<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A, B$</td>
<td>constants in Eq. (4.18)</td>
<td>[-]</td>
</tr>
<tr>
<td>$a, b, c, d$</td>
<td>constants in Eq. (4.8)</td>
<td>[-]</td>
</tr>
<tr>
<td>$A'-D', F'$</td>
<td>constants in Eq. (4.2.4)</td>
<td>[-]</td>
</tr>
<tr>
<td>$A_1$</td>
<td>constant in Eq. (4.2.7)</td>
<td>[-]</td>
</tr>
<tr>
<td>$A_2$</td>
<td>constant in Eq. (4.2.9)</td>
<td>[-]</td>
</tr>
<tr>
<td>$A_3$</td>
<td>constant in Eq. (4.2.14)</td>
<td>[-]</td>
</tr>
<tr>
<td>$a_{LG}$</td>
<td>specific liquid-gas interfacial area per unit volume reactor</td>
<td>[m$^{-1}$]</td>
</tr>
<tr>
<td>$a_{LS}$</td>
<td>specific liquid-solid interfacial area per unit volume reactor</td>
<td>[m$^{-1}$]</td>
</tr>
<tr>
<td>$a_s$</td>
<td>specific surface area of packing, $\frac{6(1 - e)}{d_p} + \frac{4}{D_e}$</td>
<td>[m$^{-1}$]</td>
</tr>
<tr>
<td>$A_w$</td>
<td>wave amplitude</td>
<td>[m]</td>
</tr>
<tr>
<td>$B_2$</td>
<td>constant in Eq. (4.2.10)</td>
<td>[-]</td>
</tr>
<tr>
<td>$B_3$</td>
<td>constant in Eq. (4.2.14)</td>
<td>[-]</td>
</tr>
<tr>
<td>$d$</td>
<td>throat diameter, $d_p$</td>
<td>[m]</td>
</tr>
<tr>
<td>$d^*$</td>
<td>diameter of pore chamber, $d_p$</td>
<td>[m]</td>
</tr>
<tr>
<td>$D_e$</td>
<td>equivalent diameter</td>
<td>[m]</td>
</tr>
<tr>
<td>$d_h$</td>
<td>equivalent diameter of the interstitial channels</td>
<td>[m]</td>
</tr>
<tr>
<td>$d_h^*$</td>
<td>hydraulic diameter, $\frac{16e^3}{9\pi(1 - e)^2} d_p$</td>
<td>[m]</td>
</tr>
<tr>
<td>$d_p$</td>
<td>effective particle diameter = equivalent spherical particle</td>
<td>[m]</td>
</tr>
</tbody>
</table>
NOMENCLATURE

d

diameter * sphericity of the particle

d_{eq} equivalent spherical diameter

\( d_{min} \) minimum throat diameter in the column,

\[ d_{min} = \left( \frac{2}{\pi} \frac{\sin \frac{\pi}{3} - \frac{1}{2}}{d_p} \right)^{0.5} \] [m]

\( E_1, E_2 \) constant of the Ergun equation for single phase flow on the packing of interest

\( E_L \) power dissipation by the liquid phase per unit volume of the bed,

\[ E_L = \frac{L}{\rho_L Z} \left[ \frac{\Delta P}{Z} \right]_{LG} \] [m]

\( E_L'' \) energy dissipation per unit mass of liquid,

\[ (V_L / \rho_L) \left( \frac{\Delta P}{Z} \right) \] [m^2 s^{-3}]

\( F^* \) factor defined by Sai and Varma 1987

\( f_G \) friction factor for gas flow over a solid

\( f_L \) friction factor for liquid flow over a solid

\( f_{LG} \) two-phase friction factor,

\[ f_{LG} = \left( \frac{\Delta P}{Z} \right)_{LG} \frac{d_p \rho_G}{2G^2} \] [-]

\( f_{LGL} \) two-phase friction factor, Eq.(4.2.1)

\( f_m \) modified friction factor for gas-phase, Eq. (3.19)

\( F_M \) factor defined by Sai and Varma 1987

\( Fr \) Froude number,

\[ F_r = \frac{V_a^2}{gd_p} \] [-]

\( Fr' \) Froude number at pulsing flow transition,

\( f_s \) shear slip factor

\( f_v \) velocity slip factor

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NOMENCLATURE

\( F_a \)  
Total drag force experienced by \( \alpha \)-phase per unit bed volume  
[Pa m\(^3\)]

\( g \)  
Acceleration due to gravity  
[mms\(^2\)]

\( G \)  
Gas superficial mass velocity  
[kgm\(^{-2}\)s\(^{-1}\)]

\( Ga \)  
Galileo number of the liquid phase, \( \frac{d_p \rho_L^2 g}{\mu_L^2} \)  
[-]

\( Ga^* \)  
Modified Galileo number, \( \frac{d_p \rho_L^2 g}{\mu_L^2} \left( \rho_L g + \frac{\Delta P}{Z} \right) \)  
[-]

\( Ga^{**} \)  
Modified Galileo number, \( \frac{d_p \rho_L^2 g}{\mu_L^2} \left[ 1 - \left( \frac{1}{\rho_L g \beta_L} \right) \frac{\Delta P}{Z} \right] \)  
[-]

\( Ga_L' \)  
Modified Galileo number of the liquid phase, \( \frac{d_p \rho_L^2 g}{\mu_L^2 \left( g \rho_L^2 \right)} \)  
[-]

\( Ga_a \)  
Galileo number of the \( \alpha \) phase, \( \frac{d_p \rho_a^2 g}{\mu_a^2} \)  
[-]

\( Ga_a^{''} \)  
Modified Galileo number of the \( \alpha \) phase, \( \frac{d_p \rho_a^2 g}{\mu_a^2 \left( 1 - \varepsilon \right)^3} \)  
[-]

\( H \)  
Hidden layer vector  
[-]

\( H_c \)  
\( \frac{K \left( \frac{3}{n} \right)^n \left( \varepsilon^2 \right)^{1-n}}{12 \left( \frac{n}{1 - \varepsilon} \right)^n} \)  
[Pa s\(^n\)]

\( H_D \)  
Dynamic liquid holdup  
[-]

\( H_S \)  
Static liquid holdup  
[-]

\( H_T \)  
Total liquid holdup, volume of liquid per unit volume of the reactor  
[-]

\( J \)  
Number of modes in hidden layer  
[-]

\( K \)  
Flow consistency index  
[Pa s\(^n\)]

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\( K'' \)
voidage correction factor

\( K_1 \)
model parameter, Eq. (4.14)

\( K_1' \)
coefficient defined by Specchia and Baldi, 1977

\( K_2' \)
coefficient defined by Specchia and Baldi, 1977

\( Ka \)
Kapitza number, \( \frac{\sigma_{L, q}^3}{\rho_L g v_L^4} \)

\( k_i \)
Blake-Kozney constant, Eq. (4.16)

\( l \)
characteristic length of the non-spherical particle or diameter of the spherical particle

\( L \)
liquid superficial mass velocity, \([\text{kg} \cdot \text{m}^{-2} \cdot \text{s}^{-1}]\)

\( L_t \)
liquid superficial mass velocity at transition, \([\text{kg} \cdot \text{m}^{-2} \cdot \text{s}^{-1}]\)

\( L_{t, neural} \)
\( L_t \) predicted by neural network correlation (Larachi et al., 1999), \([\text{kg} \cdot \text{m}^{-2} \cdot \text{s}^{-1}]\)

\( m \)
model parameter, Eq. (4.14)

\( m^* \)
minimum flow rate per unit length for complete wetting

\( MRQE \)
mean relative quadratic error, Eq. (4.1)

\( n \)
flow behavior index, Eq. (4.11)

\( n^* \)
factor defined by Sai and Varma, 1987

\( N_i \)
\( K_i v_i^m \), Eq. (4.14)

\( N_c \)
number of channels over the entire cross section

\( N_0 \)
number of circles irrespective of size per unit sectional area, \( 6(1-c)\pi d_p^2 \)

\( P_1-P_3 \)
constants in Eq. (4.3.3)

\( P_6 \)
constants in Eq. (4.3.5)

\( P_7 \)
constants in Eq. (4.3.6)

\( Q_i \)
constants in Eq. (4.3.7)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2$</td>
<td>correlation coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>$Re$</td>
<td>Reynolds number, $Ld_p/\mu_l$</td>
<td>[-]</td>
</tr>
<tr>
<td>$Re_{L,cr}$</td>
<td>Reynolds number at flow regime transition</td>
<td>[-]</td>
</tr>
<tr>
<td>$Re_{LG}$</td>
<td>$\frac{d_p G}{\mu_\ell (1-\varepsilon) \alpha_w}$</td>
<td>[-]</td>
</tr>
<tr>
<td>$Re_{LM}$</td>
<td>$\frac{d_p L}{\mu_\ell (1-\varepsilon) \alpha_w}$</td>
<td>[-]</td>
</tr>
<tr>
<td>$Re_m$</td>
<td>modified Reynolds number, Eq. (3.18)</td>
<td>[-]</td>
</tr>
<tr>
<td>$Re_M$</td>
<td>modified Reynolds number, $\frac{d_p L \gamma^2 \rho_{l}}{K}$</td>
<td>[-]</td>
</tr>
<tr>
<td>$Re_a$</td>
<td>bed Reynolds number of the $a$ phase, $V_a d_p/\gamma_d(1-\varepsilon)$</td>
<td>[-]</td>
</tr>
<tr>
<td>$S$</td>
<td>neural-network output</td>
<td>[-]</td>
</tr>
<tr>
<td>$S^*$</td>
<td>length of grain boundary per unit sectional area</td>
<td>[-]</td>
</tr>
<tr>
<td>$S_1$</td>
<td>$\frac{a_p d_p}{\varepsilon}$</td>
<td>[-]</td>
</tr>
<tr>
<td>$S_2$</td>
<td>$\left( \frac{1}{d_p} \right)^{\frac{1}{n}}$</td>
<td>[-]</td>
</tr>
<tr>
<td>$T_1$, $T_3$</td>
<td>constants in Eq.(4.3.8)</td>
<td>[-]</td>
</tr>
<tr>
<td>$U$</td>
<td>interstitial velocity</td>
<td>[ms$^{-1}$]</td>
</tr>
<tr>
<td>$u_G$</td>
<td>gas linear velocity, $G/\rho_G$</td>
<td>[ms$^{-1}$]</td>
</tr>
<tr>
<td>$u_{G,0}$</td>
<td>gas interstitial velocity, $\frac{\Gamma u_G}{\varepsilon}$</td>
<td>[ms$^{-1}$]</td>
</tr>
<tr>
<td>$U_i$</td>
<td>normalized input variable</td>
<td>[-]</td>
</tr>
<tr>
<td>$u_L$</td>
<td>liquid linear velocity, $L/\rho_L$</td>
<td>[ms$^{-1}$]</td>
</tr>
<tr>
<td>$u_{L,0}$</td>
<td>liquid interstitial velocity, $\frac{\Gamma u_L}{\varepsilon}$</td>
<td>[ms$^{-1}$]</td>
</tr>
</tbody>
</table>
NOMENCLATURE

\( V_a \)  
superficial velocity of \( a \)-phase  
\([\text{ms}^{-1}]\)

\( v_G \)  
gas actual velocity  
\([\text{ms}^{-1}]\)

\( V_{GT} \)  
superficial gas velocity at flow regime transition  
\([\text{ms}^{-1}]\)

\( v_L \)  
liquid actual velocity  
\([\text{ms}^{-1}]\)

\( V_{LT} \)  
superficial liquid velocity at flow regime transition  
\([\text{ms}^{-1}]\)

\( W_{EG} \)  
gas-phase Weber number, \( \frac{G^2 d_p}{\rho_g \sigma_L} \)  
[-]

\( W_{EL} \)  
liquid-phase Weber number, \( \frac{L^2 d_p}{\rho_l \sigma_L} \)  
[-]

\( W_G \)  
power dissipated in the gas phase referred to the volume occupied by the two phases  
\([\text{kgm}^{-1}\text{s}^{-3}]\)

\( W_i \)  
Weissenberg number, Eq. (4.17)  
[-]

\( W_T \)  
total power dissipated in reactor referred to the volume occupied by the two phases  
\([\text{kgm}^{-1}\text{s}^{-3}]\)

\( X \)  
Lockhart-Martinelli parameter, \( \left( \frac{\Delta P_l}{\Delta P_G} \right)^{0.5} \)  
[-]

\( X' \)  
modified Lockhart-Martinelli parameter, \( \left( \frac{\sigma_L}{\sigma_G} \right)^{0.5} \)  
[-]

\( X_G \)  
\( \frac{1}{X_L} \)  
[-]

\( X_L \)  
same as \( X \)  
[-]

\( Z \)  
packed bed height  
[\text{m}]

\( Z' \)  
\( \frac{\text{Re}_G^{1.167}}{\text{Re}_L^{0.757}} \)  
[-]

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**GREEK SYMBOLS**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a'$</td>
<td>$4\sqrt{1 - \beta' - (1 - \beta')}$</td>
<td>[-]</td>
</tr>
<tr>
<td>$a_m$</td>
<td>adaptive parameter, $f/f_G$</td>
<td>[-]</td>
</tr>
<tr>
<td>$a_w$</td>
<td>wall-effect correction factor, Eq. (3.20)</td>
<td>[-]</td>
</tr>
<tr>
<td>$\beta$</td>
<td>liquid saturation</td>
<td>[-]</td>
</tr>
<tr>
<td>$\beta'$</td>
<td>$\left[\frac{200}{Re_L} + 1.75\right] \frac{\nu^2_g}{gd_p} \frac{1 - \varepsilon}{\varepsilon^3}$</td>
<td>[-]</td>
</tr>
<tr>
<td>$\beta_d$</td>
<td>dynamic liquid saturation, volume of liquid that drains</td>
<td>[-]</td>
</tr>
<tr>
<td></td>
<td>from the packed bed per unit void volume of the reactor</td>
<td></td>
</tr>
<tr>
<td>$\beta_s$</td>
<td>static liquid saturation, volume of liquid that remains in the bed</td>
<td>[-]</td>
</tr>
<tr>
<td></td>
<td>after draining the liquid from the packed bed per unit void volume of the</td>
<td></td>
</tr>
<tr>
<td></td>
<td>reactor</td>
<td></td>
</tr>
<tr>
<td>$\beta_t$</td>
<td>total liquid saturation, volume of liquid per unit void</td>
<td>[-]</td>
</tr>
<tr>
<td></td>
<td>volume of the reactor</td>
<td></td>
</tr>
<tr>
<td>$\dot{\gamma}$</td>
<td>shear rate,</td>
<td>[s$^{-1}$]</td>
</tr>
<tr>
<td>$\dot{\gamma}_w$</td>
<td>shear rate at the wall, Eq. (4.16)</td>
<td>[s$^{-1}$]</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>tortuosity of the packed bed, $\frac{1}{\cos \theta_m}$</td>
<td>[-]</td>
</tr>
<tr>
<td>$\delta$</td>
<td>equivalent diameter of the gas channel</td>
<td>[-]</td>
</tr>
<tr>
<td>$\overline{\delta}$</td>
<td>dimensionless parameter Eq. (5.8)</td>
<td>[-]</td>
</tr>
<tr>
<td>$\delta_{LG}$</td>
<td>two-phase frictional pressure drop</td>
<td>[kg m$^{-2}$ s$^{-2}$]</td>
</tr>
<tr>
<td>$\delta'$</td>
<td>mean height of liquid film</td>
<td>[m]</td>
</tr>
<tr>
<td>$\Delta P$</td>
<td>two phase pressure drop</td>
<td>[Nm$^{-2}$]</td>
</tr>
<tr>
<td>$\Delta P_G$</td>
<td>pressure drop based on gas phase</td>
<td>[Nm$^{-2}$]</td>
</tr>
<tr>
<td>$\Delta P_L$</td>
<td>pressure drop based on liquid phase</td>
<td>[Nm$^{-2}$]</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>bed void fraction</td>
<td>[-]</td>
</tr>
</tbody>
</table>
NOMENCLATURE

\( e_a \) Bed holdup of \( \alpha \)-phase

\( \zeta_L, \zeta_G \) Charpentier's correlating variables

\( \zeta_{LG} \) Charpentier's correlating variable

\( \eta \) parameter defined by Rao et al. (1983)

\( \theta_m \) average settlement angle of the particles

\( \lambda \) a parameter, Eq. (4.3)

\( \lambda_{eff} \) effective fluid characteristic time, \( \frac{N_l}{2\gamma_T} \), Eq. (4.15)

\( \lambda_4 \) time constant of material

\( \mu_a \) viscosity of \( \alpha \)-phase

\( \mu_a \) apparent viscosity of liquid-phase

\( \mu_a \) \( K\gamma^n \), Eq. (4.10)

\( \xi \) \( \left( \frac{\sigma_w}{\sigma_L} \right)^{1/3} \left( \frac{\mu_{\alpha}}{\mu_w} \right)^{0.5} \left( \frac{\rho_w}{\rho_L} \right)^{1/3} \)

\( \xi' \) \( \left( \frac{\sigma_w}{\sigma_L} \right)^{1/3} \left( \frac{\mu_{\alpha}}{\mu_w} \right)^{0.5} \left( \frac{\rho_w}{\rho_L} \right)^{1/3} \), Eq. (4.12)

\( \rho_a \) density of \( \alpha \)-phase

\( \tau \) shear stress, \( K\gamma^n \), Eq. (4.11)

\( \tau_i \) shear stress at the gas-liquid interface

\( \tau_{LG} \) friction at the gas liquid interface

\( \tau_{LS} \) friction at the liquid solid interface

\( \tau_p \) interfacial drag tension at pulsing inception

\( \phi \) Lockhart-Martinelli parameter, \( \left[ \frac{\Delta P_{LG}}{\Delta P_L} \right]^{0.5} \)

---

\([-]\)

\([\text{Pa.m}^{-1}]\)

\([\text{Pa.m}^{-1}]\)

\([-]\)

\([-]\)

\([\text{Pa}]\)

\([\text{Pa}]\)

\([\text{Pa}]\)

\([\text{Pa}]\)

\([\text{kg m}^{-1} \text{s}^{-2}]\)

\([-]\)
NOMENCLATURE

\( \phi' \) modified Lockhart-Martinelli parameter, \( \frac{\sigma_{LG}}{\sigma_G}^{0.5} \) [-]

\( \phi_p \) surface shape factor of packing, \( a_s / d_p^2 \) [-]

\( \phi_s \) particle sphericity, Eq. (3.17) [-]

\( \phi \) model parameter, Larachi et al. (1993) [-]

\( \sigma \) liquid-phase surface tension [Nm\(^{-1}\)]

\( \sigma_a \) surface tension of \( \alpha \)-phase, [Nm\(^{-1}\)]

\[ \psi \equiv \left( \frac{\sigma_w}{\sigma_L} \right) \left( \frac{\mu_L}{\mu_w} \left( \frac{\rho_w}{\rho_L} \right)^2 \right)^{1/3} \] [-]

\[ \psi_{G} \] dimensionless pressure drop, \( \frac{1}{\rho_{0G}} \left( \frac{\Delta P}{Z} \right)_{LG} + \rho_{0G} \) [-]

\[ \psi_{L} \] dimensionless pressure drop, \( \frac{1}{\rho_{0L}} \left( \frac{\Delta P}{Z} \right)_{LG} + \rho_{0L} \) [-]

\( \omega \) weights [-]

SUBSCRIPTS

cal calculated

exp experimental value

G gas phase

L liquid phase

LG two-phase

mod modified

P pulse/high-interaction regime

pred predicted value
NOMENCLATURE

\[ \text{sph} \quad \text{for spherical particles} \]
\[ \text{st} \quad \text{static} \]
\[ T \quad \text{trickle/low-interaction regime} \]
\[ \text{ve} \quad \text{viscoelastic liquid-phase} \]
\[ \text{vi} \quad \text{viscoelastic liquid-phase} \]
\[ w \quad \text{water} \]
\[ \alpha \quad \text{fluid phase, liquid or gas} \]

ABBREVIATIONS

CMC \quad \text{carboxymethylcellulose} \\
HIR \quad \text{high-interaction regime} \\
HIR* \quad \text{high-interaction regime with foaming} \\
LIR \quad \text{low-interaction regime} \\
MRQE \quad \text{mean relative quadratic error} \\
PAA \quad \text{polyacrylamide} \\
PEO \quad \text{polyethyleneoxide} \\
TBR \quad \text{trickle bed reactor}