9. CONCLUSION

In the present thesis a few new results have been developed which are related to statistical inference in queues and to the design and control of queues. Though several authors have considered different aspects of control of queues, the literature on statistical inference as applied to queues is very limited. In the preceding chapters we have attempted to tackle some statistical inference problems in queues. Some of these techniques are not only important for the statistical analysis of queues but also for control of queues. Efforts have been made to establish the interrelationships between statistical inference problems and control of queues. The idea is not completely new. It stems from the relationship between control chart technique for process control, with the testing of hypothesis problem.

The sequential probability ratio test developed for the traffic intensity parameter \( \rho \) has dual advantages. First as a statistical test to draw inference on the traffic intensity parameter. Next as a control technique to detect shifts in the traffic intensity parameter as quickly as possible and then to take appropriate corrective action. In view of the optimality property of the SPRT, the expected sample size for the test would be minimum for specified probabilities of type I and type II errors. Therefore, from the point of view of cost of sampling this test can be considered economical.
The test procedure is applicable to queueing systems in which one can identify an imbedded Markov chain. Systems like $\text{M}|\text{G}|1$, $\text{GI}|\text{M}|s$, $\text{M}^{(k)}|\text{G}|1$ are a few examples. One of the limitations of the test procedure is that the state space must be restricted to a finite set for the associated Markov chain. Further, the computation of ASN and OC functions is quite tedious.

The discussion on inference about the traffic intensity in the case of $\text{M}|\text{E}_k|1$ and $\text{E}_k|\text{M}|1$ queues brings the estimation and tests of hypothesis techniques close to the standard estimation and testing procedures. For example, the m.l.e of $\rho$ has been derived and the statistic happens to be the UMVUE. Based on these results a large sample test for $\rho$ and confidence estimation is possible. Further, tests which are most powerful have been developed for one sided and two sided hypotheses on $\rho$ which are commonly encountered in practice. It is also possible to apply a SPRT for $\rho$ with the type of observations considered in Chapter 3. This overcomes the limitation of the previous discussion, namely the restriction on the queueing waiting space. In this case Wald's SPRT can be used directly which makes the computation of OC and ASN simple. However, this kind of estimation and testing procedures cannot be applied to GI|M|s or bulk queues, since the transition probabilities of the associated chains depend on the initial state of the system.

The large sample test on $\rho$ presented in chapter 4
has the merit of its simplicity and wide applicability to queueing systems with single and multiple servers and arbitrary input and service time distributions. The test makes use of complete information on interarrival and service times. It is observed that for Erlangian type input/output distributions the test statistic is a function of \( \rho \) alone and it does not require the estimates of population parameters.

The nonparametric estimation and tests for the queueing parameters such as the mean, variance and density are very useful in practice in the absence of any knowledge about the exact form of the input/service distribution. As opposed to the earlier work, where parametric hypothesis testing was used as a vehicle for the control of queues, the distribution-free test on \( \rho \) is used for maintaining the queues in stable conditions. It is interesting to note that a number of available nonparametric tests can be used for this purpose.

It may be seen that no steady state conditions are imposed while deriving all the above results.

The remaining studies in the thesis are related to some design problems in queues. The discussion is confined to only static models. Though quite a few authors have discussed this problem, here we have investigated the design problems with constraints on queueing characteristics such as mean queue length, mean waiting time, server
utilization and so on. This is of particular interest if the objective is not merely to minimize a cost function but also to satisfy certain constraints on queueing characteristics so as to improve the system performance.

In the case of $M|M|1$ queue, it is interesting to note that quite a few chance constraints could be reduced to linear form thus enabling a simple analysis. It is also possible to bring some simplicity in the constraints of a multiserver queueing optimization problem. Here only an attempt is made to investigate these types of constrained problems. However, the study requires further investigation on the convexity-concavity properties of the objective function and nonlinear constraints and conditions for the existence of optimal solutions. Multiobjective function criteria is another area that requires further study.

Some of the design problems are studied for the queueing systems arising in communication and computer systems. We have considered only a few simple models, and in no way the investigation is complete. There are a number of other queueing models which require the kind of present study in much more detail and depth.

In the final chapter some simple allocation problems have been considered in series queues. It is interesting to observe that the solution technique for these types of problems are quite straightforward. In our study we had confined to only single constraint in each case. However
such allocation problems with a larger number of constraints are generally complicated and requires a detailed investigation.

Off late, the statistical inference problems and control problems in queues have drawn the interest of many researchers. Due to the complex nature of queueing systems and due to the interdependent structure of queueing observations, this study has become a formidable task and is more challenging. There is a further need to explore this area of interest to make queueing theory more practicable.