In this chapter, we investigate the effects of two diffusive components and the vertical magnetic field on the existence of internal Alfvén gravity waves in an externally generated type of turbulent flow. Apart from few special cases, most of the fluids are stratified by more than one diffusive component. Hence it is necessary to study the effect of two diffusive components, (which produces a weak, stable stratification) on the existence of internal Alfvén gravity waves and on marginal stability when a vertical magnetic field is imposed on the turbulent conducting fluid.

7.1 INTRODUCTION

Most flows in Astrophysical and geophysical problems are expected to be turbulent because of the geometries of flow regime and/or of external forces (see Moffat 1978)
In these problems it is necessary to estimate how much energy is fed to turbulence and how much energy is transferred by this mechanism. The transfer of momentum and energy may phenomenologically be explained by turbulence generated convection or Alfven gravity waves. Because of the variation of density with position and time the system may turn out to be gravitationally stable or unstable and produces the convective instability depending upon the physical situation. In this chapter attention has been focused on the mechanism of momentum and energy transfer by internal Alfven gravity waves in a turbulent flow in the presence of an imposed magnetic field because of its importance discussed in Chapter 1. Even in this case we use the analysis and approximations discussed in the previous chapters.

The gradient diffusion model which is used as a closure leads to the appearance of turbulent diffusivities such as eddy viscosity, eddy magnetic diffusivity, eddy thermal diffusivity and eddy mass diffusivity. We expect that in the case of problems discussed in the previous chapters the assumption of constant eddy diffusivities may give reasonable results. This first order closure model is also assumed to give a physical insight on the most general MHD turbulent model with large magnetic Reynolds number.
3.2 MATHEMATICAL FORMULATION

We consider an electrically conducting fluid weakly stratified by two diffusive components, having the layer of thickness \( d \) and of infinite lateral extent. Let \((x,y,z)\) be the rectangular coordinate system with vertical \( z \)-axis, opposite to gravity, along which a uniform magnetic field \( B_0 \) is imposed on the system. The basic equations for such Boussinesq fluids, as discussed in Chapter 2, in tensorial form are:

\[
\mathbf{F} \left[ \frac{\partial q_1}{\partial t} + q_j \frac{\partial q_1}{\partial x_j} \right] = - \frac{\partial P}{\partial x_1} - \mathbf{F} \mathbf{g} \delta_{13} + \mu_m H_j \frac{\partial H_1}{\partial x_j} + \mu_f \frac{\partial^2 q_1}{\partial x_j \partial x_j}, \quad (7.1)
\]

\[
\frac{\partial T}{\partial t} + q_j \frac{\partial T}{\partial x_j} = K_T \frac{\partial^2 T}{\partial x_j \partial x_j}, \quad (7.2)
\]

\[
\frac{\partial s}{\partial t} + q_j \frac{\partial s}{\partial x_j} = K_s^* \frac{\partial^2 s}{\partial x_j \partial x_j}, \quad (7.3)
\]

\[
\frac{\partial H_1}{\partial t} + q_j \frac{\partial H_1}{\partial x_j} = K_m \frac{\partial^2 H_1}{\partial x_j \partial x_j} + H_j \frac{\partial q_1}{\partial x_j}, \quad (7.4)
\]

\[
\frac{\partial q_1}{\partial x_1} = 0, \quad (7.5)
\]
where \( q_i \) are velocity components, \( P \) is the total pressure \([= P + \mu_m H^2/2]\), \( \rho \) is the density, \( T \) is the temperature, \( g \) is the gravitational acceleration, \( \delta_{i3} \) and \( \varepsilon_{ijk} \) are second order and third order Kronecker deltas, \( J_j \) are the components of electrical current density, \( H_k \) are the components of magnetic field, \( \mu_f \) is the molecular viscosity of the fluid, \( \mu_m \) is the magnetic permeability \( K_T \) is thermometric conductivity, \( S \) is the concentration, \( K_s^\ast \) is the molecular diffusivity of concentration, \( K_m^\ast \) is the magnetic diffusivity, \( \alpha_T \) is the volumetric coefficient of expansion with respect to temperature and \( \alpha_s \) is that of concentration. The fluid is weakly stratified in respect of density which is maintained by unspecified but adequate source of heat. The basic state is a quiescent state with magnetostatic balance, given by

\[
\frac{\partial H_i}{\partial x_i} = 0 , \quad \cdots (7.6)
\]

\[
\rho = \rho_o [1 - \alpha_T(T - T_o) + \alpha_s(S - S_o)], \quad \cdots (7.7)
\]

\( q_0 = 0 \), \( P_o = -\rho_0 gz + \text{constant} \), \( \mathbf{H} = (0,0,H_0) \).

\( \cdots (7.8) \)
Superimposed on this is a horizontally homogeneous field of turbulence characterized by the fluctuations of velocity, magnetic field, pressure, density, temperature and concentration whose time averages are independent of horizontal coordinates because we have assumed that the system is homogeneous in horizontal extent.

We assume that the plane waves of small amplitude propagate in the turbulent field of external type. This type of turbulence may be generated by fluctuations in the general state of the system such as variations in the diurnal motion, variations in the path traversed by the system, variation in the gravitational field due to variation in temperature and/or concentration or due to gravitational attractions, variations in the axial rotation or orbital rotation etc. These plane waves are assumed to propagate in the x-direction with angular frequency $\omega$ and wave number $k$. Since the mean perturbed quantities are small, their products can be neglected. Also, the turbulent variables are random in phase and homogeneous in space, the $y$-average will leave only the wave field. Therefore the $y$-average of fluctuating variables are zero and the $y$-average of multiples of fluctuating quantities are non-zero. Denoting the averaged fields by overscored variables and the fluctuating
fields by primed variables the total field is given by

\[ q_i = \bar{q}_i + q'_i , \]
\[ \tilde{f} = \tilde{f}_0 + \tilde{f} + f' , \]
\[ T = T_0 + \tilde{T} + T' , \]
\[ S = S_0 + \tilde{S} + S' , \]
\[ P = P_0 + \tilde{P} + P' , \]
\[ H_j = H_0 + h_j + h'_j . \]

Substituting (7.9) into the equations (7.1) to (7.7) and taking y-average of these equations and using Boussinesq approximation and approximation on averaged Overscored and primed variables, we obtain the following equations:

\[ \frac{\partial \bar{q}_1}{\partial t} + \frac{\partial \bar{P}}{\partial \rho_0 \partial x_1} + \frac{\partial P_0}{\partial \rho_0} g \delta_{13} - \frac{\mu_m H_0}{\rho_0} \frac{\partial h_1}{\partial x_3} = \frac{\partial}{\partial x_j} \left( \gamma \frac{\partial \bar{q}_1}{\partial x_j} \right) \]
\[ - \langle q'_i q'_j \rangle - \frac{\mu_m}{\rho_0} \langle h'_i h'_j \rangle \]  

\[ \ldots (7.10) \]

\[ \frac{\partial \tilde{T}}{\partial t} + W \frac{\partial T}{\partial z} = \frac{\partial}{\partial x_1} \left( K_T \frac{\partial \tilde{T}}{\partial x_j} - \langle T' q'_j \rangle \right) \]

\[ \ldots (7.11) \]
\[
\frac{\partial \bar{S}}{\partial t} + W \frac{d\bar{S}}{dz} = \frac{\partial}{\partial x_j} \left( K_s \frac{\partial \bar{S}}{\partial x_j} - \langle S' q'_i \rangle \right),
\]
\quad \ldots (7.12)

\[
\frac{\partial \bar{h}_i}{\partial t} - H_0 \frac{\partial \bar{q}_i}{\partial x_3} = \frac{\partial}{\partial x_j} \left( K_m \frac{\partial \bar{h}_i}{\partial x_j} - \langle q'_i h'_j \rangle + \langle h'_i q'_j \rangle \right),
\]
\quad \ldots (7.13)

\[
\frac{\partial \bar{q}_i}{\partial x_3} = 0,
\]
\quad \ldots (7.14)

\[
\frac{\partial \bar{h}_i}{\partial x_1} = 0,
\]
\quad \ldots (7.15)

\[
\bar{\rho} = \rho_0 \left( - \alpha T + \alpha S \bar{S} \right),
\]
\quad \ldots (7.16)

\[
\bar{\rho}' = \rho_0 \left( - \alpha_T T' + \alpha_S S' \right),
\]
\quad \ldots (7.17)

In the above equations various stresses such as Reynolds stress \( \langle - q'_i q'_j \rangle \), magnetic stress \( \langle h'_i h'_j \rangle \), eddy heat transport term \( \langle T' q'_j \rangle \), magnetic field transport term \( \langle q'_i h'_j \rangle \), magnetic field elongation term \( \langle h'_i q'_j \rangle \) and eddy concentration transport term \( \langle S' q'_j \rangle \) are to be linearized using a suitable closure. Following Rudraiah et al (1986) we introduce the concept of eddy viscosity and eddy diffusivity as follows:
\[
\frac{\partial}{\partial x_j} < - q_i^j > = \frac{\partial}{\partial x_r} (K_i^q \frac{\delta q_i^j}{\delta x_r})
\]

\[
\frac{\partial}{\partial x_j} < - h_i^j > = \frac{\partial}{\partial x_r} (K_m^h \frac{\delta h_i^j}{\delta x_r})
\]

\[
\frac{\partial}{\partial x_j} < - T^j > = \frac{\partial}{\partial x_r} (K_h^q \frac{\delta T^j}{\delta x_r})
\]

\[
\frac{\partial}{\partial x_j} < - s^j > = \frac{\partial}{\partial x_r} (K_s^q \frac{\delta s^j}{\delta x_r})
\]

\[
\frac{\partial}{\partial x_j} < - h_i^j > = \frac{\partial}{\partial x_r} (K_e^q \frac{\delta h_i^j}{\delta x_r})
\]

\[
\frac{\partial}{\partial x_j} < - q_i^j > = \frac{\partial}{\partial x_r} (K_m^q \frac{\delta q_i^j}{\delta x_r})
\]

where \( K_i^q, K_m^h, K_h^q, K_s^q, K_e^q \) and \( K_m^q \) are eddy diffusive coefficients of viscosity, magnetic viscosity, heat, concentration, and magnetic field respectively. As the fluid is horizontally homogeneous and vertically stratified, the extra index \( r \) must be introduced to these coefficients to take their anisotropy into account. In general, they are different from one another and are assumed to be spatially uniform. This assumption of
uniformity of the coefficients with respect to space does not lead to serious error when large body of fluid is considered as in pure viscous case (see LeBlond 1966). But for practical situations this single point closure must be replaced by two point closure forms.

Substituting (7.18) into the equations (7.10), to (7.13) and eliminating all variables except \( q_3 \) \((= W)\) we get the following wave equation:

\[
\left( \frac{\partial}{\partial t} - K_{fr} \frac{\partial^2}{\partial x_r^2} \right) \left( \frac{\partial}{\partial t} - K_{hr} \frac{\partial^2}{\partial x_r^2} \right) \left( \frac{\partial}{\partial t} - K_{sr} \frac{\partial^2}{\partial x_r^2} \right) \]

\[
\left( \frac{\partial}{\partial t} - K_{er} \frac{\partial^2}{\partial x_r^2} \right) \nabla^2 W + \frac{\mu_m}{\rho_0} \left( \frac{\partial}{\partial t} - K_{hr} \frac{\partial^2}{\partial x_r^2} \right)
\]

\[
\left( \frac{\partial}{\partial t} - K_{sr} \frac{\partial^2}{\partial x_r^2} \right) \left( K_{mr} \frac{\partial^2}{\partial x_r^2} - H_0 \frac{\partial}{\partial x_r} \right) \frac{\partial}{\partial x_r}
\]

\[
- K_{mer} \frac{\partial^2}{\partial x_r^2} \nabla^2 W - \left( \frac{\partial}{\partial t} - K_{er} \frac{\partial^2}{\partial x_r^2} \right) [\alpha_T \beta_T \left( \frac{\partial}{\partial t} \right)
\]

\[
- K_{sr} \frac{\partial^2}{\partial x_r^2} + \alpha_s \beta_s \left( \frac{\partial}{\partial t} - K_{hr} \frac{\partial^2}{\partial x_r^2} \right) \right] \nabla^2 W = 0,
\]

where

\[
K_{fr} = (\nu + K_{fr}'), \quad K_{hr} = (K_T + K_{hr}'), \quad K_{mr} = K_{mr},
\]

\[
K_{er} = (K_m + K_{er}') \quad \text{and} \quad K_{mer} = K_{mer}, \quad K_{sr} = (K_s + K_{sr}').
\]
We assume the plane waves of the form:

\[ W = W(z) \exp(\omega t - ikx) \].

\((7.20)\)

Then equation (7.19) can be written as:

\[ \left\{ [\omega - (K_{f3}D^2 - K_{f1}k^2)][\omega - (K_{h3}D^2 - K_{h1}k^2)] 
\right. \\
\left. \cdot [\omega - (K_{s3}D^2 - K_{s1}k^2)][\omega - (K_{e3}D^2 - K_{e1}k^2)](D^2 - k^2) 
\right. \\
+ \frac{\mu_m}{\rho_0} [\omega - (K_{h3}D^2 - K_{h1}k^2)][\omega - (K_{s3}D^2 - K_{s1}k^2)] \\
\left. \cdot [K_{m3}D^2 - K_{m1}k^2 - H_0 D][H_0 D - (K_{me3}D^2 - K_{me1}k^2)] 
\right. \\
\left. (D^2 - k^2) + [\omega - (K_{e3}D^2 - K_{e1}k^2)][\alpha_I\beta_I (\omega - K_{s3}D^2 
\right. \\
+ K_{s1}k^2) + \alpha_s\beta_s (\omega - K_{h3}D^2 + K_{h1}k^2)] k^2 g \right\} \\
\cdot W(z) \exp(\omega t - ikx) = 0 \],

\((7.21)\)

where

\[ D^2 = \frac{\partial^2}{\partial z^2} \quad \text{and} \quad D = \frac{\partial}{\partial z}. \] The solution of this equation is useful to study the internal Alfven-gravity waves in the turbulent fluid. In order to obtain the solution of (7.21) we need the boundary conditions at both
the bounding surfaces. To reduce the complexity, we assume that the bounding surfaces are isothermal, isoconcentrated and stress free; so the perturbations vanish.

\[ W(\mathbf{r}, t) = \mathbf{T}(\mathbf{r}, t) = \mathbf{S}(\mathbf{r}, t) = h(\mathbf{r}, t) = 0, \]

at \( z = 0, d \),

and

\[ \frac{\partial^2 W}{\partial z^2} = 0 \quad \text{at} \quad z = 0, d. \]  

(7.22)

we have assumed \( h(\mathbf{r}, t) = 0 \) at the boundaries as well as in the interior of the conducting fluid when compared to \( H_0 \) at small magnetic Reynolds numbers. This assumption reduces the wave equation to a simpler form which is analytically tractable. Hence the term \( \langle h_i^I h_j^J \rangle \) in equation (7.10) and the terms \( \partial h_i^I / \partial t \), \( (\partial / \partial x_j) \langle -q_j^J h_i^I \rangle \) and \( \langle h_i^I q_j^J \rangle \) in equation (7.13) are negligible. This brings about the result that

\[ [\omega - (K_{e3} \delta^2 - K_{el} k^2)] \]

is reduced to \( (-K_{e3} \delta^2 + K_{el} k^2) \)

where \( K_{e3} = K_{el} = K_m^* \) (magnetic viscosity); \( (K_{me3} \delta^2 - K_{mel} k^2) \ll 0 \); and \( (K_{m3} \delta^2 - K_{ml} k^2) \ll 0 \) (see Nihoul, 1966). Now the governing equation reduces to the following simpler form:
By stress-free boundary condition we arrive at

\[
\frac{d^{2N-2}W}{dz^2} = D^{2N-2}W = 0, \quad N = 0, 1, 2, 3, 4, \text{ at } z = 0, d.
\]  
\[\text{(7.24)}\]

7.3 WAVE SOLUTION

Making the equation (7.23) dimensionless using

\[
z' = \frac{z}{a}, \quad a = kd, \quad K_f = \frac{1}{3} \sum_{j=1}^{3} K_{fj} \text{ and for } K_h, K_s, K_e; \quad \sigma = \frac{\omega a^2}{K_f},
\]

we get the wave equation of the form:

\[
[(\omega + K_{f1}k^2 - K_{f3}D^2)(K_{e1}k^2 - K_{e3}D^2) - \frac{\mu_m H^2}{\rho_o} D^2]
\]
\[\cdot (\omega + K_{h1}k^2 - K_{h3}D^2)(\omega + K_{s1}k^2 - K_{s3}D^2)(D^2-k^2) \bar{W}
\]
\[+ k^2g (K_{e1}k^2 + K_{e3}D^2) [\alpha_T\beta_T (\omega + K_{s1}k^2 - K_{s3}D^2)
\]
\[+ \alpha_s\beta_s (\omega + K_{h1}k^2 - K_{h3}D^2)] \bar{W} = 0 . \quad \text{(7.23)}
\]
Since equation (7.25) has constant coefficients, the solution of it satisfying the boundary conditions (7.24) is of the form:

\[
\{[\sigma - P_{43}D^2 + P_{41}a^2](P_{31}a^2 - P_{33}D^2) - A^2] \\
\cdot (\sigma - P_{23}D^2 + P_{21}a^2)(\sigma - P_{13}D^2 + P_{11}a^2)(D^2 - a^2) \\
+ a^2 [ R_1(\sigma - P_{23}D^2 + P_{21}a^2) + R_2(\sigma - P_{13}D^2 + P_{11}a^2)] \\
\cdot (P_{31}a^2 - P_{33}D^2) \} \overline{w} = 0, \quad \ldots (7.25)
\]

in which

\[
P_{ij} = K_f^{-1}(K_{ij}\delta_{i4} + K_{ej}\delta_{i3} + K_{hj}\delta_{i2} + K_{s,j}\delta_{i1} + K_f\delta_{i0}), \quad i = 0, 1, 2, 3, 4,
\]

\[
A^2 = \frac{\mu \beta_0 H_0^2 d^2}{\beta_0 K_f^2}, \quad R_1 = \frac{gd^4a_T\beta_T}{K_f^2},
\]

\[
R_2 = \frac{gd^4a_s\beta_s}{K_f^2}, \quad R = R_1 + R_2.
\]

Since equation (7.25) has constant coefficients, the solution of it satisfying the boundary conditions (7.24) is of the form:

\[
W = \text{Const.} \sin (n\pi z'), \quad n = 1, 2, \ldots \quad \ldots (7.26)
\]

Substituting (7.26) in (7.25) and letting

\[
\eta_k = (P_{k3}n^2\pi^2 + P_{k1}a^2), \quad k = 0, 1, 2, 3, 4 \quad \text{and}
\]

\[
\phi = R_1/R_2,
\]
we get the dispersion relation of the form:

\[
\sigma^3 + \sigma^2 \left[ \eta_1 + \eta_2 + \eta_4 + \frac{A^2 \pi^2}{\eta_3} \right] + \sigma \left[ \eta_1 \eta_2 + \eta_2 \eta_4 \right. \\
+ \eta_2 \frac{A^2 \pi^2}{\eta_3} + \eta_1 \eta_4 + \eta_1 \frac{A^2 \pi^2}{\eta_3} - \frac{a^2_R}{\eta_0} \left. \right] + [\eta_1 \eta_2 \eta_4 \\
+ \eta_1 \eta_2 \frac{A^2 \pi^2}{\eta_3} - \frac{a^2_{R_2} (\phi_1 + \eta_1)}{\eta_0} ] = 0.
\] ..(7.27)

Equation (7.27) leads to oscillatory solutions when its cubic discriminant is positive. The wave number where the transition from internal waves to critically damped motions occurs is now a function of \((\eta_2/\eta_1)\).

The locus of vanishing frequency in any case is continuous. This result strongly indicates the non-existence of short waves. This is an interesting situation when compared to the result obtained in the absence of magnetic field, i.e., for pure viscous case. (Refer LeBlond 1966 and see figure 7.1).

As it was shown by Stern (1960), and Walin (1964), due to the difference in the diffusivities of the two components, a gravitationally stable density field may turn out to be dynamically unstable whenever \(\sigma_I > 0\) and \(\sigma_I \neq 0\). This criterion yields the condition
Taking the value of $\phi$ in this range, we can draw marginal stability curves in the dynamically unstable range (refer figure 7.2) by solving the equation obtained by marginal stability condition using Newton–Raphson iterative technique discussed in Chapter 2.

\[
-1 \leq \phi \leq \frac{[ 1 + \frac{\eta_2}{\eta_4} + \frac{A^2 n^2 \pi^2}{\eta_3 \eta_4} ]}{[ 1 + \frac{\eta_1}{\eta_4} + \frac{A^2 n^2 \pi^2}{\eta_3 \eta_4} ]}.
\] ..(7.28)

Taking the value of $\phi$ in this range, we can draw marginal stability curves in the dynamically unstable range (refer figure 7.2) by solving the equation obtained by marginal stability condition using Newton–Raphson iterative technique discussed in Chapter 2.

7.4 CONCLUSIONS

The effect of weak stratification of two diffusive components and the vertical magnetic field on the existence of internal Alfvén gravity waves is depicted in the figure 7.1 using the values $\Delta \rho_o / \rho_o = 10^{-3}$, $n = 1$, $K_{f3} = 0.01$ m$^2$/Sec, $K_{f1} = K_{f2} = 10^4$ m$^2$ Sec$^{-1}$, $d = 1000$ m, $B_o = 0.64 \times 10^{-4}$ Web/m$^2$, $\sigma^1 = 3 \times 10^5$ U, $\rho_o = 10^4$ Kg/m$^3$, $\eta_1 / \eta_4 = 0.1$. It is clear that there is no existence of cut-off frequencies for any value of $\phi$. This means that only the long waves could exist and propagate in the turbulent conducting fluid in the presence of a magnetic field at small magnetic Reynolds numbers. It is evident that only these waves are responsible for
the maximum transport of momentum and energy in the medium. In comparison with ordinary viscous flow the extra force acting on the system is the Lorentz force. The reason for the non-existence of short waves may be that the magnetic field may stretch the existing short waves by the virtue of Lorentz force which brings about magnetic tension and friction.

The marginal stability curves for a two component stratification in the presence of a magnetic field is plotted in figure 7.2 for \( \phi = -0.95 \) and \(-0.97\) in the dynamically unstable range. The other parameters are assigned the values: \( d = 10^3 \) m and \( 10^2 \) m, \( \eta_2/\eta_1 = 0.1 \) and \( 0.01 \) with \( \Delta \beta / \beta_0 = 10^{-3} \), \( n = 1 \), \( K_{f3} = 10^{-2} \) m\(^2\)/Sec, \( K_{f1} = K_{f2} = 10^4 \) m\(^2\)/Sec, \( d = 1000 \) m, \( B_0 = 0.64 \times 10^{-4} \) Web/m\(^2\), \( \sigma ' = 3 \times 10^5 \) U, \( f_0 = 10^4 \) Kg m\(^{-3}\), \( \eta_1/\eta_4 = 0.1 \). In comparison of this result with that of pure viscous case we notice that the effect of magnetic field is to suppress the region of instability. From this investigation the following conclusions are drawn.

(i) The effect of the stratification of two diffusive components and the magnetic field is to control the momentum and energy transport by avoiding the existence of short internal Alfven gravity waves.
(ii) The loci of vanishing frequency are all continuous. Hence the imposed magnetic field do not yield the cut-off frequency.

(iii) The long internal gravity waves are not controlled by the mere imposition of magnetic field; so the transport of momentum and energy by long waves cannot be controlled.

(iv) The imposed magnetic field together with the two components stratification suppress the region of instability. Hence the effect of two components stratification and vertical magnetic field stabilize the system.
FIG. 74—THE LOCI OF VANISHING FREQUENCY FOR A TWO COMPONENT STRATIFICATION IN THE PRESENCE OF A WEAK MAGNETIC FIELD.

\( n = 1.0, \ \Delta g_0 / g_0 = 10^3, \ K_{f3} = 0.01 \text{ m}^2 \text{ sec}^{-1} \)

\( K_{f1} = K_{f2} = 10^4 \text{ m}^2 \text{ sec}^{-1}, \ \delta = 10^3 \text{ m}, \ \theta_0 = 6.4 \times 10^5 \text{ Web/m}^2 \)

\( \sigma = 3 \times 10^5 \text{ V}, \ \eta_0 = 10^4 \text{ kg/m}^3, \text{ Curves (1): } \psi = 3.0, \)

\( \psi = -0.5, \text{ (2): } \psi = -2.0, \text{ (3): } \psi = -4.0 \)
FIG. 7.2-MARGINAL STABILITY CURVES FOR A TWO COMPONENT STRATIFICATION IN THE PRESENCE OF A WEAK MAGNETIC FIELD WITH A DYNAMICALLY UNSTABLE RANGE. 

\[ d = 10^3 \text{ m}, \quad d = 10^2 \text{ m} \]

Labels: \( \text{1:} d = -0.95, \quad \text{2:} \phi = -0.97, \quad \text{3:} \eta_2 / \eta_1 = 0.1 \)
\[ \quad : \eta_2 / \eta_1 = 0.01 \]