CHAPTER 1
INTRODUCTION

1. Importance of Reliability

Reliability in recent years has been formulated as the science of predicting, estimating, or optimizing the probability of survival, the mean life, or, more generally, the life distribution of components or systems. During the past decades, development of reliability as a discipline has been rapid because of its ever increasing applications in government sectors, industries, business enterprises, research and development fields, etc. In our day to day life, we encounter several instances in which quality of the product or service plays a vital role. Hence, it is necessary to enforce exceptionally high standards on their performance and evaluate its reliability for an intended period of time. In view of this fact, application of reliability has become top priority to all stake holders of the society, as violation of this aspect will lead to serious disturbance in terms of cost, time and psychological effect of inconvenience.

It is universally recognized that lifetime of individuals, components, systems, etc., are unpredictable and random in nature, hence acquiescent to probabilistic and statistical laws. The development of models and methods to deal with such random variables took place in the second half of twentieth
century, although certain explicit and implicit results are from earliest times as well. With reference to the above mentioned facts, study of reliability can be broadly classified into two major aspects, that is, reliability modeling and reliability assessment.

Reliability modeling primarily deals with development of reliability model under certain predefined assumptions. Several reliability models formulated till date have obtained significant space in literature, namely stress-strength models, shock models, competing risk models, repairable models, proportional hazard models, reliability growth models, etc. This aspect also deals with configuration of associated components in the system, that is, whether components are placed in series or parallel or combination. On the counter part reliability assessment involves processes such as identification of various modes of system failure, life testing experiments, assigning parameter values to life time distribution/failure time distribution; stress-strength distributions; damage distribution; repair distribution and inferential procedures, used to evaluate the formulated reliability model. In the present study reliability modeling and assessment is carried out for stress-strength, shock and cascade models.
2. Reliability models under study and motivation

2.1. Stress-Strength Model

Stress-Strength relation is regarded as universal relation, which can be easily adapted to various fields of human endeavors and natural phenomena. It is a powerful tool for comparing and dissecting interrelated situations. The concept initially originated from a classical non-parametric problem of testing equality of two distribution functions. Further, it was realized that the same can be fruitful to examine the probability of inequality-type relation existing between two or more random variables under variety of conditions/situations.

In the simplest term, stress-strength relation can be described as an assessment of reliability for a random variable ‘X’ representing strength of the component/system and random variable ‘Y’ representing stress experienced by the corresponding component/system. Accordingly, it implies that, if the stress exceeds the strength then the corresponding entity fails or vice versa. Reliability function under stress-strength relation is defined as probability that the component/system survives \[ P(X > Y) \].

Stress-strength models have wide range of applications in areas of engineering, medicine, psychology, etc., where reliability function is evaluated in terms of various inbuilt qualities of a component/system
(strength) and the external environmental conditions (stress) which constantly act upon them. In other words, reliability evaluation is carried under the influence of external factors (stress) and by discounting the time dependent factor. This makes the model more realistic and applicable to various scenarios. Problems of deriving theoretical expression for \( P(X > Y) \) with its modifications and extensions under various distributional assumptions are challenging. Further, estimation of associated probabilities based on sample of various structures opens up new avenues in deriving approximations to variances and confidence bounds. These facts have motivated our study on stress-strength model. In the present study, we have considered estimation of reliability function for a two-component survival stress-strength model with stress and strength following exponential and gamma distributions respectively.

### 2.2. Shock Model

Among the many approaches for modeling deteriorating systems, shock models have found favor with reliability analysts as it has wide range of applications in diverse areas like stochastic clearing systems, drug administration, fatigue failure, etc. The simple problems of our day to day life as well as complicated problems of research, administration and commercial
applications brought to our attention by researchers, industrialist, health scientist, administrators, etc., have motivated our study on shock models.

Shock models provide a realistic base for modeling reliability of a system situated in random environment. According to time between consecutive shocks, the shock models are classified into four types-

- Shocks arriving as events of homogeneous Poisson process.
- Shocks arriving as events of non-homogeneous Poisson process.
- Shocks arriving as events of non-stationary pure birth process.
- Shocks arriving as events of renewal process.

Further, based on the nature of the damages caused due to the shocks, they are classified as accumulative damage shock model, non-accumulative damage shock model, general cumulative damage shock model, etc. In the present study, we have considered shock models with shocks arriving as events of a homogeneous Poisson process under accumulating as well as non-accumulating setup.

2.3. Cascade Model

The redesign approach has added a new dimension to the study of reliability modeling. Evaluation of system reliability has become an essential and integral part in deciding its performance. This is in view of the fact that
reliable and consistent performance of any system is of utmost importance not only in civilian applications but also in military, business and industrial applications, where performance can be critical most of the times.

As the complexity of a system increases, its reliability decreases unless compensatory measures are taken. System reliability can be increased by increasing the reliability of its associated components, but sometimes this cannot be achieved beyond certain limits. An alternative way to increase the reliability in such situation is to have redundant configuration of components in the system.

Cascade system is one special type of standby system. Cascade redundancy is a hierarchical standby redundancy, where an array of components (finite in number) are arranged in an order of activation. Here, the first component is active and the remaining components are at standby. The brunt of attack, in the first instance is borne by the active component. If it survives the attack, the system also survives with no loss and is ready to face the next attack. However, if the active component fails then the next component in the array has to face and withstand the ‘cushioned’ attack on it. The stress acting on the subsequent active component will be ‘$k$’ times the stress of the previous failed components, where ‘$k$’ denotes stress attenuation factor. In most of the works on cascade systems as mentioned in review of
literature, study is restricted to only stress-attenuated reliability, this featured gap has motivated us to make an attempt in defining reliability for a cascade system under the joint effect of stress as well as strength attenuation factors.

Quality improvement is a continuous process, where the limitations encountered during the functioning of a system needs to be rectified. Based on the experience of earlier failures (nature of failure, reasons for failure, frequency of failure, etc.), one can notice the weak points (resulted due to the stress) and accordingly undertake remedial measures to fine tune the factors contributing towards strength of the standby component associated with the stress-strength cascade system. This phenomena of fine tuning the factors contributing towards strength of the standby component, supports incorporation of strength attenuation factor for the cascade models under study.

3. Statistical Inference

Major part of the inference carried out for the proposed models in various chapters is dedicated to reliability modeling and estimation of reliability function. Maximum Likelihood Estimator (MLE), Bayes estimator and Uniformly Minimum Variance Unbiased Estimator (UMVUE) of the
parameters are used to obtain estimators of reliability function. Asymptotic distribution of MLE of the parameters is also obtained in a couple of chapters.

The method of maximum likelihood corresponds to one amongst the many well-known estimation methods in statistics. MLE has many optimal properties in estimation such as sufficiency (sufficient information about the parameter of interest), consistency (true parameter value used to generate data can be recovered asymptotically), efficiency (lowest possible variance of parameter estimates achieved asymptotically) and parameterization invariance (maximum likelihood solution is independent of parameterization used). In light of these properties, MLE has gained special place in reliability assessment.

Bayesian philosophy involves a completely different approach to statistical inference. The classical approach treats the parameters to be fixed but unknown, whereas Bayesian approach treats the parameters to be random following certain probability distributions. Bayes estimators are based on certain prior information obtained through some pilot study. This helps in synthesizing the information to be generated for the system under function. Bayes estimators exhibit properties like admissibility and asymptotic efficiency, which minimizes posterior expected loss and in turn maximizes posterior expected utility.
Uniformly Minimum Variance Unbiased Estimator (UMVUE) is an unbiased estimator that has lower variance than any other unbiased estimator for all possible values of the parameter. This concept has led to substantial development of statistical theory related to the problem of optimal estimation. The existence of UMVUE supports fulfillment of optimal inferential properties.

4. Review of Literature

Several authors have considered estimation of system reliability based on stress-strength model. Here are a few references of contributions towards these models. Church and Harris (1970) considered estimation of reliability for stress-strength relationship. Downton (1973) considered estimation of reliability for a stress-strength model under normal distribution. Wani and Kabe (1971) have worked on the problem of estimation of system reliability where life time of each component has gamma distribution. Constantine et.al (1986) considered estimation of stress-strength relationship under the assumption that stress-strength random variables follow gamma distribution with known shape parameter. Bhattacharya and Johnson (1974) have studied estimation of reliability for a multi-component stress-strength model. Kunchur and Munoli (1993) have considered estimation of reliability for a multi-component survival stress-strength model based on exponential

There are several shock models formulated with various natures of damages acting on the component/system. Among these Gumbel (1960), Freund (1961), Downton (1970), Hawkes (1972), Esary, Marshall and Proshan (1973), Ross (1981) have studied life distribution of a component subjected to random shocks occurring according to a Poisson process. Barlow
and Proschan (1975) have considered shock models yielding bivariate distributions. Kunchur and Munoli (1993) have considered estimation of reliability for a two-component cumulative damage shock model with damages following exponential distribution. Munoli and Surangi (2006) and (2008) have considered estimation of reliability for a two-component non-accumulative damage shock model with damages following (i.i.d and non-i.i.d) exponential and Weibull distributions.

There are studies available in literature which deal with cascade models, to name a few Pandit and Sriwastav (1975) have featured relevance of geometric distribution in the study of behavior of a cascade system. Raghavachar et.al (1983) presented a closed form solution of stress attenuated reliability function for n-cascade system when both stress and strength follow identical distributions. Uma Maheshwari et.al (1993) studied stress attenuated reliability for n-cascade system whose stress and strength follow normal and exponential distributions respectively. Rekha and Shyam Sunder (1997) have also highlighted a similar cascade system where stress and strength follow gamma and exponential distributions respectively. They showed that for higher parametric values and lower attenuation factors a high degree of reliability could be attained. Rekha and Chechu Raju (1999) endeavored to present a closed form solution of stress attenuated reliability function for n-
cascade system with exponential stress and standby strengths following Rayleigh and exponential distributions. Shyam Sunder (2012) has studied stress attenuation for cascade system when both stress and strength follow Rayleigh distribution. Gogoi and Borah (2012) have obtained inference for cascade models by considering one-parameter exponential strength and two-parameter gamma stress.

5. Summary of the Thesis

The thesis consists of six chapters, the first one being introduction. Construction of reliability models and estimation of reliability function is carried out for all the chapters in this thesis. In the present thesis, reliability modeling and reliability assessment is carried out for stress-strength, shock and cascade models. Maximum Likelihood Estimator (MLE), Bayes estimator and Uniformly Minimum Variance Unbiased Estimator (UMVUE) of the parameters are used to obtain estimators of reliability function. Asymptotic distribution of MLE of the parameters is also obtained in Chapter 2 and Chapter 6.

In Chapter 2, we have considered estimation of reliability for a two-component survival stress-strength model. The reliability function is derived in Section 2. Life testing experiment is explained in Section 3. The Maximum
Likelihood Estimator (MLE) of reliability function is obtained in the same section. Asymptotic distribution of MLE of the parameters is derived in Section 4. Bayes estimator of reliability function is obtained in Section 5. Computation and comparison of estimators of reliability function (MLE and Bayes) along with findings of the study are discussed in Section 6. The inference procedures developed for the proposed reliability model are validated through data generated by simulation techniques (using programming language ‘C’ and MATHCAD Software). These results are published in Munoli and Mutkekar (2011a).

In Chapter 3, we have considered estimation of reliability in an accumulating damage shock model for a two-out-of-three (i.i.d) component system. In this chapter, an attempt is made to estimate reliability for the proposed shock model when damages due to shocks follow exponential and geometric distributions. Section 2 deals with estimation of reliability in an accumulating damage shock model (with fixed threshold) for a two-out-of-three component system, when damages follow exponential distribution. The model is described in Section 2.1. The life testing experiment is described in Section 2.2. The MLE of reliability function is obtained in the same section. Bayes estimator of reliability function is obtained in Section 2.3. Computation and comparison of estimators of reliability function (MLE and Bayes) along
with findings of the study are discussed in Section 2.4. The inference procedures developed for the proposed reliability model are validated through data generated by simulation techniques (using programming language ‘C’). These results are incorporated in Munoli and Mutkekar (2012). Section 3 deals with estimation of reliability in an accumulating damage shock model (with fixed threshold) for a two-out-of-three component system, when damages follow geometric distribution. The model is described in Section 3.1. The life testing experiment is described in Section 3.2. The MLE of reliability function is obtained in the same section. Bayes estimator of reliability function is obtained in Section 3.3.

In Chapter 4, we have considered estimation of reliability in a non-accumulating damage shock model for a two-out-of-three (i.i.d) component system. In this chapter, an attempt is made to estimate reliability for the proposed shock model when damages due to shocks follow exponential and Weibull distributions. Section 2 deals with estimation of reliability in a non-accumulating damage shock model (with fixed threshold) for a two-out-of-three component system, when damages follow exponential distribution. The model is described in Section 2.1. The life testing experiment is described in Section 2.2. The MLE of reliability function is obtained in the same section. Bayes estimator of reliability function is obtained in Section 2.3. Computation
and comparison of estimators of reliability function (MLE and Bayes) along with findings of the study are discussed in Section 2.4. The inference procedures developed for the proposed reliability model are validated through data generated by simulation techniques (using programming language ‘C’). These results are presented in Munoli and Mutkekar (2011b). Section 3 deals with estimation of reliability in a non-accumulating damage shock model (with fixed threshold) for a two-out-of-three component system, when damages follow Weibull distribution. The model is described in Section 3.1. The life testing experiment is described in Section 3.2. The MLE of reliability function is obtained in the same section. Bayes estimator of reliability function is obtained in Section 3.3.

In Chapter 5, we have considered estimation of reliability in a non-accumulating damage shock model for a two (i.i.d) component system when shocks are arising from two sources. In this chapter, an attempt is made to estimate reliability for the proposed shock model when damages due to shocks follow exponential and Weibull distributions. Section 2 deals with estimation of reliability in a non-accumulating damage shock model (with fixed threshold) for a two-component parallel system, when damages follow exponential distribution. The model is described in Section 2.1. The life testing experiment is described in Section 2.2. The MLE of reliability function
is obtained in the same section. Bayes estimator of reliability function is obtained in Section 2.3. Section 3 deals with estimation of reliability in a non-accumulating damage shock model (with fixed threshold) for a two-component parallel system, when damages follow Weibull distribution. The model is described in Section 3.1. The life testing experiment is described in Section 3.2. The MLE of reliability function is obtained in the same section. Bayes estimator of reliability function is obtained in Section 3.3.

In Chapter 6, we have considered estimation of reliability for stress-strength cascade model. Cascade model with more number of standby components is not recommended in the present study as the strength goes on depleting with the order of standby. In view of this fact, we have considered estimation of reliability for a (1+1) cascade model where the first component is active (working) and the second component is standby. The reliability function for a (1+1) cascade system is derived under the joint effect of stress as well as strength attenuation factors. Section 2 deals with estimation of reliability for a (1+1) cascade system. The model is described in Section 2.1. The life testing experiment is described in Section 2.2. The MLE of reliability function and reliability estimator using Uniformly Minimum Variance Unbiased Estimators (UMVUEs) of the parameters is obtained in the same section. Asymptotic distribution of MLE of the parameters is derived in
Section 2.3. Computation and comparison of estimators of reliability function (using MLEs and UMVUEs of the parameters) along with the findings of the study are discussed in Section 2.4. The inference procedures developed for the proposed reliability model are validated through data generated by simulation techniques (using programming language ‘C’ and MATHCAD Software). These results are included in Mutkekar and Munoli (2014). Section 3 deals with estimation of reliability for a (2+1) cascade system. The model is described in Section 3.1. The life testing experiment is described in Section 3.2. The MLE of reliability function is obtained in the same section. Bayes estimator of reliability function is obtained in Section 3.3.