the cylinder expansion and fracture. Framing camera views the whole cylinder, whereas, the streak camera views only a small annular ring through 0.1mm wide camera slit. An argon gas in a cardboard container was explosively shocked to produce the back light, thus to record the optical shadowgraph of the expanding metal cylinder. As the cylinder expands its wall thickness decreases till the onset of fracture and rupture.

Figure 1: Sequential framing camera photograph of an expanding aluminium cylinder initiated from both the ends.

The rupture in the wall was identified with the outburst of detonation gaseous products through the cracks in cylinder wall. Figure 1 shows the framing camera photograph of an expanding aluminum cylinder detonated from both the ends. The expansion velocity of the
cylinder wall, \( V \), is calculated by measuring the angle made by the expanding wall with the cylinder axis along with the detonation velocity. Figure 2a shows the streak camera record of an expanding outer wall of the metal cylinder. The initial outer diameter of cylinder is shown as the masked patch on the argon flash. The distance time plot of an expanding wall is shown in figure 2b. After some finite expansion the cylinder ruptures.

From the measurements of cylinder wall velocity, \( V \), and radius at fracture, \( r_f \), the strain and strain rate have been calculated from the relations [3]

\[
\varepsilon = \frac{r_f - r_o}{r_o} = \frac{\Delta r}{r_o} \quad \text{and} \quad \varepsilon = \frac{V}{r_o} = \frac{1}{r_o} \frac{dr}{dt}
\]

The wall thickness of the cylinders and explosive compositions were changed to deform the metal under different strain rates.

4. RESULTS & DISCUSSION

4.1. Radial Deformation of Aluminium Cylinder

The results of the rupture strain of aluminium cylinder under different loading conditions are plotted in figure 3(a). The results indicate that the aluminium cylinder ruptures at the strain of 70-150\%, depending upon the wall thickness and the strain rate. Three explosive compositions, Baratol, TNT and Octol were used to deform the metal at different velocities. For each explosive composition, the wall thickness of the cylinder was changed to alter the strain rate. The results exhibit a maxima in the rupture strain for a wall thickness of 3-5mm. This type of rupture criteria, showing a maximum strain at some value of strain rate, was put forward by Ivanov [10]. By applying Ivanov criteria and assuming a visco-elastic relation, \( \sigma = \sigma_0 + \eta_0 \varepsilon \), for flow stress, we have calculated macroscopic viscosity coefficient, \( \eta_0 \), for aluminium to be 0.55-0.87x10^3 Pas, in the strain rate region 3-7 x 10^3 s^{-1}. However, when the wall thickness of the cylinder was kept constant and the explosive composition was changed to produce the different strain rate, then the rupture strain was found to increase with the strain rate, as shown in figure 3(b). This trend of increasing failure strain with strain rate was also shown by Slate et al [11]. He expanded thin walled spherical shell of various metals by detonating a sphere of explosive located at the center of shell. He found out rupture strain in the range of 30-90\%, under strain rate of 6-45x10^3 s^{-1}. Higher rupture strains observed in the present studies are may be due to higher strain rates.
4.2. Radial Deformation of Copper Cylinder

The experimental value of rupture strain found out in copper cylinder of different wall thickness, are given in Table 1. The process of rupture in copper cylinder has been found to be different than that of aluminium. Aluminium cylinder expands uniformly till it ruptures and the point of rupture is clearly identified in most of the experiments. In copper cylinder it is observed that the crack is initiated at the outer wall and then it propagates inward through the expanding cylinder wall, till the rupture is complete. From the measurements of the time between crack initiation and rupture, the crack propagation velocity through copper cylinder wall has been found to be 270-290m/s. Wall thinning up to 60% has been assumed for calculating this velocity. This phenomenon could be observed only in few experiments as the crack observation requires a very high optical resolution of the streak camera. Fracture strain, the strain at which the crack initiates, has been observed to be 30-70%. Rupture strain up to 300% has been observed in copper cylinder when loaded with powerful octol explosive.

Table 1

<table>
<thead>
<tr>
<th>Explosive</th>
<th>Inner Diameter (mm)</th>
<th>Wall Thickness (mm)</th>
<th>Expansion Velocity (mm/ms)</th>
<th>Strain Rate (x 10^-3)</th>
<th>Fracture Strain (%)</th>
<th>Rupture Strain (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RDX/TNT</td>
<td>25</td>
<td>1</td>
<td>1.45</td>
<td>5.80</td>
<td>160</td>
<td>160</td>
</tr>
<tr>
<td></td>
<td>52</td>
<td>2</td>
<td>2.5</td>
<td>4.80</td>
<td>67</td>
<td>---</td>
</tr>
<tr>
<td>TNT</td>
<td>52</td>
<td>1</td>
<td>2.49</td>
<td>4.79</td>
<td>50</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>52</td>
<td>2</td>
<td>1.93</td>
<td>3.72</td>
<td>30</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>52</td>
<td>3</td>
<td>1.75</td>
<td>3.36</td>
<td>40</td>
<td>200</td>
</tr>
<tr>
<td>Octol</td>
<td>52</td>
<td>1</td>
<td>3.1</td>
<td>5.96</td>
<td>---</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>52</td>
<td>3</td>
<td>2.5</td>
<td>4.80</td>
<td>102</td>
<td>200</td>
</tr>
<tr>
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<td>4</td>
<td>2.2</td>
<td>4.23</td>
<td>---</td>
<td>325</td>
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<tr>
<td>Baratol</td>
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<td>1</td>
<td>1.82</td>
<td>3.50</td>
<td>29</td>
<td>155</td>
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<tr>
<td></td>
<td>52</td>
<td>2</td>
<td>1.34</td>
<td>2.57</td>
<td>57</td>
<td>190</td>
</tr>
</tbody>
</table>

The observed high rupture strain indicates that the copper metal is undergoing rupture by total necking. Very large rupture strain at high strain rates has also been observed by Taylor [12]. The rupture preceded by a complete necking was confirmed by Stelly [13] who performed the metallographic examination of the recovered copper fragments. A very pronounced grain
elongation with twinning was also observed. Some defects, which will produce early instability at low strain rate, will appear to be more stable at the strain rates exceeding $10^4 s^{-1}$.

4.3. Recovery

Figure 11 shows the recovered fragments when a 45mm internal diameter aluminium tube was deformed by RDX/TNT (60:40) explosive. The thickness of the recovered fragments was $0.9 \pm 0.05mm$, for a 2mm initial wall thickness. This indicates that the phenomena of metal extrusion takes place in the expanding wall to produce wall thinning up to 60%.

Figure 4: Recovered fragments after rupturing of 2mm thick and 45mm internal diameter aluminium tube.

Close examination of the fragments reveals that the shear fracture dominates the radial fracture in high strain rate deformation of aluminium. Under explosive loading the outer layers of the cylinder are in tension as a result of the circumferential stretching or hoop strains. However, in the inner parts of the cylinder wall, a state of hydrostatic pressure is produced which is sufficient to prevent the development of tensile stresses. Tensile crack can not propagate inward during that phase and the strains are accommodated by the propagation of $45^\circ$ shear failure. In general, the proportion of shear ($45^\circ$) to tensile ($90^\circ$) failure will increase with the increase in expansion velocity.

REFERENCES