Chapter-V

SUPER RESOLUTION IMAGE RECONSTRUCTION WITH AIT
5.1 Introduction

After acquiring the low resolution image from MRI machine or low resolution observation model, the next step is to super resolve the images by extracting its features. This chapter provides detail experimentation, methodology and implementation techniques of super resolution reconstruction of image sequences.

It also presents Adaptive iterative transformation based model for low resolution image reconstruction and its results.

5.2 Adaptive Signal Processing

Adaptive filters are class of filters which changes their characteristics according to the values of the grayscale under the mask. It acts like a median filter or average filter depending on their position within the image. Filters are used to clean Gaussian noise by using local statistical properties of the values under the mask. This nonlinear spatial filter, is implemented by applying a function to the gray scale values under the mask.

5.3 Adaptive Iterative Transformation

The process of adapting the new threshold value with number of iterations for reconstruction of image is called as adaptive iterative transformation. It used for reducing computational load by adapting new value of threshold alpha with number of iterations every time for reconstruction of image while producing restored image of high visual quality. There are specific advantages of using adaptive algorithm; i) It used to avoid complexity of large scale equations. ii) The iterative process does not destroy the structure of coefficient matrix. Here proposed degradation model is given by an equation 5.1.

\[ Y = Dx + h \] (5.1)
Where the vector \( x \) and \( y \) represent original image and degraded image respectively and \( h \) is the observational noise. In our work matrix \( D \) represent a space in variant distortion and it is assumed to be block circulant. The equation 5.1 involves a circular convolution and can be implemented in the discrete frequency domain using discrete Fourier transform to find an image as close as possible to the original image. With the subject to a suitable criterion \( y \), \( D \) gives knowledge about noise. The Adaptive iterative approach is followed in solving the reconstruction problems.

### 5.4 Post Processing

Post processing is applied on an image after the preprocessing steps. This is basically essential to improve the quality of output image. The detailed processing steps are explained as below.

#### 5.4.1 MR image with Fast Fourier Transform (FFT)

The Fourier transform has fundamental importance to image processing. It allows us to perform tasks which would be impossible to performed by any other way. Its efficiency allows us to perform other task more quickly. The basic aim of this algorithm is to find the energy of pixel and high and low frequency components in an image. This algorithm is applied after smoothing the image. Figure 5.1 (a) (b) (c) shows high, low frequency components and FFT of Sample image respectively.

**Algorithm:**

1. Find real and imaginary part of image
2. Ignore imaginary part
3. Apply mask of 3x3 \([-1 0 1; -1 4 -1; 0 -1 0]\)
4. Find low and high frequency component
5.4.2 Thresholding and construction of new value

Here the adaptive value is called as alpha. Alpha is used to adjust data items, its value directly affects the recovery image results. If alpha value is too small it will not effectively remove high frequency noise, which causes unclean image.

The alpha value is given by equation

\[ \alpha = K \frac{\mu}{\sigma} \]

Where, \( \mu \) = Variance
\( s \) = Standard deviation
\( K \) = Scaling factor

We have applied constant value which is defined by \( K \), where \( K \) is scaling factor. It varies in the range of 1 to 45. It may be fixed and or vary approximately up to 25. The better performance is achieved for value of \( k \) in the range of 20 to 25.

Our task is to find new alpha value which can be calculated by \( k \) value and masking to resulted value, considering as new alpha. New alpha ranges from 0.001 to 0.2. By this new alpha value final output image is reconstructed that is the SR image.
Figure 5.2 shows an example of alpha images created when alpha varies from minimum to maximum value with number of iterations. The resultant sample images with different values of alpha and $K$ are shown in figure 5.2 (a) and (b).

![Figure 5.2](image)

(a) (b)

**Figure 5.2: Different alpha value images, (a) at alpha=0.12, k=15 (b) at alpha=0.18 and k=25.**

### 5.4.3 Convolution of FFT

The result provides one of the most powerful advantages of convolution method by using DFT. Here we convolve an image $M$ with a spatial filter $S$. Our method places $S$ over each pixel of $M$ in turn, calculate the product of all corresponding gray values of $M$ and elements of $S$, and add the results. The results is called digital convolution of $M$ and $S$, and is denoted as $M * S$. This method of convolution can be very slow, especially if $S$ is large. That’s why we have used here a adaptive iterative methods for correct results. This convolution theorem states that the result $M * S$ can be obtained by the following sequence of steps
5.4.4 Image Reconstruction by Separability

By adapting new value which is used to reconstruct the image by 3x3 mask. Separability is a time saving process. A 3x3 mask $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$ is separable in two smaller filters by rows and columns. The averaging filter can be implemented by first applying a 3x1 averaging filter and then applying 1x3 averaging filter to result matrix.

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \end{bmatrix}$$

\]

Figure 5.3: Separability (a) Original image, (b) Rows Separability of (a), (c) Column Separability of (a)
The above matrix shows the separability process applied to given mask and figure 5.3 (a) (b) and (c) respectively shows how rows and column multiplied with new reconstructed alpha values.

A complete result of pre and post processing of two sample images are as shown in figure 5.4. The details of images are (a) Low resolution Image (Blur) (b) Low resolution image (noisy) (c) FFT of image (d) Super resolved image of 256x256 (e) Super resolved image of 512x512.

Even after changing the size of the super resolved image up to multiplier factors 2 we are getting the same resolution as shown in figure 5.4(e).
5.5 **Super Resolution Model**

Generating a high-resolution image from a single low-resolution image with the help of one or more training set of images from scenes of the same or different types. Commonly this is referred as the single-image super-resolution problem. Proposed work is divides in to two models. They are as follows,

a) Observation model
b) Super resolution reconstruction model

Observation model is explained in detail in previous chapter IV. Here the detail explanation about SRR model is as given below.

5.5.1 **Super Resolution Reconstruction Model**

![Figure 5.5 Super Resolution Reconstruction Model](image)

The above Figure 5.5 shows super resolution reconstruction model. Here Observed low resolution image sequence $g(x, y)$, is reconstructed with the help of adaptive iterative transformation algorithm and compared the output images with other methods. An adaptive iterative transformation expression for obtaining $f^*(x, y)$ is high resolution image as follows

$$f^*(x, y) = g(x, y) - ((s^2_h/s^2_L)[g(x, y) - M_L])$$  \hspace{1cm} (5.2)

Our proposed algorithm operates on a local region $S_{xy}$. Response of the algorithm is based on four quantities they are as follows.
i) \( g(x,y) \) the value of noisy image at \((x,y)\)

ii) \( s^2 \) the variance of the noise corrupting \( f(x,y) \) to form \( g(x,y) \)

iii) \( M_L \) the local mean of the pixels in \( S_{xy} \)

iv) \( s^2_L \) the local variance of the pixels in \( S \)

The various steps in SR reconstruction are described in the following sections.

i) **Registration**

In super resolution reconstruction, the preprocessing task of utmost importance is accurate registration of acquired images. It is process of overlaying two or more images of the same scene taken at different time, different viewpoints and or different machines. Typically one image called the base image is considered as reference, to which the other images called input images are compared. The objective is to bring the input image in alignment with the base image by applying spatial transformation to the input image. Spatial transforms maps location in one image into new location in another image. Image registration is an inverse problem as it tries to estimate from sampled image.

It also depends on properties of the machine used for image acquisition like sampling rate, imperfection of the lens that adds blur and the noise in acquired image. As the resolution decreases, local two dimensional structure of an image degrades and registration of two low resolution of images becomes difficult. Super resolution reconstruction requires registration of high quality. The registration technique considered in our work is based on fast Fourier transform.

For the registration of the images block techniques are used. The block matching techniques interpolates to given input image \( N \) times, compare image block of one MRI image to the
other MRI image and then determines the displacement of 1/N pixels accuracy which gives the best similarity between the two blocks.

**ii) Restoration**

Image restoration is the process of the removal or reduction of the degradation which occurred during the acquisition process. It may include noise, error in the pixel values or optical effect or machine effect such as out of focus or blurring or blurring due to patient movement or breathing. There are different methods here we have discussed for restoration using neighborhood operation wavelet etc. But these techniques are complex and time consuming so here we have proposed the adaptive iterative transformation technique for restoration. The detail steps and algorithm is explained below.

**5.6 Adaptive Iterative Transformation based SRR Algorithm**

The proposed algorithm is explained below:

1) Register input image
2) Find FFT of image
3) Calculate the alpha values
4) Update or calculate new alpha values to next iteration
5) With new alpha perform block by block operation
6) Apply IFFT to reconstruct image with new alpha
7) Convert image values from double to unit 8

The details of above algorithms steps are explained below

**Step-1:** Low resolution (blurred, noisy, under sampled) images are considered as an input.

**Step-2:** The images are registered using FFT algorithm and calculate the FFT of image

1-D DFT of a signal x[n] over the interval [0: N-1]
$$X(k) = X(2\pi\frac{k}{N}) = \sum_{n=0}^{N-1} x[n] e^{-j2\pi\frac{k}{N}}$$

Where \(k=0,1,2,\ldots,N-1\)

1-D IDFT of sequence \(x(k)\) gives a sequence \(x(n)\) on the interval [0:N-1]

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j2\pi\frac{k}{N}}$$

Apply this to an image \(f(x,y)\) of size \(MxN\)

$$S(k_x, k_y) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f[x,y] e^{-j2\pi\left(\frac{k_x x}{M} + \frac{k_y y}{N}\right)}$$

The 2D DFT is

$$f(x,y) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} S[k_x, k_y] e^{j2\pi\left(\frac{k_x x}{M} + \frac{k_y y}{N}\right)}$$

Where \(k_x, k_y\) are frequency variables and \(x, y\) are spatial variables

**Step 3:-** Calculate Fourier spectrum, phase angle, power spectrum of an image

1) Fourier spectrum

$$|s(k_x, k_y)| = |R^2(k_x, k_y) + I^2(k_x, k_y)|$$

2) Phase angle

$$\phi(k_x, k_y) = \tan^{-1}\left[\frac{I(k_x, k_y)}{R(k_x, k_y)}\right]$$

3) Power Spectrum

$$P(k_x, k_y) = |s(k_x, k_y)|^2$$
Step 4:- Calculate threshold value \( a \)

\[
\alpha = K \frac{\mu}{\sigma}
\]

Where, \( \mu \) = Variance

\( s \) = Standard deviation

\( K \) = Scaling factor

Step 5:- Construct new threshold (\( a \)) value for next iteration

New \( a = a \times 1 - cc (i, j) / \text{Maxcc} \)

Step 6:- Use property of separateability for block by block process for image reconstruction

\[
S (k_x, k_y) = \sum_{x=0}^{M-1} e^{-j2\pi \frac{k_x x}{M}} \sum_{y=0}^{N-1} f [x, y] e^{-j2\pi \frac{k_y y}{N}}
\]

\[
= \sum_{x=0}^{M-1} S [x, k_y] e^{-j2\pi \frac{k_y y}{N}}
\]

\[
S (k_x, k_y) = \sum_{y=0}^{N-1} f [x, y] e^{-j2\pi \frac{k_y y}{N}}
\]

Step 7:- Apply inverse FFT to reconstruct image with New Alpha

Step 8:- Rescale the image to Super Resolution level

Step 9:- Calculate the parameters of image PSNR, MSE, Time, etc [29].
\[ \text{PSNR} = 10 \log_{10} \left( \frac{p^2}{\sqrt{MSE}} \right) \]

\[ \text{MSE} = \frac{1}{MN} \sum_{m=1}^{M} \sum_{n=1}^{N} \left[ f^\wedge(x, y) - g(x, y) \right]^2 \]

5.7 Experimentation

We have created two databases namely (i) MRI_DB1 with 100 images, 10 each of same patient with different views (Top view, side view, back side view). Each image is JPEG, 8 bit gray scale, 512x512 with 96 dpi resolution. (ii) MRI_DB2 of 50 images, 10 each of same patient with different views (Top view, side view, back side view). Each image is JPEG, 8 bit gray scale, 256x256 with 96 dpi resolutions.

All these images are collected from Asian Heart Institute Mumbai, approximately 90% of the images are of good quality, while about 10% of the images are found of poor quality which are mainly due to limitation of MRI machine or cuffing and breathing problem of patient. Set of a few sample images acquired with 1.5T MRI machine from the databases MRI_DB1 and MRI_DB2 are given in appendices 4.2 and 4.3 respectively. Implementation of various processing algorithm is discussed in chapter IV and V.

After applying Observed Low resolution image, pre processing and post processing algorithms on these image, the final image is marked with the super resolved image and is ready for diagnosis or verification. Additional results of pre and post processing on few sample images from the data bases MRI_DB1 and MRI_DB2 are given in appendices 5.1 and 5.2 respectively. The figure 5.6 shows front end of our system.
• GUI of proposed system

![GUI of proposed system](image)

**Figure 5.6: Typical front end of our system**

- An Iterative Adaptive based algorithm is developed implemented and the results are compared.
- Out of number of pixels and interpixels are extracted and marked on image which is used for resolution.
- Steps for resolution: we first apply initial level filtering and cropping the area of interest from image. Afterwards selecting the scaling factor and zooming the image for super resolution.
- The results are prepared in the form of tables to compare with different methods.
- The results of the images after application of particular algorithm are also prepared with numerical data and graphs.

**5.8 System Environment**

The system requirements related to work are as given below:

- The Machine: Hardware computing platform used is ACER P-IV, Dual core, desktop PC with 512 MB RAM with standard accessories.
- The software development platforms used is Matlab7.9 with Image processing Toolbox and its associated development tools/environment.
- After proper understanding the mathematical intricacies, the algorithms are implemented in software for various operations like pre-processing, noise removal, and post processing.
5.8.1 Parameter estimation

To compare different algorithms of super resolution several schemes of measuring the quality of image have been used. In this study we have used MRI images which force us to use different performance measure so as to benchmark our result. The evaluation of performance for super resolution algorithm includes PSNR, MSE, SNR and threshold value. The computational complexity of algorithm is part of practical implementation consideration.

i) Peak Signal to Noise Ratio

In this study PSNR and MSE as quality measuring parameter in case of MR images.

Formula to calculate PSNR is given as:

$\text{PSNR} = 10 \log \left( \frac{p^2}{\text{MSE}} \right)$

For MxN images $p$ is maximum possible value of an element of image i.e. 255 in an 8 bit image.

ii) Mean Square Error

$\text{MSE} = \frac{1}{MN} \sum_{m=1}^{M} \sum_{n=1}^{N} \left[ f^*(x, y) - g(x, y) \right]^2$

Where $g(x, y)$ is source image and $f^*(x, y)$ is reconstructed image which contains MxN pixels. MSE is Mean Squared Error between original $g(x, y)$ and reconstructed $f^*(x, y)$. 
5.8.2 Results of Proposed Model

The Adaptive Iterative Transformation based Super Resolution Reconstruction Model used for restoration of low resolution images captured from 1.5T machines, gives the PSNR, SNR, MSE as given in table 5.1 and 5.2 respectively for different threshold alpha values.

- Input image of 512x512 & Reconstructed image of 1024x1024 by taking Mean = 0, Variance = 0.005 and $\sigma = 2$

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Adaptive Iterative Transformation Based Super Resolution Reconstruction of Medical Image Sequences

(a)

(b)

(c)
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