3.1 Introduction

This chapter describes the development of a theoretical model for simulating localized defects on inner and outer race of 6205 deep groove ball bearing using curve fitting technique. A dynamic model is presented for predicting the characteristic frequencies and associated amplitudes of ball bearing vibrations under the influence of localized defects on the outer race and inner race. The pulse generated by the ball striking the defect on the races is modelled by using the blending functions of the cubic hermite spline. The effect of change in the angular position of the defect, size of the defect, presence of multiple defects and the variation of load on the vibration amplitude is predicted by this model. A computer program in MATLAB is developed and the governing equation of motion is solved by Euler’s method.

3.2 Formulation of The Mathematical Model of System

At the end of the shaft of rotor-bearing system being analyzed, a deep groove ball bearing (designation: DGBB 6205) is mounted as a test bearing as shown in Fig. 3.1.
The inner race of the bearing is rigidly fixed to the shaft and the outer race is fixed in a rigid support. The elastic deformation between the races and each point of contact with the ball is assumed to be Hertzian. In the mathematical model, the ball bearing is considered as non-linear spring-mass system. The model presented here considers the specific case of a non-rotating outer race, loaded radially.

The assumptions and considerations while deriving the model are summarized as follows:

- The ball bearing model has equi-spaced balls rolling on the surface of the inner and outer race and there is no interaction between them.
- Slipping of the balls during rolling on races is neglected.
- The motion of race and balls occur in the plane of the bearing only.
- The inner race of the bearing is rigidly fixed to the shaft and the outer race is fixed in a rigid support.
- Deformations at the contacts are Hertzian contact deformations.
- The bearing operates under isothermal conditions.
- The shape of the pulse generated by impact at the defect as shown in Fig.3.7 models the defect.

### 3.2.1 Internal Speeds, Motions and Load Distribution in a Ball Bearing

Ball bearings are used to support various kinds of loads while permitting rotational motion of a shaft. The expressions for internal rotational speeds of rolling element bearings are developed by Harris [60]. When a bearing mounted on a shaft rotates at some speed, the rolling elements orbit the bearing axis and simultaneously revolve about their own axes (refer Fig. 3.2).

The rotational speed of the cage is given by

\[ n_c = \frac{1}{2} \left[ N(1 - \frac{D}{d_m}) \right] \]  \hspace{1cm} (3.1)

The angular velocity of the cage is

\[ \omega_c = \frac{1}{2} \left[ \omega_s \left( 1 - \frac{D}{d_m} \right) \right] \]  \hspace{1cm} (3.2)

Angular velocity of balls is defined by the following relation:

\[ \omega_R = \frac{1}{2} \frac{d_m}{D} N \left( 1 - \frac{D^2}{d_m^2} \right) \]  \hspace{1cm} (3.3)
Bearing load distribution (refer Fig. 3.3) with respect to angular position of ball is calculated by the Eq. (3.4):

\[
Q_\theta = Q_{\text{max}} \left[ 1 - \frac{1}{2\epsilon} \left( 1 - \cos \Psi \right) \right]^{3/2}
\]

where \( \epsilon = \frac{1}{2} \left( 1 - \frac{y}{2\delta_{\text{max}}} \right) \)

### 3.2.2 Calculation of Restoring Force

In general the deflection of the \( i \)th ball located at any angle \( \theta \) is calculated by following expression (refer Fig. 3.4):

\[
\delta = (\cos \theta_i + \sin \theta_i) - (\gamma)
\]

\( x \) and \( y \) are the deflections along X and Y directions respectively and \( \gamma \) is the internal radial clearance which is the clearance between an imaginary circle, which circumscribes the balls and the outer race. At the time of impact at the defect, a pulse of short duration is produced and it is accounted for by the term \( \Delta \) i.e. additional deflection. Hence Eq. (3.5 a) is modified by adding \( \Delta \) to the internal radial clearance and is given by

\[
\delta = (\cos \theta_i + \sin \theta_i) - (\gamma + \Delta)
\]

The restoring force generated by ball-race contact deformation of the ball is of nonlinear nature because of the Hertzian contact.

The local Hertzian contact force and deflection relationship for bearing may be written as

\[
F_{\theta_i} = K(\delta)^r, \ r = 3/2
\]

where \( K \) is the constant for Hertzian contact elastic deformation which depends on the contact geometry.

Substituting \( \delta \) from Eq. (3.5b) in Eq. (3.6), we get

\[
F_{\theta_i} = K \left[ (\cos \theta_i + \sin \theta_i) - (\gamma + \Delta) \right]^{3/2}
\]

For the contact of each ball in the non-defective region, referred as normal race contact, additional deflection \( \Delta \) is zero. The restoring force is resolved along directions X and Y.
Fig. 3.2 Rolling speeds and velocities

Fig. 3.3 Bearing load distribution
The components of the restoring force are

\[
F_x = \sum_{i=1}^{z} K\left[(x\cos \theta_i + y\sin \theta_i) - (y + \Delta)\right]^{3/2} \cos \theta_i
\]  
(3.8)

\[
F_y = \sum_{i=1}^{z} K\left[(x\cos \theta_i + y\sin \theta_i) - (y + \Delta)\right]^{3/2} \sin \theta_i
\]  
(3.9)

where \(z\) = No. of balls

Fig. 3.5 shows the contact between mating surfaces of revolution. Under no load condition, point contact exists between the ball and races which changes to area contact which has the shape of an ellipse. The two terms curvature sum and curvature difference describes the contact between the mating surfaces. Curvature sum \(\sum \rho\) is calculated using the radii of curvature in a pair of principal planes passing through the point contact and using curvature difference \(F(\rho)\), dimensionless contact deformation \(\delta^*\) is calculated. The effective elastic modulus \(K\) for the bearing system is written as:

\[
K = \left[\frac{1}{\left(\frac{1}{K_i}\right)^{1/n} + \left(\frac{1}{K_o}\right)^{1/n}}\right]^{1/n}
\]  
(3.10)

The elastic modulus for the contact of a ball with the inner race is

\[
K_i = 3.587 \times 10^{7} (\sum \rho_i)^{-1/2} (\delta_i^*)^{-3/2}
\]  
(3.11)
Similarly elastic modulus for the contact of a ball with the outer race is

\[ K_0 = 3.587 \times 10^7 (\Sigma \rho_0)^{-1/2} (\delta_0')^{-3/2} \]  

\[(3.12)\]

![Fig. 3.5 Geometry of contacting bodies](image)

The value of K for 6205 bearing is 49,582 N/mm [61].

<table>
<thead>
<tr>
<th>Table 3.1 Input data for the model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner race diameter</td>
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<tr>
<td>Outer race diameter</td>
</tr>
<tr>
<td>Pitch diameter</td>
</tr>
<tr>
<td>Ball diameter</td>
</tr>
<tr>
<td>Internal radial clearance</td>
</tr>
<tr>
<td>Radial load</td>
</tr>
<tr>
<td>Mass of rotor</td>
</tr>
<tr>
<td>No of balls</td>
</tr>
<tr>
<td>Speed of rotor</td>
</tr>
<tr>
<td>Damping factor</td>
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<tr>
<td>Contact angle</td>
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</tbody>
</table>
3.3 Simulation of Localized Defects Using Blending Functions of Cubic Hermite Spline

This section describes mathematical simulation of localized defects on outer race and inner race of rolling element bearing. Cracks, pits, spalls are included in the class of localized defects. When such a defect on one surface strikes its mating surface, a pulse of short duration is produced. Different pulse forms being investigated in the earlier models are rectangular, triangular, half-sine pulse. This pulse represents the severity, extent and age of the damage. Patil et al. [11] have modeled the defect itself as a half sinusoidal pulse. Different pulse forms are shown in Fig. 3.6.

The dimensions of the pulse affect the vibration amplitude. As the defect become more severe with aging, the dimension of the pulse changes. This pulse being of short duration, there is tendency to neglect the pulse width. In real situation, the pulse generated due to impact at the defect may not be of such a regular shape.

![Different shapes of pulse forms](image.png)

(a) half-sine (b) rectangular (c) triangular pulse

**Fig. 3.6** Different shapes of pulse forms

Rectangular, triangular and sinusoidal pulse forms are the approximations for the shape of the real pulse. The rectangular and triangular forms are the two extremities (non-realistic) in between which the shape of the defect pulse lies in practical situation. Half sine pulse is one of the cases lying in between these two extreme forms. For the pulse of regular shape, the variation in displacement is gradual throughout which may not happen in real situation depending on the form of the defect. Secondly, this assumption is based on the fact that the displacement at all intermediate points between the two instances; entry of ball in the defective region and exit at the defect are known. In real situation, displacement at intermediate points are dictated by the shape of the defect, depth of the defect and width of the defect, based on which the real pulse will assume some shape.

The simulation of the defect pulse using Cubic Hermite Spline is based on the assumption that the displacements at intermediate points are difficult to predict and the more realistic assumption would be to assume displacement behaviour at the entry and
exit edge of the defect. In case of Cubic Hermite Spline, displacements at the control points (intermediate points) are not known. Instead displacements at the start and end points are known.

![Simulated pulse generated by impact at the defect](image)

**Fig. 3.7** Simulated pulse generated by impact at the defect

Normal race contact refers to the normal geometrical movement of the race i.e. when the defect lies between the neighboring balls and is not getting struck. When the defect gets struck, the movement of the race geometry changes resulting in generation of the pulse. At the starting edge of the defect, ball retards due to loss of contact with the race and at this instant the displacement is assumed to be -1 as shown in Fig. 3.7. As the ball leaves the defect, it accelerates due to regaining of contact with the race and the displacement is assumed to be +1. Hence, the displacement law is retardation followed by acceleration. As opposed to this, in case of regular shapes, the displacements are gradual which is far from reality. As every ball establishes and breaks the contact with the defective region of the races shown in Fig.3.8, the geometry of race movement will change. In case of defective outer race, the angular position of the defect is stationary and as such physically the rolling elements try to catch a defect which is stationary. In case of defective inner race, the defect itself rotates along with the inner race and as such the rolling elements try to catch a moving defect. While modelling the defect pulse for inner race, the relative velocity between the cage and the inner race is taken into account in the computer program. The behaviour of ball establishing and breaking the contact with the races is similar to cam (follower) jump phenomenon. Essentially the ball orbiting the inner race acts as a cam and the race acts as the follower. For normal race contact, there is hardly any change in the geometry of the movement. During the travel of the ball in the defective region of the races, the locus of race centre position as a function of angle $\theta$
and its time derivatives are to be obtained through the mathematical modelling of the system.

(a) Different positions of ball no. 2 relative to the outer race defect

(b) Different positions of ball no. 2 relative to the inner race defect

Fig. 3.8 Different positions of ball no. 2 relative to the outer and inner race defect

While developing the computer program, it is ensured that the ball at $\theta$ is the only one in the defective region. The ball ahead of it (leading) has lost its contact with the defective region and the ball lagging behind is waiting for its contact with the defective region.

In mathematics, spline is a sufficiently smooth polynomial function that is piecewise defined and it possesses a high degree of smoothness at the places where the polynomial pieces connect. Splines are curves which are usually required to be continuous and smooth. Cubic spline interpolation is a fast, efficient and stable method of function interpolation between key points. In spline interpolation, the interpolation interval is divided into small subintervals. Each of these subintervals is interpolated by using a third-degree polynomial. The polynomial coefficients are chosen to satisfy certain conditions. General requirements are function continuity passing through all given points and continuity of higher derivatives.
In hermite interpolation, the function value and the value of the first derivative are known at each interpolating point. Interpolation with derivative values is also known as osculatory interpolation. Hermite curves are very easy to calculate and also very powerful.

To calculate a hermite curve, the following vectors are needed:
1. \( P_0 \) = the start point of the curve
2. \( P_0' \) = the tangent at the start point describing how the curve leaves point \( P_0 \)
3. \( P_1 \) = the endpoint of the curve
4. \( P_1' \) = the tangent at the endpoint of the curve

These four vectors are simply multiplied with four hermite basis functions shown in Fig. 3.9 and added together. These blending functions are given by Eqs. (3.13)- (3.16):

\[
H_0(t) = 2t^3 - 3t^2 + 1 
\]
\[
H_1(t) = -2t^3 + 3t^2 
\]
\[
H_2(t) = t^3 - 2t^2 + t 
\]
\[
H_3(t) = t^3 - t^2 
\]

The function \( H_0(t) \) starts at 1 and goes slowly to 0. Function \( H_1(t) \) starts at 0 and goes slowly to 1. Multiply the start point with \( H_0(t) \) and the endpoint with \( H_1(t) \). Let \( t \) go from 0 to 1 to interpolate between known start and end points. \( H_2(t) \) and \( H_3(t) \) are applied to the tangents in the same manner. They make sure that the curve blends in the desired direction at the start and end point.

Blending functions point wise can be written as

\[
P(t) = (2t^3 - 3t^2 + 1)P_0 + (-2t^3 + 3t^2)P_1 + (t^3 - 2t^2 + t)P_0' + (t^3 - t^2)P_1' 
\]

\[
(3.17) 
\]
\( P'(t) = (6t^2 - 6t) P_0 + (-6t^2 + 6t) P_1 + (3t^2 - 4t + 1) P_0' + (3t^2 - 2t) P_1' \) \hspace{1cm} (3.18)

Substituting \( P_0 = -1 \) and \( P_1 = 1 \) in Eq. (3.17), equation of the curve is

\[ S = -4t^3 + 6t^2 - 1 \] \hspace{1cm} (3.19)

As an example of calculation of ‘t’ in MATLAB program for 1mm defect size on outer race

\[ t = 0:0.043478:1 \] \hspace{1cm} (3.20a)

where time Step = 0.043478 = 1/stay instants of ball in outer race defect (23 for 1mm defect size). Stay instants of ball refers to the no. of control points (intermediate points) of the cubic hermite spline. For 1mm defect size on inner race ‘t’ in MATLAB program is calculated as

\[ t = 0:0.028571428:1 \] \hspace{1cm} (3.20b)

For 1mm defect on inner race, no. of control points are 35.

The stay instants of ball in the defective region of races changes linearly depending on the defect size. Accordingly, ‘t’ in MATLAB program is calculated while simulating the effect of defect size on the vibration amplitude. The effect of bearing degradation is simulated in the mathematical model by considering different defect sizes such as 0.5 mm, 1 mm, 1.5 mm and 2 mm. The shape of the pulse remains same but the size varies when there is change in the defect size. This is one of the features of the defect pulse modelled by cubic hermite spline.

### 3.4 Equation of Motion and Computational Procedure

Taking \( x \) and \( y \) as the displacements along X and Y directions, the governing equations accounting for inertia, damping and restoring forces and constant vertical force in X direction for a two degree of freedom system are formed.

\[ M\ddot{x} + C\dot{x} + \sum_{i=1}^{2} K[(x \cos \theta_i + y \sin \theta_i) - (y + \Delta)]^{3/2} \cos \theta_i = W \] \hspace{1cm} (3.21)

\[ M\ddot{y} + C\dot{y} + \sum_{i=1}^{2} K[(x \cos \theta_i + y \sin \theta_i) - (y + \Delta)]^{3/2} \sin \theta_i = 0 \] \hspace{1cm} (3.22)

The system of Eqs. (3.21) and (3.22) are two coupled non-linear ordinary second order differential equations. Here \( \Delta \) term corresponds to additional deflection for the travel of ball in the defective region of each race. This \( \Delta \) term is calculated based on Eqs. 3.19, 3.20a, 3.20b and is graphically represented in Fig. 3.7. For normal race contact, \( \Delta \) i.e. additional deflection will be zero. The damping in this system is represented by an equivalent viscous
damping C. Using Euler’s method Eqs. (3.21) and (3.22) are solved and the displacements in X and Y direction and their time derivatives are obtained. Initial conditions of x and y are $10^6$. The time step in the computation corresponds to $0.1^0$ bearing rotation increment. This is to ensure that the rolling elements strike the defect to produce impulsive time waveform with the rotation of the cage. If the time step corresponding to larger rotational increment is chosen, the rolling elements may advance in angular direction without striking the defect which may result in health assessment of bearing as healthy for a defective bearing. A computer program in MATLAB simulates the effect of defect size, effect of load, effect of multiple defects for stationary and moving defects and effect of change in angular position for stationary defect on vibration amplitude.

### 3.5 Summary

In this chapter a dynamic model is presented for predicting the vibration behavior of a ball bearing under the influence of localized defects on the outer and inner race. The calculation of contact force is based on Hertzian contact deformation theory. The pulse generated by the ball striking the defect is modeled by using the blending functions of the cubic hermite spline. The modeling of the defect pulse for defective inner race differs from the defective outer race because of the fact that the defect itself changes its angular position in case of inner race as opposed to a stationary defect on outer race. The model predicts the effect of change in the angular position of the defect, size of the defect, presence of multiple defects and the variation of load on the vibration amplitude. A MATLAB based programming environment is used for solving the governing equation of motion.