Chapter 2

Transmission of longitudinal waves through a plane interface between two dissimilar porous elastic solid half-spaces

2.1 Introduction

Employing Cowin-Nunziato theory of elastic material with voids, Ciarletta and Sumbatyan (2003) studied the reflection of obliquely incident plane elastic waves (longitudinal/transverse) from the stress-free boundary of an elastic half-space. They showed that only the transverse wave can propagate without attenuation after having been reflected from the free boundary surface. They have derived the formulae for reflection coefficients asymptotically in closed form for high and low frequency incident transverse wave. They have compared their results with those earlier known in classical elastic material and gave an estimate of the difference of the reflected angles and the amplitude of the free surface vibrations. It is found that the reflection coefficient and the vibration amplitude are typically less than that in classical media without voids. However, for relatively large transverse wave speed and high porosity, free boundary oscillation can exceed the classical one.

In this chapter, the reflection and transmission phenomena of an obliquely incident plane longitudinal wave at a plane interface between two dissimilar elastic half-spaces

containing pores have been studied. Using potential method and imposing appropriate boundary conditions at the interface, the equations giving the reflection and transmission coefficients are presented. It is found that for the case of limitly low frequency incident longitudinal wave, the presence of voids have significant effect on reflection and transmission coefficients. For the case of limitly high frequency incident longitudinal wave, the reflection and transmission coefficients exactly match with those already known in classical elasticity for the corresponding problem. The energy partitioning due to reflected and transmitted waves at the interface have also been presented and balance of energy has been verified at each angle of incidence.

2.2 Plane wave propagation

For time harmonic wave motion \( \exp\{-i\Omega t\} \), the equations of motion in the absence of body forces given by (1.145) and (1.146) reduce to

\[
\mu \nabla^2 u + (\lambda + \mu) \nabla(\nabla \cdot u) + \beta \nabla \phi + \rho \Omega^2 u = 0, \tag{2.1}
\]

\[
\alpha \nabla^2 \phi - \xi \phi - \beta \nabla \cdot u + (\omega \Omega + \rho K \Omega^2) \phi = 0. \tag{2.2}
\]

First, we shall discuss the plane wave propagation in an infinite porous elastic medium. Let the plane of vibrations be \((x, y)\) plane and \(z\)-axis be orthogonal to this plane. Taking

\[
u = (0, 0, w(x, y)), \quad \phi = 0, \tag{2.3}
\]

and substituting this values of \(u\) and \(\phi\) into equations (2.1) and (2.2), we get

\[
(\nabla^2 + k_s^2) w = 0, \quad k_s = \frac{\Omega}{c_2} > 0, \tag{2.4}
\]

where \(c_2\) is the phase speed of transverse wave and is defined in Chapter 1. Equation (2.4) is well known Helmholtz equation whose simple solution is given by

\[
w(x, y) = \exp\{ik_s(x \sin \gamma + y \cos \gamma)\}, \tag{2.5}
\]
where \( \gamma \) is an arbitrary angle of propagation. Therefore, from equation (2.4), we can say that horizontal polarized plane wave does not attenuate with distance and propagates with classical wave speed \( c_2 \). This shows that the propagation of shear horizontal polarized wave is independent of the presence of voids in the medium.

Now let us look at what happens in the case of propagation of plane wave with vertical polarization. In this case, the displacement vector \( \mathbf{u} \) and the variable \( \phi \) are taken as

\[
\mathbf{u} = (u(x, y), v(x, y), 0), \quad \phi = \phi(x, y). \tag{2.6}
\]

Using Helmholtz representation of vector, the displacement components in terms of classical wave potentials \( p(x, y) \) and \( q(x, y) \) can be written as

\[
u = \frac{\partial p}{\partial x} - \frac{\partial q}{\partial y}, \quad v = \frac{\partial p}{\partial y} + \frac{\partial q}{\partial x}. \tag{2.7}
\]

Making use of equations (2.6) and (2.7) into equation (2.1), we obtain

\[
(\nabla^2 + k_x^2)p = -H \phi, \tag{2.8}
\]

\[
(\nabla^2 + k_y^2)q = 0. \tag{2.9}
\]

where \( H = \frac{\beta}{\lambda + 2\mu} \) and \( k_p = \frac{\Omega}{c_1} \), \( c_1 \) being the phase speed of longitudinal wave defined in Chapter 1.

Substituting the values of \( u \) and \( v \) from equation (2.7) into equation (2.2), we have

\[
(a\nabla^2 - \xi + \omega \Omega + \rho K \Omega^2)\phi - \beta \nabla^2 p = 0. \tag{2.10}
\]

We note from equations (2.4) and (2.9) that the quantities \( w \) and \( q \) satisfy the same differential equation. This means that both the transverse waves (i.e. horizontally polarized shear wave and vertically polarized shear wave) propagate with the same speed \( c_2 \). While, equations (2.8) and (2.10) are coupled in \( p \) and \( \phi \). Eliminating \( \phi \) from these equations, we obtain

\[
\{(\nabla^2 + k_p^2)((l_p^2 \nabla^2 - 1 + \omega^2 k_p^2 + k^*2 k_p^2) + N \nabla^2)\}p = 0, \tag{2.11}
\]
where

\[ 0 < N = \frac{l_2^2}{l_1^2} H < 1, \quad \omega^* = \frac{\omega c_1 l_2^2}{\alpha}, \quad \text{and} \quad k^* = l_2 c_1 \sqrt{\frac{\rho K}{\alpha}}. \]

\( l_1, l_2, \omega^* \) and \( k^* \) are the quantities with dimension of length. The other symbols are already defined in Chapter 1.

One may take the potential corresponding to plane longitudinal wave in the following form

\[ p = A \exp\{i\psi(x \sin \gamma + y \cos \gamma)\}, \quad (2.12) \]

where \( \psi \) is the wavenumber, \( A \) is the amplitude and \( \gamma \) is the angle of propagation. Substituting the potential \( p \) from equation (2.12) into equation (2.11), we obtain the following equation

\[ (k_p^2 - \psi^2)(-l_2^2 \psi^2 - 1 + \omega^* k_p + k^* k_p^2) - N \psi^2 = 0. \quad (2.13) \]

This is a bi-quadratic equation in \( \psi \). Puri and Cowin (1985) have shown that in general, the equation (2.13) would give two real valued solution for each case of limitly high and low frequencies. The limitly low and high frequency cases are recalled here as follows:

In limitly high frequency case, when \( l_2 k_p >> 1 \), equation (2.13) can be written as

\[ l_2^2 \psi^4 - [(l_2^2 + k^*^2) k_p^2 + \omega^* k_p] \psi^2 + (k^*^2 k_p^2 + \omega^* k_p) k_p^2 = 0. \]

In writing this equation, the main asymptotic term in real and imaginary parts have been retained (see Ciarletta and Sumbatyan, 2003). Thus the two solutions of this equation for limitly high frequency case are given by

\[ \psi_1 = k_p = \frac{\Omega}{c_1}, \quad \psi_2 = \frac{\Omega}{c_3} + \frac{\omega^*}{2l_2 k_p} = \frac{\Omega}{c_3} + \frac{\omega^* c_3^*}{2\alpha}. \quad (2.14) \]

\( c_3^* \) is defined in Chapter 1. In limitly low frequency case \( l_2 k_p << 1 \), the two solutions are given by

\[ \psi_1 = \frac{k_p}{\sqrt{1 - N}} = \frac{\Omega}{c_1 \sqrt{1 - N}}, \quad (2.15) \]
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\[ \psi_2 = \frac{\omega k_p}{2l_2 \sqrt{1 - \eta}} + \frac{i \sqrt{1 - \eta}}{l_2} = \frac{\Omega}{c^*_1} + \frac{i \sqrt{1 - \eta}}{l_2}. \] 

(c) is defined in Chapter 1. If we look at the roots of equation (1.155) of Chapter 1 and compare with (2.13), it is noted that there is a sign difference in the imaginary part of the second root obtained in both the cases of high and low frequency. This difference in sign had occurred due to the consideration of opposite sign in time harmonic term of the wave potential. Puri and Cowin (1985) have also defined that \( \psi_1 \) and \( \psi_2 \) represent the wavenumber for predominantly elastic wave and predominantly volume fractional wave respectively. Equation (2.13) and its solution given by equations (2.14) - (2.16) for limitly low and limitly high frequency cases have been extensively studied by Ciarletta and Sumbatyan (2003). From equations (2.14)-(2.16), they concluded that (a) the first reflected longitudinal wave for high frequency in porous medium propagates with the classical wave speed \( c_1 \) at angle \( \gamma_1 \) and for low frequency case, it propagates with the phase speed \( c^*_1 \) at the same angle \( \gamma_1 \). (b) The behavior of the second reflected wave, i.e., void volume fractional wave at an angle \( \gamma_2 \) decay with distance, for both the high and low frequency cases. The attenuation coefficients \( \frac{\omega c_3}{2\alpha} \) and \( \frac{\sqrt{1 - \eta}}{l_2} \) for high and low frequencies respectively, are independent of frequency \( \Omega \). Therefore, the change in the frequency \( \Omega \) will not bring any change in the attenuation coefficients.

2.3 Problem and its solution

Let us consider two dissimilar homogeneous and isotropic porous elastic half spaces \( M \) and \( M' \) in perfect contact. Let us take the \( x- \) axis along the interface between two half spaces \( M(0 < y < \infty) \) and \( M'(\infty < y < 0) \) in such a way that \( y- \) axis is pointing vertically downwards into the medium \( M \). We denote the relevant physical quantities in the medium \( M' \) with primes and the corresponding quantities in medium \( M \) without primes. Let a unit amplitude plane longitudinal wave propagating through the lower medium and making an angle \( \gamma_1 \) with the normal be incident at the interface \( y = 0 \). The geometry of the problem is shown in Figure 2.1. We postulate that this incident longitudinal plane wave at the interface will give rise to:

Reflected waves:

(i) a longitudinal plane wave of amplitude \( P_1 \) propagating in medium \( M \) and making an angle \( \gamma_1 \) with the normal.
(ii) a longitudinal void volume fractional wave with amplitude $P_2$ propagating in medium $M$ and making an angle $\gamma_2$ with the normal.

(iii) a transverse wave of amplitude $Q$ propagating in the medium $M$ and makes an angle $\theta$ with the normal.

**Transmitted waves:**

(i) a longitudinal plane wave of amplitude $P'_1$ propagating in the medium $M'$ and making an angle $\gamma'_1$ with the normal.

(ii) a longitudinal void volume fractional wave with amplitude $P'_2$ propagating in the medium $M'$ and making an angle $\gamma'_2$ with the normal.

(iii) a transverse wave propagating in the medium $M'$ with amplitude $Q'$ and making an angle $\theta'$.

The full wave structure of reflected and transmitted waves can be written as:

In medium $M$ :

$$p = \exp\{i\psi_1(x \sin \gamma_1 - y \cos \gamma_1)\} + P_1 \exp\{i\psi_1(x \sin \gamma_1 + y \cos \gamma_1)\}$$
\[ q = Q \exp \{ i k_y (x \sin \theta + y \cos \theta) \}. \] (2.18)

In medium \( M' \):

\[ p' = P'_1 \exp \{ i \psi'_1 (x \sin \gamma'_1 - y \cos \gamma'_1) \} + P'_2 \exp \{ i \psi'_2 (x \sin \gamma'_2 - y \cos \gamma'_2) \}, \] (2.19)

\[ q' = Q' \exp \{ i k'_y (x \sin \theta' - y \cos \theta') \}, \] (2.20)

where \( \psi_1 \) and \( \psi_2 \) are respectively, the wavenumbers of reflected longitudinal and void volume fractional waves given by (2.14)-(2.16), while \( \psi'_1 \) and \( \psi'_2 \) are respectively, the wavenumbers of transmitted longitudinal and void volume fractional waves. The expressions of \( \psi'_1 \) and \( \psi'_2 \) are similar to those of \( \psi_1 \) and \( \psi_2 \) and are written bellow as

\[ \psi'_1 = \frac{k'_p}{\sqrt{1 - N'_p}} = \frac{\Omega'}{c'_1 \sqrt{1 - N'_1}}, \] (2.21)

\[ \psi'_2 = \frac{\omega' k'_p'}{2 \ell'_2 \sqrt{1 - N'_p}} + \frac{\ell \sqrt{1 - N'_1}}{\ell'_2} = \frac{\Omega'}{c'_4} + \frac{\ell \sqrt{1 - N'_1}}{\ell'_2}, \] (2.22)

which are valid for limitly low frequency case \( (\ell'_2 k'_p' \ll 1) \). The amplitudes, namely \( Q' \), \( Q'_1 \), \( P'_1 \), \( P'_2 \) and \( P'_1' \) corresponding to various reflected and refracted waves will be determined in the subsequent section, using appropriate boundary conditions at the interface \( y = 0 \) between two media.

### 2.3.1 Boundary conditions

The appropriate boundary conditions at the plane interface \( y = 0 \) between two porous elastic half spaces are given by

(i) Continuity of tangential stress, i.e.,

\[ \tau_{xy} = \tau'_{xy}. \] (2.23)
(ii) Continuity of normal stress, i.e.,
\[ \tau_{yy} = \tau_{yy}', \]  
(2.24)

(iii) Continuity of normal component of the gradient of the change in void volume fraction, i.e.,
\[ \frac{\partial \phi}{\partial y} = \frac{\partial \phi'}{\partial y}, \]  
(2.25)

(iv) Continuity of tangential displacement, i.e.,
\[ u = u', \]  
(2.26)

(v) Continuity of normal displacement, i.e.,
\[ v = v', \]  
(2.27)

(vi) Continuity of change in void volume fraction, i.e.,
\[ \phi = \phi'. \]  
(2.28)

Using equations (1.136), (2.7) and (2.8), the above boundary conditions at the interface \( y = 0 \) can be written as
\[ \mu \left( 2 \frac{\partial^2 p}{\partial x \partial y} + \frac{\partial^2 q}{\partial x^2} - \frac{\partial^2 q}{\partial y^2} \right) = \mu' \left( \frac{\partial^2 y'}{\partial x \partial y} + \frac{\partial^2 q'}{\partial x^2} - \frac{\partial^2 q'}{\partial y^2} \right), \]  
(2.29)

\[ \mu \left\{ 2 \left( \frac{\partial^2 p}{\partial x^2} - \frac{\partial^2 q}{\partial x \partial y} \right) + k_p^2 p \right\} = \mu' \left\{ 2 \left( \frac{\partial^2 y'}{\partial x^2} - \frac{\partial^2 q'}{\partial x \partial y} \right) + k_p^2 p' \right\}, \]  
(2.30)

\[ \frac{1}{H} \left( \nabla^2 + k_p^2 \right) p = \frac{1}{H'} \left( \nabla^2 + k_p^2 \right) p', \]  
(2.31)

\[ \frac{1}{H} \frac{\partial}{\partial y} \left( \nabla^2 + k_p^2 \right) p = \frac{1}{H'} \frac{\partial}{\partial y} \left( \nabla^2 + k_p^2 \right) p', \]  
(2.32)
On plugging the potentials \( p, q, p', \) and \( q' \) from (2.17)-(2.20) into equations (2.29)-(2.34), and employing the Snell’s law given by,

\[
\psi_1 \sin \gamma_1 = \psi_2 \sin \gamma_2 = k_s \sin \theta = \psi'_1 \sin \gamma'_1 = \psi'_2 \sin \gamma'_2 = k'_s \sin \theta', \tag{2.35}
\]

and at \( z = 0, \Omega = \Omega' \), the following system of six simultaneous equations are obtained as

\[
\begin{align*}
\mu k_s^2 \cos 2\theta Q - \mu' k'_s^2 \cos 2\theta' Q' &- \mu \psi_1^2 \sin 2\gamma_1 P_1 \\
-\mu \psi'_2 \sin 2\gamma'_2 P'_2 &- \mu' \psi'_1 \sin 2\gamma'_1 P'_1 - \mu' \psi'_2 \sin 2\gamma'_2 P'_2 = -\mu \psi_1^2 \sin 2\gamma_1, \\
\mu k_s^2 \sin 2\theta P_2 + \mu' k'_s^2 \sin 2\theta' P'_2 + \mu k_s^2 \cos 2\theta P_1 &+ \mu' k'_s^2 \cos 2\theta' P'_1 = -\mu k_s^2 \cos 2\theta, \\
k_s \cos \theta Q + k'_s \cos \theta' Q' &- k_s \sin \theta P_1 \\
-k_s \sin \theta P_2 + k'_s \sin \theta' P'_1 + k'_s \sin \theta' P'_2 = -k_s \sin \theta, \\
k_s \sin \theta Q - k'_s \sin \theta' Q' &+ \psi_1 \cos \gamma_1 P_1 \\
+\psi_2 \cos \gamma_2 P_2 + \psi'_1 \cos \gamma'_1 P'_1 + \psi'_2 \cos \gamma'_2 P'_2 = \psi_1 \cos \gamma_1, \\
\end{align*}
\]

\[
\begin{align*}
(k_p^2 - \psi_1^2) P_1 + (k_p^2 - \psi_2^2) P_2 - \frac{H}{H'}[(k_p^2 - \psi_1^2) P'_1 + (k_p^2 - \psi_2^2) P'_2] &- (k_p^2 - \psi_1^2), \\
(k_p^2 - \psi_1^2) \psi_1 \cos \gamma_1 P_1 + (k_p^2 - \psi_2^2) \psi_2 \cos \gamma_2 P_2 + \frac{H}{H'}[(k_p^2 - \psi_1^2) \psi'_1 \cos \gamma'_1 P'_1 \\
+ (k_p^2 - \psi_2^2) \psi'_2 \cos \gamma'_2 P'_2] &- (k_p^2 - \psi_1^2) \psi_1 \cos \gamma_1. \\
\end{align*}
\]

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These equations enable us to provide the formulae for the reflection and transmission coefficients corresponding to various reflected and transmitted waves.

2.4 Energy partition

To find out the distribution of energy carried along the incident wave into reflected and refracted waves, we shall consider the energy flux of incident, reflected and transmitted waves at the interface. The energy flux for the waves mentioned above, can be obtained by multiply the total energy per unit volume (double the mean of kinetic energy density) with the velocity of propagation and the area of the wave front involved. Thus, the energy of incident plane longitudinal wave is given by

\[ E_{inc} = \frac{1}{2} \rho \psi_1^2 \Omega^2 \{ \exp [\psi_1 (x \sin \gamma_1 - y \cos \gamma_1) - \Omega t] \}^2. \quad (2.42) \]

The energy ratios of the reflected and transmitted transverse waves to that of the incident longitudinal wave are given by

\[ E_1 = \frac{k_2^2}{\psi_2^4} |Q|^2, \quad E_2 = \frac{\rho' k_2^2}{\rho \psi_1^2} |Q'|^2. \]

The energy ratios of the reflected and transmitted longitudinal waves to that of the incident longitudinal wave are given by

\[ E_3 = |P_1|^2, \quad E_4 = \frac{\rho' \psi_2^2}{\rho \psi_1^2} |P_1'|^2, \quad E_5 = \frac{\rho' \psi_2^2}{\rho \psi_1^2} |P_2'|^2, \quad E_6 = \frac{\rho' \psi_2^2}{\rho \psi_1^2} |P_2'|^2. \]

Since both the elastic half-spaces are non-dissipating, therefore at the interface the energy of incident wave must be equal to the sum of energies of reflected and transmitted waves. Therefore, we obtain the following relation for the balance of energy at the interface \( y = 0 \)

\[ \frac{k_2^2}{\psi_1^4} |Q|^2 + \frac{\rho' k_2^2}{\rho \psi_1^2} |Q'|^2 + |P_1|^2 + \frac{\rho' \psi_2^2}{\rho \psi_1^2} |P_1'|^2 + \frac{\rho' \psi_2^2}{\rho \psi_1^2} |P_2'|^2 = 1. \quad (2.43) \]

This relation must hold at each angle of incidence.
2.5 Special cases

(i) For longitudinal wave incident normally at the interface, that is, when $\gamma = 0^\circ$, the above system of equations reduces to

\[ \mu k_s^2 Q - \mu' k'_s^2 Q' = 0, \quad (2.44) \]

\[ \mu k_s^2 P_1 + \mu k_s^2 P_2 - \mu' k'_s^2 P'_1 - \mu' k'_s^2 P'_2 = -\mu k_s^2, \quad (2.45) \]

\[ k_s Q + k'_s Q' = 0, \quad (2.46) \]

\[ \psi_1 P_1 + \psi_2 P_2 + \psi'_1 P'_1 + \psi'_2 P'_2 = \psi_1, \quad (2.47) \]

\[ (k_p^2 - \psi_1^2) P_1 + (k_p^2 - \psi_2^2) P_2 - \frac{H}{H'} [(k_p^2 - \psi_1^2) P'_1 + (k_p^2 - \psi_2^2) P'_2] = -(k_p^2 - \psi_1^2), \quad (2.48) \]

\[ (k_p^2 - \psi_2^2) \psi_1 P_1 + (k_p^2 - \psi_2^2) \psi_2 P_2 + \frac{H}{H'} [(k_p^2 - \psi_1^2) \psi'_1 P'_1 \]

\[ + (k_p^2 - \psi_2^2) \psi'_2 P'_2] = -(k_p^2 - \psi_1^2) \psi_1. \quad (2.49) \]

In this case, it is obvious from equations (2.44) and (2.46) that the reflection and transmission coefficients $Q$ and $Q'$ corresponding to transverse waves vanish. Thus, no transverse wave appears and only the reflected and transmitted longitudinal waves appear in case of normally incident longitudinal wave.

(ii) For limitly high frequency longitudinal wave striking at the plane interface, we substitute $\psi_1 = k_p$ and $\psi'_1 = k'_p$ into the boundary conditions given by equations (2.40) and (2.41), we obtain

\[ (k_p^2 - \psi_1^2) P_1 - \frac{H}{H'} (k_p^2 - \psi_1^2) P'_1 = 0, \quad (2.50) \]

\[ (k_p^2 - \psi_2^2) \psi_2 \cos \gamma P_2 + \frac{H}{H'} (k_p^2 - \psi_2^2) \psi'_2 \cos \gamma P'_2 = 0. \quad (2.51) \]
It is obvious from these two equations that $P_2 = P'_2 = 0$. Now, on substituting $P_2 = P'_2 = 0$ into the boundary conditions (2.36)-(2.39), the following system of equations are obtained

$$
\mu k_s^2 \cos 2\theta Q - \mu' k'_s^2 \cos 2\theta' Q' - \mu \psi_1^2 \sin 2\gamma_1 P_1 - \mu' \psi'_1^2 \sin 2\gamma'_1 P'_1 = -\mu \psi_1^2 \sin 2\gamma_1, \quad (2.52)
$$

$$
\mu k_s^2 \sin 2\theta Q + \mu' k'_s^2 \sin 2\theta' Q' + \mu k_s^2 \cos 2\theta P_1 - \mu' k'_s^2 \cos 2\theta' P'_1 = -\mu k_s^2 \cos 2\theta, \quad (2.53)
$$

$$
k_s \cos \theta Q + k'_s \cos \theta' Q' - k_s \sin \theta P_1 + k'_s \sin \theta' P'_1 = -k_s \sin \theta, \quad (2.54)
$$

$$
k_s \sin \theta Q - k'_s \sin \theta' Q' + \psi_1 \cos \gamma_1 P_1 + \psi'_1 \cos \gamma'_1 P'_1 = \psi_1 \cos \gamma_1. \quad (2.55)
$$

These equations exactly match with those given in Kolskey (1963) for the case of incident longitudinal wave at the plane interface between two uniform elastic half-spaces. Thus, we note that there is no effect of the presence of voids in case of limitly high frequency incident longitudinal wave.

(iii) If the presence of voids in both the half-spaces are neglected then the problem reduces to the problem of reflection and transmission of longitudinal waves at the plane interface between two uniform elastic half spaces in perfect contact. In this case, we put $H = H' = 0$ into equations (2.40) and (2.41), we see that these equations are identically satisfied. Moreover, since voids of both the half-spaces are neglected, therefore the amplitudes $P_2$ and $P'_2$ corresponding to reflected and refracted longitudinal void volume fractional waves will vanish. Thus, putting $P_2 = P'_2 = 0$ into the remaining boundary conditions given by (2.36)-(2.39), we obtain equations (2.52)-(2.55). These equations are exactly the same equations as given in Kolsky (1963) for the classical elasticity.

### 2.6 Numerical results and discussion

Since for the limitly high frequency incident longitudinal wave, the analysis is same as in case of classical elasticity, therefore, we shall seek the effect of presence of voids on the reflection and transmission coefficients numerically only for limitly low frequency
incident longitudinal wave. We shall study the variations of modulus of reflection and transmission coefficients and their associated energy with angle of incidence. For this purpose, the following numerical values of relevant parameters are taken.

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</tbody>
</table>

We have solved the equations (2.36) - (2.41) using Gauss Elimination method through a FORTRAN program developed for this purpose. The values of reflection and transmission coefficients are computed at two different values of $\Omega$ namely, $\Omega = 5$ rad/sec and $\Omega = 10$ rad/sec and the results obtained are depicted graphically.

Figure 2.2 depicts the variation of modulus of reflection coefficients with respect to the angle of incidence at $\Omega = 5$ rad/s. In this figure, we have plotted the graphs of reflection coefficients $|Q|$ and $|P_3|$ by magnifying their original values with the factors $10^3$ and $10^5$ respectively. It is clear from this figure that the amplitude $|P_3|$ of reflected volume fractional wave is quite small as compared to the amplitudes $|Q|$ and $|P_3|$ of the reflected transverse and longitudinal waves respectively. It is observed that the amplitude $|Q|$ is different for different angle of incidence and almost possesses increasing behavior, while the amplitudes $|P_1|$ and $|P_3|$ are independent of the angle of incidence. Figure 2.3 depicts the same at $\Omega = 10$ rad/sec and we see that the behavior of reflection coefficients is similar as exhibited in Figure 2.2.

Figure 2.4 depicts the variation of modulus of transmission coefficients with the angle of incidence for $\Omega = 5$ rad/sec.
Figure 2.2: Variation of modulus of reflection coefficients with angle of incidence (When $\Omega = \Omega' = 5$, Curve - I: $|Q| \times 10^3$, Curve - II: $|P_1|$, Curve - III: $|P_2| \times 10^5$).

Figure 2.3: Variation of modulus of reflection coefficients (When $\Omega = \Omega' = 10$, Curve - I: $|Q| \times 10^3$, Curve - II: $|P_1|$, Curve - III: $|P_2| \times 10^4$).

Figure 2.4: Variation of modulus of transmission coefficients with angle of incidence (When $\Omega = \Omega' = 5$, Curve - I: $|Q'| \times 10^3$, Curve - II: $|P_1'| \times 10$, Curve - III: $|P_2'| \times 10^5$).

Figure 2.5: Variation of modulus of transmission coefficients with angle of incidence (When $\Omega = \Omega' = 10$, Curve - I: $|Q'| \times 10^3$, Curve - II: $|P_1'| \times 10$, Curve - III: $|P_2'| \times 10^4$).

We have plotted the graphs of the transmission coefficients $|Q'|$, $|P_1'|$ and $|P_2'|$ by magnifying their original values with the factors $10^2$, $10$ and $10^5$ respectively. We note that the amplitude $|P_1'|$ of longitudinal wave is more than the amplitude $|Q'|$ of reflected
transverse wave and also quite large than the amplitude $|P_2|$ of reflected void volume fractional wave. Here, we note that the values of amplitudes $|Q|$ and $|P_1|$ are different at different angles of incidence and increase monotonically in the range $0 < \gamma_1 < 90^\circ$. However, the amplitude $|P_2|$ remains constant, though very small, in the entire range of $\gamma_1$.

Figure 2.5 depicts the same for $\Omega = 10 \text{ rad/sec}$. We note that as $\Omega$ increases from value 5 to 10, the magnitude of transmission coefficient increases, however the nature of dependence on the angle of incidence $\gamma_1$ remains same. Thus, the transmission coefficients are continuous functions of $\gamma_1$ and $\Omega$.

Figure 2.6: Variation of energy ratios with angle of incidence (When $\Omega = \Omega' = 5$, Curve - I: $E_1 \times 10^4$, Curve - II: $E_3$, Curve - III: $E_4 \times 10^4$, Curve - IV: $E_2 \times 10^4$, Curve - V: $E_5 \times 10^3$, Curve - VI: $E_6 \times 10^5$).

Figure 2.7: Variation of energy ratios with angle of incidence (When $\Omega = \Omega' = 10$, Curve - I: $E_1 \times 10^5$, Curve - II: $E_3$, Curve - III: $E_4 \times 10^4$, Curve - IV: $E_2 \times 10^4$, Curve - V: $E_5 \times 10^3$, Curve - VI: $E_6 \times 10^5$).

Figures 2.6 and 2.7 depict the variation of energy ratios of various reflected and transmitted waves with the angle of incidence at $\Omega = 5 \text{ rad/sec}$ and $10 \text{ rad/sec}$ respectively. In both these figures, it is found that $\sum_{i=1}^{6} E_i \approx 1$. We also note from these figures that the reflected and refracted longitudinal waves carry maximum amount of incident energy as compared to other reflected and refracted waves.
2.7 Conclusions

In the present chapter, a problem of reflection and transmission of plane longitudinal wave at a plane interface between two different elastic solid half-spaces with voids is analyzed. It is found that the reflection and transmission coefficients depend on the angle of incidence, frequency of the incident wave and also on the elastic properties of the media through which they travel. We conclude that

(i) The effect of the presence of voids on the reflection and transmission coefficients is observed only in the case of limitly low frequency incident longitudinal wave. For limitly high frequency incident longitudinal wave, the results of classical elasticity for the relevant problem are obtained, showing that there is no effect of the presence of voids in the media.

(ii) The reflected and transmitted longitudinal waves dominate as their amplitudes are very large as compared to other reflected and transmitted transverse and longitudinal void volume fractional waves.

(iii) The maximum amount of incident energy is found to carry along the reflected and refracted longitudinal elastic waves.

(iv) Although, the maximum amount of incident energy goes with the reflected and refracted longitudinal waves, the energies carried by the reflected and refracted transverse and longitudinal void volume fractional waves are significant in the sense that they are influenced by the presence of voids in the media.

(iv) In the absence of voids (pores) in the media, the longitudinal void volume fractional wave does not appear at all.