CHAPTER IV

ESTIMATION OF ELASTIC SCATTERING CROSS-SECTIONS OF GAMMA RAYS FROM DIFFERENT ELEMENTS.

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CHAPTER IV

Estimation of Elastic Scattering Cross Section of Gamma Rays of Various Energies from Different Elements.

1. Introduction.

The exact numerical calculations (2-6) of Rayleigh scattering are available only for gamma rays of energies 0.32, 0.64, 1.28, 2.56 and 5.12 meV from K-electrons in mercury. Various form factor (7-10) calculations have been proposed to estimate the Rayleigh scattering cross sections from different elements. Unfortunately, these form factor calculations have been found to be in error. It has been pointed out in chapter I, that the experimental measurements of the Z-dependence of elastic scattering of gamma rays can be combined with the refined calculations of Brown and Mayers to give a reliable estimate of the elastic scattering cross sections of gamma rays of various energies from different elements at various scattering angles. The method of estimation and comparison of the results so obtained with the available experimental data are described in this chapter.


The refined calculations of Brown and Mayers (2-6) give scattering cross sections for Rayleigh scattering from the K-shell electrons in mercury in the form

\[ \left( \frac{d\sigma}{d\Omega} \right)_R = \gamma \frac{\left[ |a_{ik}|^2 + |b_{ik}|^2 \right]^2}{\left[ |a_{ik} + ib_{ik}|^2 \right]} \] 4.1
where subscripts \( l_k \) and \( 2_k \) denote the \( K \)-shell amplitudes with no polarization change and with polarization change respectively and \( r_0 \) is the classical electron radius. The contributions of the imaginary parts are particularly small at low values of momentum transfer \( q = \frac{2h\nu}{c} \sin \theta/2 \). Neglecting the imaginary parts, Bernstein and Mann (14) have shown by plotting the real parts of the amplitudes that these may be written as,

\[
\alpha_{1k} = F_{1k}(q) \frac{1 + \cos \theta}{2} \\
\alpha_{2k} = F_{2k}(q) \frac{1 - \cos \theta}{2}
\]

\( F_{1k} \) is a smooth function of \( q \) for values of \( q \geq 0.6mc \) and it is a multivalued function depending on the gamma ray energy for values of \( q \) less than 0.6 mc. \( F_{2k} \) is a function of \( q \) only, for all values of \( q \) up to the maximum value for which an exact calculation was made, and indeed is closely equal in magnitude to the Bethe's \( K \)-shell form factor, and is consistently larger than \( F_{1k} \). By using above equation, \( K \)-shell amplitudes of Rayleigh scattering can be calculated for gamma rays of any energy and at any scattering angle. Bernstein and Mann (14) have calculated the contributions of \( L \)-shell electrons to the elastic scattering by assuming...
that the L-shell amplitudes with no polarization and with polarization changes contribute in the same ratio as the corresponding K-shell amplitudes. However, Brown and Mayers (5) suggest that this assumption may not be valid at large values of momentum transfer $q$ and the experimental data suggest that it is better to neglect the contribution of L-shell scattering amplitude for no polarization change at large values of momentum transfer. In the present calculations, the contribution of L-shell scattering amplitude for no polarization change is neglected for values of $q$ greater than $q_0$. The L-shell amplitudes were taken from the work of Bernstein (60). The contribution of nuclear Thomson scattering, which interferes constructively with Rayleigh scattering, was calculated from the equation 1.1 (1st chapter).

Combining the contributions of Rayleigh scattering from K-shell and L-shell electrons in mercury, and of nuclear Thomson scattering, differential scattering cross section for mercury at a scattering angle $\theta$ is given by,

$$\left( \frac{d\sigma}{d\Omega} \right)_{el} = \frac{\lambda^2}{4} \left[ (F_{1K}+F_{1L}+a)^2 (1+\cos^2\theta) + (F_{2K}+F_{2L}+a)^2 (1-\cos^2\theta) \right]$$

where $a = \frac{q^2}{m^2}$ which includes the contribution of
nuclear Thomson scattering. Here Z is the atomic number of the scatterer, m is the electronic mass and M is the nuclear mass. If A, B, C and D are defined by the relations,

\[
\begin{align*}
A + iB &= F_{1k} + F_{1L} + a \\
C + iD &= F_{2k} + F_{2L} + a
\end{align*}
\]

one can express the differential elastic scattering cross section as,

\[
\left( \frac{d\sigma}{d\Omega} \right)_{el} = \frac{2}{q^2} \left[ \left( A^2 + B^2 \right) \left( 1 + \cos \theta \right) + \left( C^2 + D^2 \right) \left( 1 - \cos \theta \right) \right]
\]

\( F_{1k}(q) \) and \( F_{2k}(q) \) contain both real and imaginary parts as given in the papers of Brown and Nayers (3-6) and

\[
\begin{align*}
F_{1k}(q) &= 2 \left( \alpha_{1k} + i \beta_{1k} \right) / 1 + \cos \theta \\
F_{2k}(q) &= 2 \left( \alpha_{2k} + i \beta_{2k} \right) / 1 - \cos \theta
\end{align*}
\]

\( F_{1k} \) and \( F_{2k} \) were calculated as functions of momentum transfer by taking the values of \( \alpha_{1k} + i \beta_{1k} \) and \( \alpha_{2k} + i \beta_{2k} \) at different scattering angles from the data of Brown and Nayers (5) for 1.23 and 2.56 meV.

The values of \( F_{1L} \) and \( F_{2L} \) were taken from the thesis of Bernstein (6) for different values of momentum transfer. The values of \( (A^2 + B^2) \) and \( (C^2 + D^2) \) for mercury, so obtained, are plotted as function of momentum transfer in figure 4.1. By means of these
FIG. 4.1 PLOTS OF \( (A+B)^2 \) AND \( (C+D)^2 \) AGAINST MOMENTUM TRANSFER FOR MERCURY.
curves, the elastic scattering cross section can be calculated for mercury at various gamma ray energies and scattering angles.

In order to estimate the elastic scattering cross section for gamma rays of given energy from any desired element \((Z > 47)\) at a particular scattering angle, the corresponding value of cross section for Hg was first evaluated from the figure 4.1 and the equation 4.3. The elastic scattering cross section for the desired element of atomic number \(Z\), was estimated from the following relation,

\[
\left( \frac{d\sigma}{d\Omega} \right)_{el}^{Z,q} = \left( \frac{d\sigma}{d\Omega} \right)_{el}^{80,q} \left( \frac{Z}{80} \right)^n
\]

4.7

where \(\left( \frac{d\sigma}{d\Omega} \right)_{el}^{80,q}\) is the elastic scattering cross section for Hg and \((n)\) is the index to the power of \(Z\) on which the elastic scattering cross section depends. The index \((n)\) was obtained from the experimental \(Z\)-dependence curve in the figure 3.11 (Chapter III), corresponding to the particular value of momentum transfer. This curve has been obtained from a study of the variation of the elastic scattering cross sections with the atomic number of the scatterer for 0.320, 0.412 and 0.562 MeV gamma rays from various scatterers at different scattering angles.
3. Results and Discussion.

Comparison of the values of cross sections estimated by the above method with the available experimental data is illustrated in figures (4.2-4.5). Figures (4.2a and 4.2b) show a comparison of the estimated cross sections of Pb, Pt, Ta and Sn for 0.662 MeV gamma rays at different scattering angles with the experimental data of Narasimhamurty et al. (61, 62). In these figures, values of cross sections obtained by extrapolating the refined calculations (5) for mercury to any desired Z value, as reported by Narasimhamurty et al. who have assumed the same Z-dependence as predicted by Bethe's form factor calculations, are also shown. The experimental values of cross sections agree better with the present estimated results than the extrapolated values. In figures (4.3a and 4.3b), a comparison of the estimated cross sections for Pb, Pt, Ta and Sn at 1.12 MeV is made with the experimental results of Hara et al. (45), Cindro and Ilakovac (44) and Narasimhamurty et al. (61, 62) and with the extrapolated theoretical values of the calculations of Brown and Mayers (5) (as reported by Narasimhamurty (61, 62)). The agreement between the present estimated values of cross sections and the experimental results of Hara et al. (45) and
FIG. 4.2a COMPARISON OF ESTIMATED AND EXPERIMENTAL ELASTIC SCATTERING CROSS SECTIONS OF 662 KeV GAMMA RAYS FOR Pb AND Sn.
FIG 4-2b COMPARISON OF THE ESTIMATED AND EXPERIMENTAL ELASTIC SCATTERING CROSS SECTIONS OF 662 KeV GAMMA RAYS FOR Pt AND Ta.
FIG. 4-3a COMPARISON OF ESTIMATED AND EXPERIMENTAL ELASTIC SCATTERING CROSS-SECTIONS OF 1.2 MeV GAMMA RAYS FOR Pb AND Sn.
FIG. 43b  COMPARISON OF THE ESTIMATED AND EXPERIMENTAL ELASTIC SCATTERING CROSS SECTIONS OF 142 MeV GAMMA RAYS FOR Pt AND Ta.
Marasimhamurty is quite good, while the results of Cindro and Ilakovic (44) are somewhat lower than the estimated values. The extrapolated values of cross sections are also lower than the present estimated values.

The comparison of the estimated values of cross sections for gamma rays of energy 1.33 MeV from U, Pb and Sn with the experimental results of Standing and Jovanovich (49), Goldzahi and Eberhard (48), Bernstein and Mann (14) and Hara et al. (45) at various scattering angles is shown in figure 4.4. The experimental results of Standing and Jovanovich (49) and Hara et al. (45) agree well with the estimated values of cross section. But the experimental results of Bernstein and Mann (14) and Goldzahi and Eberhard (48) are appreciably higher. This discrepancy is resolved, if as pointed out by Standing and Jovanovich (49), the experimental results of Bernstein and Mann (14) are assumed to include some contributions of inelastic scattering.

In figure 4.5, the estimated values of cross sections of Pb, Sn, In and Cd for gamma rays of different energies at 90° are compared with the experimental results of Burkhardt (47), Hara et al. (45) and Standing and Jovanovich (49). The experimental
FIG. 4-4 a. COMPARISON OF ESTIMATED AND EXPERIMENTAL ELASTIC SCATTERING CROSS SECTIONS OF 1.33 MeV GAMMA RAYS FOR U AND Sn.

--- = ESTIMATED
+ = BERNSTEIN & MANN
X = GOLDZAHL AND EBERHARD
△ = HARA et al.
O = STANDING AND JOVANOVICH
FIG. 4-4b COMPARISON OF ESTIMATED AND EXPERIMENTAL ELASTIC SCATTERING CROSS-SECTIONS OF 1.33 MeV GAMMA RAYS FOR LEAD.
FIG. 4-5 COMPARISON OF ESTIMATED AND EXPERIMENTAL ELASTIC SCATTERING CROSS-SECTIONS FOR Pb, Sn, In, AND Cd AT 90° FOR DIFFERENT GAMMA RAY ENERGIES.
results of Burkhardt (4?) are found to be consistently higher than the estimated values of the cross sections but the more reliable experimental data of Hara et al. (45) and Standing and Jovanovich agree well with the estimated values of cross sections. Burkhardt's results may be higher due to the presence of an appreciable amount of the inelastic scattering in the measurements.

Thus, there exists an overall agreement between the estimated values and the reliable experimental values of the differential scattering cross sections for different elements at various scattering angles and gamma ray energies. Moreover, the estimates of the scattering cross sections made by Narasimhamurty et al. (61) from an extrapolation of the calculations of Brown et al. (15) are lower than the present estimated and experimental values. This is particularly noticeable for tin where any deficiency in the method of extrapolation should be clearly exposed.

The extrapolation of the cross sections from Brown and Mayers' calculations for mercury to other elements, as made by Narasimhamurty (61), is found to be in error because of the following reasons:

1. Narasimhamurty et al. (61) have not included the contribution of L-shell scattering in the calculations,
which is quite important at small scattering angles. 

(ii) In extrapolating the cross sections, they have 
assumed the same Z-dependence as that of Bethe's form 
factor calculations, which is not correct. This 
assumption may hold for small change of Z, as from 
mercury to lead (90 to 92) but not for large change 
of Z as from mercury to tin (90 to 50).

4. **Conclusions.**

The estimated values of cross sections for 
different elements (Z > 47), for gamma rays of various 
energies and at different scattering angles are found 
to be in agreement with the available experimental 
data. The estimated values of cross sections of non-
resonant elastic scattering can now be used with 
confidence in the determination of the cross sections 
of resonant scattering of gamma rays when the geometry 
of the experimental set up needed to restore the 
resonance condition is complicated (for example see 
Moon (63)).