CHAPTER V

POLARIZATION OF ELASTICALLY SCATTERED GAMMA RAYS

1. Introduction

2. Theoretical considerations
   i) Polarization of Rayleigh scattering from K-shell electrons
   ii) Contribution of Nuclear Thomson scattering
   iii) Contribution of L-shell electrons

3. Gamma ray Polarimeter
   i) Compton scattering as the analysing process
   ii) Polarization analyser
   iii) Sensitivity of the analyser
   iv) Calibration of the analyser
   v) Design of the Polarimeter
   vi) Performance of the Polarimeter

4. Experimental measurement of the polarization of elastically scattered gamma rays
   i) Experimental method
   ii) Contribution of elastic and inelastic scattering to the scattered radiation
   iii) Method and results
   iv) Influence of inelastic scattering on the polarization of elastic scattering of gamma rays.

5. Conclusions

List of Author's publications

Bibliography

Acknowledgements
CHAPTER V

Polarization of Elastically Scattered Gamma Rays.

1. Introduction.

It was pointed out in chapter I, that even though the results of the previous experimental measurements on the linear polarization of the elastically scattered gamma rays showed the superiority of the refined calculations over those of the form factor methods, yet the experimental results were found to be consistently lower than those predicted by the refined calculations. This discrepancy was attributed by the previous workers (50, 51) to the contribution of L-shell electrons, for which exact calculations do not exist. In an effort to resolve this discrepancy between the experiment and theory, the polarization of the elastic scattering of 662 KeV gamma rays from lead at scattering angles $64^\circ$, $90^\circ$ and $120^\circ$ was measured and the actual measurements of the polarization are described in this chapter.

2. Theoretical Considerations.

1) Polarization of Rayleigh Scattering from K-shell Electrons.

The percentage polarization as defined by:

$$P = \frac{J_\perp - J_\parallel}{J_\perp + J_\parallel} \times 100$$
can be written in terms of the scattering amplitudes of Rayleigh scattering as,

\[ P = -\frac{2(M(1,2') + M(2',1))}{[M(1,2') + M(2',1)]^2 + [M(1,2') + M(2',1)]^2} \times 100 \]  

where \( J_\parallel \) and \( J_\perp \) are the intensities of the linear polarization with the electric vector parallel and perpendicular to the scattering plane respectively. The values of \([M(1,2') + M(2',1)]\) and \([M(1,2') + M(2',1)]\) which represent the scattering amplitudes of Rayleigh scattering with and without the polarization change, respectively, are given in the papers of Brown et al. (5) both for the refined and the form factor calculations for gamma rays of energies 1.29 and 2.55 MeV as a function of the scattering angle.

The percentage polarization of the Rayleigh scattering from the K-shell electrons in mercury as calculated from equation 5.1, using refined calculations for gamma rays of energies 1.29 and 2.55 MeV at various scattering angles is compared with that obtained from the form factor calculations in figure 5.1. It is seen that the two calculations give quite different variation of the percentage polarization with the scattering angle and gamma ray energy. According to the form factor calculations, polarization of the Rayleigh scattering of gamma rays is independent of energy but is a
FIG-54 PERCENTAGE POLARIZATION OF RAYLEIGH SCATTERING
OF GAMMA RAYS AS A FUNCTION OF SCATTERING ANGLE
CALCULATED ON THE BASIS OF FORM-FACTOR CALCULATIONS (CURVE a) AND REFINED CALCULATIONS FOR 0.64 mc²
(CURVE b), 1.28 mc² (CURVE c) AND 2.56 mc² (CURVE d).
function of the scattering angle, whereas according to the refined calculations the linear polarization of the Rayleigh scattering is a function of both the scattering angle and incident gamma ray energy. It can easily be seen from figure 5.1, that on the basis of the form factor calculations, gamma rays scattered at 90° are plane polarized with electric vector perpendicular to the scattering plane. However, refined calculations show that the gamma rays scattered at 90° are partially polarized and that the percentage polarization decreases with the increase of the incident gamma ray energy.

11) Contribution of Nuclear Thomson Scattering.

It has been pointed out in Chapter I, that for gamma ray energies and scattering angles under consideration, the process other than the Rayleigh scattering which contributes to the non-resonant elastic scattering of gamma rays is nuclear Thomson scattering. It interferes constructively with Rayleigh scattering, and the gamma rays scattered at 90° are linearly polarized with electric vector perpendicular to the scattering plane. The theory of nuclear Thomson scattering is well known and fully established. The linear polarization amplitudes with electric vector parallel and perpendicular to the scattering plane are \( a \cos \theta \)
and \((a')\) respectively in the units of \(X\), where \(a = \frac{Z^2}{M}\) and \(r_0\) is the classical electron radius.

For mercury atom, for instance, \(a = 0.0174\). The amplitudes of scattering from mercury atom including the contribution of Rayleigh and nuclear Thomson scattering in the plane of scattering and perpendicular to it are,

\[
\begin{align*}
\sqrt{J_1} &= A + B + a \cos \theta = M(\frac{1}{2}) + M(\frac{3}{2}) + M(\frac{5}{2}) + M(\frac{7}{2}) + a \cos \theta \\
\sqrt{J_0} &= A - B + a = M(\frac{1}{2}) + M(\frac{3}{2}) + M(\frac{5}{2}) + M(\frac{7}{2}) + a
\end{align*}
\]

where \(A = M(\frac{1}{2}) + M(\frac{3}{2})\) and \(B = M(\frac{3}{2}) + M(\frac{5}{2})\).

The percentage polarization \(P = \frac{\sqrt{J_0} - \sqrt{J_1}}{\sqrt{J_0} + \sqrt{J_1}} \times 100\) of the elastic scattering of gamma rays can be written as,

\[
P = \frac{(A - B + a)^2 - (A + B + a \cos \theta)^2}{(A - B + a)^2 + (A + B + a \cos \theta)^2} \times 100
\] 5.3

Figure 5.2 shows the effect of the nuclear Thomson scattering on the polarization of Rayleigh scattering of gamma rays. The form factor predictions remain unaffected by the contribution of Thomson scattering.

iii) Contribution of L-shell electrons.

The refined calculations of Brown and Mayers are available only for K-shell electrons of mercury. In calculating the percentage polarization of the elastic
FIG. 5-2 EFFECT OF NUCLEAR THOMSON SCATTERING ON THE CALCULATED VALUES OF POLARIZATION OF RAYLEIGH SCATTERING OF GAMMA RAYS. CURVE a—FORM FACTOR RESULTS WITH AND WITHOUT CONTRIBUTION OF NUCLEAR THOMSON SCATTERING.

CURVES b AND b'—RESULTS OF REFINED CALCULATIONS AT 1.28 mc² WITH AND WITHOUT NUCLEAR THOMSON SCATTERING.

CURVES c AND c'—RESULTS OF REFINED CALCULATIONS AT 2.56 mc² GAMMA RAYS WITH AND WITHOUT NUCLEAR THOMSON SCATTERING.
scattering of gamma rays, the contribution of L-shell electrons has to be taken into account. Following the suggestion of Bernstein and Mann (14) that the non-spin flip and spin flip contributions due to the L-shell electrons are in the same ratio as the corresponding contributions for the K-shell, it is found that L-shell electrons do not affect the percentage polarization values. On the other hand, if the effects of non-spin flip amplitudes are neglected for large values of momentum transfer as suggested by Brown and Mayers (5), the final values of the polarization are affected but the deviations are very small. The values of percentage polarization, as calculated from the predictions of the form factor and the refined calculations including the contribution of nuclear Thomson scattering and L-shell electrons for 932 keV gamma rays at scattering angles 60°, 90° and 120°, are given in table 9.

3. Gamma ray Polarimeter.

1) Compton scattering as the analyzing process.

The measurement of linear polarization of the elastically scattered gamma rays is made with a gamma ray polarimeter which uses Compton scattering as the analyzing process. This interaction is particularly suitable for the measurements of polarization of gamma rays because of its large cross section of
### Table 2

Polarization of elastic scattering of 662 KeV gamma rays:

<table>
<thead>
<tr>
<th>Scattering angle (degrees)</th>
<th>Percentage Polarization (R + T)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Form Factor Calculations.</td>
</tr>
<tr>
<td>64</td>
<td>67</td>
</tr>
<tr>
<td>90</td>
<td>100</td>
</tr>
<tr>
<td>120</td>
<td>60</td>
</tr>
</tbody>
</table>

- (a) Contains L-shell contribution after Bernstein and Mann.
- (b) Contains L-shell contribution after Brown and Mayer.
- (c) Without L-shell contribution.
interaction and high sensitivity to polarization.

The Compton scattered photon and the recoil electron tend to be ejected at right angles to the direction of the electric vector of the incident photon. The differential scattering cross section \( \frac{d\sigma}{d\Omega} \) for the scattering of a photon of energy \( k_1 \) through an angle \( \theta_2 \), when averaged over all the polarizations of the scattered radiation, can be expressed by the Klein Nishina (56) formula as,

\[
\frac{d\sigma}{d\Omega} = \frac{\gamma_2}{2} \left( \frac{k_2}{k_1} \right)^2 \left[ \frac{k_1}{k_2} + \frac{k_2}{k_1} - 2 \sin^2 \theta_2 \cos^2 \phi_2 \right]
\]

where \( k_2 = \frac{k_1}{1 + \frac{k_1^2}{m_c^2 c^2 (1 - \cos \theta_2)} \} \) is the energy of the scattered photon, \( \phi_2 \) is the angle between the scattering plane and the plane of incident polarization. For a fixed value of scattering angle \( \theta_2 \), the differential scattering cross section is maximum and minimum for values of \( \phi_2 = 90^\circ \) and \( 0^\circ \) respectively and hence the scattered photon has the tendency to go in a direction at right angle to the electric vector of the incident photon. This is the basic principle upon which the polarimeter operates. The radiation to be analyzed is allowed to undergo Compton scattering and the Compton scattered radiation is measured as a function of angle \( \phi_2 \). For the plane polarized
radiation, maximum of intensity of the scattered radiation occurs in a direction perpendicular to the electric vector and minimum of the intensity occurs in a direction parallel to the electric vector.

ii) The Polarization Analyzer.

The polarimeter used for the present investigations of polarization of the elastically scattered gamma rays is similar to the one used by Sood (50). The actual analyser of the polarimeter is shown in figure 5.3. It consists of two detectors A and B in coincidence. The detector A is fixed while B can rotate about it. S is the source of gamma radiation under investigation and Sc is the scatterer, the scattered radiation from which is to be analysed. Gamma rays from the source S, are scattered through an angle $\theta_1$, and the scattered radiation under study, is again scattered through an angle $\theta_2 = 90^\circ$ from the fixed detector A into the neighbouring movable detector B. The scattered radiation is detected in B while the recoil electron is detected in A and both give rise to a coincidence. Counters A and B are both shielded against the direct radiation from the source S and counter B is further shielded from the scatterer Sc. The use of the counter A instead of an ordinary scatterer enables one to select
FIG. 5-3 - POLARIZATION ANALYSER
S- SOURCE, Sc- SCATTERER, A- PLASTIC CRYSTAL
AND B- NaI(Tl) CRYSTAL
the significant events only and thus reduce the background effects.

The percentage polarization of the scattered radiation can be obtained in terms of $J_{||}$ and $J_{\perp}$, the intensities of the linear polarization with electric vector parallel and perpendicular to the scattering plane respectively, by recording the coincidences with line AB parallel and perpendicular to the scattering plane, so as to get the maximum response to $J_{\perp}$ and $J_{||}$ respectively.

For plane polarized radiation the degree of polarization $p = \frac{J_{||}}{J_{\perp}}$ is equal to zero or infinity depending upon whether the electric vector is perpendicular or parallel to the scattering plane. If $N_{||}$ is the coincidence counting rate when the line AB is parallel to the scattering plane ($\phi_2 = 0^\circ$) and $N_{\perp}$ is the coincidence counting rate when line AB is perpendicular to the scattering plane ($\phi_2 = 90^\circ$), then

$$\frac{N_{||}}{N_{\perp}} = \frac{J_{||} \left( \frac{d\sigma}{d\Omega} \right)_{\phi_2=0} + J_{\perp} \left( \frac{d\sigma}{d\Omega} \right)_{\phi_2=90^\circ}}{J_{\perp} \left( \frac{d\sigma}{d\Omega} \right)_{\phi_2=0} + J_{||} \left( \frac{d\sigma}{d\Omega} \right)_{\phi_2=90^\circ}}$$

\[5.5\]
\[
\frac{N_{ll}}{N_{l}} = \frac{J_{II}}{J_{I}} + \frac{(d\sigma/d\Omega)_{\phi_2=\phi_0}}{1 + \frac{J_{II}}{J_{I}} (d\sigma/d\Omega)_{\phi_2=\phi_0}} \frac{(d\sigma/d\Omega)_{\phi_2=\phi_0}}{d\sigma/d\Omega_{\phi_2=0}}
\]

or
\[
\frac{N_{II}}{N_{L}} = \frac{p - 1 + R}{R}
\]

where
\[
R = \frac{(d\sigma/d\Omega)_{\phi_2=\phi_0}}{d\sigma/d\Omega_{\phi_2=0}}
\]

and
\[
p = \frac{J_{II}}{J_{I}}
\]
is the degree of polarization.

The experimental determination of \(N_{II}/N_{L}\) and the knowledge of \(p\) allows the evaluation of the degree of polarization \(p\) and the percentage polarization \(P\). The percentage polarization,
\[
P = \frac{J_{II} - J_{I}}{J_{II} + J_{I}} \times 100
\]
can now be written as,
\[
P = \frac{R+1}{R-1} \cdot \frac{N_{II} - N_{L}}{N_{II} + N_{L}} \times 100
\]

iii) Sensitivity of the Analyser.

For plane polarized radiation (\(p = 0\) or \(\infty\)), \(R\) respectively is equal to \(N_{II}/N_{L}\) or \(N_{L}/N_{II}\) and is called the sensitivity of the analyser. For the ideal geometry, \(R\) is simply the ratio of the two differential cross sections,
The sensitivity of the analyser \( R \) depends on the incident energy of gamma rays \( (E_i) \) and the scattering angle \( (\theta_2) \). It is equal to unity for \( \theta_2 = 0 \) and \( \theta_2 = \pi \) and reaches a maximum value at an angle \( \theta_2 \) somewhat smaller than \( \pi/2 \). Figure 5.4, shows the values of the scattering angle \( (\theta_2)_{\text{max}} \) for which \( R \) has maximum value for different energies of gamma radiation, in the case of ideal geometry. The energy dependence of the sensitivity of the analyser \( R \) for the ideal geometry is shown in fig. 5.5 when \( R \) is calculated for a given energy using \( \theta_2 = 90^\circ \). The value of \( R \) is found to decrease with the increase of gamma ray energy.

In actual practice, due to the finite size of source, scatterer and detector, both the angles \( \theta_2 \) and \( \phi_2 \) have finite spreads which tend to decrease the polarization sensitivity of the analyser. In order to have practical coincidence counting rates, one has to use the finite spreads in angles. The sensitivity of the polarimeter \( R \) with the finite
FIG. 5-4 SCATTERING ANGLE ($\theta$)$_{\text{MAX}}$ FOR MAXIMUM VALUE OF SENSITIVITY OF POLARIZATION $R_\theta$ AS A FUNCTION OF INCIDENT ENERGY OF GAMMA RADIATION IN CASE OF IDEAL GEOMETRY
FIG. 5-5 DEPENDENCE OF SENSITIVITY $R$ OF THE POLARIZATION ANALYSER ON THE ENERGY OF INCIDENT GAMMA RADIATION FOR IDEAL GEOMETRY.
spreads $\Delta \theta_2$ and $\Delta \Phi_2$ in angles $\theta_2$ and $\Phi_2$ in the experimental arrangement, becomes,

$$R = \int_{q_0-\Delta \theta_2/2}^{q_0+\Delta \theta_2/2} \int_{q_0-\Delta \Phi_2/2}^{q_0+\Delta \Phi_2/2} \frac{\left( \frac{d\sigma}{d\Omega} \right) d\Omega}{\left( \frac{d\sigma}{d\Omega} \right) d\Omega}$$

5.10

Substituting for $\frac{d\sigma}{d\Omega}$ from the Klein–Nishina formula and integrating one gets,

$$R = 1 + \frac{2C \sin \Delta \Phi_2}{\Delta \Phi_2 (A + B - C) - C \sin \Delta \Phi_2}$$

5.11

where

$$A = \frac{1}{2\alpha} \left[ \frac{1}{(1 + \alpha - \alpha \sin \Delta \theta_2/2)} - \frac{1}{(1 + \alpha + \alpha \sin \Delta \theta_2/2)} \right]$$

$$B = \frac{1}{\alpha} \log \frac{1 + \alpha + \alpha \sin \Delta \theta_2/2}{1 + \alpha - \alpha \sin \Delta \theta_2/2}$$

$$C = -\frac{2}{\alpha^2} \sin \frac{\Delta \theta_2}{2} \left[ \log \frac{(1 + \alpha)^2 - \alpha^2}{(1 + \alpha - \alpha \sin \Delta \theta_2/2)(1 + \alpha + \alpha \sin \Delta \theta_2/2)} \right. + \left. 2 \left( \frac{1 + \alpha}{\alpha^2} \right) \cdot B \right]$$

and $\alpha = \frac{h^2}{m_0c^2}$.

In the present experimental arrangement, spreads
Δθ₂ and Δθ₂ are equal. An attempt to increase the counting rates by increasing the spreads, reduces the sensitivity of polarization, therefore, in an actual experiment, a compromise between the two has to be made.

iv) Calibration of the Analyzer.

For the finite geometry of the experimental arrangement and the energy of gamma rays under investigation, the values of the spreads Δθ₂ and Δθ₂ are needed, in order to calculate the values of R. Because of the complicated geometry, it becomes difficult to make a reliable estimate of the spreads by the scale drawing. The spreads Δθ₂ and Δθ₂ in the scattering angles θ₂ and θ₂ are obtained experimentally by the method used by Metzger and Deutsch (64).

A collimated beam of 662 KeV gamma rays from the source 137Cs was scattered from copper scatterer at a scattering angle θ₁ = 90°. The scattered gamma rays have energy 285.9 KeV and degree of polarization p = \( \frac{J_{||}}{J_{\perp}} = 0.27 \). This scattered beam of gamma rays of known energy and degree of polarization was analysed by the polarimeter and the ratio of coincidence counting rates \( N_{||} \) and \( N_{\perp} \) corresponding to the two positions of counter \( \phi_2 = 0° \) and \( \phi_2 = 90° \) was determined. Substituting the values of p and \( N_{||} / N_{\perp} \)
in the equation 5.7, $R$ (experimental) was determined. Corresponding to this value of $R$, the value of spreads $\Delta \theta_2 = \Delta \phi_2$ were calculated from the equation 5.11. Having determined the values of the spreads of $\Delta \theta_2 = \Delta \phi_2$, the value of $R$ could be calculated for gamma rays of any energy.

v) **Design of the Polarimeter.**

The actual arrangement of the counters and shielding in the polarimeter is illustrated in figure 5.6. The detector $A$, with its plastic crystal projecting downwards and facing the scatterer $S_c$ was held vertical in a ring $C$ which was fixed at the centre of a circular ring $P$ with the help of a three legged iron stand $E$. The counter $B$, with its NaI ($Tl$) crystal facing the detector $A$ was held horizontal by a lever $L$, which was attached to another ring $D$ and was capable of rotation about $A$ along the circular ring $P$. The circular ring helped to keep the distance between $A$ and $B$ the same for all positions of the detector $B$. The angular spreads $\Delta \theta_2$ and $\Delta \phi_2$ could be adjusted by varying the distance between the detectors $A$ and $B$.

The lead shielding $S_h$ was arranged to be symmetrical about the main axis of the apparatus which coincided with the axis of detector $A$. Gamma rays from the
FIG. 5-6
DESIGN OF GAMMA RAY POLARIMETER
source were collimated to the scatterer and the scattered radiation was allowed to enter the detector A only.

The plastic crystal and NaI(Tl) crystal used as fluorescent materials in the detectors A and B, were mounted on a Dumont 9292 and RCA 6342-A photomultipliers respectively. Silicon oil was used to provide the optical coupling between the crystals and the photocathodes of the photomultipliers.

A conventional arrangement was used to record the coincidence counting rate between the output pulses of detectors A and B. The block diagram is shown in figure 5.7. The output pulses from the photomultipliers were fed to the amplifiers through the pre-amplifiers. The amplified pulses were sorted by the single channel pulse height analysers. The selected pulses were fed to the coincidence unit. Three scalers were used, two for recording the pulses from individual counters and one for recording the coincidence counts. The coincidence unit was 1036C, Dyntron. The resolving time of the unit could be varied from 0.1 μsec to 4 μsec. For the present measurements, the resolving time was kept at 1 μsec and it was measured from time to time during the
FIG. 5.7. BLOCK DIAGRAM OF THE COUNTING EQUIPMENT.
experiment to see if it remained constant. This was done by observing the random coincidence counting rate \( N \) and the individual counting rates \( N_A \) and \( N_B \) in the two counters. The resolving time can be calculated from the equation:

\[
N = 2 N_A N_B \gamma
\]

where \( N_A \) and \( N_B \) are the individual counting rates from the counters A and B respectively and \( \gamma \) is the resolving time.

vi) Performance of the Polarimeter.

The performance of the polarimeter depends upon the characteristics of the two scintillation detectors A and B. The resolution and linearity of the detectors A and B were tested using different radioactive sources.

The working of the polarimeter was checked by measuring the polarization and azimuthal variation of the Compton scattering of 662 KeV gamma rays at scattering angles 60°, 90° and 120°, and by comparing the experimental results with the theoretical calculations of Klein and Mishina. The gamma rays of energy 662 KeV were obtained from source \(^{137}\)Cs. A collimated beam of gamma rays from source S was scattered from Cu scatterer at a scattering angle \( \Theta_1 \) and the scattered radiation was allowed to undergo Compton scattering
through $\Theta_2 = 90^\circ$ by a plastic crystal A. The recoil electrons detected in crystal A and the Compton scattered photons detected in crystal B, gave rise to a coincidence.

To set the channels of the two detectors so as to count only the required events, the detector B was set to accept the energy corresponding to the second scattered photons and a coincidence spectrum was taken by varying the bias of the detector A. The position of the peak in the coincidence spectrum was found to correspond to the energy of the recoil electrons in the second scattering. The channel of the plastic counter A was set to cover the full width at half height of the maximum of the coincidence spectrum. With this setting of the plastic counter A, another coincidence spectrum was taken by varying the bias of the NaI (Tl) counter to check the channel setting of the latter. The position of the maxima in the second coincidence spectrum agreed with the channel of NaI (Tl) counter. Some of the spectra are shown in figure 5.B.

a. Polarization of Compton Scattering

The gamma rays of energy 662 KeV when Compton scattered at angles 60°, 90° and 120° have energies 421,
Fig. 5-8 Performance of the polarimeter.

Curve a is Compton scattered spectrum of 0.662 MeV gamma rays at 90°
taken with counter B. Curves b and c are coincidence spectra of scattered
radiation taken with A and B respectively and channel on the other being kept fixed.
and 194 KeV respectively and are partially polarized. The experimental determination of the percentage polarization of Compton scattered gamma rays was made by measuring the coincidence counting rates \( \bar{N}_1 \) and \( \bar{N}_2 \) when line AB is parallel and perpendicular to the scattering plane, corresponding to \( \phi_2 = 0^\circ \) and \( 90^\circ \) respectively.

The counts in three scalers were recorded for five minutes in each position of B for \( \phi_2 = 0^\circ \) and \( 90^\circ \), firstly with scatterer in its position and then by removing it. The coincidence counts were corrected for the background and chance coincidences. During the experiment, the calibrations of the two counters were checked off and on and the channels were reset.

Spreads \( \Delta \Theta_2 \) and \( \Delta \phi_2 \) in the scattering angles \( \Theta_2 \) and \( \phi_2 \) were obtained by the method described in the section 3.4. Knowing the spreads \( \Delta \Theta_2 = \Delta \phi_2 \), the values of \( \Theta_2 \) were calculated for gamma rays of energies 421, 288 and 194 KeV corresponding to scattering angles 60°, 90° and 120°, respectively, from the equation 5.11. The percentage polarization of Compton scattered gamma rays was calculated from equation 5.3.

The experimental results are shown in table 10.
Table 10

Polarization of Compton Scattering of 662 KeV gamma rays.

<table>
<thead>
<tr>
<th>Scattering Angle (degrees)</th>
<th>Percentage Polarization</th>
<th>Experimental</th>
<th>Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>64</td>
<td>51.6 ± 1.5</td>
<td>53.8</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>58.2 ± 1.8</td>
<td>57.6</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>29.8 ± 1.3</td>
<td>29.8</td>
<td></td>
</tr>
</tbody>
</table>

and the agreement between the experimental results and the theoretically calculated values from the Klein-Nishina formula, confirms the correct performance of the polarimeter.

b. Azimuthal Variation of Compton Scattering of Gamma Rays.

Hoover et al. (65) have measured the azimuthal variation of the Compton scattering of gamma rays at 90° that are partially polarized by a previous Compton scattering of 1250 KeV gamma rays through 23° and 50°. Though their results show an overall agreement with the theory, yet there are appreciable deviations at some azimuthal angles, these large deviations were ascribed to low counting rates and drifts in the electronics during long counting times needed in their
experiments. A similar experiment has been performed for gamma rays of energy 662 keV at scattering angles $\theta_1 = 64^\circ$, $90^\circ$ and $120^\circ$.

When incident gamma rays scattered from Cu at scattering angle $\Theta_1$ having energy $k_1$ and the degree of polarization $p_1$, are scattered again through a scattering angle $\Theta_2$, from the crystal A, energy of a second scattering reduces to $k_2$ and the number of gamma rays scattered at an azimuthal angle $\Phi_2 = 0$ with respect to the number scattered in the first scattering plane, i.e., $\Phi_2 = \Phi$, can easily be shown to be,

$$\frac{N_{k_2=0}}{N_{k_2=\Phi}} = \frac{1 + b \cdot R_1}{b \cdot R_2 + R_3}$$

where

$$b_1 = \frac{\int_{\Delta \theta_1, \Delta \Phi_1} \left( \frac{d\sigma_1(\theta_1)}{d\Omega} \right) \Phi_1 = 0^\circ \ d\Omega}{\int_{\Delta \theta_1, \Delta \Phi_1} \left( \frac{d\sigma_1(\theta_1)}{d\Omega} \right) \Phi_1 = 90^\circ \ d\Omega}$$

$$R_1 = \frac{\int_{\Delta \theta_2, \Delta \Phi_2} \left( \frac{d\sigma_2(\theta_2)}{d\Omega} \right) \Phi_2 = 0^\circ \ d\Omega}{\int_{\Delta \theta_2, \Delta \Phi_2} \left( \frac{d\sigma_2(\theta_2)}{d\Omega} \right) \Phi_2 = 90^\circ \ d\Omega}$$
\[ R_2 = \oint_{\Delta \theta_1, \Delta \phi_1} \left( \frac{d\sigma_1(\theta_1)}{d\Omega} \right) \phi_1 \ d\Omega \]

\[ R_3 = \oint_{\Delta \theta_1, \Delta \phi_2} \left( \frac{d\sigma_2(\theta_2)}{d\Omega} \right) \phi_2 \ d\Omega \]

Here \( \Delta \theta_1, \Delta \theta_2, \Delta \phi_1 \) and \( \Delta \phi_2 \) are the spreads in the scattering angles, \( \theta_1, \theta_2, \phi_1 \) and \( \phi_2 \) respectively, and \( \frac{d\sigma_1(\theta_1)}{d\Omega} \) and \( \frac{d\sigma_2(\theta_2)}{d\Omega} \) are Compton scattering cross sections averaged over polarizations of the scattered photons, given by the Klein-Nishina formula.

The measured values of \( \frac{N\phi_1}{\phi_2} \) (\( \phi_2 \) varying from 30° to 90°) are compared with the theoretically calculated values from the equation 5.13, in figure 5.9. The angular spreads \( \Delta \theta_1 \) & \( \Delta \phi_1 \) were about 2°. However, the spreads \( \Delta \theta_2 \) and \( \Delta \phi_2 \) were kept quite large to get good counts within a reasonable time to reduce the effect of drift in the electronics.

The consistent agreement of the experimental results with the theoretical values provides another check on the performance of the polarimeter.
FIG. 5-9 COMPARISON OF THE EXPERIMENTAL AND THEORETICAL RESULTS.
THE ERRORS SHOWN ARE DUE TO STATISTICAL ERRORS IN THE COUNTING RATE.

1) Experimental Arrangement.

The polarimeter described in section 3, was used to study the polarization of the elastically scattered gamma rays. The experimental set-up is shown in figure 5.10. A collimated beam of 662 KeV gamma rays from 200 mc. 137Cs source S was scattered from lead scatterer held in its position by the wooden stand through an angle $\Theta_1$. The elastically scattered gamma radiation was allowed to enter the counter A. The counter B was so shielded that it could only receive the Compton scattered radiation from the crystal A. Both the counters A and B were shielded against the direct radiation from the source. The shielding was arranged to be symmetrical with respect to the main axis of the apparatus in order to avoid any spurious coincidence counting in the experiments due to scattering from the shield. The symmetry of the lead shield was checked by placing a small source of known energy in place of the scatterer and measuring the coincidence counting rates in four symmetrical positions of the counter B. A lead filter F was placed between the scatterer and counter A to reduce the
FIG. 10 EXPERIMENTAL SET UP FOR THE MEASUREMENT OF
POLARIZATION OF ELASTICALLY SCATTERED GAMMA RAYS.
S-SOURCE, Sc-SCATTERER, A-PLASTIC CRYSTAL, B-NaI(Tl)
CRYSTAL AND F-LEAD FILTERS
Intensity of the inelastic scattering from the scatterer in comparison to the elastic scattering of gamma rays.

11) Contribution of Elastic and Inelastic Scattering to the Scattered Radiation.

Gamma radiation scattered from heavy elements, consists mostly of Compton scattering of well-defined energy, a continuum extending from the Compton peak to almost full energy elastic peak, due to the inelastic scattering from the bound atomic electrons and the elastic peak having the same energy as the incident energy of gamma rays which sits at the tail of the continuum and is generally masked by it. It was seen in Chapter III, that even within the channels corresponding to the elastic peak, there is always some contribution of the inelastic scattering. The method of using an equivalent thickness of aluminium scatterer as used by previous workers (51) to eliminate the effect of the inelastic scattering is in error because only the effect of free electrons is eliminated while a residual continuum due to the inelastic scattering from the bound atomic electrons persists.

The isolation of the elastic scattering from the
inelastic scattering was achieved with the help of lead filters which reduced the intensity of lower energy inelastic scattering more than that of higher energy in elastic scattering and by using suitable channels of the two counters. The spectrum of 662 KeV gamma rays scattered from lead at 90° with 10 gm/cm² of lead filter in front of the detector is shown in figure 5.11. The Compton scattering which is 400 times more intense than the elastic scattering is reduced by a factor of 20 in comparison to the elastic scattering. The scattered spectrum is analysed by subtracting different fractions of the direct spectrum (a) from the scattered spectrum (b) so as to obtain a continuous spectrum. Difference spectra are plotted as curves (c, d, e and f). Curve (e) seems to be the suitable difference continuum which varies continuously with energy and thus represents the contribution of the inelastic scattering process. This analysis of the spectrum shows that beyond 630 KeV, the contribution of inelastic scattering is negligibly small. The percentage polarization of the elastically scattered gamma rays can be measured by setting the channels to accept the energy region of spectrum beyond 630 KeV.
FIG. 5-11 ANALYSIS OF SCATTERED SPECTRA OF 662 keV GAMMA RAYS FROM LEAD AT 90° WITH 10 g/cm² OF LEAD FILTERS IN FRONT OF DETECTOR (CURVE b). CURVE a IS DIRECT SOURCE SPECTRUM. CURVES (c, d, e & f) ARE DIFFERENCE SPECTRA OBTAINED WHEN VARIOUS FRACTIONS OF THE DIRECT SPECTRUM (a) ARE SUBTRACTED FROM SCATTERED SPECTRUM (b).
Method and Results.

The first step in the measurement of the polarization of the elastically scattered gamma rays at a particular scattering angle \( \Theta_1 \) consisted of taking the scattered spectra with the different thicknesses of lead filters. A proper thickness of lead filter was adjusted with which the contribution of inelastic scattering was negligible under the elastic peak, by analysing the scattered spectra as described in section 4(ii).

The elastically scattered gamma rays of energy 0.692 MeV undergoing Compton scattering in crystal A and being subsequently absorbed in B, lose 0.374 MeV in A and 0.999 MeV in B. The discriminator channels of A and B were set to correspond to gamma rays of these energies. To ensure the correct setting of the channels, a small source of \(^{137}\text{Cs}\) was placed in the position of the scatterer. A coincidence spectrum was taken by varying the bias of the elastic counter A, fixing the channel of NaI (TI) counter B so as to accept only the energy corresponding to 90°, the Compton scattered gamma rays from A (0.288 MeV).

The peak in the coincidence spectrum corresponded to
the energy of the recoil electrons. The channel of the plastic counter A was then set to cover the full width at half height of peak in the coincidence spectrum. With this setting of the channel of the plastic counter, coincidence spectrum was taken by varying the bias of the NaI (Tl) counter B in order to check the channel setting of the latter. Some of the spectra are shown in figure 5.12. After calibrating the polarimeter as described above, the channels were adjusted so that the contribution of inelastic scattering to elastic scattering was negligible.

The polarization of the elastically scattered gamma radiation was measured with the polarimeter in terms of $J_{||}$ and $J_{\perp}$, the intensities of plane polarization with the electric vector parallel and perpendicular to the scattering plane, respectively. The method of measurement of the percentage polarization consisted of determining the sensitivity $R$ of the polarimeter and by measuring the coincidence counting rates $N_{||}$ and $N_{\perp}$ when line AB is parallel ($\phi_2=0^\circ$) and perpendicular ($\phi_2=90^\circ$) to the scattering plane. The percentage polarization $P$ can be calculated from the equation 5.8.
FIG. 5.12 DIRECT AND COINCIDENCE SPECTRA OF 662 KeV GAMMA RAYS TAKEN WITH COUNTERS A AND B OF THE POLAR IMETER. IN (a) THE SPECTRA ARE TAKEN EITHER DIRECTLY WITH COUNTER A, (S) OR BY VARYING ITS CHANNELS AND KEEPING THOSE ON B FIXED AT APPROPRIATE ENERGY. IN (b), THE SPECTRA ARE TAKEN EITHER DIRECTLY WITH COUNTER B OR BY VARYING ITS CHANNEL AND KEEPING THE CHANNELS OF 'A' FIXED AT APPROPRIATE ENERGY (C).
For the ideal geometry, $(\Delta \theta_2 = \Delta \phi_2 = 0)$ at $\theta_2 = 90^\circ$
the value of polarization sensitivity ($R$) for
incident gamma rays of energy 0.632 MeV, as calculat-
ed from the Klein-Nishina formula, is 3.7. In
practice this value is reduced by the angular
spreads $\Delta \theta_2$ and $\Delta \phi_2$. The angular spreads $\Delta \theta_2 = \Delta \phi_2$
were determined by analysing 0.362 MeV gamma rays
Compton scattered at 90° and the details of the method
have been described in the section 3. Knowing the
spreads $\Delta \theta_2 = \Delta \phi_2$, the value of $R$ was calculated
for the elastically scattered gamma rays of energy
0.632 MeV from the equation 5.11. Its value for the
present experimental spreads was found to be 3.0 as
compared to 3.7 for ideal geometry. It is found that
$\frac{R+1}{R-1}$ is not very sensitive to the spreads in the
geometry. A change of about 10% in the angular
spreads changes the value of $\frac{R+1}{R-1}$ by 3% for the
energy of gamma rays and spreads used in the experiment.

The coincidence counting rates $N_{11}$ and $N_{1}$ in
the main experiment were rather low because of the
narrow energy range accepted in the experiment and
the absorption in the lead filters. Consequently the
experiment had to be run for several months to get
good statistics. The sequence of taking observations
was so arranged as to minimize the effect of drifts in electronics. The equipment was calibrated off and on to keep a check on its stability. The consistency of the experimental data provided an indirect check on the stability of the electronics equipment during the period of observations. The final experimental results after correcting for background and chance coincidences are given in table 11. The errors shown are the probable errors in the final results. The theoretical values have been corrected for the contribution of nuclear Thomson scattering and the contribution of L-shell electrons. The experimental results are found to agree with the predictions of the refined calculations of Brown et al. and contradict the results of the form factor calculations, and hence the superiority of the refined calculations with regard to the polarization is also established. The previous assumption that the discrepancy between theory and experiment is due to the contribution of L-shell electrons, the exact calculations for which do not exist, is also not valid.
Table II

Polarization of 662 KeV gamma rays elastically scattered from lead.

<table>
<thead>
<tr>
<th>Scattering angle (degrees)</th>
<th>Lead filters used (gm/cm²)</th>
<th>% N₁/N₁</th>
<th>Experimental</th>
<th>Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>64</td>
<td>24.0</td>
<td>2.32±0.22</td>
<td>96±8</td>
<td>67 91</td>
</tr>
<tr>
<td>90</td>
<td>10.0</td>
<td>2.28±0.18</td>
<td>72±8</td>
<td>100 72.5</td>
</tr>
<tr>
<td>120</td>
<td>2.0</td>
<td>1.33±0.10</td>
<td>83.4±6.5</td>
<td>60 25.3</td>
</tr>
</tbody>
</table>

a) Contains L-shell contribution after Bernstein and Mann.
b) Contains L-shell contribution after Brown and Mayers.
c) Without L-shell contribution.

5. Influence of Inelastic Scattering on the Polarization of Elastic Scattering.

An experiment was performed to investigate the reasons for the lower results of the previous experiments (50, 51) than the theoretical calculations by investigating the influence of inelastic scattering on the polarization of the elastic scattering. The measurements were made with 662 KeV gamma rays scattered from lead at 64°. At this scattering angle and energy of gamma rays, it is possible to...
analyse the scattered spectrum and get a good estimate of the contributions of the elastic and inelastic scattering. By the choice of suitable filters in front of the detector, the relative contributions of the elastic and the inelastic scattering to the observed spectrum can be varied. With a lead filter of thickness 24 gm/cm², the contribution of inelastic to the observed spectrum in the region beyond 640 KeV is negligible as shown in figure 3.5 (Chapter III). The value of the percentage polarization ($96 \pm 3$), measured by accepting the energy region beyond 640 KeV and using lead filter of thickness 24 gm/cm², agrees well with the theoretical value (table II).

The thickness of lead filter was then reduced to 2 gm/cm² and 0 gm/cm² to get the ratios of the elastic to inelastic scattering within the experimental channels from 520 KeV to 760 KeV as 1.13 and 1.30 respectively and the polarization was measured. The values of percentage polarization obtained were $66 \pm 5$ and $54 \pm 4$ which are lower than the measured value of the percentage polarization of the elastic scattering. Hence, this clearly indicates that the incomplete isolation of inelastic scattering from
the elastic scattering lowers the percentage polarization of the elastic scattering.

Knowing the contribution of inelastic scattering to elastic scattering, in the experimental channels, the experimental value of the percentage polarization can be corrected for the contribution of the inelastic scattering provided, its polarization within the same channels is known. This correction was calculated by making use of the fact that the elastic and inelastic contribution add in terms of their intensities. In an effort to make this correction, the percentage polarization of the inelastic scattering in the regions before the elastic peak was measured to be $45 \pm 3$. The experimental value of the percentage polarization was then corrected from the following relation,

$$P_T = \frac{(J_{\parallel})_{el} + (J_{\parallel})_{in}}{(J_{\perp})_{el} + (J_{\perp})_{in}}$$

where $P_T$ is the measured degree of polarization, $J_{\parallel}$ and $J_{\perp}$ are the intensities of polarization with the electric vector parallel and perpendicular to the scattering plane. Subscripts 'el' and 'in' stand for the elastic and inelastic components. The above equation can be written as,
Knowing the ratio of the contributions of the elastic to the inelastic scattering intensities \( \frac{\sigma_{el}}{\sigma_{in}} \) and the polarization of the inelastic scattering, the experimental value of percentage polarization was corrected by using the above formula. The corrected values of the percentage polarization were found to be 94 ± 10 and 99 ± 10 which agree fairly well with theoretical value as well as with the value (96 ± 3) measured by completely isolating the contribution of inelastic scattering.

6. **Conclusions.**

The experimental results show that when the contribution of the inelastic component to the scattering under investigation is negligibly small, the experimental values agree fairly well with the predictions of the refined calculations but disagree with those of the form factor calculations and the
incomplete elimination of inelastic scattering tends
to lower the experimental values. In contradiction
with the earlier suggestions, the effect of L-shell
electrons on the observed values of polarization, if
any, is comparatively small. The discrepancy between
the theory and experiment with regard to the polari-
sation of the elastic scattering is thus removed.