CHAPTER III

SECOND QUANTIZED REPRESENTATION OF OPERATORS: ISOSPIN OPERATOR, HAMILTONIAN OPERATOR, SINGLE NUCLEON TRANSFER OPERATOR, ELECTROMAGNETIC TRANSITION OPERATORS; AND MULTISHELL BASIS WITH GOOD ISOSPIN.

This chapter is divided into two sections. Section I deals with the second quantized version of the isospin operator $T^2$ as well as the construction of multishell basis states having definite isospin value $T$, in $(n-p)$ representation.

In Section II the Hamiltonian operator $H$, single nucleon transfer operator, and the electromagnetic transition operators for $M1$ and $E2$ transitions are expressed in terms of the coupled products of one-body and two-body state operators. Construction of matrix elements of a given operator in the final good $JT$ basis is also discussed.

SECTION I. $T^2$ Operator in $(n-p)$ Representation and Construction of Good $T$ States.

A. Isospin Operator in Neutron-Proton Representation.

In isospin representation, for a system having $n$ nucleons, the isospin operator is defined as

$$ T = \sum_{i=1}^{n} t_i $$  \hspace{1cm} (3.1)

where $t_i$ is the isospin operator for a single nucleon, having eigenvalue 1/2. For a proton, we define, the $z$ component of $t_i$, $t_z = -1/2$ and for a neutron $t_z = +1/2$. Thus

$$ T^2 = \frac{3n}{4} + 2 \sum_{i<j} t_i \cdot t_j $$  \hspace{1cm} (3.2)
As we shall be working in neutron-proton representation we must express $T^2$ operator in terms of operators for protons and neutrons. We can express $(t_i \cdot t_j)$ in terms of the space-spin exchange operator $P_{ij}$ by using the following relationship,

$$H_{ij} = \frac{1}{4} (1 + 4 t_i \cdot t_j) = -P_{ij}$$  \hspace{1cm} (3.3)

Here $H_{ij}$ is Heisenberg operator (isospin exchange operator).

$T^2$ operator can now be written as

$$T^2 = \frac{n(4-n)}{4} - \sum_{i<j} P_{ij}$$  \hspace{1cm} (3.4)

For a system having $n_1$ neutrons and $m_1$ protons ($n = n_1 + m_1$) one can directly write equation (3.4) in neutron-proton representation,

$$T^2 = \frac{1}{4} [(n_1-m_1)^2 + 2(n_1+m_1)] - \sum_{i<j} P_{ij}(n-p)$$  \hspace{1cm} (3.5)

Here $P(np)$ interchanges the neutron number and the proton number while the label remains the same. For the simplest case, $P(np)$ operating on the one neutron and one proton state, the neutron being in orbit $r$ while the proton occupies the orbit $s$,

$$P(np) \psi_{rs}^{J}(np) = (-1)^{j_r + j_s - J} \psi_{sr}^{J}(np)$$  \hspace{1cm} (3.6)

Obviously non-vanishing matrix elements of $P(np)$ are

$$\langle \psi_{rs}^{J}(np) | P(np) | \psi_{sr}^{J}(np) \rangle = (-1)^{j_r + j_s - J}.$$  \hspace{1cm} (3.7)

In state operator language

$$\psi_{rs}^{J}(np) = j_m \sum_{A} \beta_{rs}^{J}(np) \mid 0 \rangle = Z_{rs}^{J}(np) \mid 0 \rangle$$  \hspace{1cm} (3.8)
Combining equations (3.6), (3.7) and (3.8) we write down the matrix element form of the \( P(np) \) operator.

\[
\sum P(np) = \sum [j_j^J (-1)^{3-r+3s-J} (Z_{rs}(np) x Z_{sr}(np)) 0] 
\]

The \( T^2 \) operator in (n-p) formalism can now readily be written in its second quantized form as

\[
T^2 = \frac{1}{4} [ (n_1-m_1)^2 + 2 (n_1+m_1) ]
\]

A recoupling of the operator (2) transforms the equation to give the multipole form of the operator,

\[
T^2 = \frac{1}{4} [ (n_1-m_1)^2 + 2 (n_1+m_1) ]
\]

Here \( U_{rs}^{k} \) is the Racah's unit tensor.

It is to be noticed from the multipole form that the \( T^2 \) operator does not in general conserve the angular momenta and seniorities of single shell identical nucleon groups comprising the multishell wave function. It does, however, conserve the final angular momentum as well as the overall seniority of a given single orbit nucleon group.

B. Construction of Single Shell good T States.

For a system having \( n_l \) neutrons and \( m_l \) protons occupying a single orbit \( j_r \),
Let the antisymmetrised and normalised wave functions for the neutron group and the proton group be $\psi_{v_{1n}}^{J_{1n}}$ and $\psi_{v_{1p}}^{J_{1p}} (m_{1})$.

Here $v_{1n}$ and $v_{1p}$ are the seniority quantum numbers for the neutron group and the proton group respectively, whereas $\alpha_{1n}$ and $\alpha_{1p}$ stand for extra quantum numbers, if any.

As mentioned in chapter II, we can write down the properly antisymmetrised and normalised (n-p) wave functions by simply vector coupling the wave functions $\psi_{v_{1n}}^{J_{1n}}$ and $\psi_{v_{1p}}^{J_{1p}} (m_{1})$.

The coupled neutron-proton states so constructed do not in general have definite isospin or a good overall seniority.

The matrix element of $T^2$ operator connecting two such neutron-proton states is given by:

$$
\langle n_{1} v_{1n} \alpha_{1n} J_{1n} | m_{1} v_{1p} \alpha_{1p} J_{1p} | \frac{2}{T} | n_{1} v_{1n} \alpha_{1n} J_{1n} | m_{1} v_{1p} \alpha_{1p} J_{1p} \rangle
$$

Single shell m.e.'s of $U^k$ give a non-vanishing contribution only when $J_{1n}$, $k$, $J'_{1n}$, and $J'_{1p}$, $k$, $J'_{1p}$ satisfy the triangular
conditions. Besides that, $U^k$ connects identical nucleon states that differ from each other by 0 or ± 2 units in seniority quantum number.

One sets up the $T^2$ matrix in the space spanned by single orbit (n-p) states, obtained by vector coupling allowed antisymmetrised neutron states to the allowed antisymmetrised proton states, and diagonalizes it to obtain the states labelled by definite isospin quantum number.

Consider for example two neutrons and one proton in $j_1$ orbit with neutrons in seniority zero state. The (n-p) wave function looks like

$$\Psi_{j_1}^{\frac{1}{2}}(j_{ln}^0 \times j_{lp}) = \frac{A_{j_1}}{Z_{j_{ln}}(2) \sqrt{v_{j_{ln}}}} Y^*_{j_1} x (i x-j) = -N_{A_{j_1}}$$

Obviously the system has a definite overall seniority $\nu=1$. But as such it is not an eigen function of $T^2$ operator.

$T^2$ operator connects this state to the one in which we have a zero coupled (n-p) pair coupled to a single unpaired neutron. This is expected since in going from (n-p) representation to isospin representation, the distinction between (p-p), (n-n), and (n-p) pairs is lost.

In state operator language we expect the n-p wave function, having a zero coupled neutron-proton pair coupled to a single neutron, to look like

$$\Psi_{j_1}^{j_1} [j_{ln} (j_{ln}^0 x j_{lp})_0] = -N_{A_{j_{ln}}}$$

(3.15)
here $N$ is the normalisation constant. $\mathcal{V}^{j_l} \Psi_{j_l n} (j_{ln} x j_{lp})_0$, defined as above, has non-zero overlap with $\mathcal{V} [j_{ln}]_0 x j_{lp}$, pointing to the fact that it has a component having two neutrons coupled to zero. We construct a linear combination

$$\Psi^j_1 [(j_{ln}) x j_{lp}] = A \mathcal{V}^{j_l} [(j_{ln})_0 x j_{lp}]$$

$$+ B \mathcal{V}^{j_l} [j_{ln} x (j_{ln} x j_{lp})_0]. \quad (3.16)$$

The Amplitudes $A$ and $B$ are determined by the conditions that ensure the orthonormality of the wave function $\Psi^{j_l} [(j_{ln}) x j_{lp}]$ i.e.,

$$\langle \Psi^j_1 [(j_{ln}) x j_{lp}] | \Psi^j_1 [(j_{ln})_0 x j_{lp}] \rangle = 0 \quad (3.17a)$$

$$\langle \Psi^j_1 [(j_{ln})_0 x j_{lp}] | \Psi^j_1 [(j_{ln}) x j_{lp}] \rangle = 1 \quad (3.17b)$$

$T$ operator is now diagonalized in the two-dimensional space spanned by basis vectors $\mathcal{V}^{j_l} [(j_{ln})_0 x j_{lp}]$ and $\mathcal{V}^{j_l} [(j_{ln}) x j_{lp}]$. We obtain the following two eigenfunctions of $T$ operator corresponding to the eigenvalues $T = \frac{1}{2}$ and $T = \frac{3}{2}$ respectively.

$$\Psi^{j_l, T = 3/2}_{v_1 = 1} = \frac{\zeta^{j_l - 1}}{3 (2j_{ln} + 1)} \mathcal{V}^{j_l} [(j_{ln})_0 x j_{lp}]$$

$$+ \frac{2 (2j_{ln} + 2)}{3 (2j_{ln} + 1)} \mathcal{V}^{j_l} [(j_{ln}) x j_{lp}] \quad (3.18)$$
\[
\psi_{j_1, T_1; \frac{1}{2}}^{(j_3^3)} = \begin{cases} 
\frac{2(2j_1+2)}{3(2j_1+1)} & \text{if } j_1 \geq 1 \\
\frac{(2j_1-1)}{3(2j_1+1)} & \text{if } j_1 < 1 
\end{cases} \psi_{j_1}^{\frac{3}{2}} [(j_{1n})_0 \times j_{1p}] 
\]

\[
T \text{ operator commutes with the symplectic Casimir operator } G, 
G = 2 \sum_{k \text{ odd}} [k] (U \cdot U) 
\]

This fact can be used to further remove the \( T \) degeneracy. The \( G \) matrix is set up in the good \( T \) basis obtained earlier and diagonalized to obtain states labelled by quantum numbers \( T, v \) and \( t \). Here \( v \) is the overall seniority of the single orbit nucleon groups (the number of unpaired nucleons) and \( t \) is the reduced isotopic spin, defined as the isotopic spin due to the unpaired nucleons. The eigenvalues of \( G \) are a function of \( v \) and \( t \), being given for the nucleon group in orbit \( j \) by the value,\([ (2j + 2)v - \frac{3}{2} v - 2t (t+1) ]\).

The final wave functions, characterised by definite isospin value \( T \), seniority \( v \), reduced isotopic spin \( t \) and angular momentum \( J \) can be expressed in terms of the coupled products of the wave functions for the neutron group in orbit \( j \) and the wave functions for the proton group in orbit \( j \), through the following expansion,

\[
\psi_{j T}^{(n + m)} = \sum_{\alpha} A_{(n + m, \alpha) \cdot (n_1, \alpha_1 \cdot J_n \cdot n \cdot J_n \cdot J)} \left( \psi_{n, \alpha_1}^{(n)} \times \psi_{p, \alpha_2}^{(p)} \right) 
\]

\[
(3.21) 
\]
The expansion coefficients $\mathbf{A}^{JT}$ are the matrix elements of the unitary transformation that simultaneously diagonalizes $T$ and $G$ in the coupled neutron-proton basis and hence produces states having good $T$ and good symplectic symmetry.

C. Construction of Good $T$ Multishell States in (n-p) Representation.

We now come to the generation of good $T$ states having $n = n_1 + n_2 + \ldots + n_k$ and $m = m_1 + m_2 + \ldots + m_k$ protons distributed over $k$ orbits. Firstly, single shell states having definite isospin, seniority and reduced isospin are generated for the desired distributions of neutrons and protons in each of the active orbits. These are now successively vector coupled to obtain the set of basis vectors that span the vector space in which the isospin operator $T^2$, for the $(n+m)$ nucleon system, is to be diagonalized.

The eigen vectors obtained after the diagonalization of $T^2$ operator are simultaneously eigenfunctions of the operators $T_1^2$, $T_2^2$, $T_3^2$ and $T^2$.

In order to illustrate the procedure, we consider the case where $n$ neutrons and $m$ protons are distributed over two orbits having angular momenta $j_1$ and $j_2$. We confine ourselves to the good $T$ states generated by a distribution $[n_1, n_2 + m_2]$ of the nucleons. The basic (n-p) states generated by vector coupling $\mathbf{v}_{\text{inn}}^{j_1 n_1}$ and $\mathbf{v}_{\text{inn}}^{j_2 n_2}$ are denoted by $\mathbf{v}_{1p}^{j_1 n_1}$ and $\mathbf{v}_{1p}^{j_2 n_2}$. 
Here $p$ simply numbers the state $\chi$, a member of the $(n-p)$ dimensional $(n-p)$ basis. For the distribution $[j_1^L]$ of nucleons in orbit $j_1$, the states labelled by definite values of $T_1, v_1, t_1$ and $a_1$ are given by,

$$\varphi_p (n_1^L m_1) = \left( \sum_{p=1}^{d_1} A_{pq} (n_1^L m_1) \chi_p (n_1^L m_1) \varphi_q (n_1^L m_1) \right), \quad (3.23)$$

$q$ also takes values 1 to $d_1$.

Similarly the states which are eigenfunctions of the operators $T_2^2, g_2$ for the distribution $[j_2^L]$ on the nucleons in orbit $j_2$ are

$$\varphi_s (n_2^L m_2) = \sum_{r=1}^{d_2} B_{rs} (n_2^L m_2) \chi_r (n_2^L m_2), \quad (3.24)$$

Suffix $s$ takes values 1 to $d_2$.

The coupled vectors

$$\varphi_t (n^L m) = \sum_{p=1}^{d_1} \sum_{r=1}^{d_2} A_{pq} (n_1^L m_1) B_{rs} (n_2^L m_2) \chi_p (n_1^L m_1) \chi_r (n_2^L m_2) \quad (3.25)$$

span the $d = (d_1 d_2)$ dimensional space. Isospin operator $T_1^{n_1^L m_1 n_2^L m_2}$ connects the states generated by distribution $[j_1^L, j_2^L]$. 
of neutrons and protons to those generated by all possible distributions \([ j_1^{n_1 \pm \delta n_1}, j_2^{n_2 \pm \delta n_2}]\), obtained from the former by interchanging proton in orbit \(j_1\) with a neutron in orbit \(j_2\) and vice versa. The product vectors \(\varphi_j^{(n + m)}\) are generated for all possible distributions of nucleons in the two orbits as described above. The \(T^2\) matrix is set up in the basis so generated and diagonalized to obtain good \(T\) multishell basis states. The coefficients that enter as a result of diagonalizing \(T^2\) operator in the basis obtained by angular momentum coupling of single shell good \(T\) states, happen to be Clebsch-Gordon coefficients in the isospin space having the form

\[
\begin{pmatrix}
T_1 & T_2 & T \\
C_{T_1^{T_2^T}} & z_1 & z_2 & z
\end{pmatrix}
\]

A. Hamiltonian Operator in (n-p) Representation.

One of the basic assumptions of shell model is, the two-body nature of the interaction between the nucleons inside the nucleus. With this assumption, the Hamiltonian for a nuclear system can be written as a sum of a one-body part, corresponding to the kinetic energy of the system and a two-body part which corresponds to the potential energy of the system.

\[ H = H^{(1)} + H^{(2)} \] (3.26)

The expansion of the Hamiltonian operator \( H^{(1)} \) in terms of the second quantized spherical tensors is obtained by using eq. (2.24)

\[
H = \sum_{\mathbf{r}\mathbf{s}} \varepsilon_{\mathbf{r}\mathbf{s}} \left[ \mathcal{B}_\mathbf{r}^\dagger \right] \delta_{\mathbf{r}\mathbf{s}} \left( A^0_x B^0_y \right) \\
+ \sum_{\mathbf{r}\mathbf{s}\mathbf{t}\mathbf{u}} \left[ \mathcal{A}^\dagger \right] \mathcal{B}_{\mathbf{r}\mathbf{s}\mathbf{t}\mathbf{u}} (Z_{\mathbf{r}\mathbf{s}} \times Z_{\mathbf{t}\mathbf{u}})^0 \] (3.27)

where

(i) \( \mathcal{B}_{\mathbf{r}\mathbf{s}\mathbf{t}\mathbf{u}} = \langle \mathcal{B}_{\mathbf{r}\mathbf{s}}^\dagger \mathcal{B}_{\mathbf{t}\mathbf{u}} | H^{(2)} | \mathcal{B}_{\mathbf{r}\mathbf{s}}^\dagger \mathcal{B}_{\mathbf{t}\mathbf{u}} \rangle \), is the anti-symmetrised two-body matrix element.

(ii) \( \varepsilon_{\mathbf{r}\mathbf{s}} = \langle \mathcal{B}_{\mathbf{r}\mathbf{s}} | H^{(1)} | \mathcal{B}_{\mathbf{r}\mathbf{s}}^\dagger \rangle \) is the one-body matrix element.
(iii) \( Z^{rs}(\alpha) \equiv \frac{1}{r_{rs}} (A_{r} x A_{s})^{T} \) is the state operator for a two nucleon system characterised by quantum numbers \( \Gamma \) (JT or LST or J, depending on the coupling scheme being used).

(iv) We also define \( \frac{1}{r_{rs}} = \frac{1}{\sqrt{1+\delta_{rs}}} \).

The condition \( r \leq s \) and \( t \leq u \) ensures that there is no double counting. The superscript \( a.m. \) on Kronecker delta stands for that angular momentum part of \( r_{s} \) and \( r_{f} \) is involved here. The expression does not necessarily vanish when the radial quantum numbers are not the same.

In (n-p) formalism the interaction Hamiltonian can be split up into two distinct parts

\[ H = H_{nn} + H_{pp} + H_{np} = H(\text{identical nucleons}) + H_{np} \quad (3.28) \]

here \( H_{np} \) is the interaction between neutrons and protons.

The second quantized version of \( H \) in (n-p) formalism is essentially a special case of the general eq. (3.27). The labels \( rstu \) characterise the orbits ordered in a definite "fashion. Here \( r \) stands for the quantum numbers \( n_{r}, \ell_{r} \) and \( j_{r} \).

Suffixes \( n \) and \( p \) are added to the orbit labels to show whether the nucleon occupying a particular single-particle orbit is a neutron or a proton. The final angular momentum of the two-particle state is \( \Gamma = J \) in the present case.
H(Identical nucleons) = \sum_{s} \left\{ \left[ \begin{array}{c} 1 \\ \hline 1 \\ \hline s \\ \end{array} \right] \right\}^{\frac{1}{2}} \epsilon_{rs}(n) B^{j_{sn}} + \frac{1}{2} \left[ j_{rs} \right] \epsilon_{rs}(p) B^{j_{sp}} \right\}

+ \sum_{s \leq t \leq u} \left[ J_{n} \right]_{n \leq s, t \leq u} W_{rstu}^{(nn)} Z_{rs} Z_{tu}^{(tu)}

+ \sum_{s \leq t \leq u} \left[ J_{p} \right]_{n \leq s, t \leq u} W_{rstu}^{(pp)} Z_{rs} Z_{tu}^{(tu)}

H(n-p) = \sum_{s \leq t \leq u} \left[ J_{n} \right]_{n \leq s, t \leq u} W_{rstu}^{(np)} Z_{rs} Z_{tu}^{(tu)}

here Z_{np}^{(rs)} = \triangle A_{np}^{rs} \text{ is the state operator for a single neutron and a single proton coupled to final angular momentum } J.

\text{W}^{J}_{rstu}^{(np)} = \left\langle j_{rn}^{j_{sp}}; J \right| W_{rstu}^{J_{n}} + W_{rstu}^{J_{p}} \left| j_{tn}^{j_{up}}; J \right\rangle \text{ is the two body (n-p) matrix element. } W_{n}^{J}(nn), W_{p}^{J}(pp) \text{ and } W_{np}^{J} \text{ are related to the isospin representation two body matrix elements, } W_{J}^{T}, \text{ where } T \text{ can take values zero and one, through the following equation}

\begin{align*}
W_{rstu}^{J}(np) &= \frac{1}{Z_{rs} Z_{tu}^{(tu)}} \left[ W_{rstu}^{J_{0}} + W_{rstu}^{J_{1}} \right] \quad (3.30a) \\
W_{rstu}^{J}(nn) &= W_{rstu}(pp) = W_{rstu}^{J_{1}} \quad (3.30b)
\end{align*}

B. Single - Nucleon Transfer Operator.

The operator appropriate for the addition of a single nucleon (nucleon-hole) to the orbit \((n, \ell, j)\) of the target state \(\psi_{x}^{(n)}(n, \ell, j)\), having \(n_{(n+1)}\) active nucleons, is the creation tensor \(A^{\rho}\) (destruction tensor \(B^{\rho}\)). The reduced matrix element of the operator between the target state and an
eigenstate of the residual nucleus $\psi_x^{T_z}(n+1)(\psi_{x_0}^{T_z}(n))$ is a measure of the reduced single nucleon (nucleon-hole) transition amplitude and is related to the spectroscopic amplitude (the square root of the usual spectroscopic factor) through

$$S_{JT}^{(+)}(n_0 x_0 \rightarrow \Gamma x) = (-1)^{T_0 + p - \Gamma} (2T_0 + 1)^{-1/2} \langle \Gamma x || A^p || n_0 x_0 \rangle$$

We have therefore

$$S_{JT}^{(+)}(n_0 x_0 \rightarrow \Gamma x) = (-1)^{T_0 - p - \Gamma} (2T_0 + 1)^{-1/2} \langle \Gamma x || A^p || n_0 x_0 \rangle$$

$$S_{JT}^{(-)}(n_0 x_0 \rightarrow \Gamma x + p) = (-1)^{T_0 + p - \Gamma} (2T_0 + 1)^{-1} \langle \Gamma x + p || A^p || n_0 x_0 \rangle$$

(+) and (-) stand for nucleon stripping and pickup respectively.

The spectroscopic strengths are defined as

$$G_{JT}^{(+)} = (C_{T_0}^{\frac{1}{2}})^2 \frac{(2J_0 + 1)}{(2J_0 + 1)} S_{JT}^{(+)}(n_0 x_0 \rightarrow \Gamma x)$$

$$G_{JT}^{(-)} = (C_{T_0}^{\frac{1}{2}})^2 S_{JT}^{(-)}(n_0 x_0 \rightarrow \Gamma x + p)$$

The sign of $t_z$ determines whether the added nucleon
(nucleon-hole) is a proton (proton-hole) or a neutron (neutron-hole).

C. M1 Transition Operator in (n-p) Representation

The magnetic dipole operator \( \mu \) for a system having neutrons as well as protons can be split as

\[
\mu_{\text{op}} = \mu_{\text{n}} + \mu_{\text{p}}
\]

(3.34)

It can now be expressed in terms of coupled product of one-body operators and single particle matrix elements of \( \mu_{\text{n}} \) and \( \mu_{\text{p}} \):

\[
\mu_{\text{op}}^{(n-p)} = \frac{1}{\sqrt{3}} \sum_{\nu} \langle j_{\text{op}}^{\text{op}} | j_{\nu}^{\text{n}} \rangle | j_{\nu}^{\text{p}} \rangle A_{\nu}^{\text{B}} B_{\nu}^{\text{n}} j_{\nu}^{\text{p}}
\]

(3.35)

Single-particle reduced matrix element of \( \mu \) in units of \( \mu_0 = \frac{e}{2M_c} \) (nuclear magneton) is given by (4,5)

for \( j = \ell + \frac{1}{2} \)

\[
\langle j | \mu_{\text{op}} | j \rangle = \left( \frac{2j+1}{j+1} \right) \left( \frac{\ell_2 + \ell_3}{\epsilon_2 + \epsilon_3} \right)
\]

(3.36a)

for \( j = \ell - \frac{1}{2} \)

\[
\langle j | \mu_{\text{op}} | j \rangle = \left( \frac{2j+1}{j+1} \right) \left[ (\ell_2 + \ell_3) + (\ell_2 + \ell_3) \right] \left( \epsilon_2 - \epsilon_3 \right)
\]

(3.36b)

In addition we also have

\[
\langle \ell + \frac{1}{2} | \mu_{\text{op}} | \ell - \frac{1}{2} \rangle = \left[ 2(\ell + 1)/(\ell + 1) \right]^{\frac{1}{2}} (\epsilon_2 - \epsilon_3)
\]

(3.36c)

\[
\langle \ell - \frac{1}{2} | \mu_{\text{op}} | \ell + \frac{1}{2} \rangle = -\left[ 2(\ell + 1)/(\ell + 1) \right]^{\frac{1}{2}} (\epsilon_2 - \epsilon_3)
\]

(3.36d)
The gyromagnetic ratios, $g$, for neutrons and protons have the values

$$g_s(p) = 5.585 \quad , \quad g_s(n) = -3.826$$

The reduced transition strength, $B(M1)$, for $M1$ transition from a state $i$ to the final state $f$ is defined as

$$B(M1) = \frac{2}{4\pi(2J_i+1)} \left| \left\langle f \left| \mu \right| i \right\rangle \right|^2$$

In units of $[(3/4\pi)\mu_0^2]$, $B(M1)$ is given by

$$B(M1) = \frac{1}{(2J_i+1)} \left| \left\langle f \left| \mu/\mu_0 \right| i \right\rangle \right|^2$$

D. E2 Transition Operator in (n-p) Representation.

The electric quadrupole operator $Q$ is given by

$$Q = \sum_{\lambda} e_i r_i^2 Y_2(\sigma_i \varphi_i)$$

The quadrupole moment of a neutron is attributed to an effective charge $e_n$ carried by it. Similarly, the deviation of the proton quadrupole moment from the value calculated by taking $e$ as the actual protonic charge is accounted for by assigning an effective charge $e_p$ to the proton, which is generally found to be somewhat higher than $e$. For a system having $n$ neutrons and $m$ protons

$$Q = \sum_{\lambda} e_n r_i^2 Y_2(\sigma_i \varphi_i) + \sum_{\lambda} e_p r_i^2 Y_2(\sigma_i \varphi_i)$$

The second quantized representation of the quadrupole operator in (n-p) representation is,
The reduced single-particle matrix element of the operator \( Q \) can be expressed as a product of an angular momentum dependent part and the radial integral

\[
\langle j_r \mid e^r Y^2_{2}(\theta, \varphi) \mid j_s \rangle = \sqrt{\frac{5}{4\pi}} \left( (2j_r+1)(2j_s+1)(2\ell +1) \right)^{\frac{1}{2}} C(\ell_{r} \ell_{s} \ell_{0}; 000) W(j_r \ell_s, j_r \ell_s) \langle r^2 \rangle
\]

\[
\langle r^2 \rangle = \int r^2 R^n_{\ell_{r}}(r) r^2 R^n_{\ell_{s}}(r) dr
\]

being the radial part of the single-particle wave function. Harmonic oscillator wave functions with oscillator constant \( \hbar \omega = 41 A^{-1/3} \) MeV have been used for numerical evaluation of the radial integral; \( A \) is the total number of nucleons in the nucleus.

\( W(\ell_{r} \ell_{s}; j_r \ell_s) \) is unnormalised Racah coefficient.

The values of effective charge used by us are \( e_n = 0.5 e \) and \( e_p = 1.5 e \).

The reduced E2 transition rate \( B(E2) \), for a transition \((i \rightarrow f)\) is defined as

\[
B(E2) = \frac{5}{4\pi(2j_{i}+1)} \left| \langle f \mid q \mid i \rangle \right|^2
\]

\( B(E2) \) are finally expressed in Weisskopf units defined as
E. Evaluation of Matrix Elements of an Operator in (n-p) Basis and Good JT Basis.

A computer program has been written to evaluate the matrix elements of a given operator in the given (n-p) basis, using the general formula (2.26). The standard form expansions of $T^2$ operator, $H$ operator, $B(\text{M1})$ operator and $B(\text{E2})$ operator are supplied by separate subroutines. The identical nucleon single orbit matrix elements used have been calculated by Mr. S.K. Gandhi and loaded on tape.

The matrix elements of the transformation that simultaneously diagonalises the operators $T_1^2$, $T_2^2$, ..., $T_k^2$, $G_1$, $G_2$, ..., $G_k$ and $T^2$ in the multishell (n-p) basis are evaluated for a given system having $n$-neutrons and $m$ protons by explicitly diagonalising these operators in the basic representation following the procedure outlined in Section I of this Chapter.

The transformation matrix is later used to obtain the representation of $H_{\text{cp}}$, $\text{M1}$ operator and $B2$ operator in the good JT basis, starting from the representation of these operators in the neutron-proton basis.

As a check on the computer programs, the results of the calculation by Bloom et al. (7), of the $3/2^-$ states of $^{49}$Sc, taking $[(1f_{7/2}^8(n)\times 2p_{3/2}^2(p))]$ and $[(1f_{7/2}^7(n)\times 1f_{7/2}^7(p))\times 2p_{3/2}^2(n)]$ as the basic neutron-proton configurations and using Kuo and Brown interaction, (8) have been reproduced.
REFERENCES TO CHAPTER III


