CHAPTER: 2
LITERATURE REVIEW

2.1 BACKGROUND

In recent years, light steel structures have been extensively used as being the most effective in practical applications. The main advantages of such kind of structures are the effective usage of material and faster erection as well as their good service characteristics. Over the past three decades, solution of the buildings with frames comprising the web tapered I-columns and web tapered I-beams, manufactured from high tensile steel, have been become a standard practice. The use of automatic welding techniques minimizes the fabrication cost of such tapered members. Their cross sectional profiles are intended to match the flexural strength close to the bending moment diagram, so that the requirement of the cross sections is well optimized. Such structural members are quite efficient in steel rigid frames of medium and large span with no in-plane bracing system, so that the increase in the fabrication cost is more than compensated by the decrease in weight when compared to rolled sections. With this type of frames the web depth-to-thickness ratio will be in the range of 180 and there won’t be any need of additional vertical or horizontal stiffeners in the web regions of these structural members.
Despite several advantages of tapered structural members, the use of tapered structural members suffers the lack of appropriate simple and accurate design formulae in most codes of practice. Then the design solutions for tapered structural members are limited, because the available approaches and/or recommendations consist elastic design formulae, where taper effects are not properly accounted for and analysis of such frames is rather complicated. The stability of loaded tapered columns/rafters was investigated, with proposals to calculate tapered columns/rafters as uniform prismatic members, using additional factors. It is obvious that results of buckling analysis for a tapered column / rafter under the combination of an axial force and a bending moment cannot be obtained by just adding the solutions obtained for those loads separately because the interactive behavior is non-linear. It is therefore observed that there is a need for the development of design formulae for tapered structural members. These formulae also have to be checked for both accuracy and safety supported by experimental results or a specific numerical tool (i.e., through Finite element method). Once such a set of reference results would be made available, the accuracy of behavioral knowledge of tapered structural members can be improved to acceptable levels.

The limited availability of well-documented test results on tapered structural members is another constraint for a detailed investigation. Therefore, the present investigation resorted to
numerical simulations by using the finite element modeling and analysis, for establishing the behavior of such structural members.

For instance, discretizing a member elements (i.e., flange plates and web plates) with shell finite elements is likely to provide more accurate results, if the software takes due account of geometrical and material non-linearties. A convenient alternative would then lie in a discretisation with finite elements for simplification and ease of data input. Further, it facilitates to study the non-linear behavior of web tapered I-beams up to their collapse load. Such a numerical tool can also be used for the calibration of design formulae. Earlier investigations on tapered beam finite elements are limited to elastic behavior.

The present investigation aims at extending their application to simulate the non-linear behavior up to collapse.

Traditional analysis and design practices do not consider 3D structural behavior of metal frame buildings. Normally, practice is to consider only an internal frame (2D analysis), an end frame, purlin and girt systems, and profiled steel claddings, all of which are designed independently, based on simple assumptions of load transfer from one structural member to another structural member. As per codal practice and specification, a structural framing is designed as a combination of 2D vertical and horizontal planes of framing. The main parameters affecting both the strength and deflection behavior of metal buildings under the given load conditions are described as:-
• Cladding action
• Rigidity of end wall frames and the interior frames.
• Bracing schemes
• Joint action (at base, either pinned or fixed)
• Knee/haunch flexibility

Ignoring the effects of these elements in the modeling and analysis of metal frames give conservative sections for the main frame design, and conservative premature failure criteria including deflections, and side sway due to lateral loading. The presence of end wall frames and the diaphragm action of profiled steel roof and wall claddings causes part of the loads to be transferred to the end walls. As a result, actual calculated maximum frame stresses and deflections are much less than in the bare frame. This cladding action which resists in-plane deflection by shear is known as stressed skin or diaphragm action and has been extensively studied by Davies and Bryan [5] and co-workers. Their results are used in the design of metal buildings in the Europe, U.K, and the United States.

However, the claddings do carry in-shear/racking forces, whether their effect is acknowledged or not. Therefore, it is necessary that cladding action and end frame rigidity be taken into consideration in the design of metal frame buildings using a 3D modeling approach. Similarly, joint action also should be taken into account. The 2D model that ignores the effects mentioned above is likely to be governed by the serviceability criteria more than the ultimate strength criteria.
The two characteristics that are very important for any form of bracing system are strength and stiffness. The stiffness of the bracing acts in conjunction with the out of plane stiffness of the web tapered I-beams/I-columns, to increase the resistance to lateral buckling and to improve its load carrying capacity. Bracing stiffness is a function, not only of the geometrical and material properties of the bracing itself, but also the way in which it is fastened to the primary members. The bracing must also be capable of maintaining its stiffness up to the stage at which collapse is imminent.

Lateral torsional buckling is a failure mode that can often control the design of flexural members in metal building systems. The buckling capacity of the members can be increased by using intermediate bracing that reduces the unbraced length. The IS code, IS: 800-1984[15], specifies that, for the beams which are provided with members giving effective lateral restraint of the compression flange at intervals along the span, the effective restraint shall be capable of resisting a force of 2.5% of the maximum force in the compression flange taken as divided equally between the number of points at which the restraint members are provided. The newly released code IS: 800-2007[16] specifies for the beams which are provided with members giving the effective lateral restraint at intervals along the span, the effective lateral restraint shall be capable of resisting a force of 2.5% of the maximum force in the compression flange taken as divided equally between the points at which the restraint members are provided. Further, each restraint point should
be capable of resisting 1% of the maximum force in the compression flange.


In recent times, improved technologies in structural analysis, design and detailing with advanced materials, fabrication technologies and construction methodologies have in most cases led to lighter buildings, for which strength and serviceability criteria have become more critical. The existing design methods are based on isolated element behavior. In case of this type of structures, the interactions between the primary and secondary members/components, as well as flexibility of connections significantly influence this behavior/response.

The buckling strength of these structural members is directly influenced by the lateral restraints. Sensitivity to in plane second order effects, the global imperfections, and the combined effect of local and overall buckling may also influence their behavior/response.

It is well known that an isolated metal frame /main frames loaded in its own plane may suddenly deflect laterally and twist out of the plane of loading, and so fail in a flexural torsional buckling mode. However a single, main frame standing alone is seldom used in
practice. A main frame is often connected with other members such as purlins, girts, lateral braces, bracing members, roof sheeting, wall sheeting, that produce restraining actions to the main frame and significantly influence the buckling resistance of the main frame. Continuous restraints provided by the members, such as roof sheeting systems, wall sheeting systems along with purlin and girt members are usually considered to be uniform along the length of the main frame, and their influence can be treated to be similar to the action of the discrete restraining members with close spacing. When the spacing between the restraints is not close, the restraints should be considered to be discrete. Discrete restraints act at the points where other members are connected to the main frame and induce actions which resist buckling, deflections, rotations, twists and warping displacements. These restraints are usually assumed to be elastic, in which case their design is influenced by their elastic stiffness’s.

In some cases discrete restraints may be considered to be rigid, so that they fully inhibit one or more buckling deformations. When the restraints prevent lateral deflections and rotations due to the twist of cross-sections at which they act, then the main frame may be described as a braced rigid frame that consists of main frame segments between the points of restraints.
2.2 TERMINOLOGY

Buckling during the flexure may be categorized as lateral torsional buckling, local buckling and distortional buckling.

2.2.1 Lateral-Torsional Buckling (LTB)

Is a limit state of the structural usefulness where the deformation of a beam changes and the failure occurs by a combination of lateral deflection and twist. LTB is the first limit state and occurs when the compression portion of a cross section is restrained by the tension portion of the cross section and the deflection due to flexural buckling is accompanied by lateral torsion.

The Lateral torsional buckling can be avoided by properly spaced and designed lateral bracings. The other factors affecting the LTB are the proportions of the beam, support conditions, initial imperfections of the geometry, the type and the application of the loading.

For each cross section of the beam, it is possible to compute limits of the interval of the bracing for LTB.

Figure 2.1 (Buckling Modes)- Local, Lateral and distortional buckling
2.2.2 Flange Local Buckling and Web Local Buckling

Is generally the plate buckling in the flange and/or web of the beam with no overall deflection in the direction perpendicular to the application of the loading. This type of failure may be controlled by selecting suitable width to thick ratio of flanges and the web components.

The third limit state for beams is Web Local Buckling (WLB). This type of buckling occurs when the width-thickness ratio is beyond the limit to withstand the moment on the beam. The way to prevent this type of buckling is to limit the width-thickness ratio. The limits can be computed for web local buckling from different codal provisions. The fourth limit state for beams is Flange Local buckling (FLB). It is exactly the same as Web Local Buckling, except the width-thickness ratio is in terms of the flange and not the web. This type of buckling occurs when the width-thickness ratio is beyond the limit to withstand the moment on the beam. The way to prevent this type of buckling is to limit the width-thickness ratio. The limits can be computed for flange local buckling from different codal provisions.

2.2.3 Distortional Buckling

Is a mode involving the characteristics consistent with a combination of the lateral torsional buckling and the local buckling. Shorter length beams with slender webs are quite susceptible to this kind of buckling.
2.3 GENERAL BEAM BEHAVIOR

The generalized beam behavior of singly or double symmetric beam bent about the strong axis is illustrated in figure 2.2.

The beam ultimately fails by lateral torsional buckling, by local plate buckling of the compression flange, or web buckling. Because the grades of steel generally used in the metal building construction industry posses sufficient ductility, failure by tensile rupture will not occur prior to buckling type of failure associated with compression.

![Figure 2.2 Generalized beam behavior](image)

Figure 2.2 Generalized beam behavior [67]

The behavior shown in figure 2.2 is classified in to three regions:
:- (General beam behavior is classified as under one of the three response categories:- Plastic, Inelastic and Elastic).
(I) **Plastic Region:** In the plastic range, the beam has the capability of reaching the plastic moment, $M_p$, and maintaining its strength through a rotation capacity, sufficient to ensure that moment distribution may take place in indeterminate structures.

(II) **Inelastic Range:** In the Inelastic range, a portion of the entire cross section will yield with a small amount of inelastic deformation. In this range, the plastic moment, $M_p$, may or may not be reached before unloading. The unloading is due to the instabilities occurring in the form of local or global buckling.

(III) **Elastic Range:** In the elastic range, buckling takes place while the beam cross section is still elastic.

Practical beams usually falls in the ranges I and II, while the range III often become of importance only during erection before all the bracing is in place.

In the plastic range not only the strength counted on in design, but also inelastic rotation capacity, so plastic analysis may be used to determine the bending moment distribution as a plastic mechanism forms. In range II and III no appreciable rotation capacity exists, and so the forces in the members must be obtained by an elastic analysis.

The order of importance, in terms of frequency of occurrence and optimum utilization of the material, is: Ranges I, II and III.
2.4 LATERAL BRACINGS

Bracings are usually considered to be perpendicular or inclined for the structural members to be braced. The brace strength (moment or force) and the brace stiffness (moment per unit rotation or force per unit displacement) shall be computed based on the inclination angle of the bracing member. The evaluation of the brace strength and stiffness shall include its material properties, geometric properties and its end connections.

Generally two types of bracing systems are considered and they are "relative and "nodal" (discrete). A relative brace controls the movement of the brace point with respect to adjacent points along the length of the beam or column. A discrete (nodal) brace point controls the movement at the braced point without the direct interaction with the adjacent brace points.

Figure 2.3 Relative bracing system
A relative brace system controls the relative lateral movement between two points along the span of the beam. The top flange horizontal beam system as shown in figure 2.3 is an example of a relative brace system. The system relies on the fact that if the individual beams buckle laterally, points “x” and “y” would move different amounts. Since the diagonal brace prevents points “x” and “y” from moving different amounts, lateral buckling cannot occur except between the brace points. Typically, if a perpendicular cut anywhere along the span length passes through one of the bracing members, the brace system is a relative type.

A nodal (discrete) brace system can be represented by individual lateral springs along the span length. The system is shown in figure 2.4.
2.5 REVIEW OF PREVIOUS BRACING STUDIES

The majority of the structural members of metal building systems are subjected to considerable restraints under the working conditions, either inherent in the construction of the systems or deliberately applied as a precaution against buckling. These restraints have the effect of increasing the stability of the structural members. In many cases it is found that the structural members will fail due to yielding under the plane bending stresses before collapsing under the laterally. Thus provided that the degree of the restraint required to achieve the stability may be determined, it will be frequently be possible to design a slender structural members on the basis of the material yield stresses alone.

It is evident from the literature that increase in efficiency of the design of structural members can be effected if the structural engineers allow for the influence of the restraints, and realize how bracing members of low weight can afford considerable economics in material saving by raising the allowable stresses, reduction in fabrication costs, easiness and simplicity in erection of the metal building systems.

Bracing can be categorized in to two main types, lateral bracing and torsional bracing. Lateral bracing increases the buckling strength of the structural members by restraining the lateral movement of the structural members. Since most of the buckling problems involve twisting about a point near or below the tension flange as shown in figure 2.5.
The structural members braced with secondary members like purlins, girts and flange braces (fly braces) may be regarded as a structural members laterally supported. Investigations into the buckling of laterally braced beams were carried out from the 1950’s by A.R Flint [21]. Studies were continued by D.A Nethercot and K.C Rockey [55], D.A. Nethercot [57], B.R Mutton and N.S. Trahair [58] and later by P.E. Cuk and N.S. Trahair [75], G.S Tong and S.F Chen [85, 86] and B. Gosowski [105]. The assumption of elastic lateral and/or torsional bracing was employed in these investigations. From these studies, it can be seen that the required bracing is very small to increase the buckling moment of beams to a required value in common cases and many bracing members in practice can easily satisfy this requirement. So the braced point is often taken as a rigid support to the beams in practice.
If the bracing is assumed as a rigid support to the beams, the beams are laterally continuous although they may be simply supported in their vertical plane. A.J. Hartmann [44] and N.S. Trahair [50] carried out studies on the buckling of laterally continuous beams.

The origin of the 2.5 % rule appears to have been in engineering intuition. Throop [20] in 1947 stated that the 2% figure had been in use in his design office for many years and a more conservative 2.5% value appeared in the original AREA (American Railway Engineer’s Association) specification in 1925.

A.R Flint [21] presented experimental data for the buckling of a beam with a lateral brace located at mid span on the top flange, bottom flange and at the shear center. He has also presented the graphical solutions for the torsional brace at mid span. In his solutions, the critical moment increased to a maximum moment of twice the unbraced beam moment at a brace stiffness of infinity. The inaccuracy of the solution can be seen by observing that the critical moment will increase nearly linearly with the brace stiffness until the second mode “S” shape is reached at finite brace stiffness. Flint indicated that a brace can be effective by only preventing the twist of the cross section while allowing lateral movement can occur.

A.R. Flint [21] also studied the stability of the beams loaded through the secondary members. He examined the restoring effect of a load that is applied through a relatively stiff secondary member resting on the top flange of the critical beam. Ignoring the effects of cross section distortion, Flint found that no lateral buckling can occur.
in the first mode unless the beam has an initial bow greater than the half the flange width.

W. Zuk [23], who was the first to consider strength requirements, found that the “two percent” rule was an adequate design criterion. This rule states that the force in the bracing is 2% of the compression force in the compression flange of the beam. Unfortunately, W. Zuk [23] used the wrong amplification factor on lateral deflections by assuming it would be the same as for initially crooked columns. Schmidt [35] suggests that design should proceed as follows: 1) specify the permissible deflections 2) solve for required stiffness and 3) check the required strength.

W. Zuk [23] analytically studied the bracing forces for the elastic beams and obtained values between 2.0 % and 2.4%. The results assumed certain initial imperfections and rigid supports. G. Winter [25, 27] extended this elastic study to include the axial stiffness of the braces. In addition to knowledge of initial imperfections, it was also necessary to know the deflections at failure. G. Winter's [25, 27] analysis assumed that once the supports are above a certain axial stiffness the buckling load will be identical to the load for rigid supports. The stiffness required was found to be small.

G. Winter [25, 27], presented the methods to compute the lower limits for the strength and rigidity of one or more intermediate lateral supports in order to provide the full bracing to beams and columns and also proposed the formulas for the strength and rigidity of the continuously braced compression members.
C. Massey[32] applied a similar approach to the analysis of post elastic beams, but returned to Zuk's[23] initial assumptions of rigid supports. It was specified as necessary to assume an initial imperfection pattern.

Massey [32] also presented experimental results on beams with L/r_y values between 40 and 120. For these tests at L/r_y = 40, the bracing forces averages 0.011 P_yf (P_yf =the flange force). However Massey has indicated that the recorded bracing forces do not apply to the point of unloading, but to some earlier stage in the hinge formation.

G.C. Lee, A.T. Ferrara and T.V. Galambos [37] conducted the experiments on braced wide flange beams, these tests indicated that the usual methods of purlins attachment are adequate and the partial depth stiffener is effective at brace points. They also showed how stronger braces are needed when only one side of a beam is braced. Unfortunately, the purlins were used were much more than adequate with respect to axial strength and stiffness, and therefore do not provide any conclusive information on these problems. Also stated that the action of the purlins which were attached to the compression flange of each test beam at the end of the critical span acted as effective lateral braces, because each beam was able to sustain the full plastic moment through a certain amount of inelastic rotation. They indicated that the action of the purlins were twofold: 1) they prevent the lateral movement of the compression flange at the bracing points, and they 2) helped, along with the elastic compression flange
of the adjacent beam span, to restrain the lateral rotation of the compression flange at the ends of the critical span.

Samuel J. Errera, George Pincus and Gordon P. Fisher [42] presented theory and test results for the columns and beams braced by diaphragms and concluded when the diaphragms are properly attached to beams and columns effectively behave as a lateral bracing and reliably increase the load carrying capacity up to the limit load.

T.V.S.R. Appa Rao, Samuel J. Errera and Gordon P. Fisher [48] conducted the theoretical and experimental investigations to determine the load carrying capacity for ideal and imperfect columns with the combined diaphragm and girt bracing and concluded that the relative movement of the girts parallel to each other produces shear in the diaphragm and this shear is transferred through the girts as forces resisting the lateral movement of the columns as a result the combined diaphragm and girts can reliably and sizably increases the load carrying capacity of the columns.

R.S Barsoum and R.H Gallagher [51] derived a finite element method applied by D.A Nethercot [57]. D.A Nethercot [57] has proposed a design method for lateral and torsional restraints which does not, however consider strength requirements. D.A Nethercot and K.C. Rockey [55] studied the stiffness needed to force the second mode and concluded that this requirement is easily met in practical situations.
D.A. Nethercot and N.S. Trahair [64] proposed the expression for calculating the necessary strength and diaphragm rigidity which allows a beam to reach its yield moment.

Leroy A. Lutz and James M. Fisher [79] presented the bracing stiffness required from continuous point bracing to single point bracing and developed the expressions for the point bracings and also established the relationship between the brace stiffness and the buckling behavior of the compression member. They have also presented some numerical examples.

Tong Geng-Shu and Chen Shao–Fan [85] conducted a study on the buckling behavior of simply supported beams which are braced laterally and torsionally and subjected to uniform moment. They have presented a closed form solution, the relation between buckling moments and bracing stiffness. They have also presented the formulae for the critical stiffness required for the full bracing. They have indicated that the combined lateral and torsional bracing is more effective than either lateral or torsional bracing alone for beams under uniform moment.

G.S. Stanway, J.C. Chapman and P.J. Dowling [89] examined the behavior of an initially imperfect column with an intermediate elastic restraint at any position. They have studied the variations in the restraint position, column slenderness, column imperfection shape and its magnitude. They have concluded that the column flexure can be a significant contributor to the restraint force.
G.S. Stanway, J.C. Chapman and P.J. Dowling [90] examined the behavior of an initially imperfect column with an intermediate elastic restraint at any position by considering the effects of column span ratio, stiffness of column, stiffness of restraint, column imperfection shape and its magnitude. They have presented the restraint stiffness criteria for effective subdivision of column, and a procedure for determining the column strength when the restraint stiffness is predetermined. They have also suggested that a bracing stiffness which is sufficient to reach 90% of the rigidly braced structural system is a suitable criteria for design of bracing.

Yura et al. [92] presented a typical interaction solution for a 24 feet W12 X 14 section braced at midspan with combined lateral and torsional bracing and indicated that a lateral brace stiffness of one half the ideal stiffness is used, the solution indicates that an additional torsional brace with a stiffness one quarter of the ideal stiffness will force the beam to buckle between the braces. They have presented the equations considering the influence of combined lateral and torsional bracing on the elastic buckling resistance of the steel I-section members.

Todd A. Helwig, Karl H. Frank and Joseph A. Yura [99] presented the results of a finite element analysis on the stiffness requirements for the shear diaphragms which are used for the beam bracing. In their research they have considered single symmetric and doubly symmetrical cross sections. The parameters that were
investigated in their research include diaphragm stiffness, load type, load position, cross sectional shape and the web slenderness. They have concluded that the type of loading had a significant effect on the bracing response of the shear diaphragms and also concluded that the current solutions in the literature are based upon uniform moment solutions and often overestimate the capacity of the diaphragm braced beams. They have presented the solutions that can be used to determine the diaphragm stiffness requirements to prevent the lateral torsional buckling of the beams with general load applications.

Heungbae Gil and Joseph A.Yura [107], presented the experimental and analytical studies to determine the bracing requirements for the inelastic columns. In their experimental research, columns with a brace at mid height were loaded to their maximum limit with the brace stiffness at the mid height as the main variable. They have kept the tangent modulus as constant, which governs the inelastic buckling of columns. They have concluded that the bracing requirements for inelastic columns depend upon the number of braces, buckling load and the length of the columns but not on the material state.

bracing provisions. The provisions include both stiffness and strength criteria/requirements for the bracing. Many of these provisions were developed utilizing finite element analyses (FEA) to consider the numerous parameters that affect the bracing system. To effectively study the bracing behavior, a clear understanding of the modeling geometry for the members in the structural system as well as the modeling techniques is necessary to obtain the stiffness and strength criteria/requirements. The stiffness requirements are typically found by conducting an eigen value buckling analysis on the system. In most situations, the eigen value analysis is conducted on a perfectly straight system since this type of an analysis is relatively insensitive to geometrical imperfections. Such an analysis defines the ideal brace stiffness requirements, which is the minimum stiffness that is necessary so that a perfectly straight member will buckle between the braces as shown in Figure 2.6.

Figure 2.6 Buckled shaped for member with full bracing
Previous studies (Winter [27]; Yura [110]) have shown that providing the ideal stiffness results in relatively poor behavior for real structures due to the effects of geometrical imperfections. Winter [27] developed a simple model consisting of rigid links and hinges at the lateral brace points that demonstrated the stiffness requirements for column bracing. His model also demonstrated the effects of imperfections on the bracing behavior. If the ideal stiffness is provided in an imperfect structure, the member to be braced will generally not reach a load level corresponding to buckling between the brace points. This is demonstrated in Figure 2.7 using Winter’s simple model for a column with a single lateral brace at mid height.

Figure 2.7 Effect of Brace Stiffness on deformations using Winter’s model [110]

The applied load is graphed versus the total lateral displacement, $\Delta_T$, at the brace location. The applied load has been normalized by $P_E$, which is the load corresponding to buckling between the brace points while the total lateral displacement has been
normalized by the initial imperfection, $\Delta_0$. The graphs show that if the ideal stiffness, $\beta_i$, is used on the imperfect system, the deformations become very large and $P_e$ is not reached. The magnitude of the force in the braces $F_{br}$ for the system shown in Figure 2.7 can be determined using the following expression:

$$F_{br} = \beta_i \cdot \Delta \quad \text{..................................................} \quad 2.1$$

Where $\beta_i =$ brace stiffness and $\Delta =$ lateral displacement at the brace location.

Since the brace forces are a direct function of the deformation at the brace location, the forces become very large if the ideal stiffness is used. However, figure 2.7 shows that providing a brace stiffness larger than the ideal value results in much better behavior. For example, providing a stiffness equal to the $2\beta_i$ results in a deformation at the brace point that is equal to the magnitude of the initial imperfection and likewise does a much better job of controlling brace forces since the deformation, $\Delta$, in Equation 2.1 is much smaller. To control deformations and brace forces, the bracing provisions in the AISC LRFD Specification 2001[9] recommend providing at least $2\beta_i$.

Determining the brace force requirements for a system generally requires a large displacement analysis on an imperfect system. Most bracing studies usually focus on determining the maximum brace forces that are likely to occur in typical applications. However, in
many cases, determining the critical shape imperfection can be difficult to establish. This is particularly true for beam bracing in which the brace location and distribution of the loading can have a significant effect on the brace forces [103].

For a bracing system with a given stiffness and applied loading, the two main factors that affect the magnitudes of the brace forces are the size and distribution of the initial imperfection. Figure 2.7 demonstrated that providing at least twice the ideal stiffness limits the amount of deformation at the brace location to a value equal to the magnitude of the initial imperfection. For the design of bracing for many members, the magnitude of the initial lateral imperfection is often taken as $L_b/500$, which comes from AISC Code of Standard Practice (2000) for erection tolerances.

![Figure 2.8 Cross-sectional Imperfections](image)

However, equation 2.1 shows that stability brace forces are directly proportional to the magnitude of the deformation that occurs at the brace location. Therefore, if the stiffness requirements of the
brace are satisfied, brace forces can generally be scaled accordingly for systems that do not match the assumed out-of-straightness.

Assuming one flange has the maximum lateral sweep in one direction with the other flange remaining straight was deemed reasonable in this study.

In the design process of the steel-I-beams, the most crucial and often the most difficult stage is the decision as to whether or not the beam is adequately laterally braced. Lateral Bracing may be provided either by means of a continuous medium such as metal roof sheeting attached to roof purlins, metal wall sheeting attached to girts, metal deck/sheet attached to I-beams to form the mezzanine floor or by one or more discrete braces to I-beams. In the case of discrete braces, rules exists in most structural design codes for assessing the strength and stiffness required for the braces, which will ensure that they act as a completely rigid. The assessment of the beam capacity in reducing the lateral torsional buckling with the available stiffness is much more difficult and the present research aims at fulfilling this gap.

Liqun Wang and Todd A. Helwig[117] studied the effect of different imperfection schemes on the magnitude of the stability brace forces. They have presented the results, which demonstrates the impact of several imperfection parameters on the bracing behavior and they have made the recommendations for selecting the imperfection shape that maximizes the stability brace forces for beam bracing systems.
Todd A. Helwig and Joseph A. Yura [129] presented the results of a finite element analysis on the strength and stiffness behavior of shear diaphragm bracing for the beams. They have recommended the factors that will permit the estimates of the shear diaphragm bracing that are representative of the types of the loading and the web slenderness values. They have demonstrated that providing twice the ideal stiffness, which has been the requirement for many bracing systems, resulted in relatively large deformations and shear diaphragm brace forces and concluded that providing four times the ideal stiffness limited the deformations and provided the better control on the brace forces. They have also concluded that the presence of intermediate discrete bracing, such as cross frames, reduced the strength and stiffness requirements of the shear diaphragm bracing.

Todd A. Helwig and Joseph A. Yura [130] presented the design requirements for the shear diaphragm bracing and focused on the strength and stiffness requirements for beam stability. They have presented the design expressions for the strength and stiffness and also presented a model/method which can be used to estimate the forces in the fasteners that connects shear diaphragm to the top flanges of the beams. A numerical example is illustrated, which demonstrates the usage of the proposed design expressions.
2.6 REVIEW OF PREVIOUS TAPERED MEMBER STUDIES

Tapered steel members with I-sections have become very popular in building construction. Tapered sections can resist a maximum stress at a single location. While in the rest of the member, the stresses are considerably lower. This results in appreciable saving in material, fabrication time and in construction.

The study of tapered elements and in particular the study of tapered columns has been the interest of many researchers throughout the years. Among the first was Dinnik [19] who in 1929 analytically solved the differential equations of buckling and published formulas and coefficients of stability which enabled the design engineers to select the proper column sections without having to set up the differential equations and their integration.

The use of tapered steel members in a steel structure was first proposed by Amirkian [22] during 1952 for reasons of economy. The elastic analysis of tapered I-sections has been the focus of investigations by numerous researchers.

W.J. Krefeld, D.J. Butler and G.B. Anderson [26] conducted experimental studies of steel cantilever beams having tapered flanges and tapered webs of I-section and channel sections with various dimensions, span length and degree of taper to predict the load carrying capacity of these sections. They presented a numerical example, which indicates that tapered section can results in saving of material as long as failure is produced by yielding and the capacity is not limited by elastic buckling.
James M. Gere and Winfred O. Carter [28] presented the graphs by considering the tapered columns are initially perfectly straight, elastic and subjected to axial compressive loads and the same enables to compute the critical buckling loads for uniformly tapered columns with different boundary conditions.

Charles M. Fogel and Robert L. Ketter [30] developed the elastic strength of tapered columns by considering two different cross sections and variations in the stiffness along the length of the members, one cross section with material concentrated in the flanges and the distance between the flanges varies linearly along the length of the member and the other section is having a tapered rectangular cross section with constant thickness. By using these cross sections, they have developed the interaction formulas and the curves between the axial force, end bending moments and the slenderness with the pinned boundary conditions. They have considered only the in plane bending of the member in the plane of the applied moments. i.e., the effect of lateral torsional buckling is not considered.

B.J. Vickery [31], experimentally investigated the collapse behavior of steel portal frame structures with tapered beams and columns made from standard rolled steel joists. The investigations were carried out on small scale and on full scale tests. Conclusion was drawn on material saving by using the tapered members as against the uniform members.
Boley [33] studied the behavior of tapered rectangular beams and concluded that for members with angles of taper less than 15 degrees, the usual approximation of structural mechanics can still be applied. Based on this finding, the analysis of tapered members is considerably simplified, and he suggested that design guidelines for prismatic members can be extended to tapered members as well.

D.J. Butler and G.B. Anderson [34] conducted the experimental research for the confirmation of H. Nakagawa’s theoretical analysis for the elastic buckling of tapered columns (Tapered flanges and tapered webs of I-section and channel section) and established the bending and thrust interaction curve and which is independent of the amount of tapering. They also concluded that the interaction formula suggested by Salvadori [24] for uniform beam-columns was applicable to a wide range of tapered I-sections. They have not reported any interaction between the flange and web tapering.

D.J. Butler [38] conducted the experimental tests on several tip loaded tapered cantilever I-beams using the rigid lateral braces and torsional braces, they have studied the influence of lateral brace location and the torsional brace stiffness on the elastic buckling strength and concluded a relatively modest torsional bracing effectively inhibits the lateral buckling.

Charles G. Culver and Stephen M. Preg [46] investigated the tapered beam-columns with axial force and with unequal end moments. They have considered tapered beam-columns with respect to both the flange width and the web depth and assumed to be a
linear function of the length of the member and also assumed to be initially straight. They have established the differential equation for determining the critical combination of axial forces and unequal member end moments of the tapered beam-columns. The solutions for the mathematical model were obtained using finite difference method and established the curves, tables and interaction formula to compute the applied moments which are required to cause the lateral torsional buckling.

Sritawat Kitipornchai and Nicolas S.Trahair[53] studied the elastic stability of simply supported double symmetric I-beams with tapered flanges/ tapered webs and proposed a general method for deriving the elastic lateral torsional buckling load of tapered symmetric I-beams subjected to different loading conditions. The experimental results of tapered aluminum I-beams were found to correlate very well with the predicted values of critical loads.

G.C. Lee, M.L. Morrell and R.L.Ketter [54] investigated the behavior of tapered members extensively and developed the guidelines for the design of tapered members by considering the five sections which represent the column members, sections with thick and shallow for beam members and also the thin and deep sections for the beams. They have assumed the maximum bending stresses at both the ends of the beam member are equal. The beams considered in their research have been assumed that the end conditions at both the ends of the tapered member are simply supported for both bending action and warping torsion. The elastic critical buckling strength of
these tapered beams are computed for various beam lengths and various degrees of tapering by using the Rayleigh-Ritz method and proposed the length modification factors that account for the web tapered I-beams in the pure torsion and warping resistance respectively. They have presented the design formulas for the axial compressive strength, flexural strength and the combined axial compression and flexure interactions.

It is pointed out that, using a prismatic beam element approach to model the web tapered beam elements, will exhibit a number of significant shortcomings as compared with a more detailed modeling approach. The most significant shortcoming is the fact that such an approach is unable to correctly duplicate the effect of the sloping flange bi-moment (a quantity frequently used to compute warping normal stresses in a cross section) on the buckling response.

M.L.Morrell and G.C.Lee [59] conducted the research on the allowable stresses for the web tapered I-beams with lateral restraints and they have developed the provisions for determining the flexural strength of web tapered I-beams with lateral restraints. They have investigated the two aspects of the effect of the adjacent spans on the critical span, namely, the adjacent span length and the number of adjacent spans. The flexural strength formulas presented by Dr. Lee et al [54] were improved by incorporating the total resistance to lateral buckling and the restraining effects of adjacent spans. The restraint factor is defined as which relates the restrained elastic critical buckling stress to the unrestrained stress for the particular case
when the compression flange is braced at nearly equal intervals and the loads were applied to the web tapered I-beams in the plane of the bending at these braced points.

S.P. Prawel, M.L. Morrell and G.C. Lee [60] conducted experimental research on inelastic stability of I-shaped tapered members in order to determine the bending and buckling strength of these members and also to determine the effect of the method of cutting the plates on their response to the loading. They have concluded that, because of the different residual stress patterns, the oxygen cut members are subjected to have higher inelastic bending stiffness and correspondingly a higher inelastic lateral buckling strength and also concluded that the fabrication by one side welding of the I-shaped tapered members does not indicate to influence the static strength of the laterally supported members as long as they are proportioned to satisfy the requirements of local buckling.

Sritawat Kitipornchai and Nicolas S. Trahair [63] studied the behavior of mono symmetric tapered I-beams and a general theory of bending and torsion of mono symmetric tapered I-beams has been proposed. They have derived the simultaneous bending and torsion differential equations by considering the flange deflections and concluded that the bending and the torsion resistances are interdependent, and the behavior of the beam cannot be separated into independent bending and torsion actions.
M.T. Horne et al [69] developed expressions for estimating the critical loads of tapered members subjected to arbitrary bending moment distributions.

J.B. Salter, D. Anderson and I.M. May [71] conducted experiments on steel columns with web tapered I-sections with pinned boundary conditions at both the ends and with torsion and warping prevented. These members are subjected to axial load and the major axis bending moment at the larger end. They have concluded that the failure occurred by lateral or lateral torsional inelastic buckling and the ultimate loads predicted by design codes were found to be conservative and suggested the modifications to the British draft limit state code for structural steel work in buildings.

T.G Brown [72] performed numerical solutions for tapered I-shaped cantilever and simply supported beams.

D.J. Fraser [74] conducted the parametric study in order to determine the buckling capacity of the tapered member frames and the haunched member frames for the rectangular and pitched roof frame profiles with pinned and fixed base conditions. He has proposed the formulas that would allow the designers to convert the non uniform frames in to an equivalent uniform frame which carries the same buckling load. This study is limited to symmetrical frames only.

O. Olowokere [78] presented the finite element method for determining the lateral torsional buckling load for linearly tapered I-section members with unequal flange areas, laterally supported tension flange only and unsupported members and they have
obtained the solutions for different flange area ratios and tapering ratios and are used to develop an interaction relationship for tapered unequal flanged steel structural columns subjected to both axial and bending stresses.

J.W. Wekezer [80] derived numerical results for tapered I-beams using membrane shell theory and are in good agreement with the results obtained by S. Kitipornchai and N.S. Trahair [53].

Y.B. Yang and J.D Yau [82], M.A. Bradford and P.E. Cuk [83] and S.L. Chan [87] developed and used tapered beam finite elements in their analysis.

Steen Krenk [88] investigated the lateral torsional buckling behavior of I-beam gable frames restrained at the top flange and subsequently generalized his solution to include web distortion effects.

The lateral torsional and local buckling of web-tapered I-beams has been investigated by D. Polyzois and Q. Li [96] using the finite element method.

Hamid Reza Ronagh and Bradford, M.A [97], presented the method, which was used to study some of the parameters that affect the elastic buckling capacity of tapered elements and the amount of distortion of the cross section, they have found that the loading away from the shear center and the presence of the restraints increases the distortion and the use of thicker webs is recommended to reduce the degree of the distortion. Due to the high degree of complexity of the problem and the lack of the general model to provide the cross
sectional property to quantify distortion, they have presented a specific parametric research of special cases that demonstrates the effect of distortion.

D.Polyzois and I.G. Raftoyiannis [101] reexamined the modification factor, $B$, which accounts for both the stress gradient and the restraint provided by the adjacent spans of a continuous web tapered I-beam. The usage of the recommended AISC [6] values for $B$ factor implies that both parameters, stress gradient and continuity, are equally important. If lateral restraint supports have insufficient stiffness or the supports are improperly applied to the web tapered I-beam, the continuity effect of the modification factor, $B$, may need to be ignored. By using a finite element computer program, D. Polyzois and I.G. Raftoyiannis [101] developed separate modification factor equations for the stress gradients and continuity for various load cases.

Gabriel Jimenez Lopez and T.V. Galambos [102], presented results concerning the inelastic stability of pinned ended tapered columns and compared them with the effective length factor results from the AISC Specifications for the pinned end tapered columns and recommended the modifications to the effective length factors.

Jin-Jun Li and Guo –Qiang Li [114], conducted the large scale testing of steel portal frames comprising web tapered I-columns and web tapered I-beams to examine the non linear effects in order to include in the theoretical research of similar frames. The tested
frames were identical in dimensions and also identical with respect to web tapered I-beam and web tapered I column sizes. One frame was subjected to incremental vertical loads and the other was subjected to simultaneous constant vertical load and the incremental horizontal loads. They have concluded that the in plane limit load bearing capacity for the frame with only vertical loads was obtained, where as the frame which was subjected to constant vertical load and incremental horizontal loads, the out of plane instability was occurred and the actual in plane limit load capacity was not obtained and also concluded that the partial restraint of pinned base condition significantly influences the strength and stiffness of these steel portal frames. In the test, they have considered the out of plane restraints.

I.G. Raftoyiannis, John Ch. Ermopoulos [123] discussed the elastic stability of eccentrically loaded steel columns with tapered and stepped cross sections and initial imperfections. The stability problem is formulated in a manner covering most cases met in design applications. Their formulations are based on the exact solution of the governing equation for buckling of columns with variable cross sections. A parabolic shape is assumed for the initial imperfections according to Euro code. They have applied the plasticity criterion to determine the material failure in the buckled configuration and concluded that the presence of the imperfections is responsible for a significant reduction of the carrying loads.
Anisio Andrade, Dinar Camotim, P.Borges Dinis [126] discussed the global performance and the underlying assumptions of a recently developed one dimensional model [118] characterizing the elastic lateral-torsional buckling behavior of single symmetric tapered thin walled open beams, which was able to account for the influence of the pre buckling deflections. A comparative study involving the critical load factors and the buckling mode factors yielded by the one dimensional model and the two dimensional shell finite element analysis is presented. They concluded that in general one dimensional model found to agree well with the shell finite element model results. They have also noted that some discrepancies for the shorter beams, which are due to the occurrence of relevant cross section distortion or due to the localized buckling behavior.

Zhang Lei and Tong Geng Shu [132] presented a new theory on lateral buckling of the web tapered I-beams, in which the deformations of the web tapered I-beams are investigated, based on the assumptions for thin walled members, and the relationships between the displacements of the flanges and web are developed and new equivalent sectional properties are considered. Based on the classical variational principle, they have proposed the total potential for the lateral buckling of the web tapered I-beams. Based on the proposed total potential, the lateral buckling of the web tapered cantilever beams and simply supported beams of I-sections are compared with FEA using the shell elements and they have concluded
that the equivalent method of using prismatic beam elements to consider the tapered members yielded unreliable buckling loads.

A.H. Salem, M.EI Aghoury, M.N. Fayed and I.M. EI Aghoury [133] have presented the non linear finite element model which allows for geometric nonlinearities to study the effect of interaction between the flange and web width-thickness ratios along with the member slenderness and the tapering ratio’s on the behavior of tapered I-section columns fabricated from the thin plates in order to define the ultimate strength and slenderness ratio curves and to study the different modes of failure. They have developed a series of curves for the design purposes and also developed an empirical equation to determine the ultimate axial capacity of the taper slender I-section members considering the entire cross section.

G.I.B. Rankin, J.C. Leinster and D.J. Robinson[134], conducted the experimental investigations of two full web tapered member steel portal frames with I-shaped cross sections. These portal frames are restrained by purlins and girts in out of plane sense. They have concluded that the deflections are favorable when compared with the predictions given by the elastic analysis by BS 5950. They also concluded that the method of modeling the tapered members by a series of prismatic members provided good comparison with respect to experimental results.
2.7 PRACTICAL DESIGN AND FABRICATION OF WEB TAPERED STEEL RIGID /MAIN FRAMES

A major goal of a structural analysis and design of a metal building system is to achieve the harmony between maximum safety with minimum cost. The metal building frame (rigid frame/main frame) is inefficient and uneconomical in the cases where the prismatic sections are used (example- standard hot rolled sections) as compared to the tapered sections and the same is due to the distribution of moments. The cross sections for the columns and the rafters has to be selected in order to satisfy the controlling bending moments at the eaves, apex point and at the base for the fixed base columns, the locations where the interior columns are having field moment connections with the roof beams, away from these defined locations, stresses in beams/columns are low. By using the web tapered columns and web tapered beams (using the three plated sections), the economy in the material will be achieved. By using the tapered sections/segments, an economical strength envelope can be achieved as a close fit to the bending moment diagram and the bending stresses are high throughout the metal frame profile.

The structural engineer has the liberty to select the following geometric parameters in the case of web tapered I-members design.

- Member Shape( variation in depth around the frame)
- Web thickness (possibly varying around the frame)
- Top Flange width/Bottom flange width and thickness of top and bottom flanges (possibly varying around the frame)
Of course minimizing weight does not necessarily minimize cost. In the instance, considerable economics will be achieved in the fabrication and in the construction, if the fabrication is automated.

Recent technological advances in this context are furnished below.

The behavior of built up cross sections which are fabricated from the slender plate elements is naturally complex than the rolled sections, due to cross sectional distortions and due to the local buckling. Stabilizing the metal frame by suitable lateral bracings from the secondary elements requires special attention during the design process. Efficient computer aided design software should be adopted to handle complexities in the analysis and strength checks and to obtain the desired optimization level and the turnaround time. Thus, before web tapered metal frames can be contemplated, a considerable investment is required in both the computer aided design and the automatic fabrication. Given this investment, one can see fair advantages in the usage of welded tapered metal frames in contrast to frames fabricated from rolled sections.

- They can achieve significant weight and cost economics.
- They are inherently stiffer because the optimized welded sections are considerably deeper than the rolled sections having the same resistance.
• Automated design and fabrication can readily be incorporated into a fully computerized manufacturing system, with very rapid and accurate cost estimation, computerized stock control and waste minimization, computerized drawings and computer controlled equipment.

The potential economic advantage of tapered frames can only be realized if the structural sophistication is synchronized by efficient office procedures and the fabrication techniques. Estimation, design, drafting and detailing, material stock control needs to be computerized. The estimation and design procedures need to encompass analysis, strength, serviceability and optimization based on the design guide lines and plant /workshop limitations. Drafting and detailing should produce full drawings based on the detailing guide lines and also based on the plant /workshop limitations. As a byproduct the data is generated and is suitable for all numerically/computer controlled machines in the plant/workshop and as a result the accurate estimation/quotation for the new projects follows readily from the available data with precise cost and time information. Stock control can be utilized to minimize the waste; this is particularly vital for the efficient tapered webs cutting. The usage of the stock materials can be made as a constraint on the design optimization. The automatic fabrication techniques/procedures are used for welding of the flanges to the web by means of submerged arc welding.
However, in order to achieve the overall efficiency, the manufacturing process needs to be supported by:

- Efficient materials handling for the web and flange components.
- Numerically controlled cutting for the web and also for the flanges if they are stripped from plates.
- Automatic butt welding for flange and web joints.

In metal buildings, both the columns and rafters are generally tapered to place the structural steel material where it is most actually required. The columns and rafters connections are typically designed as fully restrained moment end plate connections using available design procedures.
2.8 REVIEW OF CURRENT WEB TAPERED BEAM SPECIFICATIONS (ASD and LRFD)

The development of the web tapered member research by Dr. Lee et al [54] was restricted to only doubly symmetric web tapered “I” sections. The basic reasoning for this restriction was the inability to uncouple the torsional and flexural deformations due to varying location of the shear center for single symmetric web tapered beam sections. This is illustrated in figure 2.9, which is considered from Dr. Lee et al [54].

![Figure 2.9 Variable location of shear center for single symmetric shapes [54].](image)

B.A. Boley [33] studied the behavior of tapered rectangular beams and concluded that for members with angles of taper less than 15 degrees, the usual approximation of structural mechanics can still be applied. Based on this finding, the analysis of tapered members is considerably simplified, and he suggested that design guide lines for prismatic members can be extended to tapered members as well.
Accordingly the development of web tapered members was limited to small tapering angles, \( \theta = \frac{d_L - d_0}{L} \) is less than 15 degrees. The same was used as a codal provision. The tapering ratio of the web tapered members was defined as below:

\[
\gamma = \frac{d_L - d_0}{d_0} \quad \text{and the limiting value,} \quad \gamma \leq 0.268 \frac{L}{d_0}
\]

Figure 2.10 Geometry of typical web tapered I-beam [101]

Where

- \( d_L = \) Depth of the tapered section at the larger end, inches
- \( d_0 = \) Depth of the tapered section at the smaller end, inches
- \( \gamma = \) Tapering Ratio
- \( L = \) Length of the tapered member, inches
For practical design considerations, the limiting value was further limited to 6. (Dr. Lee et al)[54].

Often a Structural Engineer always attempts to select a particular tapered member such that the maximum bending stress is nearly constant along the length of the tapered member. In the case of a simply supported web tapered I-beam subjected to end moments alone, one possible optimum design would require equal extreme fibre bending stresses at the both the ends i.e., \((M/S_x)_o=(M/S_x)_L\). This implies that, \((M_o/M_L)=(S_{x0}/S_{xL})\), which is the limiting moment gradient, \(\alpha=+k\). Due to the varying moment of inertia, it is recognized that the maximum bending stress under this limiting moment gradient does not necessarily occur at the ends of the web tapered I-beam. The range of variable considered for this condition \((M_o/M_L)=(S_{x0}/S_{xL})\) is an adequate design consideration for the web tapered I-beam subjected to end moments at the both the ends producing single curvature deformations.

For the elastic torsional buckling of a prismatic beam, the critical extreme fibre elastic buckling stress will be calculated from the following formula:

\[
\sigma_{cr} = \frac{1}{S_x} \sqrt{\frac{\pi^2 E I_y G K_T}{L^2} + \frac{\pi^4 E I_y E I_w}{L^4}} \tag{2.2}
\]

Where

\(\sigma_{cr}=\) Elastic critical buckling stress of a prismatic member, Ksi
$S_x =$ Strong axis elastic section modulus, in$^3$

$E =$ Elastic modulus, Ksi

$I_y =$ Weak axis moment of Inertia, in$^4$

$K_T =$ Pure Torsional Effective length factor

$I_w =$ Warping constant

$G =$ Shear Modulus, Ksi

$L =$ Unbraced Length, Inches

The above equation is made applicable to web tapered I-members by introducing a length modification factor, and the corresponding critical extreme fibre elastic buckling stress will be calculated from the following formula:

$$
\left( \sigma_{cr} \right)_y = \frac{1}{S_{xo}} \sqrt{\frac{\pi^2 E I_{yo} G K_{TO}}{(hL)^2} + \frac{\pi^4 E I_{yo} E I_{WO}}{(hL)^4}} 
\tag{2.3}
$$

Figure 2.11 Equivalent Prismatic member for Lateral Torsional Buckling of web tapered I-members
Where

\((\sigma_{cr})_\gamma\) = Elastic critical buckling stress of a tapered member, Ksi

\(S_{xo}\) = Strong axis elastic section modulus at the smaller end, \(In^3\)

\(I_{yo}\) = Weak axis moment of inertia at the smaller end, \(In^4\)

\(K_{TO}\) = Pure torsional effective length factor

\(I_{WO}\) = Warping constant at the smaller end

\(h\) = Tapered member length modification factor

Solving the above equation for the \(h\)

\[
h = \sqrt{\frac{\pi^2 E I_{yo} G K_{TO}}{L^2 S_{xo} (\sigma_{cr})^2}} + \left[1 + \sqrt{1 + \left[\left(\frac{S_{xo} d_0}{G K_{TO}}\right)^2\right]}\right] \]

Dr. Lee et al [54] used five sections and represent the smaller end of the tapered members. Figure 2.12, which was considered from Dr. Lee et al [54]. For these sections with different lengths of the tapered members, \((\sigma_{cr})_\gamma\) values were calculated by using the Rayleigh-Ritz procedure with the most severe end moment ratio. The end moment ratio is defined as the ratio which causes the maximum bending stress to be equal at the larger end and the smaller end of the web tapered I-member.

The resulting \((\sigma_{cr})_\gamma\) were substituted in to the equation 2.4 to give 128 data points for the \(h\), were quite scattered, but showed a strong dependence on the tapering ratio\(\gamma\) (Dr. Lee et al)[54].
The main objective of Dr. Lee et al [54] associates was to modify the existing allowable bending stress equations of prismatic members to account for the tapering ratio through a length modification factor. The bending stress equations for the prismatic members are as follows:

\[ F_b = \frac{170000C_b}{(L/r_T)^2} \]  
\[ \text{Equation 2.5} \]

\[ F_b = \text{Critical Lateral –Torsional Buckling stress considering only warping resistance, (Ksi)} \]

\[ C_b = \text{Moment Gradient Coefficient for the prismatic sections} \]

\[ r_T = \text{Weak axis radius of gyration considering the compression flange and one-third of the compression web, Inches} \]

\[ F_b = \frac{12000C_b}{(Ld/A_f)} \]  
\[ \text{Equation 2.6} \]

\[ F_b = \text{Critical Lateral –Torsional Buckling stress considering only pure torsional resistance, (Ksi)} \]

\[ d = \text{Depth of the prismatic member, Inches} \]

\[ A_f = \text{Area of the compression flange, In}^2 \]

The Equations 2.5 and 2.6 represents the warping resistance and the pure torsional resistance components respectively, of the total lateral torsional bending strength. A factor of safety 5/3 is included in the above equations 2.5 and 2.6.

The standard design procedure was to calculate the higher stress values from the above equations and make it use in regular
design practice. The equation 2.5 controls for the thin and deep sections which have little pure torsional resistance, but having significant warping resistance, where as the equation 2.6 controls for the thick and shallow sections which have high pure torsional resistance and with little warping resistance. (Dr. Lee et al)[54].

Figure 2.12 Sections considered for calculating \( (\sigma_{cr})_\gamma \) [54].

Different types of sections will develop different amounts of their torsional resistance from pure torsion and warping torsion, it seems reasonable that two different length modification factors would be required (Dr. Lee et al)[54]. For the thick and shallow member sections (Section type: I and III from figure 2.12), the Rayleigh-Ritz
procedure was adopted to solve for the \((\sigma_{cr})_\gamma\) and equation 2.4 was solved to determine the corresponding values of “\(h\)”. The section types: II and IV are very clearly indicates the thick and shallow sections and were excluded because they are much more representative of typical column sections than the beam sections. A entirely new section, i.e., section type: V which represents the thin and deep sections and are which usually exits in the usual design practices of metal building web tapered columns and beams of the metal frame. The Rayleigh-Ritz procedure was adopted to solve for the \((\sigma_{cr})_\gamma\) and equation 2.4 was solved to determine the corresponding values of “\(h\)”. Using the curve fitting methods resulted in the following two equations for “\(h\)”, and the same will be applied to equation 2.5 and 2.6 respectively.

\[
h_w = 1.00 + 0.00385\gamma \sqrt{L/r_T} \hspace{1cm} 2.7
\]

\[
h_s = 1.00 + 0.0230\gamma \sqrt{Ld_0/A_f} \hspace{1cm} 2.8
\]

Where:

\(h_w\) = Web Tapered member length modification factor considering the warping resistance only.

\(h_s\) = Web Tapered member length modification factor considering the pure torsional resistance only.

The allowable bending stress for the Web tapered beam for \(\alpha = +k\) is larger of the values from the following equations:
To 170000
\[
F_{by} = \frac{170000}{(h_w L/r_{to})^2} \tag{2.9}
\]

\[
F_{by} = \frac{12000}{(h_s L d_o/A_t)} \tag{2.10}
\]

Where

- $F_{by}$ = Critical lateral torsional buckling stress considering warping torsional resistance and pure torsional resistance, respectively (Ksi)
- $r_{to}$ = Weak axis radius of gyration of smaller end of the Web tapered section considering the compression flange and one-third of the compression web (Inches)
- $d_o$ = Depth at the smaller end of the tapered section (Inches)

The tapering ratio, “$\gamma$”, has a much larger effect on the length modification factor in equation 2.8 than in equation 2.7, which indicates it is having the larger effect for the short and the thick sections (Dr. Lee et al)[54]. The difference between the “$h_w$” and “$h_s$” is partially counteracted, because the “$h_w$” is squared in equation 2.9 and the “$h_s$” is not squared in equation 2.10.

To arrive at a moment gradient coefficient for the $\alpha=0$ case, the following is considered.

\[
C_{by} = \frac{\sigma_{Taper|\alpha=0}}{\sigma_{Taper|\alpha=+k}} \tag{2.11}
\]
$$C_{by} = \frac{1.75}{1.00 + 0.25\sqrt{\gamma}}$$ ................................. 2.12

$$F_{by} = \frac{170000C_{by}}{(h_wL/r_{to})^2}$$ ................................. 2.13

$$F_{by} = \frac{12000C_{by}}{(h_sLdo/A_f)}$$ ................................. 2.14

Dr. Lee et al [54] chose to handle the inelastic lateral torsional buckling with a transition curve. They have considered that the inelastic lateral torsional buckling would occur when the total stresses (Applied stresses + residual stresses) in the web tapered section reaches half the yield stress. They multiplied equation 2.9 by $C_{by}$, removed the factor of safety and then rearranged the same to solve for the lower limit on $(h_wL/r_{to})$.

The limiting value, $C_r = \sqrt{\frac{510000C_{by}}{F_y}}$, where $F_y = $ yield stress of the material in Ksi.

The following transition equation was chosen for $h_wL/r_{to} \leq C_r$

$$F_{by} = \frac{2}{3} \left[ 1.0 - \frac{(h_wL/r_{to})^2}{2C_r^3} \right] F_y$$ ................................. 2.15
2.8.1 Coefficient for continuity past supports, B

The allowable bending stresses were developed for a single unbraced length for the web tapered beams. This condition very rarely occurs in the metal buildings frame design. The web tapered columns and the web tapered beams (rafters) is continuous past several lateral supports such as side wall girts and roof purlins. The boundary conditions for an unbraced length which is continuous past supports are more advantageous due the available restraint of rotation about the vertical axis provided by the adjacent segments.

M.L. Morrell and G.C. Lee [59] developed the coefficients to account for the advantageous boundary conditions. Based on their research, the AISC[6] considered these provisions as a part of the codal provisions/specifications.

Figure 2.13 Four cases considered in AISC Specifications [101]
**Case-1:**

When the maximum moment $M_2$ in three adjacent segments of approximately equal unbraced length is located within the central segment and $M_1$ is the larger moment at one end of the three segment portion of the member. Where $M_1/M_2$ is considered as negative when the critical segment is bent in single curvature. If $M_1/M_2$ is positive, the specification recommends this ratio be taken as zero.

$$B = 1.0 + 0.37 \left( 1 + \frac{M_1}{M_2} \right) + 0.50\gamma \left( 1 + \frac{M_1}{M_2} \right) \geq 1.0 \hspace{1cm} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldOTS
\[ B = 1.0 + 0.55 \left( 1 + \frac{F_{b1}}{F_{b2}} \right) + 2.20 \left( 1 + \frac{F_{b1}}{F_{b2}} \right) \geq 1.0 \quad \ldots 2.18 \]

**Case-4:**

When the computed bending stress at the smaller end of the tapered member or segment is equal to zero.

\[ B = \frac{1.75}{1.00 + 0.25 \sqrt{\gamma}} \quad \ldots \ldots \ldots \ldots \ldots 2.19 \]

The critical stress can be determined by using the following equation:

\[ F_{by} = B \sqrt{F_{Sy}^2 + F_{Wy}^2} \quad \ldots \ldots \ldots \ldots \ldots 2.20 \]

Where:

- \( B \) = Factor accounts for the Moment gradient and continuity past several lateral supports

\[ F_{Wy} = \frac{170000C_{by}}{\left( h_w L/r_w \right)^2} \quad \ldots \ldots \ldots \ldots \ldots 2.21 \]

\[ F_{Sy} = \frac{12000C_{by}}{(h s L d_0 / A_r)} \quad \ldots \ldots \ldots \ldots \ldots 2.22 \]

The equation 2.20 contains the AISC ASD Factor of safety 1.67. When the first edition of the AISC LRFD Specification was published, the below equation was used to compute the nominal moment capacity of the tapered section.

\[ M_n = \frac{5}{3} S_x \sqrt{F_{Sy}^2 + F_{Wy}^2} \quad \ldots \ldots \ldots \ldots \ldots 2.23 \]

The critical bending stress for the tapered beams, \( F_{by} \), was calculated using the equation 2.20 or equation 2.24. Equation 2.24
provides results for the inelastic case where as the equation 2.20 provide for the elastic case.

\[ F_{by} = \frac{2}{3} \left[ 1.0 - \frac{F_y}{6B \sqrt{F_{sy}^2 + F_{wy}^2}} \right] F_y \leq 0.60F_y \] 

Unless \( F_{by} \leq \frac{F_y}{3} \), in which case

\[ F_{by} = B \sqrt{F_{sy}^2 + F_{wy}^2} \]
2.9 REVIEW OF CURRENT RELATIVE AND NODAL BRACING REQUIREMENTS FOR BEAMS (ASD and LRFD)

In general, the bracing may be divided into two main categories, i.e., Lateral bracing and Torsional bracing. Lateral bracing, which is the content of this research program, restrains lateral displacement. This lateral bracing is further classified as relative and nodal. When a simply supported beam is subjected to uniform loading, the top compressed flange will move laterally much more than the bottom flange. From this, the lateral support attached to the bottom flange of simply supported beam with loading on the top flange is totally ineffective. The inflexion point shall not be considered as a brace point in the case of a double curvature bending. Bracing should be attached near the compression flange and to the both the flanges (compression and tension flanges) in the case of beams subjected to double curvature.

Relative Bracing:

The required brace strength, \( P_{br} = \frac{0.008 \cdot M_r \cdot C_d}{h_o} \)

The required brace stiffness,

\[
\beta_{br} = \frac{1}{\phi} \left( \frac{4 \cdot M_r \cdot C_d}{L \cdot h_o} \right) \text{ (LRFD)}; \quad \beta_{br} = \Omega \left( \frac{4 \cdot M_r \cdot C_d}{L \cdot h_o} \right) \text{ (ASD)}
\]

Where, \( \phi = 0.75(\text{LRFD}) \) and \( \Omega = 2.00(\text{ASD}) \)

\( h_o = \text{Distance between the flange centroids} \)
$C_d = 1.0$ for bending in single curvature; and $2.0$ for double curvature

$L_b =$ Laterally unbraced length (inches)

$M_r =$ Required flexural strength using LRFD/ASD Load combinations

**Nodal Bracing:**

The required brace strength, \[ P_{br} = \frac{0.02 M_r C_d}{h_o} \]

The required brace stiffness, \[
\beta_{br} = \frac{1}{\phi} \left( \frac{10 M_r C_d}{L_b h_o} \right) \quad (LRFD) ; \quad \beta_{br} = \Omega \left( \frac{10 M_r C_d}{L_b h_o} \right) \quad (ASD)
\]

Where, $\phi = 0.75(LRFD)$ and $\Omega = 2.00(ASD)$

$h_o =$ Distance between the flange centroids

$C_d = 1.0$ for bending in single curvature; and $2.0$ for double curvature

$L_b =$ Laterally unbraced length (inches)

$M_r =$ Required flexural strength using LRFD/ASD Load combinations

When $L_b$ is less than $L_q$, the maximum unbraced length for $M_r$, then $L_b$ in above equation shall be permitted to be equal to $L_q$.

The AISC commentary states the lateral bracing requirements for beams as follows.

Lateral brace stiffness requirements:

\[
\beta_{br} = \frac{1}{\phi} \left( \frac{2 Ni (C_b P_\beta C_i C_d)}{L_b} \right) \quad (LRFD)
\]

$Ni = 1.0$ for relative bracing
\[ n = \text{number of intermediate braces} \]

\[ P_t = \text{beam compression flange force} \left( \frac{\pi^2 EI_{yc}}{L_b^2} \right) \]

\[ I_{yc} = \text{out of plane moment of inertia of compression flange} \]

\[ C_b = \text{moment modifier} \]

\[ C_t = \text{accounts for top flange loading (} C_t = 1.0 \text{ for centroidal loading)} \]

\[ C_d = \text{double curvature factor (compression in both flanges)} \]

\[ = 1 + \left( \frac{M_s}{M_L} \right)^2 \]

\[ M_s = \text{smallest moment causing compression in each flange} \]

\[ M_L = \text{largest moment causing compression in each flange} \]

Lateral brace strength requirements:

\[ P_{br} = \frac{0.004 M_r C_t C_d}{h_o} \quad \text{(For relative bracing)} \]

\[ P_{br} = \frac{0.01 M_r C_t C_d}{h_o} \quad \text{(For nodal bracing)} \]
2.10 METAL DIAPHRAGM BEHAVIOR

The primary function of the metal cladding is to act as a metal envelope for the metal building. Basically the metal cladding is designed for flexure to carry the roof live loads to purlins and wind loads to both purlins and girts. The metal diaphragms usually possess a good shear stiffness and the same can be used along with the secondary members (like purlins/girts) to act as a lateral bracing system to avoid lateral buckling of the web tapered I-members in the plane of the metal diaphragm.

The shear stiffness of the metal diaphragm as shown in figure 2.14 can be defined as follows.

\[
G' = \frac{\tau}{\gamma} = \frac{P/X}{\Delta/Y} = \frac{PY}{\Delta X}
\]

Where

\( G' \) = Effective Shear Modulus
\( \tau \) = Effective Shear stress of the corrugated metal panel

\( \gamma \) = Shear strain

\( P \) = Shear load applied to the metal diaphragm

\( X \) = Metal panel width

\( Y \) = Metal panel span/ length

\( \Delta \) = Shear deflection

The practical way to establish the \( G' \) is to perform the physical laboratory tests or by using the finite element analysis. Once the \( G' \) is known, the shear stiffness per unit of shear deformation can be evaluated.

From the available data, reference [84], the value of \( G' = 4.50 \) Kips/Inch and the allowable shear strength \( V = 0.120 \) Kips/feet are considered to estimate the lateral bracing stiffness for the compression flange of the web tapered I-members. Here the compression flange is assumed to be at the outer flange, where the metal diaphragm exists. Here it should be noted, that the shear stiffness and the allowable shear strength of the metal diaphragm varies from panel to panel.
2.11 AVAILABLE STIFFNESS THROUGH THE SECONDARY MEMBERS, METAL CLADDING AND FLANGE BRACES.

The following are the contributions from girts /purlins and flange braces with metal sheeting toward the elastic lateral restraint.

a) When the web tapered I- member exterior flanges are in compression, thus laterally braced directly by girts/purlins along with the metal sheeting. This is illustrated in the below figure 2.15. The lateral bracing forces from the outside flanges of the web tapered I-members are transferred through the connections to the girts/purlins and induce axial forces in to the girts/ purlins along with metal sheeting. The girts/ purlins with metal sheeting offers the diaphragm action and the same is effective in offering the lateral restraint to the outside flanges of the web tapered I-members.

![Figure 2.15 Member with lateral braces](image)

The following is the calculation to determine the available lateral stiffness/strength of the lateral brace system.

Consider the wall/roof sheeting as screw down system with girts /purlins along with the metal sheeting, based on the reference [84] the $G' = 4.50$ Kips/Inch and the allowable shear strength,
V = 0.120 Kips/feet is considered. Consider a building with bay spacing as 25 feet c/c and secondary members are placed at 60 inches c/c to support the metal sheeting.

Using the equation,

\[ G = \frac{\tau}{\gamma} = \frac{P/X}{\Delta/Y} = \frac{PY}{\Delta X} \]

**Stiffness,** \( k = \frac{G X}{Y} \frac{4.5 \times 25 \times 12}{60} = 22.5 \text{kips / inch} \)

**Strength,** \( F = 0.120 \times 25 = 3.0 \text{kips} \)

This stiffness and strength available in the system will be used to verify the lateral bracing requirements for outer flanges (assumed outer flanges are in compression) of the web tapered I-members by means of the relative lateral bracing concepts.

b) When the web tapered I-members interior flanges are in compression, thus laterally braced by the flange braces from the girts/purlins along with the metal sheeting. This is illustrated in the below figure 2.15 and 2.16. The lateral bracing forces from the inside flanges of the web tapered I-members are transferred through the flange braces which are connected to purlins/girt, where they are resolved into axial and transverse forces. The effectiveness of these flange braces depends up on the flexural stiffness of girts/purlins.
Consider a building with bay spacing as 25 feet c/c and secondary members are placed at 60 inches c/c to support the metal sheeting. Assuming the secondary members as cold formed Z-section and the flange brace as 75 x 75 x 6.0 mm and also assuming the depth of the web tapered -I- member as 45 inches as case-1) and 60 inches as case-2) at the brace point. The flange braces are positioned at 60 degrees to horizontal.

Consider the secondary members are continuous (5 bays) over the main frame members as shown in figure 2.17

The vertical stiffness of the secondary members are converted to horizontal stiffness using the formula below.

Consider the vertical stiffness of the secondary members with the load application as defined in figure 2.17 as $K_{Sec} (v)$. 

Figure 2.16 Behaviour of members with flange braces

Figure 2.17 Continuous beam model
The horizontal stiffness from the secondary members at the inner flanges of the member where the flange braces are connected as defined as follows.

\[ K_{\text{Sec} (H)} = \frac{K_{\text{Sec} (v)}}{\tan^2 \theta} \], where “\( \theta \) “ is the inclination of the flange brace with the horizontal.

The horizontal stiffness, \( K_{\text{Sec} (H)} \) from the secondary members is further designed as \( K_{\text{SEC}} \).

The following calculations demonstrate the available lateral brace stiffness from the secondary members along with the flange braces.

\[ \frac{1}{K_{\text{BRACE}}} = \frac{1}{K_{FB}} + \frac{1}{K_{SEC}} \]

**Case: 1** : Applying a point load of 1 kip at a distance of 2.2 feet from the center line of the mainframe as shown in figure 2.17.

For flange brace, 75 x 75 x 6.0 mm (hot rolled angle)

Cross sectional area, \( A = 8.66 \text{ cm}^2 = 1.342 \text{ in}^2 \)

Length of the flange brace, \( L_{FB} = (26.42 + 45^2) = 52.17 \text{ Inches} \).

Flange brace stiffness, \( K_{FB} = \frac{AE}{L_{FB}} \times \cos^2 \theta \)

\[ = \frac{(1.342 \times 29000)/52.17}{60.0} = 186.50 \text{ kips/inch} \]

STAAD Software is utilized to calculate the vertical deflections at the point loads, the vertical stiffness values, horizontal stiffness values and the combined stiffness of the system with secondary members and the flange braces are calculated and presented in the table 2.1. (The design of flange brace is not verified here, actually this should be verified).
Table 2.1 Stiffness calculations case-1

<table>
<thead>
<tr>
<th>Section ID</th>
<th>Strong axis Moment of Inertia (In ^4)</th>
<th>Vertical Deflection (In)</th>
<th>Vertical Stiffness (Kip/In)</th>
<th>Horizontal Stiffness (Kip/In)</th>
<th>Flange brace stiffness (Kip/In)</th>
<th>Combined stiffness (Kip/In)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8Z2.5X105</td>
<td>13.90</td>
<td>0.09</td>
<td>10.78</td>
<td>3.59</td>
<td>186.50</td>
<td>3.52</td>
</tr>
<tr>
<td>8Z2.5X090</td>
<td>12.01</td>
<td>0.11</td>
<td>9.31</td>
<td>3.10</td>
<td>186.50</td>
<td>3.05</td>
</tr>
<tr>
<td>8Z2.5X075</td>
<td>10.10</td>
<td>0.13</td>
<td>7.83</td>
<td>2.61</td>
<td>186.50</td>
<td>2.57</td>
</tr>
<tr>
<td>8Z2.5X060</td>
<td>8.15</td>
<td>0.18</td>
<td>5.58</td>
<td>1.86</td>
<td>186.50</td>
<td>1.84</td>
</tr>
<tr>
<td>8Z2.5X048</td>
<td>6.56</td>
<td>0.22</td>
<td>4.50</td>
<td>1.50</td>
<td>186.50</td>
<td>1.49</td>
</tr>
</tbody>
</table>

Case: 2 : Applying a point load of 1 kip at a distance of 2.92 feet from the center line of the mainframe as shown in figure 2.17.

For flange brace, 75 x 75 x 6.0 mm (hot rolled angle)

Cross sectional area, \( A = 8.66 \text{ cm}^2 = 1.342 \text{ in}^2 \)

Length of the flange brace, \( L_{FB} = (35.04^2 + 60^2) = 69.5 \text{ Inches} \)

Flange brace stiffness, \( K_{FB} = \frac{(AE)}{L_{FB}} \times \cos^2 \theta \)

\[ = (1.342 \times 29000/69.5) \times \cos^2 60.0 = 140.00 \text{ kips/inch} \]

STAAD Software is utilized to calculate the vertical deflections at the point loads, the vertical stiffness values, horizontal stiffness values and the combined stiffness of the system with secondary members and the flange braces are calculated and presented in the table 2.2. (The design of flange brace is not verified here, actually this should be verified).

Table 2.2 Stiffness calculations case-2

<table>
<thead>
<tr>
<th>Section ID</th>
<th>Strong axis Moment of Inertia (In ^4)</th>
<th>Vertical Deflection (In)</th>
<th>Vertical Stiffness (Kip/In)</th>
<th>Horizontal Stiffness (Kip/In)</th>
<th>Flange brace stiffness (Kip/In)</th>
<th>Combined stiffness (Kip/In)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8Z2.5X105</td>
<td>13.90</td>
<td>0.16</td>
<td>6.22</td>
<td>2.07</td>
<td>140.00</td>
<td>2.04</td>
</tr>
<tr>
<td>8Z2.5X090</td>
<td>12.01</td>
<td>0.19</td>
<td>5.37</td>
<td>1.79</td>
<td>140.00</td>
<td>1.77</td>
</tr>
<tr>
<td>8Z2.5X075</td>
<td>10.10</td>
<td>0.22</td>
<td>4.52</td>
<td>1.51</td>
<td>140.00</td>
<td>1.49</td>
</tr>
<tr>
<td>8Z2.5X060</td>
<td>8.15</td>
<td>0.31</td>
<td>3.22</td>
<td>1.07</td>
<td>140.00</td>
<td>1.07</td>
</tr>
<tr>
<td>8Z2.5X048</td>
<td>6.56</td>
<td>0.39</td>
<td>2.60</td>
<td>0.87</td>
<td>140.00</td>
<td>0.86</td>
</tr>
</tbody>
</table>
2.12 OTHER CONTRIBUTIONS FOR THE INNER FLANGE RESTRAINT:

Other contributions towards the lateral restraint for the inside flanges of the members (i.e. without flange braces) because of the presence of the girts /purlins at outside flanges.

Dooley [41] states that the stiffness of the elastic restraint against rotation, $\beta_n$, comprises two component parts, namely the stiffness of the sheeting rail against flexure at each attachment point to the column, $\beta_f$ and the local stiffness of the column section against the distortion at each attachment point under a concentrated torque at one flange, $\beta_c$.

When a series of columns buckle and assuming the columns are equally loaded, they will twist in alternate direction (twisting alternately left and right) or in the same direction.

If all the columns twist in the alternate direction, a situation of single curvature will develop in the sheeting rails and stiffness against the rotation is as follows.

$$\beta_f \text{ (single curvature)} = 2EI/L + 2EI/L = 4EI/L$$

![Figure 2.18 Twisting of columns in alternate direction](image)

The above rotational stiffness can be converted to lateral stiffness by the following way, $K_f = \beta_f / d^2$
If all the columns twist in the same direction, a situation of double curvature will develop in the sheeting rails and stiffness against the rotation is as follows.

\[ \beta_f \text{ (double curvature)} = 3EI/L + 3EI/L = 6EI/L \]

![Figure 2.19 Twisting of all columns in same direction](image)

The above rotational stiffness can be converted to lateral stiffness by the following way.

\[ K_f = \frac{\beta_f}{d^2} \]

For the design purpose a conservative approach of all the columns twist in the alternate direction may be followed.

The local stiffness of the column section against the distortion, for an infinitely long I-section with a concentrated torque applied to one flange can be estimated as:

\[ \beta_c = \frac{GJ}{3d} \]

The above rotational stiffness can be converted to lateral stiffness by the following way.

\[ K_c = \frac{\beta_c}{d^2} \]

Finally, the stiffness of elastic restraint against rotation at each attachment point can be estimated as:

\[ 1/ \beta_a = 1/ \beta_f + 1/ \beta_c \]
2.13 OBJECTIVES AND SCOPE OF RESEARCH

In reality the inherent connection stiffness and the stiffness of the roof system components and wall system components may provide some degree of lateral resistance. This resistance may prevent the large deformation from taking place in the metal building systems.

The stiffness from roof panel to purlin, purlin to main frame rafter/beam, wall panel to girt, girt to main frame columns and connections between the frame rafters and columns, are making a contribution towards the stiffness of the metal building system as a whole.

However, there has never been an attempt reported to evaluate the extent of this contribution. This may be due to the inhibitive cost that would be associated with an experimental set up to investigate it, and also the difficulty involved in developing a computer model that would accurately reproduce the behavior of the structural members and their interaction with each other. Nevertheless recent advances in structural analysis software’s have made it possible to model and study the behavior of complex structural systems and assemblies with reasonable effort and accuracy.

Majority of the structural members of a metal building system are subjected to considerable restraints under the working conditions, either inherent in the construction of the system or deliberately applied as a precaution against buckling. These restraints have the effect of increasing the stability of these structural members.
It is understood that an elastic lateral brace (by means of secondary elements like purlins/girts with metal sheeting and flange braces, etc) restricts the lateral buckling of web tapered I-members in a metal building frame and improves the elastic buckling strength. However, studies of the effects of elastic restraints on the flexural–torsional buckling of web tapered I-beams do not appear to have reported, and it seems extending research findings relevant to buckling of the restrained beams and columns to web tapered I-members needs further investigations.

There may be situations in design practice, where the effect of continuity is drastically diminished, especially when the lateral supports have inadequate stiffness and/or are improperly applied to the web tapered I-members. In these cases the usual design practice is to ignore the effect of continuity due to lateral restraints.

This research proposes a method to determine the optimum stiffness of the lateral restraints for the web tapered I-beams, validity of the lateral bracing requirements of the prismatic members to the web tapered I-members and the utilization of the available lateral stiffness and considering the improvement in the strength of the web tapered I-members than ignoring the effect of the lateral stiffness which is available in the metal building systems using finite element modeling and analysis.
2.13.1 OBJECTIVES:

The following are the objectives of the proposed research.

1. To investigate the effect of the lateral bracing stiffness (Nodal lateral bracing) for the compression flange on the inelastic flexural torsional buckling of the web tapered –I-members and to propose a method to calculate the strength of the member with smaller lateral brace stiffness (Nodal lateral bracing) using the optimum stiffness. The optimum stiffness is the one which corresponds to attaining equal to or more than 90 % of the capacity of the members with full lateral bracing (i.e., rigid lateral brace system).

2. At present there are no specific lateral bracing (stability bracing) requirements for the web tapered I-members and the metal building industry design practice is to consider the stability bracing requirements of prismatic members and applying to the web tapered I-members by considering the depth at the brace point of the web tapered I-members as equivalent to the depth of the prismatic sections. Because of this consideration the stability bracing requirements for the web tapered I-members may lead to a conservative designs or design deficiencies. Considering the importance of these lateral bracing requirements it is decided to investigate the application of the stability bracing requirements of prismatic members to the web tapered I-members and to propose the necessary recommendations. This is accomplished by studying the effect
of smaller lateral bracing stiffness (nodal lateral brace stiffness) in order to reach 90% of the capacity of the members with full lateral bracing (i.e., rigid lateral brace system) and to compare with the nodal lateral bracing provisions and to recommend for the design practice.

3. To investigate the effect of the available lateral bracing stiffness (nodal bracing) by means of purlins/girts/metal sheeting/flange braces which are available in the metal building systems and considering the improvement in the system than ignoring the available lateral stiffness.
2.13.2 SCOPE OF RESEARCH

This Research proposes a 3D finite element modeling and analysis using the ABAQUS software to investigate the effect of the elastic lateral bracing stiffness on the inelastic flexural–torsional buckling of the web tapered I-beams with an elastic lateral restraint at mid span of the compression flange and to study the lateral bracing requirements for the web tapered I-members with simply supported boundary condition and subjected to pure moment. This involves the modeling of the web tapered I-beams and its interaction with various secondary structural elements, namely-purlins, girts, panels and lateral braces though its stiffness characteristics.

More importantly, the proposed research will make it possible to study the behavior of the web tapered I-beams with lateral restraints and its interactions in a simulated realistic environment, which may lead to the better component design and performance.

To fulfill the objective -1, the cross section originally proposed by Dr. Lee (54) with spans ranging from 240 inches (6096 mm) to 384 inches (9753.60 mm) with lateral bracing stiffness ranging from 0.00 Kips/ Inch to 4.00 Kips/Inch and also a rigid lateral brace considered at the mid span of the compression flange and subjected to pure moment based on the capacity of the cross section is investigated.

To fulfill the objective -2 and 3, a selected practical sectional sizes specified in the table 6.1 considered with a span of 360 inches (9144 mm). The considered cross sections are generally used in the metal building’s frame design. These sections are verified with
different lateral bracing stiffness ranging from 0.00 Kips/ Inch to 12.00 Kips/Inch and also a rigid lateral brace considered at the mid span of the compression flange and subjected to pure moment based on the capacity of the cross section is investigated.
2.14 THESIS ORGANIZATION

Chapter-3 describes the finite element modeling methods and techniques used in the current research. Chapter-4 discusses the verification study to validate these same finite element modeling techniques.

The results of the parametric study- part I (to fulfill the objective -1), the capacity, lateral bracing stiffness and slenderness for a number of the more practically useful parametric combinations resulting in web tapered I- member response are illustrated in chapter-5. The parametric study and the results are obtained from more than 120 finite element models.

The results of the parametric study- part II ( to fulfill the objective -2 and 3 ) the capacity, lateral bracing stiffness and slenderness for a number of the more practically useful parametric combinations resulting in web tapered I- member response are illustrated in chapter-6. The parametric study and the results are obtained from more than 392 finite element models.

Conclusions, recommendations, design proposal from this study are provided, with future scope of research, in chapter-7.

Appendix-A includes example ABAQUS input file utilized during the verification study-1.

Appendix-B includes example ABAQUS input file for the buckling analysis, which is utilized during the parametric study-I.

Appendix-C includes example ABAQUS input file for the non linear analysis, which is utilized during the parametric study part-I.
Appendix-D includes the technical paper presented by Mr. P. Ravindra Murthy, Dr.V.V.V.S. Murty and Dr.N.V. Ramana Rao[116] and the same got published through the Institute for steel development and growth (INSDAG).