5.1 Introduction:

This Chapter deals with reactions of 4.8 - 8 MeV/nucleon $^{208}$Pb beam bombarded on many light targets of $^{26}$Mg, $^{27}$Al, $^{48}$Ca, $^{50}$Ti, $^{52}$Cr, $^{58}$Fe and $^{64}$Ni. In these reactions, recently, the cross-sections for making the fused systems are measured (called, the capture cross-sections) and the fused systems are then found to disintegrate with fragment mass distributions centred around zero mass asymmetry /2/. The measured excitation energies are also large — about 70 to 80 percent of the Coulomb barrier heights at 8 MeV/nucleon.

We have analyzed the above mentioned data as a two step process of "symmetric mass fragmentation following capture" using the "fusion model" of Aroumougame et al /29/. The "fusion model" is based on the dynamical fragmentation theory, wherein the fused or the captured system is formed by the crossing over of an adiabatic interaction barrier and depending on the excitation energy, which is shown to increase as mass asymmetry increases, the captured system proceeds to form a cool compound nucleus or the fusion - fission process occurs. Thus, for very asymmetric reaction partners (as is the case in the present reactions), the second possibility of fusion - fission is expected to
happen and the symmetric mass fragmentation is given by the adiabatic fission of the excited composite system formed.

The process of capture and subsequent symmetric fragmentation is also given by the 3-dimensional time-dependent Hartree-Fock calculations of Stocker et al /83/. Also, Swiatecki /84/ has proposed a similar two-step model of 'nuclear coalescence and reseparation' for these reactions.

In the following, we present our calculations, first for the angular momentum dependent adiabatic scattering potentials in section 5.2. Calculations of the first step (the capture cross-sections) and the second step (mass distribution yields) of our model are then presented in sections 5.3 and 5.4 respectively. Finally, summary of our results is presented in section 5.5.

5.2 Angular Momentum Dependent Adiabatic Scattering Potential:

First of all, we argue the possibility of this phenomena of "fusion and subsequent fission" in reactions of $^{208}$Pb on various light targets, from the point of view of the amount of angular momentum $l$ a fused system can carry. This angular momentum is an interesting parameter which depends on the entrance channel and particularly on the bombarding energy. Figure 5.1 shows, as an illustrative example, the
adiabatic scattering potentials for $^{208}\text{Pb} + ^{50}\text{Ti} \rightarrow ^{258}\text{104}$, calculated for different angular momentum $\ell$ values. Some of the associated nuclear shapes for this and other colliding systems are shown in Fig. 5.2 (these are used for calculating the moment of inertia $I_\ell$ in the rotational energy term $V_\ell$ (Eqs. (2.34) and (A5))).

We first notice from Fig. 5.1 that the barrier exists for $\ell=0$ and the effective barrier height decreases as $\ell$ increases. For some $\ell = \ell_{\alpha}$ the barrier vanishes completely ($\ell_{\alpha} = 268$ for this system). Since the lowest incident bombarding energy used for this reaction is greater than the height of the Coulomb barrier, $E_{\text{cm}}/V_0 > 1$, as shown in Table 5.1, the incoming system gets captured in the pocket behind the barrier and form a compound system. However, the incoming system being very asymmetric, the compound system is very much excited /29/ (see also Table 5.1) and fissions rather than proceeding to the ground state for forming a cool compound nucleus. The effective barrier against fission is apparently less as $\ell$ increases. For $\ell > \ell_{\alpha}$, since the barrier reduces to zero, we say that the probability of forming the compound system is zero. The $\ell_{\alpha}$ values, for all the systems studied here, are calculated and given in Table 5.1.

Next, since the capture cross-sections $\sigma_\ell$ are measured /2/, the angular momentum $\ell_{\alpha}$ carried by the
compound system formed at each bombarding centre – of mass energy $E_{cm}$, can be calculated by using the sharp cut-off approximation.

$$\sigma = \frac{\pi}{2\mu} \frac{h^2}{E_{cm}} l_c^2$$

(5.1)

This means that at a given $E_{cm}$, the compound system formed with cross-section $\sigma$ can carry an angular momentum $l_c$. These numbers are also given in Table 5.1 for all the reactions of $^{208}$Pb on $^{50}$Ti, $^{52}$Cr, $^{58}$Fe and $^{64}$Ni. The scattering potentials given in Fig. 5.1 are for the $l$ -values greater than or equal to these $l_c$ -values. As already observed, for each of $l = l_c$ the barrier exists for capturing the incoming nuclei to form the compound system and in each case the barrier is low enough for the compound system formed to be able to fission. This is exactly what has been observed in the experiments of Bock et al /2/ for the bombarding energies upto 8 MeV/nucleon. Furthermore, we notice in Table 5.1 that the critical angular momentum $l_{cr}$, for all these reactions, correspond to $E_{lab}/A >> 8$ MeV/nucleon. Hence it will be of interest to see what happens beyond the present range of the experiments /2/.

In the following sections, we analyze further these two steps of capture and subsequent fission more quantitatively.

5.3 Capture Cross-sections:

For the first step of our model, we have calculated the capture cross-sections by using the sharp cut-off model expression.
\[ \sigma_c = \pi R_I^2 \left( 1 - \frac{V_I}{E_{cm}} \right) \]  \hspace{1cm} (5.2)

where \( R_I \) and \( V_I \) are the positions and heights of the interaction barriers. We have seen in Fig. 5.1 that \( V_I \) varies considerably with incident energy (the \( \ell \)-value) and \( R_I \) (\( \ell \)) remains almost constant (\( R_I = 7.74 \text{ fm} \) for \( ^{208}\text{Pb} + ^{50}\text{Ti} \)). In view of this result, for the calculations of \( \sigma_c \) as a function of \( E_{cm} \), shown in Fig. 5.3, we have used (i) the barrier for \( \ell = 0 \) only (dashed lines) and (ii) the \( \ell \)-dependent barriers (solid lines). For comparisons, we have normalized the calculations to the experimental data /2/ (shown as dots with error bars) at one point (the lowest \( E_{cm} \) value). This is essential because the interaction (or fusion) barriers are known /26.85/ to lie higher and at much smaller \( R \)-values than the Coulomb barriers. We notice in Fig. 5.3 that our calculations show a reasonable agreement with experiments, in particular for the low energy region and for the angular momentum dependent interaction barriers.

5.4 Mass Fragmentation Distribution Yields:

The second step of our model is to calculate the mass fragmentation distribution yields for the fission of the excited composite systems formed. Figs. 5.4-5.7 show the calculated adiabatic fragmentation potential \( V(\eta) \) and the adiabatic cranking masses \( B_{xy}(x,y = \eta, R) \) for all the four composite systems \( ^{258}\text{104}^*, \text{260}^{106}^* \),
and 272 respectively. Under the assumption that the main behaviour of the distributions is fixed at a point just after the barrier penetration /20,24/, the $\Lambda$ value is chosen near the top of the barrier and the effect of varying this choice is then studied. We notice in Figs. 5.4-5.7 that in each case the liquid drop potential $V_{\text{LDM}}(\eta)$ is smooth, like a simple harmonic oscillator, and the shell effects $\delta U$ contribute to both the potential and mass parameters. The effect of temperature $\Theta$ on the potential, given by Eq. (2.19), is shown in Fig. 5.4 for the illustrative case of $^{258}104$. We notice that the shell effects $\delta U(\Theta)$ reduce as $\Theta$ increases and for $\Theta = 1.90 \text{ MeV}$, which corresponds to 6.5 MeV/nucleon for $^{208}\text{Pb}$ bombarded on $^{50}\text{Ti}$, the shell effects are nearly zero such that the total potential $V(\eta)$ reduce almost to $V_{\text{LDM}}(\eta)$. Similar effects are expected for the mass parameters but, as stated in Chapter 2, to-date one does not know how to calculate the mass parameters at finite temperatures.

The mass fragmentation yields, calculated by using the potential $V_{\text{LDM}} + \delta U(\Theta)$ and the masses $\eta(\eta)$ of Figs. 5.4-5.7 at various $\Theta$ values, corresponding to the available experimental data /2/, are presented in Figs. 5.8-5.11, respectively, for fission of $^{258}104^*$, $^{260}106^*$, $^{266}108^*$ and $^{272}110^*$. The temperature effects
are also included by allowing, through Eq. (2.45), the fission from excited states. The experimental data /2/ is shown as dots at the three incident energies of 5.5, 5.9 and 6.5 MeV/nucleon. The data at 5.2 MeV/nucleon is not considered here because it is similar to that at 5.5 MeV/nucleon and is less precise. It is also relevant to mention here that the data on the shoulders at lower energies i.e. the peaks at \(A_2 \approx 200\) and \(A_1 = A - A_2\), seen in the experiments, is not certain /2/ because the measurements are unreliable for \(\eta > 0.04\). In other words, experimentally only the symmetric fragmentation can be said to be seen at all the incident energies /2/. In the following paragraph we shall see that this result might have an important consequence for the dynamical fragmentation process. We shall first analyze the case of \(^{258}\text{I}^{104}\) (refer to Fig. 5.8) in detail and then give our results for other systems.

Fig. 5.8(a) refers to the calculations and the experimental data at \(E_{cm} = 5.5\) MeV/nucleon (\(\theta = 1.42\) MeV) for the composite system \(^{258}\text{I}^{104}\). Curve 1 gives the results of our calculation for \(V_{\text{LDM}} + \delta U(\theta = 1.42\) MeV) and \(B\eta\eta(\eta)\). We notice here strong peaks at \(A_2 = 199\) and \(A_1 = 59\) and some other structure. The interesting point about this structure as well as the peaking effect is that, except for the decrease or increase of amplitudes, no change occurs when \(\delta U = 0\)
i.e. only $V_{\text{LDM}}$ is used (curve 1) or $\theta = 0$ i.e. no temperature dependence at all (Curve 1'). Apparently then, both the peaks and other structure in the distribution are not due to shell effects in the potential. In order to study the role of large structure in mass parameter $B_{\eta}(\eta)$, which might get reduced with the addition of temperature in it, we have calculated the mass yields using the averaged constant mass $\overline{B}_{\eta}$ ($= 5 \times 10^3$ fm$^{-2}$, in units of nucleon mass, for the present case of $^{258}104$ at $\lambda = 1.55$). This is shown by curves 2 and 2' calculated, respectively, for $V_{\text{LDM}} + SU(\theta = 1.42$ MeV) and $V_{\text{LDM}}$ alone. We notice that these two distributions are identical and completely smooth, without any peaking effect (curve 2 is slightly broader because of the additional $SU(\theta = 1.42$) energy). Thus, it seems that the peaking effect as well as the other structure in the distributions arose due to the large structure in masses $B_{\eta}(\eta)$. A realistic temperature dependence on masses, however, might not be able to keep this result and one has to then look for their source somewhere else. Fig. 5.8(b) and 5.8(c) (referring to $E_{\text{cm}} = 5.9$ and 6.5 MeV/nucleon or $\theta = 1.62$ and 1.90 MeV, respectively), give exactly the same result. Now, comparing our calculations with experiments, we notice that, if the shoulders or peaks observed in the experimental data are disregarded /2/, the symmetric mass fragmentation is simply the (dynamical) liquid drop effect. The calculated mass
distributions are symmetric for the averaged constant mass parameter and with or without shell effects in the potential energy, thereby reproducing successfully the gross features of the experimental data /2/, and their absence, if confirmed, could help in determining the effect of dynamical role of the mass parameters. This demands, however, the use of temperature-dependent cranking masses.

In view of our observations in the last paragraph above, Figs. 5.9, 5.10 and 5.11 give for the systems $^{260}106$, $^{266}108$ and $^{272}110$, respectively, a comparison of the experimental data with calculated mass distribution yields for only two cases of $V_{\text{LDM}} + \delta U (\Theta)$ with (i) $B/\eta (\eta)$ and (ii) constant $B/\eta \eta$ (curves 1 and 2). Apparently, all the results obtained above for $^{258}104$ are supported, except that no shoulders are predicted for $^{266}108$ and $^{272}110$. There is however, still enough structure in the calculated distributions of $^{266}108$ and $^{272}110$ also, not present in experimental data, which is sensitive to the details of the mass parameters used.

Finally, in order to study the effect of varying the chosen $\lambda$-value, we have shown in Figs. 5.12 and 5.13, the calculated potential and mass parameters for a choice of a smaller and a larger $\lambda$-value, respectively, for the composite systems $^{258}104$ and $^{272}110$. The corresponding calculated yields are
presented in Figs. 5.14 and 5.15. We notice that, in conformity with our earlier calculations \cite{20,25}, the main results of the last two paragraphs are still obtained, independent of the choice of \( \lambda \)-value. The distribution are again symmetric. The important point of difference, however, lies in the predictions of detailed structure and shoulders. The shoulders now disappear in \(^{258}104\) but are predicted in \(^{272}110\). Thus, the question of shoulders etc. in experimental data points out not only to the problems of temperature dependence of mass parameters but also to the dynamical coupling between relative motion and mass asymmetry for these reactions. This apparently calls for further experiments with refined measurements in the region of mass asymmetry \( \eta > 0.4 \).

5.5 Summary of Results:

In this Chapter, an application of "fusion model" is studied. We have seen that in the reactions of 4.8 - 8 MeV/nucleon \(^{208}\)Pb on \(^{50}\)Ti, \(^{52}\)Cr, \(^{58}\)Fe and \(^{64}\)Ni, the colliding systems overcome the adiabatic interaction (or fusion) barriers and get captured in the pockets behind the barriers and form composite systems \(^{258}104\), \(^{260}106\), \(^{266}108\) and \(^{272}110\), respectively. Being strongly asymmetric systems, the excitation energies of the composite systems formed are large, so that they fission back adiabatically. The fusion (or capture) cross sections are shown to compare reasonably well
with experiments up to 8 MeV/nucleon and the gross features of the mass yields, i.e., the symmetric mass fragmentation are reproduced systematically, independent of the choice of relative separation distance $R$ and the detailed structure in the cranking mass parameters. The symmetric fission is shown to be the (dynamical) liquid drop effect and the other detailed structures in the mass distributions, including the shoulders, are found to depend on (i) how the temperature would affect the variation of masses with mass asymmetry, and (ii) the dynamical coupling of mass asymmetry with the relative motion of the separating systems in these reactions. Therefore, refined measurements of the data for larger mass asymmetry ($\eta > 0.4$) are of great importance. The calculations of the critical angular momentum for the vanishing of the fusion barrier also suggest extension of the present experiments to still higher energies.

Hence, in these compound - nucleus - fission reactions a two step process of "symmetric fragmentation following capture", which is given by the dynamical fragmentation theory, is clearly prevalent.
Table 5.1
Some of the characteristic quantities for the reactions with $^{208}$Pb

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$E_{\text{lab}}/A$ (MeV/nucleon)</th>
<th>$V_C$ (MeV)</th>
<th>$E_{\text{cm}}$ (MeV)</th>
<th>$E_{\text{cm}}/V_C$</th>
<th>$E^a$ (MeV)</th>
<th>$\ell_C$ (fm)</th>
<th>$\ell_{\text{cr}}$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{208}\text{Pb}+^{50}\text{Ti}$</td>
<td>4.77</td>
<td>190.5</td>
<td>192</td>
<td>1.01</td>
<td>22</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.00</td>
<td>202</td>
<td>1.06</td>
<td>32</td>
<td>29</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.25</td>
<td>211</td>
<td>1.11</td>
<td>42</td>
<td>43</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.50</td>
<td>222</td>
<td>1.16</td>
<td>52</td>
<td>62</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.90</td>
<td>238</td>
<td>1.25</td>
<td>68</td>
<td>71</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.50</td>
<td>263</td>
<td>1.38</td>
<td>93</td>
<td>89</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>8.00</td>
<td>323</td>
<td>1.69</td>
<td>152</td>
<td>117</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| $^{208}\text{Pb}+^{52}\text{Cr}$ | 5.24                             | 207.3       | 218                   | 1.05                | 33         | 21          |                |
|                                | 5.50                             | 228         | 1.10                  | 44                  | 41         |             |                |
|                                | 5.90                             | 245         | 1.18                  | 61                  | 57         |             |                |
|                                | 6.50                             | 270         | 1.30                  | 86                  | 79         |             |                |
|                                | 8.00                             | 332         | 1.61                  | 149                 | 102        |             |                |

| $^{208}\text{Pb}+^{58}\text{Fe}$ | 5.24                             | 222.0       | 238                   | 1.07                | 34         | 15          |                |
|                                 | 5.50                             | 249         | 1.12                  | 46                  | 34         |             |                |
|                                 | 5.90                             | 268         | 1.21                  | 64                  | 50         |             |                |
|                                 | 6.50                             | 295         | 1.33                  | 91                  | 79         |             |                |
|                                 | 8.00                             | 363         | 1.63                  | 159                 | 107        |             |                |
| $^{208}_{\text{Pb}}$ + $^{64}_{\text{Ni}}$ | 5.24  | 236.5 | 256 | 1.09 | 32  | 3  |
| 5.50  | 269   | 1.14  | 44  | 13   |
| 5.90  | 289   | 1.22  | 64  | 42   |
| 6.50  | 319   | 1.35  | 94  | 65   |
| 8.00  | 392   | 1.66  | 167 | 107  |

a) The ground state binding energies for these compound systems were not available. We have used here the corresponding values given in Seegre's Table/86/ for $^{267}_{108}$ and $^{273}_{110}$. 
Figure Captions:

Fig. 5.1 The adiabatic scattering potentials for \( ^{208}\text{Pb} + ^{50}\text{Ti} \rightarrow ^{258}\text{104} \), calculated for different angular momentum \( l \)-values. The energy scale is normalized to the binding energy \( E_B = 2074.25 \text{ MeV} \) of \( ^{208}\text{Pb} + ^{50}\text{Ti} \) at \( R = \infty \). For the complete overlap of nuclei (\( \lambda = 1.0 \)), following earlier works /5,59/, the curves are extrapolated to the ground state binding energy \( E_{\text{g.s.}} \) of the compound system \( (E_{\text{g.s.}} + E_B = -1904.70 + 2074.25 = 169.55 \text{ MeV} \) for \( l = 0 \) with \( V_{\ell} \) added for a spherical compound nucleus. The choices of \( 0 \leq l \leq \ell_{\text{c}} \) correspond to \( l_{\text{c}} \)-values given in Table 5.1.

Fig. 5.2 The nuclear shapes in adiabatic approximations at different lengths (\( \lambda \) values) for the target projectile combinations of \( ^{258}\text{104} \).

Fig. 5.3 The capture cross-section \( \sigma_c \) as a function of the centre-of-mass energy \( E_{\text{cm}} \). Our calculations, in sharp cut-off approximation, are made for constant \( V_{\text{I}} (l = 0) \) (dashed lines) and for \( l \)-dependent \( V_{\text{I}} \) (solid lines) and compared with the experimental data of Bock et al /2/ (dots
Fig. 5.4 The adiabatic (a) fragmentation potentials and (b) cranking mass parameters (in units of the nucleon mass $M$) for the compound system $^{258}104$ at $\lambda = 1.55$. In (a), the dot dashed curve gives the liquid drop model potential $V_{LDM}$ (calculated for 24 points) that are shown as dots.

Fig. 5.5 Same as in Fig. 5.4, except for the compound system $^{260}106$ at $\lambda = 1.53$.

Fig. 5.6 Same as in Fig. 5.4, except for the compound system $^{266}108$ at $\lambda = 1.47$.

Fig. 5.7 Same as in Fig. 5.4 except for the compound system $^{272}110$ at $\lambda = 1.35$.

Fig. 5.8 The calculated mass yield distributions compared with the experimental data, for the composite system $^{258}104$ at $\lambda = 1.55$ and different temperatures, $\Theta$ -values (or incident energies). Curves 1, $1'$ and $1''$ give the calculated yields by using, respectively, the potentials $V_{LDM} + \delta U(\Theta)$, $V_{LDM}$ or $V_{LDM} + \delta U(\Theta = 0)$ and $B\eta \eta (\eta)$. Curves 2 and $2'$ give the calculated yields, respectively for $V_{LDM} + \delta U(\Theta)$ or $V_{LDM}$ and
the average constant $\overline{B_{\eta\eta}} = 5 \times 10^3 \text{ M fm}^2$. The experimental data, deduced from Ref. 2 at different energies, is shown as dots. The calculations are not normalized to experimental data.

Fig. 5.9 Same as in Fig. 5.8, except for the compound system $^{260}_{106}$ at $\lambda = 1.53$ and $\overline{B_{\eta\eta}} = 2.2 \times 10^4 \text{ M fm}^2$.

Fig. 5.10 Same as in Fig. 5.8 except for the compound system $^{266}_{108}$ at $\lambda = 1.47$ and $\overline{B_{\eta\eta}} = 1.7 \times 10^3 \text{ M fm}^2$.

Fig. 5.11 Same as in Fig. 5.8 except for the compound system $^{272}_{110}$ at $\lambda = 1.35$ and $\overline{B_{\eta\eta}} = 3 \times 10^3 \text{ M fm}^2$.

Fig. 5.12 Same as in Fig. 5.4, except at $\lambda = 1.40$.

Fig. 5.13 Same as in Fig. 5.7, except at $\lambda = 1.55$.

Fig. 5.14 Same as in Fig. 5.8, except at $\lambda = 1.40$ and $\overline{B_{\eta\eta}} = 2.4 \times 10^3 \text{ M fm}^2$.

Fig. 5.15 Same as in Fig. 5.11, except at $\lambda = 1.55$ and $\overline{B_{\eta\eta}} = 2.8 \times 10^4 \text{ M fm}^2$. 
Adiabatic

$^{258}$Pb + $^{50}$Ti
($\eta = 0.612$)

$^{194}$Pt + $^{64}$Fe
($\eta = 0.50$)

$^{174}$Yb + $^{84}$Se
($\eta = 0.349$)

$^{156}$Sm + $^{102}$Mo
($\eta = 0.20$)

$^{136}$Xe + $^{122}$Sn
($\eta = 0.054$)

Fig. 5.2
Fig. 5.3
\[ \lambda = 1.53 \]

Fig. 5.5
\[ \lambda = 1.47 \]

\[ B_{RR}(x \times 10^6) \]
\[ B_{\pi \pi}(x \times 10^3) \]
\[ V_{LDM} + \delta U(\theta = 0) \]
\[ V_{LDM} \]

Fig. 5.6
Fig. 5.7
$\lambda=1.55 \quad \theta = 1.42 \text{ MeV}$

$E_{cm}=5.5 \text{ MeV/A}$

$E_{cm}=6.5 \text{ MeV/A}$

$E_{cm}=6.0 \text{ MeV/A}$

$\theta = 1.90 \text{ MeV}$
Fig. 5.10

(a) $E_{cm} = 5.5$ MeV/A, $\theta = 1.32$ MeV, $\lambda = 1.47$

(b) $E_{cm} = 5.9$ MeV/A, $\theta = 1.55$ MeV

(c) $E_{cm} = 6.5$ MeV/A, $\theta = 1.85$ MeV
Fig. 5.11

(a) $E_{\text{cm}} = 5.5\text{ MeV/A}$
$\theta = 1.27\text{ MeV}$
$\lambda = 1.35$

(b) $E_{\text{cm}} = 5.0\text{ MeV/A}$
$\theta = 1.53\text{ MeV}$

(c) $E_{\text{cm}} = 6.5\text{ MeV/A}$
$\theta = 1.66\text{ MeV}$
Fig. 5.12
Fig. 5.13

272

\[ \lambda = 1.55 \]

\[ B_{xy}(10^4 \, \text{fm}^2) \]

\[ V(\text{MeV}) \]

\[ V_{\text{LDM}} \]

\[ V_{\text{LDM}} + \delta U(0=0) \]

\[ -0.7 \quad -0.5 \quad -0.3 \quad -0.1 \quad 0.1 \quad 0.3 \quad 0.5 \quad 0.7 \]

\[ \text{MASS ASYMMETRY} \]

\[ \text{Fig. 5.13} \]