CHAPTER V
OPTICAL-MODEL ANALYSES

5.1 INTRODUCTION

The optical model plays a crucial role in the description of nuclear reactions, providing an interpretation of elastic scattering in terms of a potential associated with a wavefunction for the relative motion of the colliding pair. These wave functions, called the distorted waves, can then be used as inputs in various reaction theories. An optical model attempts to replace the complicated many-body problem posed by the interaction of two nuclei by the much simpler problem of two particles interacting through a potential. Indeed, the optical potential for the scattering of nucleons from nuclei is closely related to a Hartree-Fock potential, and the physical interpretation of the parameters of its real part is quite direct. The optical model may be regarded as an extension of the nuclear shell model into the continuum.

A wide variety of angular distribution shapes observed for elastic scattering can be explained through an optical potential, which has a real potential of short range arising from the basic nuclear interactions between the projectile and the target. This potential is attractive and causes refraction and reflection of the incident waves. Then there is another influence, viz., absorption, resulting from non-elastic collisions and the consequent loss of flux from the elastic channel, which is described by the imaginary part of the potential. A real spin-orbit potential, parametrized in terms of the $\mathbf{L}\cdot\mathbf{S}$ operator, is also included to account for the interaction of the spin of the incoming particle with its relative angular momentum about the target nucleus. Detailed discussion of optical-model
analyses can be found in [Au 70], [Ho 71], [Ja 70], and [Sa 83a].

5.2 FORM OF THE OPTICAL POTENTIAL

The theoretical cross sections and analyzing powers are obtained from the one-body Schrödinger equation

\[
\left[ \frac{d^2}{dr^2} + k^2 - \frac{2\eta k}{r} - \frac{2\mu}{\hbar^2} U(r) - \frac{\ell(\ell + 1)}{r^2} \right] \chi_L(k, r) = 0
\]  
(5.1)

by choosing a suitable form for the potential U(r). Both the real and imaginary parts of this complex potential are energy dependent. At low energies the real part of the potential is similar to the potential used in the nuclear-shell model. The imaginary part of the potential causes attenuation of the incident wave. Its value is chosen so as to produce the required mean free path of the incident particle in the target nucleus at various incident energies. A small value of the imaginary part of the potential is connected with a long mean free path. In the optical-model analysis of experimental results, one uses both surface- and volume-absorption terms quite often.

A customary measure of the agreement between the theoretical predictions and the experimental values is the value of chi-squared defined by

\[
\chi^2 = \frac{1}{N} \sum_{i=1}^{N} \frac{(\sigma_{fit}(\theta_i) - \sigma_{exp}(\theta_i))^2}{(\Delta \sigma_{exp}(\theta_i))^2}.
\]  
(5.2)

Here, N is the number of data points in the distribution being fitted, \( \sigma_{fit}(\theta_i) \) and \( \sigma_{exp}(\theta_i) \) are the calculated and experimental values, respectively, and \( \Delta \sigma_{exp}(\theta_i) \) is the corresponding experimental uncertainty at scattering angle \( \theta_i \). Similar expressions are used for fitting the analyzing-power angular distribution data \( A_y(\theta) \). Automatic search routines are then used to minimize \( \chi^2 \) by varying one or more of the adjustable parameters of a given optical-model potential.
The optical-model analyses of the elastic scattering data discussed in this chapter were performed using the automatic search code GOMFIL developed by Leeb [Le 84]. The data analyses were performed using a complex potential of the form

\[
V(r) = V_o(r) - V_n(1 + e^{x/\alpha})^{-\alpha} - iW_o(1 + e^{y/\beta})^{-\beta} + iW_D(d/dy)
\]

\[
(1 + e^{y/\beta})^{-\beta} - 2\left(\frac{\hbar}{m_\pi c}\right)^2 (V_{so} f_{so}(r) + iW_{so} f_{so})(\vec{L} \cdot \vec{S}),
\]

(5.3)

with

\[
x = (r - r_0 A^{1/3})/a_0 \quad \text{and} \quad y = (r - r_w A^{1/3})/a_w.
\]

The Coulomb potential has the form

\[
V_c(r) = Z_p Z Te^2 (3 - (r/R_c)^2)/(2R_c), \quad \text{for} \quad r < R_c = r_c A^{1/3}, \quad \text{and}
\]

\[
= Z_p Z Te^2 /r, \quad \text{for} \quad r > R_c.
\]

(5.4)

The exponents \(\alpha, \beta\) of the nuclear potential form factors can be chosen different from unity to modify the real or imaginary central Woods-Saxon (WS) potential form (\(\alpha = 1\)) or squared Woods-Saxon (WS\(^2\)) potential form (\(\alpha = 2\)) without changing the asymptotic fall-off which remains as \(\sim e^{-r/\alpha}\).

The real and imaginary spin-orbit form factors are of the conventional Thomas type and given by

\[
f_{so}(r) = (1/r)(d/dr)(1 + e^s)^{-1}, \quad \text{with} \quad s = (r - r_{so} A^{1/3})/a_{so},
\]

where \(r_{so}, a_{so}\) may be chosen differently for \(V_{so}\) or \(W_{so}\).
In this chapter the description of the measured elastic scattering data of protons from $^{87}$Sr, deuterons from $^{86}$Sr and $^{206}$Pb, and $^3$He from $^{205}$Tl is discussed in the framework of the optical model. The data on $\vec{p} + ^{87}$Sr and $d + ^{86}$Sr are needed for a study of the $^{87}$Sr($\vec{p},d)^{86}$Sr reaction, and the data on $d + ^{206}$Pb and $^3$He + $^{205}$Tl are needed in the study of the $^{206}$Pb($d,^3$He)$^{205}$Tl reaction. The energy for the elastic scattering studies in the exit channels are chosen to match the center of mass energy of the time-reversed reaction, taking into account the reaction Q values.

The optical-model parameters are also characterized by the volume integral per nucleon defined by

$$\left(\frac{J}{A}\right) = -\frac{1}{A} \int V(r)d^3r.$$  \hspace{1cm} (5.5)

In the fitting procedure, the influence of the cross-section data on the fits would have been much greater than the influence of the analyzing-power data, because of the smaller uncertainties in the cross-section data in comparison to those for the analyzing-power data. Therefore, in the present analyses, the uncertainty $\Delta\sigma_{xp}(\theta_i)$ for the differential cross section data has been taken as 2% of the cross-section value for all those angles at which the actual error is less than 2%. On the other hand, a constant absolute error of 0.015 is assigned to all analyzing-power data for all angles at which this absolute error is less than 0.015. By using such a different weighting for $\sigma(\theta)$ and $A_y(\theta)$, the fits to $A_y(\theta)$ have been improved considerably with almost no visible change in the fits to $\sigma(\theta)$; this result is expected from the low sensitivity of the differential cross section to the spin-orbit potential parameters. Non-relativistic kinematics were used throughout the optical-model analyses.
It is always advantageous to have a reasonable set of starting values for the optical-model parameters and then proceed with their optimization in fitting the measured elastic scattering data. In practice, a good way to start an optical-model analysis is to use parameters generated from a global optical potential. The term "global" here means that the potential parameters are based on fits to a wide range of targets and bombarding energies, and are expressed as a function of incident projectile energy and/or target mass number A. Although global optical potentials show the general trends in angular distributions for a particular target or energy, they often do not fit the data and hence may not give the correct elastic wave function for the specific reaction channel.
5.3 ANALYSIS PROCEDURES

5.3.1 Proton elastic scattering from $^{87}\text{Sr}$

The angular distributions of cross section and analyzing power of elastically scattered protons from $^{87}\text{Sr}$ were measured at a proton energy of 94.2 MeV. Data were taken from $9^\circ$ to $40^\circ$ in $1^\circ$ steps and from $40^\circ$ to $90^\circ$ in $2^\circ$ steps. The experimental data are shown in Fig. 5.1. The values of $\sigma(\theta)$ show periodic oscillations with a sharp fall off at small angles and a gradual decrease in magnitude at large angles. Values of $A_\nu(\theta)$ show a sharp negative peak near $20^\circ$ and are positive beyond that angle. The actual values of $\sigma(\theta)$ and $A_\nu(\theta)$ of p+$^{87}\text{Sr}$ are listed in Appendix A-1.

The optical-model analysis of the proton elastic scattering data was carried out by choosing different sets of starting parameters from different sources in the literature until an acceptable fit was obtained. The global optical-model potential parameters from the work of Schwandt et al. [Sc 82], listed as set P0 in Table 5.1, does not reproduce the data as can be seen from Fig 5.1. This particular set of parameters does not contain the surface imaginary part of the potential. Choosing this set as starting parameter set, a search was made on all the parameters without constraint on any of them. The resulting fit is shown in Fig. 5.2, and the parameters thus obtained are listed as set P1 in Table 5.1.

As a next step, proton parameter set II of Table 1 from Ref [De 87] were used as starting parameters. This particular set contains not only the surface imaginary part of the potential, but also has different geometry parameters as compared to the volume imaginary part of the potential. The geometry for volume and surface imaginary parts were kept different and the parameters were
varied without any restriction. This in fact gave the most acceptable fit to the data. The fit to the data is displayed in Fig. 5.3 and the parameters are listed as set P2 in Table 5.1.

Another global set of proton parameters widely used are from Ref [Ba 69]. These parameters were extrapolated to 94.2 MeV, although this proton energy is slightly above the range of prescriptions of Ref [Be 69], and were then used as a starting set in order to see the influence of a different set of global potential parameters. This set does not include the imaginary spin-orbit term. The fit obtained starting with these parameters is shown in Fig. 5.4, and the parameters are listed as set P3 in Table 5.1. Both cross-section and analyzing-power data are poorly described by this potential.
Table 5.1. Optical-Model Parameters for Proton Elastic Scattering from $^{87}\text{Sr}$ at 94.2 MeV

<table>
<thead>
<tr>
<th>$V_R$ (MeV)</th>
<th>$r_0$ (fm)</th>
<th>$a_0$ (fm)</th>
<th>$W_S$ (MeV)</th>
<th>$W_D$ (MeV)</th>
<th>$r_w$ (fm)</th>
<th>$a_w$ (fm)</th>
<th>$V_{so}$ (MeV)</th>
<th>$r_{so}$ (fm)</th>
<th>$a_{so}$ (fm)</th>
<th>$W_{so}$ (MeV)</th>
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<th>$a_{ws}$ (fm)</th>
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<th>$J/A$ (MeV-fm$^3$)</th>
<th>$\sigma_r$ (mb)</th>
<th>$\chi^2$/point</th>
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<td>7.00</td>
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<td>1.424</td>
<td>0.5555</td>
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<td>0.6736</td>
<td>7.599</td>
<td>-</td>
<td>1.466</td>
<td>0.4974</td>
<td>4.93</td>
<td>1.079</td>
<td>0.656</td>
<td>-1.143</td>
<td>1.048</td>
<td>0.422</td>
<td>1.25</td>
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<td>11</td>
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<td>26.89</td>
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<td>2.80</td>
<td>1.331</td>
<td>0.5271</td>
<td>3.99</td>
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<td>0.620</td>
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<td>0.442</td>
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<td>267</td>
<td>1174</td>
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<td>0.7712</td>
<td>6.24</td>
<td>4.30</td>
<td>1.072</td>
<td>0.7146</td>
<td>2.98</td>
<td>1.302</td>
<td>0.734</td>
<td>-0.645</td>
<td>0.819</td>
<td>0.377</td>
<td>1.30</td>
<td>271</td>
<td>957</td>
<td>32</td>
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</table>

$^a$ 4$W_D$ is used in DWUCK5.

$^b$ 4$V_{so}$ is used in DWUCK5.

$^c$ 4$W_{so}$ is used in DWUCK5.

$^d$ Here $W_S$ has different geometry viz., $r_0=1.183$ fm, and $a_0=1.089$ fm.
Fig. 5.1. Optical-model fit to proton elastic scattering data from $^{87}$Sr at $E_p=94.2$ MeV, obtained using global optical potentials of Ref. [Sc 82] listed as P0 in Table 5.1.
Fig. 5.2. Optical-model fit to proton elastic scattering data from $^{87}$Sr at $E_p=94.2$ MeV, obtained using global optical potentials of Ref. [Sc 82] as starting parameters, and varying all of them without any constraints. Parameters are listed as set P1 in Table 5.1.
Fig. 5.3. Optical-model fit to proton elastic scattering data from $^{87}$Sr at $E_p = 94.2$ MeV, obtained using set II optical-model parameters of Ref. [De 87] as starting parameters and following the same strategy in keeping different geometry for volume and surface imaginary parts of the potential. Best fit parameters are listed as set P2 in Table 5.1.
Fig. 5.4. Optical-model fit to proton elastic scattering data from $^{87}$Sr at $E_p=94.2$ MeV, obtained using global optical potentials of ref. [Be 69] as starting parameters, and varying all of them without any constraints. Parameters are listed as set P3 in table 5.1.
5.3.2 Deuteron elastic scattering from $^{86}\text{Sr}$ and $^{206}\text{Pb}$

Elastic scattering measurements of both cross section and analyzing power were made out to large angles ($\sim 125^\circ$) with vector-polarized deuterons of energy 88.0 MeV incident on $^{86}\text{Sr}$ target and of 79.4 MeV incident on $^{206}\text{Pb}$ target. The deuteron energy of 88.0 MeV on $^{86}\text{Sr}$ was chosen to match the center-of-mass energy for the time-reversed $^{87}\text{Sr}(p,\alpha)^{86}\text{Sr}$ reaction, after taking into account the reaction Q value. The measured angular distributions for d+$^{86}\text{Sr}$ are shown in Fig. 5.5 and for d+$^{206}\text{Pb}$ in Fig 5.10. The pattern of the angular distributions in both cases look similar. In the case of $^{86}\text{Sr}$, the oscillations in $\sigma(\theta)$ damp out quickly around 50° while in the case of $^{206}\text{Pb}$ the oscillations prevail up to 70°. The oscillations in $A_y(\theta)$ are more pronounced in the case of $^{206}\text{Pb}$ than in the case of $^{86}\text{Sr}$ . At large angles the analyzing-power data reach large positive values. The actual values for $\sigma(\theta)$ and $A_y(\theta)$ of d+$^{86}\text{Sr}$ and d+$^{206}\text{Pb}$ are listed in Appendices A-2 and A-3 respectively.

The fits to deuteron elastic scattering using the global deuteron optical-model parameters from Ref. [Da 80] are shown in Figs. 5.5 and 5.10; these sets of parameters are listed as set D0 in Table 5.2 for $^{86}\text{Sr}$ and in Table 5.3 for $^{206}\text{Pb}$ . In both the cases the optical-model predictions overestimate the cross section and underestimate the analyzing power at large angles. However, they do reproduce the oscillations to a considerable extent. Starting with these global sets of parameters, and keeping the shape of the potential as the Woods-Saxon (WS) form, all the parameters were varied during the search. The resulting fits are shown in Figs. 5.6 and 5.11 for the two nuclei; the best-fit parameters are listed in Table 5.2 and 5.3 as set D1. As seen in the figures, using only the Woods-Saxon form of the potential, the calculated values of $A_y(\theta)$ do not reproduce the
oscillatory structure at small angles for \( d+^{86}\text{Sr} \) and fall far below the measured values for \( d+^{206}\text{Pb} \) at large angles. As a next step, an imaginary spin-orbit (ISO) term was added to the Woods-Saxon form of the potential (WS+ISO), and an extensive 13-parameter searches were made for both \( ^{86}\text{Sr} \) and \( ^{206}\text{Pb} \). This form of the potential did not change the description of \( d+^{86}\text{Sr} \) data (see Fig 5.7) to a great extent but gave a better description of both \( \sigma(\theta) \) and \( A_y(\theta) \) for the \( d+^{206}\text{Pb} \) data as shown in Fig. 5.12. The best-fit parameters obtained with WS+ISO form of potential are listed as D2 in Tables 5.2 and 5.3 respectively for \( d+^{86}\text{Sr} \) and \( d+^{206}\text{Pb} \).

In order to investigate the influence of the radial shape of the potential form on the fit to the elastic scattering data, the shape of the potential was changed from WS form to a squared Woods-Saxon (WS2) form. This alteration in the form of the potential did not produce any significant change in the quality of the fit to the data compared to that of the WS form of the potential. The optical-model parameters obtained with the WS2 shape are listed as set D3 in Tables 5.2 and 5.3 for \( d+^{86}\text{Sr} \) and \( d+^{206}\text{Pb} \) respectively, and the fits are shown in Figs. 5.8 and 5.13. Finally, an imaginary spin-orbit term was added to the WS2 form of the potential. With this form of the potential (WS2+ISO) not much of an improvement was seen in the case of \( d+^{86}\text{Sr} \), but there was considerable improvement in the quality of fit at large angles for \( A_y(\theta) \) in the case of \( d+^{206}\text{Pb} \). The best optical-model fits thus obtained with the WS2+ISO form of potential are shown in Figs. 5.9 and 5.14 respectively for \( d+^{86}\text{Sr} \) and \( d+^{206}\text{Pb} \). A common feature of all the distributions shown in the figures is the poor description of analyzing-power data at smaller angles in the case of \( d+^{86}\text{Sr} \) and at larger angles in the case of \( d+^{206}\text{Pb} \).
<table>
<thead>
<tr>
<th>Set</th>
<th>Type</th>
<th>$V_R$ (MeV)</th>
<th>$r_0$ (fm)</th>
<th>$a_0$ (fm)</th>
<th>$W_S$ (MeV)</th>
<th>$W_D$ (MeV)</th>
<th>$a_w$ (fm)</th>
<th>$V_{so}$ (MeV)</th>
<th>$r_{so}$ (fm)</th>
<th>$a_{so}$ (fm)</th>
<th>$W_{so}$ (MeV)</th>
<th>$r_{wa}$ (fm)</th>
<th>$a_{wa}$ (fm)</th>
<th>$r_c$ (fm)</th>
<th>$J/A$ (MeV-fm$^3$)</th>
<th>$\sigma_r$ (mb)</th>
<th>$\chi^2$/point</th>
</tr>
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<tr>
<td>D0</td>
<td>WS(global)</td>
<td>73.20</td>
<td>1.170</td>
<td>0.8586</td>
<td>7.81</td>
<td>6.68</td>
<td>1.325</td>
<td>0.8243</td>
<td>2.39</td>
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<td>0.660</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.30</td>
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<td>1931</td>
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<td>D2</td>
<td>WS+ISO</td>
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<td>8.45</td>
<td>7.51</td>
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<td>1.30</td>
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a $4W_D$ is used in DWUCK5.

b $4V_{so}$ is used in DWUCK5.

c $4W_{so}$ is used in DWUCK5.
Table 5.3. Optical-Model Parameters for Deuteron Elastic Scattering from $^{206}$Pb at 79.4 MeV

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<th>Set</th>
<th>Type</th>
<th>$V_R$ (MeV)</th>
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<th>$r_w$ (fm)</th>
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<th>$V_{so}$ $^b$ (MeV)</th>
<th>$r_{so}$ (fm)</th>
<th>$a_{so}$ (fm)</th>
<th>$W_{so}$ $^c$ (MeV)</th>
<th>$r_{so}$ (fm)</th>
<th>$a_{so}$ (fm)</th>
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<th>J/A (MeV.fm$^3$)</th>
<th>$\sigma_r$ (mb)</th>
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<td>0.660</td>
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<td>-</td>
<td>1.30</td>
<td>313</td>
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<td>127</td>
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<tr>
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<td>WS</td>
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<td>D2</td>
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$^a$ 4$W_D$ is used in DWUCK5.

$^b$ 4$V_{so}$ is used in DWUCK5.

$^c$ 4$W_{so}$ is used in DWUCK5.
Fig. 5.5. Optical-model fit to deuteron elastic scattering data from $^{86}$Sr at $E_d=88.0$ MeV, obtained using global optical potentials of ref. [Da 80] listed as set D0 in table 5.2.
Fig. 5.6. Optical-model fit to deuteron elastic scattering data from $^{86}{\text{Sr}}$ at $E_d=88.0$ MeV, obtained using global optical potentials of Woods-Saxon (W8) type of ref. [Da 80] as starting parameters, and varying all of them without any constraints. Parameters are listed as set D1 in table 5.2.
Fig. 5.8. Optical-model fit to deuteron elastic scattering data from $^{86}$Sr at $E_d=88.0$ MeV, obtained by using the radial shape of the optical potential as squared Woods-Saxon $(WS^2)$, and varying all the parameters without any constraints. Parameters are listed as set D3 in table 5.2.
Fig. 5.9. Optical-model fit to deuteron elastic scattering data from $^{86}\text{Sr}$ at $E_d=88.0 \text{ MeV}$, obtained by including an imaginary spin-orbit (ISO) term to WS$^2$ type (WS$^2+$ISO) of the potential and varying all the parameters without any constraints. Parameters are listed as set D4 in table 5.2.
Fig. 5.10. Optical-model fit to deuteron elastic scattering data from $^{208}\text{Pb}$ at $E_d=79.4$ MeV, obtained using global optical potentials of ref. [Da 80] listed as set D0 in table 5.3.
Fig. 5.11. Optical-model fit to deuteron elastic scattering data from $^{208}$Pb at $E_d=79.4$ MeV, obtained using global optical potentials of Woods-Saxon (WS) type of ref. [Da 80] as starting parameters, and varying all of them without any constraints. Parameters are listed as set D1 in table 5.3.
Fig. 5.12. Optical-model fit to deuteron elastic scattering data from $^{208}Pb$ at $E_d=79.4$ MeV, obtained by including an imaginary spin-orbit (ISO) term to WS type (WS+ISO) of the potential and varying all the parameters without any constraints. Parameters are listed as set D2 in table 5.3.
Fig. 5.13. Optical-model fit to deuteron elastic scattering data from $^{206}$Pb at $E_d=79.4$ MeV, obtained by using the radial shape of the optical potential as squared Woods-Saxon ($WS^2$), and varying all the parameters without any constraints. Parameters are listed as set D3 in table 5.3.
Fig. 5.14. Optical-model fit to deuteron elastic scattering data from $^{206}$Pb at $E_d=79.4$ MeV, obtained by including an imaginary spin-orbit (ISO) term to WS$^2$ type (WS$^2$+ISO) of the potential and varying all the parameters without any constraints. Parameters are listed as set D4 in table 5.3.
5.3.3 \(^3\text{He}\) elastic scattering from \(^{205}\text{Tl}\)

As there was no polarized \(^3\text{He}\) ion source at IUCF, elastic scattering measurements with \(^3\text{He}\) beam were confined only to the measurement of cross sections. \(\sigma(\theta)\) was measured on a \(^{205}\text{Tl}\) target using 78.4-MeV \(^3\text{He}\) beam between lab angles of 9° and 77°. The energy of the \(^3\text{He}\) beam was chosen to match the center-of-mass energy for the time-reversed \(^{206}\text{Pb}(\vec{d},^3\text{He})^{205}\text{Tl}\) reaction, after taking into account the reaction Q value. The magnitude of the measured cross section decreased more than eight orders of magnitude from 9° to 77°, with a smooth decrease up to 50° and then oscillations of varying magnitude up to 77°. Actual values of \(\sigma(\theta)\) are listed in Appendix A-5.

The starting parameters for the automatic searches in this case were picked from four different sources. The first set included parameters from Hyakutake et al. [Hy 80] around the mass region \(A=90\), which were extrapolated to \(A=205\); the second set from Matsuoka et al. [Ma 78] incorporated a deep potential with a surface imaginary part; the third and fourth sets from Dжалoeis et al. [Dj 78] had shallow and deep potentials with a volume imaginary part. These four different sets of parameters yielded four different best sets of parameters labelled H1, H2, H3, and H4 in Table 5.4. The fits obtained from these sets of best fit parameters are shown in Figs. 5.16 through 5.19. Besides these sets, the \(\text{WS}^2\) form of the potential was also tried; this did not alter the quality of the fit, unlike in the deuteron case. The best-fit parameters obtained with the \(\text{WS}^2\) shape of potential are listed in Table 5.4 as set H5, and the corresponding fit to the data is shown in Fig. 5.20. Although there are five different sets of \(^3\text{He}\) optical-model best-fit parameters, only one of them (H2) gave a better description of the \(^{206}\text{Pb}(\vec{d},^3\text{He})^{205}\text{Tl}\) reaction data. A more recent global \(^3\text{He}\) optical-model
parameter set of Trost et al. [Tr 87], designated as set H0 in Table 5.4, does not describe the data very well. In fact the predictions follow the data points up to 35° but go out of phase beyond 40°.
Table 5.4. Optical-Model Parameters for $^3\text{He}$ Elastic Scattering from $^{205}\text{Tl}$ at 78.4 MeV

<table>
<thead>
<tr>
<th>Set</th>
<th>Type</th>
<th>V (MeV)</th>
<th>$r_0$ (fm)</th>
<th>$a_0$ (fm)</th>
<th>$W_S$ (MeV)</th>
<th>$W_D^a$ (MeV)</th>
<th>$r_w$ (fm)</th>
<th>$a_w$ (fm)</th>
<th>$r_c$ (fm)</th>
<th>J/A (MeV.fm$^3$)</th>
<th>$\sigma_r$ (mb)</th>
<th>$\chi^2$/point</th>
</tr>
</thead>
<tbody>
<tr>
<td>H0</td>
<td>WS(global)</td>
<td>151.85</td>
<td>1.150</td>
<td>0.864</td>
<td>-</td>
<td>28.83</td>
<td>1.206</td>
<td>0.800</td>
<td>1.40</td>
<td>374</td>
<td>2652</td>
<td>189</td>
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<tr>
<td>H1</td>
<td>WS(shallow)</td>
<td>126.71</td>
<td>1.185</td>
<td>0.785</td>
<td>-</td>
<td>27.37</td>
<td>1.209</td>
<td>0.881</td>
<td>1.30</td>
<td>331</td>
<td>2813</td>
<td>2.4</td>
</tr>
<tr>
<td>H2</td>
<td>WS(deep)</td>
<td>160.89</td>
<td>1.183</td>
<td>0.738</td>
<td>-</td>
<td>30.60</td>
<td>1.169</td>
<td>0.928</td>
<td>1.30</td>
<td>412</td>
<td>2970</td>
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<tr>
<td>H3</td>
<td>WS(shallow)</td>
<td>134.57</td>
<td>1.133</td>
<td>0.860</td>
<td>15.83</td>
<td>-</td>
<td>1.570</td>
<td>0.767</td>
<td>1.30</td>
<td>318</td>
<td>2823</td>
<td>3.4</td>
</tr>
<tr>
<td>H4</td>
<td>WS(deep)</td>
<td>178.38</td>
<td>1.102</td>
<td>0.846</td>
<td>17.20</td>
<td>-</td>
<td>1.556</td>
<td>0.787</td>
<td>1.30</td>
<td>389</td>
<td>2850</td>
<td>3.3</td>
</tr>
<tr>
<td>H5</td>
<td>WS$^2$</td>
<td>138.45</td>
<td>1.319</td>
<td>0.601</td>
<td>-</td>
<td>26.72</td>
<td>1.182</td>
<td>0.951</td>
<td>1.30</td>
<td>326</td>
<td>2903</td>
<td>3.1</td>
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$^a$ $4W_D$ is used in DWUCK5.
Fig. 6.16. Optical-model fit to $^3$He elastic scattering data from $^{205}$Tl at $E_{^3\text{He}}=78.4$ MeV, obtained using global optical potentials of ref. [Tr 87] listed as H0 in table 5.4.
Fig. 5.16. Optical-model fit to $^3$He elastic scattering data from $^{205}$Tl at $E_{^3\text{He}}=78.4$ MeV, obtained using shallow optical potentials with surface imaginary part of ref. [Hy 80] as starting parameters, and varying all of them without any constraints. Parameters are listed as set H1 in table 5.4.
\( ^3\text{He} + ^{205}\text{Tl} \) Elastic Scattering; \( E_{^3\text{He}} = 78.4 \text{ MeV} \).

Fig. 5.17. Optical-model fit to \(^3\text{He}\) elastic scattering data from \(^{205}\text{Tl}\) at \( E_{^3\text{He}} = 78.4 \text{ MeV} \), obtained using deep optical potentials with surface imaginary part of ref. [Ma 78] as starting parameters, and varying all of them without any constraints. Parameters are listed as set H2 in table 5.4.
Fig. 5.18. Optical-model fit to $^3\text{He}$ elastic scattering data from $^{205}\text{Tl}$ at $E_{^3\text{He}}=78.4\text{ MeV}$, obtained using shallow optical potentials with volume imaginary part of ref. [Dj 78] as starting parameters, and varying all of them without any constraints. Parameters are listed as set H3 in table 5.4.
Fig. 5.19. Optical-model fit to $^3$He elastic scattering data from $^{205}$Tl at $E_{^3\text{He}}=78.4$ MeV, obtained using deep optical potentials with volume imaginary part of ref. [Dj 78] as starting parameters, and varying all of them without any constraints. Parameters are listed as set H4 in table 5.4.
Fig. 5.20. Optical-model fit to $^3\text{He}$ elastic scattering data from $^{205}\text{Tl}$ at $E_{^3\text{He}}=78.4$ MeV, obtained by using the radial shape of the optical potential as squared Woods-Saxon (WS²), and varying all the parameters without any constraints. Parameters are listed as set D5 in table 5.4.