Chapter 6

PROBLEM DEFINITION
AND
MODEL DEVELOPMENT
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PROBLEM DEFINITION AND MODEL DEVELOPMENT

6.0 INTRODUCTION

With the advent of Computers and the widespread of the internet and fibre optics network across the world, the huge population comprising good number of unemployed Indian youth provided a platform for MNCs to open up their ICT branches in India. And the IT intensive banking, insurance, railways, corporate houses etc. are using computer based operations in large scale. Consequently the decisions on capital investment on Computer and Computer based system became important and a need for scientific approach for the replacement decisions is felt.

6.1 THE DECISION PROBLEM

The replacement decisions in MNCs in ICT area are predominantly with the computers and computer based system. The primary decision is generally whether to replace the existing computer based system consisting a large number of computers and accessories that encompasses both hardware and software or use for some more period of time.

The replacement of the items arises in the following situations:

- The existing machinery or the system is less-efficient and demanding too much maintenance
- Breakdown of the existing machinery or the system due to accident or otherwise
- The existing machinery or the system wears out and is likely to fail shortly
- A new system with better design and capability is available in the market

To act upon the above situations, the problem is to balance the cost of the new system against the maintenance cost to be incurred to keep the existing system in efficient condition or the cost due to the loss of efficiency. It is essential to determine the optimum duration over which the present system can be used. And it is also required to decide the optimal replacement policy that covers the time at which the replacement is most economical. This policy shall aim at minimizing the annual capital and maintenance cost.

The replacement decisions are relatively complex to analyse and estimate the true cost for either a proposed new system or for an existing system. Some of the reasons for this:
- Various costs viz. capital cost, repairs & maintenance, system operator’s wage, resale price, cost of finance- associated with the present and proposed systems
- Repairs & maintenance involves a variety of repairs ranging from small to big which cannot be defined and computed exactly in specific
- The influence of various economic variables such as Inflation, value of money etc.
Random element in breakdowns and subsequent repairs: two identical systems maintained in an apparently similar manner may have different repair cost.

Change in government policies and other external factors may change the decision in an unexpected manner.

Given these inherent complexities, it is unrealistic to expect the systems maintenance manager, an IT technical guy, or his advisors to make these calculations on regular basis. Presently, the decisions being made on intuition would seem less than satisfactory.

6.1.1 Group Vs. Block Replacement

In general group replacement model is applicable to the items such as electric bulbs, tubes, tyres etc. that fail suddenly and completely on usage and the result is group replacement age for the entire group of items in the system irrespective of whether they are functioning or not.

However there are certain types of items viz. computer and computer based system, a block of air conditioners, a block of LCD televisions in hotel industry, a block of pressure gauges in filling plants, a block of cutting tools in a work shop etc. that may not fail completely on usage. They fail partially and can be brought back to the functional state by doing some repairs. They consist of many intermediate states like minor repair, major repair etc. between functional and complete failure states that are more realistic in practice. Over a period of time such items, on usage, switch from one
state to another state and can be brought back to the functional state rather allowing them directly into complete failure state. Block replacement age encompasses all these intermediate repairable states. It is also to be noted that some items may completely fail on usage or become obsolete due to the availability of new system with better design and capability.

6.1.2 Replacement Vs. Reengineering

Replacement is considered to be the regeneration point of whole life where the operating cost function initially starts. In practice such methods really work well and the life of the equipment/system is enhanced.

On the other hand, the concept of reengineering in lieu of replacement is one viable model as the operating cost increases with time. This model maximizes the gain between the operating costs before and after the reengineering. Reengineering can be perceived as the adjustment, alteration, or partial replacement of a process or equipment in order to make it to meet a new need. Successful implementation of reengineering will improve the equipment or process performance and this reduces the operating costs.
6.2 OBJECTIVES OF THE STUDY

Accordingly the objective of this study reported here is to develop an optimal replacement model for block of items using Markov chain approach that considers the influence of inflation and value of money on the optimal replacement policy. A special reference is given to the multi-state (breakdown) repairable system containing a block of similar items viz. Computer and Computer based system.

This model deals with the situation that arises when a block of multi (repairable) state items such as machines or electronic equipment need to be replaced due to their decreased efficiency or repeated repairs/maintenance or breakdown or failure.

This model aims at finding the block replacement period for a block of items (a block of computers, a block of LCD televisions in hotel industry, a block of pressure gauges in filling plants, a block of cutting tools in a work shop etc.) that may consist of many intermediate repairable states like minor repair, major repair etc. between functional and complete failure states that are more realistic in practice.

The objective of replacement is to decide best policy to determine an age at which the replacement is the most economical instead of continuing at increased maintenance costs. The fundamental objective of replacement is to direct the organization for maximising its profit (or minimizing cost).

The main objective of the model is to provide means for analyzing the behaviour of the system for the purpose of improving its
performance. Consequently this study aims at exploring and evaluation of an option of reengineering in lieu of replacement. The objective of reengineering is to improve the process performance by alteration, or partial replacement of a process and thereby reducing the maintenance costs.

The model employs a set of mathematical symbols to represent the decision variables of the system. These variables are related together by means of a mathematical equation or set of equations to describe the behaviour of the system.

6.3 MODEL DEVELOPMENT:

The formulation and development of the model is done as given below.

(a) Block Replacement Model Using First Order Markov Chain without considering the macroeconomic variable i.e. Inflation

(b) Influence of Inflation on Block Replacement decision using First order Markov Chain

(c) Influence of Inflation on Block Replacement decision using Higher order Markov Chain

(d) Reengineering Treatment on Block Replacements – Model Extension
The above models are described below:

**(a) Block replacement Model using first order Markov Chain:**

As discussed in Sec.6.1.1 usually group replacement model is applicable to the items that fail completely on usage and the result is group replacement age for the entire group of items in the system irrespective of whether they are functioning or not.

However there are certain types of items viz. a block of computers, a block of LCD televisions in hotel industry, a block of pressure gauges in filling plants, a block of cutting tools in a workshop etc. that may consist of many intermediate states like minor repair, major repair etc. between functional and complete failure states that are more realistic in practice. Over a period of time such items, on usage, switch from one state to another state that can be brought back to the functional state rather directly moved to complete failure state. It is also to be noted that some items may completely fail on usage or become obsolete due to the availability of new system with better design and capability.

So in the current model, two intermediary repairable breakdown states are introduced and stochastic process like Markov process (Chain) is made use of in generating the probabilities of items switching over to different states on usage. Using these probabilities a mathematical model is developed for replacement decision.
The Markov process is said to be the first order, if

(i) The set of possible outcomes is finite

(ii) The probability of next outcome (state) depends only on the immediately preceding outcome.

(iii) The transition probabilities are constant over time.

The second/higher order Markov process assumes that the probability of the next outcome (state) may depend in the two/ ‘l’ previous outcomes.

When intermediate states are not considered in between functioning and failure states, and then it is applicable to the class of items that fail suddenly and completely on usage.

(b) Influence of Inflation on Block Replacement decision using First order Markov Chain

Conventional models are available to evaluate different replacement strategies for a combination of similar machine tools of different ages considering and without considering money value. Here net present value criterion based on nominal interest rates does not reflect the real increase in the value of money.

Real increase in the value of money or purchasing power of money depends upon many macro economic aggregates and variables like Gross Domestic Product (GDP), Money supply, Capital formation, Inflation etc. Real interest rates are computed using Fisherman’s relation. When present worth factors are computed on it and multiplied with future money, it gives purchasing power of money.
Usually nominal interest rates will be revised based on the changes in the economic variables viz. inflation in the free market economic conditions.

Inflation is sustained rapid increase in the aggregate price level, which is very important for the policy makers. Persistent inflation has a number of undesirable economic, political and social consequences. The aggregate demand-aggregate supply framework can be employed to investigate the causes underlying observed movements of the price level. Position of the aggregate demand curve depends upon the nominal money supply, the tax rate, government expenditure and exports. The future values of inflation are predicted using suitable forecasting technique and a mathematical model is developed to forecast future values of inflation. For this purpose real time inflation data is considered.

Real interest rate is computed considering inflation. A block replacement model for multi state repairable items, considering two intermediate repairable breakdown states i.e. minor repair and major repair is developed using first order Markov chain.

Replacement model considering money value gives better realistic results than without considering money value.
(c) **Influence of Inflation on Block Replacement decision using Higher order Markov Chain**

Here the concept of higher order Markov process is made use of in generating the probabilities of items switching from one state to different states on usage. Second Order Markov Chain assumes that probability of next state depends on the probabilities of immediately two preceding states.

For higher order Markov process with more states, it is required to compute several hundreds of transition probabilities (Refer Sec. 3.10) for the current and future time periods and is a very time consuming one. Also it will be difficult to analyse and make conclusions or decisions.

Moreover as maintenance cost is in proportion of the items falling in each state at certain time, the state probabilities are fairly important rather through which intermediate states the current state has been attained. Therefore focus on developing a model that yields better forecast of the proportion of items in each state at a certain time period is justifiable.

To address this, a parsimonious model, the Weighted Moving Transition Probabilities (WMTP) method is introduced to approximate higher order Markov chains. This makes the number of Transition probabilities to be computed is far less.

Also influence of the macroeconomic variable Inflation and real interest rate is considered in the development of block replacement model using second order Markov process.
(d) Reengineering Treatment on Block Replacements – Model Extension

In nutshell, Reengineering can be perceived as the adjustment, alteration, or partial replacement of a process or product in order to make it to meet a new need.

This section looks into the possibility of reengineering the computer hardware networking process to reduce the costs of maintenance and operations; thereby extending the optimal block replacement age. Thin client technology is proposed to reengineer the computer networking process in lieu of replacement option. Thin client is a generic term for a group of emerging technologies that reduces costs of hardware, maintenance and support. Thin client solution shows dramatic experience in the “cost saving” and “Performance enhancement”.

The developed block replacement model is extended to compute the block replacement cost with reengineering. The weighted Moving Transition Probability Method (WMTP), a parsimonious model that approximates higher order Markov Chain is employed to estimate the state probabilities for future time periods.

Also influence of the macroeconomic variable Inflation and real interest rate is considered in the development of block replacement model using second order Markov process.

This kind of block replacement model with reengineering can be applied to a block of computers IT intensive banking, insurance, railways etc.
The outline of the above steps is furnished in the Fig. 6.1

Fig. 6.1: Outline diagram of the model development
6.4 DEVELOPMENT OF BLOCK REPLACEMENT MODEL USING FIRST ORDER MARKOV CHAIN:

Before presenting the development of model, few concepts viz. Markov chains, transition probability matrix, estimation of transition probabilities for future time periods are given below for ready reference.

The First Order Markov Chain (FOMC) assumes the probability of next state depends only on the immediately preceding state. Thus if \( t_0 < t_1 < \ldots < t_n \) represents the points on time scale then the family of random variables \( \{X(t_n)\} \) whose state space \( S = x_0, x_1, \ldots, x_{n-1}, x_n \) is said to be a discrete-time Markov process provided it holds the Markovian property:

\[
P[X(t_n) = x_n \mid X(t_{n-1}) = x_{n-1}, \ldots, X(t_0) = x_0] = P[X(t_n) = x_n \mid X(t_{n-1}) = x_{n-1}]
\]

for all \( X(t_0), X(t_1), \ldots, X(t_n) \)

If the random process at time \( t_n \) is in the state \( x_n \), the future state of the random process \( X_{n+1} \) at time \( t_{n+1} \) depends only on the present state \( x_n \) and not on the past states \( x_{n-1}, x_{n-2}, \ldots, x_0 \).

The simplest of the Markov Process is discrete and constant over time. A system is said to be discrete in time if it is examined at regular intervals, e.g. daily, monthly or yearly.

**Transition Probability:** The probability of moving from one state to another future state or remaining in the same state during a single time period is called transition probability.
Mathematically the probability

\[ P_{X(n),X(n-1)} = P\{X(t_n) = x_n \mid X(t_{n-1}) = x_{n-1}\} \]

is called FOMC transition probability. The transition probabilities can be arranged in a matrix of size \( m \times m \) and such a matrix can be called as one step Transition Probability Matrix (TPM), represented as below:

\[
P = \begin{bmatrix}
P_{11} & P_{12} & \cdots & P_{1m} \\
P_{21} & P_{22} & \cdots & P_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
P_{m1} & P_{m2} & \cdots & P_{mm}
\end{bmatrix}, \text{ where 'm' represents the number of states.}
\]

The matrix \( P \) is a square matrix of which each element is non-negative and sum of the elements in each row is unity i.e. \( \sum_{j=1}^{m} P_{ij} = 1 \); \( i = 1 \) to \( m \) and \( 0 \leq P_{ij} \leq 1 \).

The initial estimates of \( P_{ij} \) can be computed as, \( P_{ij} = \frac{N_{ij}}{N_{i}}, \) (i, j = 1 to m) where \( N_{ij} \) is the raw data sample that refer the number of items or units or observations transitioned from the state i to state j.

\( N_{i} \) is the raw data sample in state i.

**Estimation of future transition probabilities:**

The transition probabilities for future time periods (n-step) can be calculated by using the application of the Chapman-Kolmogorov equation (Sec. 3.6). This contains the fact that 1-step probabilities determine the n-step probabilities, for any n.
The equation can be represented as,

\[ P_{ij}^{(n+m)} = \sum_{k=1}^{M} P_{ik}^{(n)} P_{kj}^{(m)} \text{ for all } n, m \geq 0 \text{ and } \]

where \( i, j = 1, 2, \ldots, M \) (state space).

\( P_{ik}^{(n)} P_{kj}^{(m)} \) represents the probability that a process beginning from state ‘i’ will go to state ‘j’ in (n+m) transitions or steps through a path taking it into state ‘k’ at the \( n^{th} \) transition.

By the repeated application of the Chapman-Kolmogorov equations as below:

\[ P^{(1)} = P \quad (\text{Since } P^{(1)} \text{ is just } P, \text{ the 1-step Transition Probability Matrix}) \]

\[ P^{(2)} = P^{(1+1)} = P^{(1)} P^{(1)} = PP = P^2 \]

\[ P^{(3)} = P^{(2+1)} = P^{(2)} P^{(1)} = P^2P = P^3 \]

Now by induction,

\[ P^{(n)} = P^{(n-1+1)} = P^{(n-1)} P^{(1)} = P^{n-1}P = P^n \]

*Hence, the n-step transition probability may be determined by multiplying the matrix P by itself ‘n’ times.*

However, alternately, as the estimation of higher order Markov chain transition probabilities for bigger state space is much time consuming one, advantage of spectral representation (Sec. 3.9) of transition probabilities for multi state process is employed to compute the transition probabilities for future time periods.

An \( n \times n \) matrix \( P \) will always have ‘n’ eigen values, which can be arranged in more than one way to form a diagonal matrix \( D \) of size
n x n and a corresponding matrix V of non zero columns which satisfies the eigen value equation \( PV = VD \).

This results in the decomposition of \( P \) as, \( P = VD^{-1} \)

Further by squaring on both sides of above equation,

\[
P^2 = (VD^{-1})(VD^{-1})
\]

\[
= VD(V^{-1}V)DV^{-1}
\]

\[
= VD^2V^{-1}
\]

Mathematically, Spectral Decomposition can be represented as

\[
P^i = VD^iV^{-1} \text{ where } i = 1 \text{ to } n
\]

**Procedure to calculate n-step Transition Probability Matrix (TPM) of First order Markov Chain using Spectral Decomposition Method:**

If \( P \) represents the four state TPM then the higher order transition probabilities are obtained by the following procedure.

i. Determine the eigen values of the Transition Probability Matrix ‘\( P \)’ by solving \( |P-\lambda I| = 0 \)

ii. If all eigen values say \( \lambda_1, \lambda_2, \lambda_3, \ldots \lambda_k \) are distinct then obtain \( k \)-column vectors say \( X_1, X_2, X_3, \ldots X_k \) corresponding to the Eigen values by solving \( PV = VD \) or \( PX = \lambda X \) where \( X \neq 0 \)

iii. Denote these column vectors (eigen vectors by matrix \( V \)) where \( V = (X_1, X_2, X_3, \ldots X_k) \) and obtain \( V^{-1} \)

iv. Compute \( D \), a diagonal matrix formed from the eigen values of \( P \).
Higher order Transition Probability Matrix (TPM) of four state Markov chain can be computed using the equation, \( P^i = V D^i V^{-1} \)

where \( i = 1 \) to \( n \)

**State probabilities:**

If the state probabilities at time \( t = 0 \) are known, say \( X_0 \), then the state probabilities \( X_i \) at any time ‘\( n \)’ can be calculated as below:

\[
\begin{bmatrix}
X_{i}^{I} & X_{i}^{II} & X_{i}^{III} & X_{i}^{IV}
\end{bmatrix}
= \begin{bmatrix}
X_{0}^{I} & X_{0}^{II} & X_{0}^{III} & X_{0}^{IV}
\end{bmatrix} \cdot P^i, \text{ where } i = 1 \text{ to } n
\]

i.e. \( X_i = X_0 \cdot P^i \), where \( i = 1 \) to \( n \) (Sec. 3.5.1)

**Note:** Software such as MATLAB, OCTAVE etc. can be used to run this process.

**Model:**

**Assumptions:**

(i) In block replacement decisions, in addition to the functioning state and failure state, two intermediate repairable breakdown states-minor repair and major repair- are introduced. This results in four-state Markov chain.

(ii) Obsolescence of the present system, due to the availability of new system with better design and capability is available in market, is assumed as break down/complete failure state.
(iii) Instead of assuming discrete probability distribution from one period to the other period it is assumed to follow first order discrete-time Markov Chains.

(iv) This model can be applied to a system that consists of N items in four different states and to find out the block replacement period. **In this study, a special reference has been made to the computer and computer based system which is widely used by ICT industry.**

(v) Repair cost or rectification cost of any item is different for minor repairable state and major repairable state if two intermediary states are allowed. However, it is assumed constant if only one intermediary state is considered.

**Notations:**

N = Total number of items in the system

C_1 = Individual replacement cost per unit

C_2 = Minor repair cost

C_3 = Major repair cost

C_4 = item cost during Group replacement

X_0^I = proportion of units under working condition initially

X_0^{II} = proportion of units under minor repair initially

X_0^{III} = proportion of units under major repair initially

X_0^{IV} = proportion of units under complete failure initially
X_{i}^{l} = \text{proportion of units under working condition at the end of } i^{\text{th}} \text{ time period}

X_{i}^{ll} = \text{proportion of units under minor repair at the end of } i^{\text{th}} \text{ time period}

X_{i}^{lll} = \text{proportion of units under major repair at the end of } i^{\text{th}} \text{ time period,}

X_{i}^{lv} = \text{proportion of units under complete failure at the end of } i^{\text{th}} \text{ time period}

P_{ij} = \text{Probability of items switching over from } i^{\text{th}} \text{ state to } j^{\text{th}} \text{ state in a period}

TPM = \text{Transition Probability Matrix}

AC(t) = \text{average cost per period in group replacement policy}

As two intermediary states—minor and major repairable states—between functional and complete failure state are considered, the Transition Probability matrix or the generator of Markov process for a four state system can be represented as:

\[
P = \begin{bmatrix}
p_{11} & p_{12} & p_{13} & p_{14} \\
p_{21} & p_{22} & p_{23} & p_{24} \\
p_{31} & p_{32} & p_{33} & p_{34} \\
p_{41} & p_{42} & p_{43} & p_{44}
\end{bmatrix}
\]

where 1,2,3 and 4 represents functional, minor repairable, major repairable and complete failure states.

The state probabilities of items in different states can be computed as:

\[\text{X}_{n} = (\text{Probability of items in different states in initial period}) \times (\text{TPM})^{n}\]

i.e. \[\text{X}_{n} = \text{X}_{0} \times (\text{TPM})^{n}\] \hspace{1cm} ---\{(6.1)\}
At the end of the first period,

the state probabilities can be calculated from

\[
X_1 = X_0 P \quad (\therefore X_n = X_0 P^n)
\]

\[
\Rightarrow X_1^I \quad X_1^{II} \quad X_1^{III} \quad X_1^{IV} = X_0^I \quad X_0^{II} \quad X_0^{III} \quad X_0^{IV} \begin{bmatrix}
P_{11} & P_{12} & P_{13} & P_{14} \\
P_{21} & P_{22} & P_{23} & P_{24} \\
P_{31} & P_{32} & P_{33} & P_{34} \\
P_{41} & P_{42} & P_{43} & P_{44}
\end{bmatrix}
\]

---(6.2)

Therefore,

Probability of items in functional state,

\[
X_1^I = X_0^I P_{11} + X_0^{II} P_{21} + X_0^{III} P_{31} + X_0^{IV} P_{41}
\]

Probability of items in minor repair state,

\[
X_1^{II} = X_0^I P_{12} + X_0^{II} P_{22} + X_0^{III} P_{32} + X_0^{IV} P_{42}
\]

Probability of items in major repair state,

\[
X_1^{III} = X_0^I P_{13} + X_0^{II} P_{23} + X_0^{III} P_{33} + X_0^{IV} P_{43}
\]

Probability of items in irreparable state,

\[
X_1^{IV} = X_0^I P_{14} + X_0^{II} P_{24} + X_0^{III} P_{34} + X_0^{IV} P_{44}
\]

Similarly the probabilities of items falling in different states in future time periods \((i= 1 \text{ to } n)\) are to be calculated by using equation \(X_n = X_0 P^n\).

Using these state probabilities the number of items falling in different states in future time periods are to be calculated as below:
Number of individual replacements:

1st period: \( a_1 = \text{NX}_1^{IV} \)

2nd period: \( a_2 = \text{NX}_2^{IV} + a_1 \text{X}_1^{IV} \)

3rd period: \( a_3 = \text{NX}_3^{IV} + a_1 \text{X}_2^{IV} + a_2 \text{X}_1^{IV} \)

4th period: \( a_4 = \text{NX}_4^{IV} + a_1 \text{X}_3^{IV} + a_2 \text{X}_2^{IV} + a_3 \text{X}_1^{IV} \)

Number of minor repairs:

1st period: \( \beta_1 = \text{NX}_1^{II} \)

2nd period: \( \beta_2 = \text{NX}_2^{II} + \beta_1 \text{X}_1^{II} \)

3rd period: \( \beta_3 = \text{NX}_3^{II} + \beta_1 \text{X}_2^{II} + \beta_2 \text{X}_1^{II} \)

4th period: \( \beta_4 = \text{NX}_4^{II} + \beta_1 \text{X}_3^{II} + \beta_2 \text{X}_2^{II} + \beta_3 \text{X}_1^{II} \)

Number of major repairs:

1st period: \( \lambda_1 = \text{NX}_1^{III} \)

2nd period: \( \lambda_2 = \text{NX}_2^{III} + \lambda_1 \text{X}_1^{III} \)

3rd period: \( \lambda_3 = \text{NX}_3^{III} + \lambda_1 \text{X}_2^{III} + \lambda_2 \text{X}_1^{III} \)

4th period: \( \lambda_4 = \text{NX}_4^{III} + \lambda_1 \text{X}_3^{III} + \lambda_2 \text{X}_2^{III} + \lambda_3 \text{X}_1^{III} \)
Total cost = Group replacement cost + 
   Replacement cost of individual failures + 
   total minor repair cost + total major repair cost

Total cost at the end of first period = $NC_4 + C_1a_1 + C_2\beta_1 + C_3\lambda_1$

Total cost upto two periods = $NC_4 + C_1(a_1 + a_2) + C_2(\beta_1 + \beta_2) + C_3(\lambda_1 + \lambda_2)$

Total cost upto three periods = $NC_4 + C_1(a_1 + a_2 + a_3) + C_2(\beta_1 + \beta_2 + \beta_3) + C_3(\lambda_1 + \lambda_2 + \lambda_3)$

Total cost upto ‘n’ time periods = $NC_4 + C_1\sum a_n + C_2\sum \beta_n + C_3\sum \lambda_n$ ---(6.3)

where n=1,2,....

Average cost per period = $\frac{\text{Total cost for 'n' periods}}{\text{number of periods}}$ ---(6.4)

*Replacement policy:*

‘n’ is optimal when the average cost per period is minimum i.e. average cost per period should be minimum in n$^{th}$ period, to block replace in n$^{th}$ period.
6.5 INFLUENCE OF INFLATION ON BLOCK REPLACEMENT DECISION USING FIRST ORDER MARKOV CHAIN

To understand the influence of inflation on block replacement decision, the forecasted inflation is used. The forecasting of inflation is discussed below in Sec.6.5.1 and the model is discussed in Sec.6.5.2.

6.5.1 Forecasting of Inflation

In the present study, the half yearly inflation (Table 6.1) data for Computer and Computer based system in India from the second half of 2004 to the second half of 2009 which is calculated based on the Wholesale Price Index-WPI (Appendix-V) is considered. Upward linear trend with nearly sinusoidal fluctuations is observed as plotted in the Fig. #6.2.

<table>
<thead>
<tr>
<th>Year</th>
<th>Inflation ((\phi)) during</th>
<th>Jan. - June</th>
<th>July - Dec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td></td>
<td></td>
<td>-17.48</td>
</tr>
<tr>
<td>2005</td>
<td>-22.40</td>
<td>-1.18</td>
<td></td>
</tr>
<tr>
<td>2006</td>
<td>-16.44</td>
<td>-15.27</td>
<td></td>
</tr>
<tr>
<td>2007</td>
<td>10.76</td>
<td>6.03</td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td>3.41</td>
<td>16.5</td>
<td></td>
</tr>
<tr>
<td>2009</td>
<td>20.38</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table #6.1: Past data of (Half yearly) Inflation
Inflation is forecasted using Time Series and Causal forecasting models to identify the underlying model that best fits the time series data. And also an error analysis is made as a measurement of accuracy. The following models are employed for the forecasting of inflation.

1. **Moving Average Method**

\[
F_t = (\phi_{t-4} + \phi_{t-3} + \phi_{t-2} + \phi_{t-1})/4; \quad ---(6.5) \quad \text{Refer Table 6.2}
\]

2. **Weighted Moving Average Method**

\[
F_t = (\phi_{t-4} + 2\phi_{t-3} + 3\phi_{t-2} + 4\phi_{t-1})/10; \quad ---(6.6) \quad \text{Refer Table 6.2}
\]

3. **Simple Exponential Smoothing Method**

\[
F_t = \mu \phi_{t-1} + (1 - \mu)F_{t-1}; \quad ---(6.7) \quad \text{Refer Table 6.3}
\]

where \( \mu = \) Smoothing coefficient

4. **Regression Method** *with trigonometric function*

\[
F_t = a + bt + c \sin (t\pi + \pi/4); \quad ---(6.8) \quad \text{Refer Tables 6.4 & 6.5}
\]

where \( F_t = \) Forecasted inflation for a time period \( t \).

\[
\phi_t = \text{actual inflation for the time period } t.
\]

<table>
<thead>
<tr>
<th>Time period</th>
<th>Inflation rate (( \phi ))</th>
<th>Moving Average Method</th>
<th>Weighted Moving Average Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>预报值(F)</td>
<td>e</td>
<td>(</td>
</tr>
<tr>
<td>2004H-II</td>
<td>-17.48</td>
<td>-14.38</td>
<td>0.90</td>
</tr>
<tr>
<td>2005H-I</td>
<td>-22.4</td>
<td>-13.82</td>
<td>-24.58</td>
</tr>
<tr>
<td>2005H-II</td>
<td>-1.18</td>
<td>-5.53</td>
<td>-11.56</td>
</tr>
<tr>
<td>2006H-I</td>
<td>-16.44</td>
<td>-3.73</td>
<td>-7.14</td>
</tr>
<tr>
<td>2006H-II</td>
<td>-15.27</td>
<td>-15.36</td>
<td>11.33</td>
</tr>
<tr>
<td>2007H-I</td>
<td>10.76</td>
<td>1.23</td>
<td>15.36</td>
</tr>
<tr>
<td>2007H-II</td>
<td>6.03</td>
<td>3.41</td>
<td>-7.14</td>
</tr>
<tr>
<td>2008H-I</td>
<td>3.41</td>
<td>-3.73</td>
<td>-7.14</td>
</tr>
<tr>
<td>2008H-II</td>
<td>16.59</td>
<td>1.23</td>
<td>-15.36</td>
</tr>
<tr>
<td><strong>average</strong></td>
<td></td>
<td>-11.55</td>
<td>11.91</td>
</tr>
</tbody>
</table>

**Table # 6.2:** Forecasted Inflation data using Moving Average & Weighted Moving Average Methods
Table # 6.3: Forecasted Inflation data using Simple Exponential Smoothing

Regression model with trigonometric function:

A sinusoidal trigonometric function is used in the regression model to accommodate cyclical fluctuations of inflation. For this the following mathematical equation is considered.

\[
\phi = a + bt + c \sin (t\pi + \pi/4)
\]  

(6.9)

To find the constants \(a\), \(b\) & \(c\) the following set of equations are used.

\[
\sum \phi = na + b \sum t + c \sum \sin (t\pi + \pi/4)
\]  

(6.10)

\[
\sum (\phi t) = a \sum t + b \sum t^2 + c \sum [t \sin (t\pi + \pi/4)]
\]  

(6.11)

\[
\sum (\phi t^2) = a \sum t^2 + b \sum t^3 + c \sum [t^2 \sin (t\pi + \pi/4)]
\]  

(6.12)

where \(\phi\) is the inflation, \(t\) is time period, \(n\) is the number of time periods and \(a\), \(b\) & \(c\) are the coefficients. Table #6.1 gives the half yearly inflation data.

By solving the equations (6.10),(6.11),and (6.12) we get the values of regression coefficients.

\[
a = -22.51 \quad b = 9.0369 \quad c = 5.5802
\]  

---(6.13)
The final regression equation is:

\[ F_i = -22.51 + 9.0369t + 5.5802\sin(t\pi + \pi/4) \quad - - - (6.14) \]

And the calculations are tabulated in Table 6.4 and the forecasted inflation is shown in Table 6.5.
**Measures of Accuracy:**
An error analysis is made in terms of Mean error and Mean Absolute Deviation (MAD) and the results are tabulated in the Table 6.6.

| Forecasting Model                        | Mean Error = $(\sum e) / n$ | Mean Absolute Deviation$= (\sum e | e |)/n$ |
|------------------------------------------|-----------------------------|-------------------------------------------|
| Moving Average Method                    | $F_t = (\varphi_{t-4} + \varphi_{t-3} + \varphi_{t-2} + \varphi_{t-1})/4$ | -11.55                                   |
| Weighted Moving Average Method           | $F_t = (\varphi_{t-4} + 2\varphi_{t-3} + 3\varphi_{t-2} + 4\varphi_{t-1})/10$ | -5.11                                     |
| Simple Exponential smoothing method      | $F_t = \mu \varphi_{t-1} + (1-\mu)F_{t-1}$ | $\mu=0.2$ -33.64                         |
|                                           |                             |                                            | $\mu=0.4$ -7.6                           |
| Regression Model with trigonometric function | $F_t = -22.51 + 9.0369t + 5.5802\sin(\pi t + \pi/4)$ | 0.00                                      |

**Table 6.6:** Table of errors and accuracy of Inflation

Though it seems Mean absolute deviation is a little bit higher, it is evident that the regression model with trigonometric function is producing relatively minimal errors. It can be noticed from the results that the absolute deviation for the forecasted inflation is high during the starting time periods (2005 and 2006), and is very minimal during the latest time periods (2007 and 2008).

Subsequently Inflation for the forthcoming periods is predicted by using regression model with trigonometric function and are tabulated in the Table 6.7 and plotted in the Fig 6.3.
Table 6.7: Forecasted inflation data for future time periods using regression model

<table>
<thead>
<tr>
<th>Year</th>
<th>Time period (t)</th>
<th>Inflation rate (φ)</th>
<th>Forecasted inflation (F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004H-II</td>
<td>0</td>
<td>-17.48</td>
<td>-18.56424</td>
</tr>
<tr>
<td>2005H-I</td>
<td>0.5</td>
<td>-22.4</td>
<td>-14.04579</td>
</tr>
<tr>
<td>2005H-II</td>
<td>1</td>
<td>-1.18</td>
<td>-17.41886</td>
</tr>
<tr>
<td>2006H-I</td>
<td>1.5</td>
<td>-16.44</td>
<td>-12.90041</td>
</tr>
<tr>
<td>2006H-II</td>
<td>2</td>
<td>-15.27</td>
<td>-0.490441</td>
</tr>
<tr>
<td>2007H-I</td>
<td>2.5</td>
<td>10.76</td>
<td>4.0280094</td>
</tr>
<tr>
<td>2007H-II</td>
<td>3</td>
<td>6.03</td>
<td>0.6549406</td>
</tr>
<tr>
<td>2008H-I</td>
<td>3.5</td>
<td>3.41</td>
<td>5.173906</td>
</tr>
<tr>
<td>2008H-II</td>
<td>4</td>
<td>16.59</td>
<td>17.583359</td>
</tr>
<tr>
<td>2009H-I</td>
<td>4.5</td>
<td>20.38</td>
<td>22.101809</td>
</tr>
<tr>
<td>2009H-II</td>
<td>5</td>
<td></td>
<td>18.728741</td>
</tr>
<tr>
<td>2010H-I</td>
<td>5.5</td>
<td></td>
<td>23.247191</td>
</tr>
<tr>
<td>2010H-II</td>
<td>6</td>
<td></td>
<td>35.657159</td>
</tr>
<tr>
<td>2011H-I</td>
<td>6.5</td>
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<td>40.175609</td>
</tr>
<tr>
<td>2011H-II</td>
<td>7</td>
<td></td>
<td>36.802541</td>
</tr>
<tr>
<td>2012H-I</td>
<td>7.5</td>
<td></td>
<td>41.32</td>
</tr>
<tr>
<td>2012H-II</td>
<td>8</td>
<td></td>
<td>53.73</td>
</tr>
<tr>
<td>2013H-I</td>
<td>8.5</td>
<td></td>
<td>58.24</td>
</tr>
<tr>
<td>2013H-II</td>
<td>9</td>
<td></td>
<td>54.87</td>
</tr>
</tbody>
</table>

Fig. 6.3 Forecasted Vs. Actual Inflation
6.5.2 Block Replacement Decision Under The Influence Of Forecasted Inflation Using First Order Markov Process:

Assumptions:

(i) In block replacement decisions, in addition to the functioning state and failure state, two intermediate repairable breakdown states—minor repair and major repair—are introduced. This results in a four-state Markov chain.

(ii) Obsolescence of the present system, due to the availability of a new system with better design and capability, is assumed as a breakdown/complete failure state.

(iii) Instead of assuming discrete probability distribution from one period to the other, it is assumed to follow a first-order discrete-time Markov Chain.

(iv) This model can be applied to a system that consists of N items in four different states and to find out the block replacement period. In this study, a special reference has been made to the computer and computer-based system that is widely used by ICT industry.

(v) Repair cost or rectification cost of any item is different for minor repairable state and major repairable state if two intermediary states are allowed. However, it is assumed constant if only one intermediary state is considered.

(vi) The money to be spent is taken as loan for certain items at a given nominal interest rate, $r_n$. 
(vii) Nominal interest rate ‘$r_n$’ assumed to be constant during the life span of the asset

**Notations:**

$N =$ Total number of items in the system  
$C_1 =$ Individual replacement cost per unit  
$C_2 =$ Minor repair cost  
$C_3 =$ Major repair cost  
$C_4 =$ item cost during Group replacement  
$r_n =$ Nominal interest rate  
$\phi_t =$ Inflation rate at the ‘$t$’ th period  
$r_t =$ real interest rate$= \frac{(r_n - \phi_t)}{(1 + \phi_t)}$ from Fisherman’s relation  
$v = \frac{1}{(1 + r_t)} =$Present Worth Factor  
$X_{0I} =$ proportion of units under working condition initially  
$X_{0II} =$ proportion of units under minor repair initially  
$X_{0III} =$ proportion of units under major repair initially  
$X_{0IV} =$ proportion of units under complete failure initially  

$X_{iI} =$ proportion of units under working condition at the end of $i^{th}$ time period  
$X_{iII} =$ proportion of units under minor repair at the end of $i^{th}$ time period  
$X_{iIII} =$ proportion of units under major repair at the end of $i^{th}$ time period,  
$X_{iIV} =$ proportion of units under complete failure at the end of $i^{th}$ time period
\( P_{ij} \) = Probability of items switching over from \( i^{th} \) state to \( j^{th} \) state in a period

TPM = Transition Probability Matrix

AC(t) = Weighted average cost per period

As two intermediary states – minor and major repairable states - between functional and complete failure state are considered, the Transition Probability matrix or the generator of Markov process for a four state system can be represented as

\[
P = \begin{bmatrix}
P_{11} & P_{12} & P_{13} & P_{14} \\
P_{21} & P_{22} & P_{23} & P_{24} \\
P_{31} & P_{32} & P_{33} & P_{34} \\
P_{41} & P_{42} & P_{43} & P_{44}
\end{bmatrix}
\]

where 1,2,3 and 4 represents functional, minor repairable, major repairable and complete failure states.

The state probabilities of items in different states can be computed as

\[
= (\text{probability of items in different states in initial period}) \times (\text{TPM})^n
\]

i.e. \( X_n = X_0 P^n \) -----(6.15)

The values of \( P^1, P^2, ..., P^n \) can be calculated by spectral decomposition method.
At the end of the first period,
the state probabilities can be calculated from

\[ X_1 = X_0 \mathbf{P} \quad (\therefore X_n = X_0 \mathbf{P}^n) \]

\[
\Rightarrow \begin{bmatrix} X^I_1 & X^II_1 & X^III_1 & X^IV_1 \end{bmatrix} = \begin{bmatrix} X^I_0 & X^II_0 & X^III_0 & X^IV_0 \end{bmatrix} \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{bmatrix} \]

\[ (6.16) \]

Therefore,
Probability of items in functional state,

\[ X^I_1 = X^I_0 p_{11} + X^II_0 p_{12} + X^III_0 p_{31} + X^IV_0 p_{41} \]

Probability of items in minor repair state,

\[ X^II_1 = X^I_0 p_{12} + X^II_0 p_{22} + X^III_0 p_{32} + X^IV_0 p_{42} \]

Probability of items in major repair state,

\[ X^III_1 = X^I_0 p_{13} + X^II_0 p_{23} + X^III_0 p_{33} + X^IV_0 p_{43} \]

Probability of items in irreparable state,

\[ X^IV_1 = X^I_0 p_{14} + X^II_0 p_{24} + X^III_0 p_{34} + X^IV_0 p_{44} \]

Similarly the probabilities of items falling in different states in future time periods \( i = 1 \) to \( n \) are to be calculated by using equation \( X_n = X_0 \mathbf{P}^n \).
Using these state probabilities the number of items falling in different states in future time periods are to be calculated as below:

**Number of individual replacements:**

1\(^{st}\) period: \(a_1 = NX_{i}^{IV}\)

2\(^{nd}\) period: \(a_2 = NX_{2}^{IV} + a_1 X_{1}^{IV}\)

3\(^{rd}\) period: \(a_3 = NX_{3}^{IV} + a_1 X_{2}^{IV} + a_2 X_{1}^{IV}\)

4\(^{th}\) period: \(a_4 = NX_{4}^{IV} + a_1 X_{3}^{IV} + a_2 X_{2}^{IV} + a_3 X_{1}^{IV}\) \(\quad \text{--- (6.17)}\)

**Number of minor repairs:**

1\(^{st}\) period: \(\beta_1 = NX_{i}^{II}\)

2\(^{nd}\) period: \(\beta_2 = NX_{2}^{II} + \beta_1 X_{1}^{II}\)

3\(^{rd}\) period: \(\beta_3 = NX_{3}^{II} + \beta_1 X_{2}^{II} + \beta_2 X_{1}^{II}\)

4\(^{th}\) period: \(\beta_4 = NX_{4}^{II} + \beta_1 X_{3}^{II} + \beta_2 X_{2}^{II} + \beta_3 X_{1}^{II}\) \(\quad \text{--- (6.18)}\)

**Number of major repairs:**

1\(^{st}\) period: \(\lambda_1 = NX_{i}^{III}\)

2\(^{nd}\) period: \(\lambda_2 = NX_{2}^{III} + \lambda_1 X_{1}^{III}\)

3\(^{rd}\) period: \(\lambda_3 = NX_{3}^{III} + \lambda_1 X_{2}^{III} + \lambda_2 X_{1}^{III}\)

4\(^{th}\) period: \(\lambda_4 = NX_{4}^{III} + \lambda_1 X_{3}^{III} + \lambda_2 X_{2}^{III} + \lambda_3 X_{1}^{III}\) \(\quad \text{--- (6.19)}\)
Total cost = Group replacement cost + 
Replacement cost of individual failures +
  total minor repair cost + total major repair cost

Total cost upto 'n' time periods:

\[ TC(n) = C_1 \left[ \alpha_1 + \alpha_2 v + \alpha_3 v^2 + \ldots + \alpha_n v^{n-1} \right] \\
+ C_2 \left[ \beta_1 + \beta_2 v + \beta_3 v^2 + \ldots + \beta_n v^{n-1} \right] \\
+ C_3 \left[ \lambda_1 + \lambda_2 v + \lambda_3 v^2 + \ldots + \lambda_n v^{n-1} \right] + NC_4 v^{n-1} \]  

\[ = C_1 \sum (\alpha_n v^{n-1}) + C_2 \sum (\beta_n v^{n-1}) + C_3 \sum (\lambda_n v^{n-1}) + NC_4 v^{n-1} \]

where \( n=1,2,3,\ldots \)

Weighted Average cost per period= \( AC(n) = \frac{TC(n)}{\sum v^{n-1}} \)  

\( ---(6.21) \)

Replacement policy:

'\( n \)' is optimal when the weighted average cost per period is minimum
i.e. average cost per period should be minimum in \( n^{th} \) period, to block replace in \( n^{th} \) period.

This model can be applied to find the block replacement age for a class of items that can be brought back to use by repair. The developed model is novel in approach as there is no such facility of incorporating the intermediate stage i.e. minor repair and major repair states in establishing the replacement models so far in the available literature.

This type of model building will be of quite handy in the development of block replacement models and the same analogy of using higher order Markov chains technique can be extended by introducing number of finite states depending upon the nature of repairs that one can encounter in practice. This model finds its applications in the maintenance of a group of microprocessors in a big software firm, a group of similar cutting tools in production environment, a large group of pressure gauges in gas filling plants etc.
6.6 INFLUENCE OF INFLATION ON BLOCK REPLACEMENT
DECISION USING HIGHER ORDER MARKOV CHAIN

Second Order Markov Chain assumes that probability of next state depends on the probabilities of immediately two preceding states. Then we will have Second Order Markov Chain (SOMC) whose transition probabilities are

\[ P_{x(n-2),x(n-1),x(n)} = P\{X(t_n) = x_n \mid X(t_{n-1}) = x_{n-1}, X(t_{n-2}) = x_{n-2}\} \]

So as the model under study consists 4 states, the SOMC Transition Probability Matrix (TPM) (Andre Berchtold et al 2002) can be formulated as

\[
\begin{array}{cccc}
  & I & II & III & IV \\
 I & P_{11} & P_{112} & P_{113} & P_{114} \\
 II & P_{21} & P_{212} & P_{213} & P_{214} \\
 III & P_{31} & P_{312} & P_{313} & P_{314} \\
 IV & P_{41} & P_{412} & P_{413} & P_{414} \\
 I & P_{12} & P_{122} & P_{123} & P_{124} \\
 II & P_{22} & P_{222} & P_{223} & P_{224} \\
 III & P_{32} & P_{322} & P_{323} & P_{324} \\
 TPM = P = IV & P_{42} & P_{422} & P_{423} & P_{424} \\
 I & P_{13} & P_{132} & P_{133} & P_{134} \\
 II & P_{23} & P_{232} & P_{233} & P_{234} \\
 III & P_{33} & P_{332} & P_{333} & P_{334} \\
 IV & P_{43} & P_{432} & P_{433} & P_{434} \\
 I & P_{14} & P_{142} & P_{143} & P_{144} \\
 II & P_{24} & P_{242} & P_{243} & P_{244} \\
 III & P_{34} & P_{342} & P_{343} & P_{344} \\
 IV & P_{44} & P_{442} & P_{443} & P_{444} \\
\end{array}
\]

The size of the TPM will be \( m^4 \times m \) and the number of transition probabilities to be calculated in each TPM will be \( m^{t+1} \). The Table 6.8
gives the number of transition probabilities for various combinations of order (l) and states (m).

<table>
<thead>
<tr>
<th>Number of states (m)</th>
<th>Order (l) of Markov Chain</th>
<th>Size of the TPM</th>
<th>No. of Transition probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>2x2</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4x2</td>
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</tr>
<tr>
<td>3</td>
<td>3</td>
<td>8x2</td>
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</tr>
<tr>
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<td>4</td>
<td>16x2</td>
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<td>3x3</td>
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<td>2</td>
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<td>243</td>
</tr>
<tr>
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<td>1</td>
<td>4x4</td>
<td>16</td>
</tr>
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<td>2</td>
<td>16x4</td>
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<td>3</td>
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<td>256</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>256x4</td>
<td>1024</td>
</tr>
</tbody>
</table>

**Table 6.8:** Number of transition probabilities for various combinations of order (l) and states (m)

**Parsimonious modeling of Higher (Second) order Markov Chain using Weighted Moving Transition Probabilities (WMTP)**

From the Table 6.8 it is evident that for higher order Markov chain with more state space, the size of the TPM will be too large. Estimation of several hundreds of parameters is a very time consuming one. And also it will be difficult to analyse and make conclusions or decisions.

Moreover as maintenance cost is in proportion of the items falling in each state at certain time, the state probabilities are fairly important rather through which intermediate states the current state has been attained. Therefore focus on developing a model that yields
better forecast of the proportion of items in each state at a certain
time period is justifiable.

To address this, a parsimonious model, the Weighted Moving
Transition Probabilities (WMTP) method is introduced that
approximates higher order Markov chains. This makes the number of
parameters in each TPM to be computed far less. The size of the TPM
is m x m only. Each element of the TPM is the probability for the
occurrence of a particular event at time t given the probabilities of
immediate previous \( l \) (= order of the Markov Chain) time periods i.e.
at times from \( (t-l) \) to \( (t-1) \). The effect of each lag is considered by
assigning the weights.

The Weighted Moving Transition Probabilities (WMTP) can be
estimated as

\[
P\{X(t_n) = x_n \mid X(t_{n-1})=x_{n-1},\ldots,X(t_{l})=x_{l}\} = \sum_{g=1}^{l} \delta_g (P_{ij})_g
\]

subject to \( \sum_{g=1}^{l} \delta_g = 1 \) and \( \delta_g \geq 0 \).

\( \delta \) is the weight parameter corresponding with the lag ‘g’. \( P_{ij} \) are the
transition probabilities of the corresponding m x m TPM. It is based on
the premise that the most recent value is the most relevant to
estimate the future value; consequently the weights decrease as we
consider the older lags.
The Weighted Moving Transition Probabilities-WMTP (for $l = 2$, Second Order Markov process) can be written as:

$$
(p_{ij}) = p(X(t_n) = x_n \mid X(t_{n-1}) = x_{n-1}, X(t_{n-2}) = x_{n-2})

= \sum_{g=1}^{2} \delta_g (P_g) = \delta_{n-1} (P_{g-1}) + \delta_{n-2} (P_g)
$$

where $\delta_{n-1} + \delta_{n-2} = 1$ and $\delta \geq 0$.

As shown in the following Fig. 6.4, real Second Order Markov Chain (SOMC) carries the combined influence of lags whereas WMTP model analogue carries the independent influences of each lag on the present.

---

**Assumptions:**

(i) In block replacement decisions, in addition to the functioning state and failure state, two intermediate repairable breakdown states- minor repair and major repair- are introduced. This results in four-state Markov chain.

(ii) Obsolescence of the present system, due to the availability of new system with better design and capability is available in market, is assumed as break down/complete failure state.
(iii) Instead of assuming discrete probability distribution from one period to the other period, it is assumed to follow second order discrete-time Markov Chain.

(iv) TPMs for two years or the generators of the Markov chain are assumed. (In real practice these values can be taken from the past experience.)

(v) This model can be applied to a system that consists of N items in four different states and to find out the block replacement period.  

**In this study, a special reference has been made to the computer and computer based system which is widely used by ICT industry.**

(vi) Repair cost or rectification cost of any item is different for minor repairable state and major repairable state if two intermediary states are allowed. However, it is assumed constant if only one intermediary state is considered.

(vii) The money to be spent is taken as loan for certain items at a given nominal interest rate, \( r_n \).

(viii) Nominal interest rate ‘\( r_n \)’ assumed to be constant during the life span of the asset

**Notations:**

\( N = \) Total number of items in the system  
\( C_1 = \) Individual replacement cost per unit  
\( C_2 = \) Minor repair cost  
\( C_3 = \) Major repair cost
$C_4 = \text{item cost during Group replacement}$

$r_n = \text{Nominal interest rate}$

$\phi_t = \text{Inflation rate at the 't' th period}$

$r_t = \text{real interest rate} = \frac{(r_n - \phi_t)}{(1 + \phi_t)} \text{ from Fisherman's relation}$

$v = \frac{1}{(1 + r_t)} = \text{Present Worth Factor}$

$X_{0I} = \text{proportion of units under working condition initially}$

$X_{0II} = \text{proportion of units under minor repair initially}$

$X_{0III} = \text{proportion of units under major repair initially}$

$X_{0IV} = \text{proportion of units under complete failure initially}$

$X_{iI} = \text{proportion of units under working condition at the end of } i^{th} \text{ time period}$

$X_{iII} = \text{proportion of units under minor repair at the end of } i^{th} \text{ time period}$

$X_{iIII} = \text{proportion of units under major repair at the end of } i^{th} \text{ time period}$

$X_{iIV} = \text{proportion of units under complete failure at the end of } i^{th} \text{ time period}$

$P_{ij} = \text{Probability of items switching over from } i^{th} \text{ state to } j^{th} \text{ state in a period}$

$\text{TPM = Transition Probability Matrix}$

$= p_n , \text{where n represents the } n^{th} \text{ year}$

$\delta = \text{weight parameter associated with lag}$

$\text{AC(t) = Weighted average cost per period}$

As two intermediary states –minor and major repairable states-between functional and complete failure state are considered, the two
Transition Probability matrices or the generators of Markov process for a four state system for can be represented as

\[
P_{n-1} = \begin{bmatrix}
P_{11} & P_{12} & P_{13} & P_{14} \\
P_{21} & P_{22} & P_{23} & P_{24} \\
P_{31} & P_{32} & P_{33} & P_{34} \\
P_{41} & P_{42} & P_{43} & P_{44}
\end{bmatrix}_{n-1}
\quad \text{and} \quad P_{n-2} = \begin{bmatrix}
P_{11} & P_{12} & P_{13} & P_{14} \\
P_{21} & P_{22} & P_{23} & P_{24} \\
P_{31} & P_{32} & P_{33} & P_{34} \\
P_{41} & P_{42} & P_{43} & P_{44}
\end{bmatrix}_{n-2}
\]

where 1,2,3 and 4 represents functional, minor repairable, major repairable and complete failure states.

The Weighted Moving Transition Probabilities-WMTP (for \( l = 2 \), Second Order Markov process) can be written as:

\[
(p_{ij})_n = \sum_{g=1}^{2} \delta_{g} (p_{ij})_g = \delta_{n-1} (p_{ij})_{n-1} + \delta_{n-2} (p_{ij})_{n-2}
\]

\[\text{---(6.22)}\]

where \( \delta_{n-1} + \delta_{n-2} = 1 \) and \( \delta \geq 0 \).

The state probabilities of items in different states can be computed as

\[X_n = X_0 (p_{ij})_n\]

\[\text{i.e.} \quad X_n = X_0 (p_{ij})_n\]

\[\text{---(6.23)}\]

Using these state probabilities the number of items falling in different states in future time periods are to be calculated as below:

\[\text{Number of individual replacements:}\]

\[\begin{align*}
\text{1st period:} \quad & a_1 = NX_1^{IV} \\
\text{2nd period:} \quad & a_2 = NX_2^{IV} + a_1X_1^{IV} \\
\text{3rd period:} \quad & a_3 = NX_3^{IV} + a_1X_2^{IV} + a_2X_1^{IV} \\
\text{4th period:} \quad & a_4 = NX_4^{IV} + a_1X_3^{IV} + a_2X_2^{IV} + a_3X_1^{IV} \\
\end{align*}\]

\[\text{--- (6.24)}\]
Number of minor repairs:

1\textsuperscript{st} period: $\beta_1 = \text{NX}_1^\text{II}$

2\textsuperscript{nd} period: $\beta_2 = \text{NX}_2^\text{II} + \beta_1 X_1^\text{II}$

3\textsuperscript{rd} period: $\beta_3 = \text{NX}_3^\text{II} + \beta_1 X_2^\text{II} + \beta_2 X_1^\text{II}$

4\textsuperscript{th} period: $\beta_4 = \text{NX}_4^\text{II} + \beta_1 X_3^\text{II} + \beta_2 X_2^\text{II} + \beta_3 X_1^\text{II}$  \hspace{1cm} \text{---(6.25)}

Number of major repairs:

1\textsuperscript{st} period: $\lambda_1 = \text{NX}_1^\text{III}$

2\textsuperscript{nd} period: $\lambda_2 = \text{NX}_2^\text{III} + \lambda_1 X_1^\text{III}$

3\textsuperscript{rd} period: $\lambda_3 = \text{NX}_3^\text{III} + \lambda_1 X_2^\text{III} + \lambda_2 X_1^\text{III}$

4\textsuperscript{th} period: $\lambda_4 = \text{NX}_4^\text{III} + \lambda_1 X_3^\text{III} + \lambda_2 X_2^\text{III} + \lambda_3 X_1^\text{III}$  \hspace{1cm} \text{---(6.26)}

Total cost upto ‘n’ time periods:

$$
\text{TC}(n) = C_1 \left[ a_1 + a_2 v + a_3 v^2 + \ldots + a_n v^{n-1} \right] \\
+ C_2 \left[ \beta_1 + \beta_2 v + \beta_3 v^2 + \ldots + \beta_n v^{n-1} \right] \\
+ C_3 \left[ \lambda_1 + \lambda_2 v + \lambda_3 v^2 + \ldots + \lambda_n v^{n-1} \right] + NC_\nu v^{n-1}
$$

\hspace{1cm} \text{---(6.27)}

$$
= C_1 \sum (a_n v^{n-1}) + C_2 \sum (\beta_n v^{n-1}) + C_3 \sum (\lambda_n v^{n-1}) + NC_\nu v^{n-1}
$$

where \( n=1,2,3, \ldots \)

Weighted Average cost per period=AC(n) = \( \frac{\text{TC}(n)}{\sum v^{n-1}} \)  \hspace{1cm} \text{---(6.28)}

Replacement policy:

‘n’ is optimal when the weighted average cost per period is minimum

i.e. weighted average cost per period should be minimum in \( n^{th} \) period, to block replace in \( n^{th} \) period.
6.7 REENGINEERING TREATMENT ON BLOCK REPLACEMENTS – MODEL EXTENSION

In precise, reengineering can be perceived as the adjustment, alteration, or partial replacement of a process or product in order to make it to meet a new need. In this study as the special reference is given to a system of computers, reengineering of computer hardware is looked at.

In IT intensive sectors such as banking, insurance, railways etc, traditional personal computers (PCs) - spread over various divisions as stand alone systems or LAN clients - contain own hard disk and using its own operating system. As the size of computer network grows in terms of workstations, the costs of hardware, maintenance and support will become the issues and also impact the budget and daily operations.

Some times the components of existing computers may not support to run the new software. Then replacing the entire workstation is an option; but an expensive one. As an alternative reengineering the process of computing with thin client technology is economical.

Thin client is a generic term for a group of emerging technologies that reduces costs of hardware, maintenance and support. Also this helps in saving the bandwidth, reducing downtime and improving network security. The reengineering of computer network is discussed in detail in chapter 5.
Assumptions:

(i) The old systems are not replaced; but reengineered with suitable technology e.g. Thin client technology for computer hardware

(ii) In block replacement decisions, in addition to the functioning state and failure state, two intermediate repairable breakdown states-minor repair and major repair- are introduced. This results in four-state Markov chain.

(iii) Obsolescence of the present system, due to the availability of new system with better design and capability is available in market, is assumed as break down/complete failure state.

(iv) Instead of assuming discrete probability distribution from one period to the other period, it is assumed to follow second order discrete-time Markov Chains.

(v) This model can be applied to a system that consists of N items in four different states and to find out the block replacement period. In this study, a special reference has been made to the computer and computer based system which is widely used by ICT industry.

(vi) Repair cost or rectification cost of any item is different for minor repairable state and major repairable state if two intermediary states are allowed. However, it is assumed constant if only one intermediary state is considered.

(vii) The money to be spent is taken as loan for certain items at a given nominal interest rate, $r_n$.

(viii) Nominal interest rate $r_n$ assumed to be constant during the life span of the asset
### Notations:

<table>
<thead>
<tr>
<th></th>
<th>Without reengineering</th>
<th>With reengineering</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of items in the system</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>Individual replacement cost per unit</td>
<td>$C_1$</td>
<td>$C_{1r}$</td>
</tr>
<tr>
<td>Minor repair cost</td>
<td>$C_2$</td>
<td>$C_{2r}$</td>
</tr>
<tr>
<td>Major repair cost</td>
<td>$C_3$</td>
<td>$C_{3r}$</td>
</tr>
<tr>
<td>Group replacement cost</td>
<td>$C_4$</td>
<td>$C_{4r}$</td>
</tr>
<tr>
<td>Reengineering cost</td>
<td>$C_r$</td>
<td></td>
</tr>
<tr>
<td>Proportion of units under working condition initially</td>
<td>$X_0^I$</td>
<td>$X_{0r}^I$</td>
</tr>
<tr>
<td>Proportion of units under minor repair initially</td>
<td>$X_0^{II}$</td>
<td>$X_{0r}^{II}$</td>
</tr>
<tr>
<td>Proportion of units under major repair initially</td>
<td>$X_0^{III}$</td>
<td>$X_{0r}^{III}$</td>
</tr>
<tr>
<td>Proportion of units under complete failure initially</td>
<td>$X_0^{IV}$</td>
<td>$X_{0r}^{IV}$</td>
</tr>
<tr>
<td>Proportion of units under working condition at the end of $i^{th}$ time period</td>
<td>$X_i^I$</td>
<td>$X_{ir}^I$</td>
</tr>
<tr>
<td>Proportion of units under minor repair at the end of $i^{th}$ time period</td>
<td>$X_i^{II}$</td>
<td>$X_{ir}^{II}$</td>
</tr>
<tr>
<td>Proportion of units under major repair at the end of $i^{th}$ time period</td>
<td>$X_i^{III}$</td>
<td>$X_{ir}^{III}$</td>
</tr>
<tr>
<td>Proportion of units under complete failure at the end of $i^{th}$ time period</td>
<td>$X_i^{IV}$</td>
<td>$X_{ir}^{IV}$</td>
</tr>
<tr>
<td>TPM = Transition Probability Matrix</td>
<td>P</td>
<td>$P_r$</td>
</tr>
<tr>
<td>Weighted average cost per period</td>
<td>$AC(t)$</td>
<td>$AC(t)_r$</td>
</tr>
<tr>
<td>Number of individual replacements</td>
<td>$\alpha_n$</td>
<td>$\alpha_{nr}$</td>
</tr>
<tr>
<td>Number of minor repairs</td>
<td>$\beta_n$</td>
<td>$\beta_{nr}$</td>
</tr>
<tr>
<td>Number of minor repairs</td>
<td>$\lambda_n$</td>
<td>$\lambda_{nr}$</td>
</tr>
</tbody>
</table>

$r_n =$ Nominal interest rate ; $\phi_t =$Inflation rate at the ‘$t’$ th period

$r_t =$ real interest rate $= \frac{(r_n - \phi_t)}{(1 + \phi_t)}$ from Fisherman’s relation

$v = \frac{1}{(1 + r_t)} =$Present Worth Factor

$P_{ij} =$Probability of items switching over from $i^{th}$ state to $j^{th}$ state in a period

$\delta =$ weight parameter associated with lag
As two intermediary states – minor and major repairable states – between functional and complete failure state are considered, the two Transition Probability matrices or the generators of Markov process for a four state system for can be represented as

\[
P_{n-1} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{bmatrix}_{n-1} \quad \text{and} \quad P_{n-2} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{bmatrix}_{n-2}
\]

where 1, 2, 3 and 4 represents functional, minor repairable, major repairable and complete failure states.

The equations from (6.22) to (6.26) hold good for calculating the following respectively:

- The weighted moving transition probabilities
- The state probabilities of items in different states
- The number of individual replacements
- Number of minor repairs and
- Number of major repairs

Total cost upto ‘n’ time periods WITH REENGINEERING:

\[
\text{Total cost} = \text{Reengineering cost} + \text{Group replacement cost} + \text{Replacement cost of individual failures} + \text{total minor repair cost} + \text{total major repair cost}
\]

\[
TC(n)_r = C_r + C_{1r} \left[ a_{1r} + a_{2r}v + a_{3r}v^2 + \ldots + a_{mr}v^{n-1} \right] + C_{2r} \left[ \beta_{1r} + \beta_{2r}v + \beta_{3r}v^2 + \ldots + \beta_{mr}v^{n-1} \right] + C_{3r} \left[ \alpha_{1r} + \alpha_{2r}v + \alpha_{3r}v^2 + \ldots + \alpha_{mr}v^{n-1} \right] + N C_{4r} v^{n-1}
\]

\[
= C_r + C_{1r} \sum (a_{mr}v^{n-1}) + C_{2r} \sum (\beta_{mr}v^{n-1}) + C_{3r} \sum (\alpha_{mr}v^{n-1}) + N C_{4r} v^{n-1} \quad \text{---(6.29)}
\]

where \(n=1,2,3,\ldots\)

Weighted Average cost per period=\(AC(n)_r = \frac{TC(n)_r}{\sum v^{n-1}}\) \quad \text{---(6.30)}
Total cost upto ‘n’ time periods WITHOUT REENGINEERING:

\[ TC(n) = C_1 \left[ a_1 + a_2 v + a_3 v^2 + \ldots + a_n v^{n-1} \right] + C_2 \left[ b_1 + b_2 v + b_3 v^2 + \ldots + b_n v^{n-1} \right] + C_3 \left[ c_1 + c_2 v + c_3 v^2 + \ldots + c_n v^{n-1} \right] + NC_4 v^{n-1} \]

\[ = C_1 \sum (a_n v^{n-1}) + C_2 \sum (b_n v^{n-1}) + C_3 \sum (c_n v^{n-1}) + NC_4 v^{n-1} \quad ---(6.31) \]

where \( n=1,2,3,\ldots \)

Weighted Average cost per period=AC(n) = \( \frac{TC(n)}{\sum v^{n-1}} \) ---(6.32)

Replacement policy:

‘n’ is optimal when the weighted average cost per period is minimum i.e. average cost per period should be minimum in \( n^{th} \) period, to block replace in \( n^{th} \) period.

Summary: The objective of this study reported here is to develop a mathematical model using Markov process for block replacement problem that considers the influence of inflation and value of money on the optimal replacement policy. A special reference is given to the multi-state repairable system containing a block of Computers. The formulation and development of the model is done as given below.

i. Block Replacement Model Using FOMC without Inflation

ii. Influence of Inflation on Block Replacement decision using FOMC

iii. Influence of Inflation on Block Replacement decision using SOMC

iv. Reengineering Treatment on Block Replacements – Model Extension

Also this chapter deals with few more observations about the behaviour of the block replacement model under the influence of variable maintenance cost; and also under rapid up-trend and sluggish up-trends in inflation.