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1. On the possible Existence of 1.33 MeV level in Mg$^{26}$.
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2. Electron Scattering on Light Nuclei.
   Raj K. Gupta and Y.R. Waghmare;
   Nuclear Physics 48 (1963) 321.

   Yeshwant Waghmare, Raj K. Gupta and Naresh Kumar;

   Raj K. Gupta and P.C. Sood;

5. Nuclear Energy Levels in Fe$^{56}$ and Rotational Model.
   P.C. Sood and Raj K. Gupta;
   Indian J. Pure Appl. Phys. 2 (1964) 301.
On the Possible Existence of 1.33 MeV Level in Mg$^{26}$

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May 24, 1963

During the past year several investigators$^{1,2,3}$ have attempted to look for the possible existence of a new level at 1.33 MeV in Mg$^{26}$ as tentatively reported by Dearnaley and Ferguson.$^{4}$ In the present paper we present an evidence from systematics of nuclear levels in $1d-2s$ shell even-even nuclei which appears to completely rule out the existence of any such low lying level in this nucleus.

Justification for this low lying level as the first excited state in this nucleus was sought through the statements$^{4}$ that the previously known 1.84 MeV level is the highest first state excitation energy between O$^{16}$ and Si$^{28}$, and that the first excited states in the neighbouring nuclei Table I. The energies for the first excited states in all the possible nuclei with two particles or holes configuration $1d-2s$ orbit.

<table>
<thead>
<tr>
<th>Nucleus$^a$</th>
<th>First state excitation energy (in MeV)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>O$^{14}$</td>
<td>1.98</td>
<td>(a)</td>
</tr>
<tr>
<td>Ne$^{18}$</td>
<td>1.88</td>
<td>(b)</td>
</tr>
<tr>
<td>O$^{20}$</td>
<td>1.67</td>
<td>(a)</td>
</tr>
<tr>
<td>Ne$^{24}$</td>
<td>1.89</td>
<td>(c)</td>
</tr>
<tr>
<td>Mg$^{26}$</td>
<td>1.84</td>
<td>(d)</td>
</tr>
<tr>
<td>Si$^{26}$</td>
<td>1.78</td>
<td>(e)</td>
</tr>
</tbody>
</table>

(c) M. G. Silbert and N. Jarmie, Phys. Rev. 123 (1961), 221.
(e) P. M. Endt and C. Van Der Leun, Nucl. Phys. 34 (1962), 1.

Mg$^{24}$ and Mg$^{28}$ lie at about 1.4 MeV. But it is evident from Table I that 1.84 MeV excitation energy for the first excited state is in no way abnormally high; such a value is quite consistent with the first state excitation energies for all other nuclei having the same configuration. Further a comparison of the first level energies for the mirror nuclei O$^{14}$-Ne$^{18}$ and Mg$^{26}$-Si$^{26}$ clearly reveals the consistency of such an assignment. As expected from the Coulomb dependence of level shifts in isobaric multiplets$^5$ the level in Ne$^{18}$ and Si$^{26}$ are slightly depressed with respect to their possible analogue states in O$^{16}$ and Mg$^{26}$. In contrast Mg$^{24}$ and Mg$^{28}$ are nuclei with four particle-hole structure and, in common with Ne$^{22}$, Mg$^{22}$, etc., have lower excitation energies for the first excited states. Thus, contrary to the remarks of Dearnaley and Ferguson,$^6$ the occurrence of the first excited state in Mg$^{26}$ at excitation energy much lower than 1.84 MeV is not at all consistent with the known experimental data.

While Deuchars and Lawrence$^7$ and Depraz et al.$^{2}$ failed to find any $\alpha$-particle group corresponding to such a level in the study of Si$^{29}$ ($\alpha$,a) Mg$^{26}$ reaction, Ramaswamy$^3$ looked for a Y-transition between the 1.84 MeV and the postulated 1.33 MeV level and concluded from his negative results that if this level exists, it can only be a $0^+$ state to be consistent with his experi-
mental accuracy. From Fig. 1 it is seen that all the experimentally known 0+ levels lie on two distinct curves—one for the $4^n$ nuclei and the other for the $(4n^2)$ nuclei. The observed regularities appear to rule out the occurrence of a 0+ level below 3 MeV excitation energy in these light nuclei.

Certain interesting observations can be made based on Fig. 1. Level spectra for the nuclei Ne$^{20}$, Mg$^{24}$ and Si$^{28}$ have been quite satisfactorily explained on the rotational model assuming permanently deformed structures, whereas the nuclei O$^{18}$, Mg$^{26}$ and Si$^{30}$ are believed to possess collective modes of vibration. What sort of structure may be assumed for S$^{32}$ and beyond is still an open question. This behaviour is very neatly reflected in Fig. 1. The near constancy of the excitation energy for the excited 0+ level in certain nuclei is consistent with its being a member of the two phonon excitation triplet for those cases. The extension of these curves up to $A=40$ is well worth investigating. In particular the location of this 0+ level in $A^{40}$ would be quite important. It may lie at about 3.5 MeV on the smooth curve for $4^n$ nuclei extending up to $C^{40}$ or it may be the observed 4.45 MeV level thus pointing to a subshell closure at S$^{32}$. Also:

a) A 0+ level around 3.5 MeV in Ne$^{22}$ should be looked for.

b) The spin parity assignments for the 3.92 MeV level in S$^{34}$ and 4.3 MeV level in A$^{38}$ may be given as 0+ based on these empirical trends. Similarly spin-parity assignments of 0+ may be made for the 3.61 MeV level in Ne$^{18}$ and 3.4 MeV level in Si$^{34}$, these assignments being also suggested by the isobaric multiplet considerations.

The detailed analysis of the levels occurring in various isobaric multiplets is in progress and is expected to suggest or confirm various spin-parity assignments in these nuclei.

The financial help of the Department of Atomic Energy, Government of India is gratefully acknowledged.

4) G. Dearnaley and A. T. G. Ferguson, Pre­ceedings of the Rutherford Jubilee Inter­national Conference (Manchester, 1961), p. 335.
Raj K. Gupta and P. C. Sood, to be pub­lished.
ELECTRON SCATTERING ON LIGHT NUCLEI

RAJ K. GUPTA and Y. R. WAGHMARE

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Received 5 February 1963

Abstract: Calculations of the nuclear scattering of high energy electrons are made, with a Saxon-Woods distribution of charge, for light nuclei having large quadrupole moments. It is shown that the contribution due to quadrupole scattering forms a sizable part of the total elastic scattering cross section. A variation of the deformation parameter is found in conformity to expectation.

1. Introduction

In recent years the scattering of high energy electrons by atomic nuclei has been used to elicit the details of the charge distribution. In this paper we apply a Saxon-Woods type of charge distribution to light nuclei having large quadrupole moments, viz. Be$^9$, B$^{11}$ and N$^{14}$.

2. Analysis

The differential scattering cross section for a high energy electron incident on a massive nucleus is given by $^1$)

$$\frac{d\sigma}{d\Omega} = \left(\frac{2e^2}{2E}\right)^2 \frac{\cos^2 \frac{1}{2} \theta}{\sin^2 \frac{1}{2} \theta} |F(q)|^2,$$

(1)

here the form factor $F(q)$ is expressed as

$$F(q) = \int_0^\infty e^{iqr} \rho(r) dr$$

(2)

in terms of the four-momentum transfer $q$, with

$$hq = \frac{2E}{c} \sin \frac{1}{2} \theta,$$

(3)

considering the deformation of a nucleus, we expand the charge distribution $\rho(r)$ in the form of a series as

$$\rho(r) = \rho_0(r) + \frac{\rho_1(r)}{r} rY_0^0(\cos \theta) + \cdots = \rho_0(r) + \rho_1(r)Y_2^2(\cos \theta) + \cdots$$

(4)

$^1$ Now at Dept. of Physics, University of California, La Jolla, California.
The form factor in this case is then expressed as

$$|F|^2 = |F_0|^2 + |F_2|^2,$$

where

$$F_0(q) = 4\pi \int \rho_0(r) j_0(qr) r^2 dr,$$

$$F_2(q) = -\sqrt{20\pi} \int \rho_2(r) j_2(qr) r^2 dr.$$  \hspace{1cm} (5a)

The quadrupole moment is defined as

$$Q = \sqrt{16\pi} \int \rho_2(r) r^4 dr.$$  \hspace{1cm} (6)

For the charge distribution $\rho_0(r)$ we assume the Saxon-Woods type

$$\rho_0(r) = \rho_0 \left[ 1 + \exp \left( \frac{r - c}{z} \right) \right]^{-1},$$

with the well-known meanings of the parameters.

3. Results and Discussion

The results for scattering of 300 MeV electrons by Be$^9$ have been discussed earlier \(^1\), and we shall just quote them briefly. Although the monopole part of the charge distribution (corresponding to a spherical nucleus) gives satisfactory results with the

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$c$ (fm)</th>
<th>$z$ (fm)</th>
<th>$s$ (fm)</th>
<th>$Q$ (fm$^2$)</th>
<th>$\langle r^2 \rangle^{1/2}$ (fm)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Be$^9$</td>
<td>1.40</td>
<td>0.60</td>
<td>2.64</td>
<td>0.256</td>
<td>2.48</td>
<td>3)</td>
</tr>
<tr>
<td>0.90</td>
<td>0.79</td>
<td>3.48</td>
<td></td>
<td>2.49</td>
<td>2.25</td>
<td>3)</td>
</tr>
<tr>
<td>1.80</td>
<td>0.45</td>
<td>2.00</td>
<td></td>
<td>2.25</td>
<td>2.25</td>
<td>3)</td>
</tr>
<tr>
<td>B$^{11}$</td>
<td>1.60</td>
<td>0.70</td>
<td>3.08</td>
<td>0.458</td>
<td>2.40</td>
<td>4)</td>
</tr>
<tr>
<td>2.00</td>
<td>0.45</td>
<td>2.00</td>
<td></td>
<td>2.23</td>
<td>2.40</td>
<td>4)</td>
</tr>
<tr>
<td>N$^{14}$</td>
<td>2.60</td>
<td>0.72</td>
<td>3.17</td>
<td>0.140</td>
<td>3.38</td>
<td>3)</td>
</tr>
<tr>
<td>2.30</td>
<td>0.50</td>
<td>2.20</td>
<td></td>
<td>3.38</td>
<td>2.46</td>
<td>4)</td>
</tr>
</tbody>
</table>

The quantity $z$ is the “surface thickness” as conventionally defined for the Saxon-Woods distribution.

parameters $c = 1.4$ fm and $z = 0.6$ fm up to an angle of 80°, the calculations showed a deviation from the experimental results for the higher values of $\theta$ as is usually the case with Born approximation. However, the quadrupole part of the form factor gives, for the deformation parameter $\varepsilon = 0.256$ calculated by assuming the quadrupole
moment of Be$^9$ to be 2.0 fm$^2$, the required cancelling of the discrepancy. For these values of the parameters, the root mean square radius was obtained to be 2.48 fm. A comparison with the parameters given by other authors is made in table 1. Fig. 1

![Graph 1](image1.png)

Fig. 1. Saxon-Woods charge distribution for Be$^9$. Curve A is for $c = 1.4$ fm, $z = 0.6$ fm, while curve B is for $c = 0.9$ fm, $z = 0.79$ fm (ref. 3). The density $\rho(r)$ is in units of 100 fm$^3$ while $r$ is in fm.

![Graph 2](image2.png)

Fig. 2. Elastic scattering of 300 MeV electrons by Be$^{11}$. The continuous curve represents the monopole scattering, the dotted curve the quadrupole contribution and the dots are the experimental points. ows the difference in charge distribution densities between our analysis and that of Meyer-Berkhout et al.$^3$. The latter is slightly more extended towards the surface than ours.
In $^{11}$B it is interesting to note (fig. 2) that even for smaller angles the quadrupole part plays an important role. The parameters obtained $^{5)}$ from the $^{11}$B analysis were $c = 1.6$ fm, $z = 0.7$ fm and $\varepsilon = 0.458$. The root mean square radius obtained was 2.40 fm.

Next, the results of elastic scattering $^{3)}$ of 420 MeV electrons by $^{14}$N were analysed, and it was found that a satisfactory agreement can be obtained with the parameters $c = 2.60$ fm, $z = 0.72$ fm and $\varepsilon = 0.138$, giving the value of the root mean square radius as 3.38 fm. It should be remarked that a still better fit for this nucleus could be obtained by a slight variation of the parameters, which, however, would not change the value of the deformation parameter $\varepsilon$ to any considerable extent.

It is interesting to note that the consideration of the deformation of a nucleus in calculating the charge density does not affect the root mean square radius to any sizable amount. However, it does affect the values of $c$ and $z$ to an appreciable extent and gives a proper contribution to filling up the diffraction minimum.

The value of the parameter $c$ is larger for $^{11}$B than for $^{9}$Be or $^{14}$N: this is expected as $^{11}$B lies in the middle of the p-shell while the $^{9}$Be nucleus lies before $^{11}$B and $^{14}$N almost at the end of the p-shell, which also suggests that $\varepsilon_{^{11}B} < \varepsilon_{^{9}Be}$.\[\]

The authors are grateful to Dr. S. P. Pandya and Dr. P. C. Sood for various stimulating discussions. They also gratefully acknowledge the encouragement given to them by Professor B. M. Anand during the preparation of results. One of us (R.K.G.) is grateful to the Department of Atomic Energy, Government of India, for the financial support during the period of the work.

References

1) R. Hofstadter, Ann. Revs. Nucl. Sci. 7 (1957) 231
2) Y. R. Waghmare, Prog. Theo. Phys. 24 (1960) 681
5) Y. R. Waghmare, Ph. D. Thesis, University of Bombay, India (1962)
S-State Interactions in Effective Two-Body Forces

Yeshwant WAGHMARE, Raj K. GUPTA and Naresh KUMAR

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S-State Interactions in Effective Two-Body Forces

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(Received August 17, 1963)

The energy level spectrum of Ni$^{58}$ has been analyzed on the simple considerations of pure singlet $s$-state forces. By taking the single-particle energy (p$_s$f$_s$ spacing) as a free parameter, the best fit is obtained for $J=0.9$ MeV which is in fair agreement with the experimentally suspected value of 0.85 MeV for the single particle level of Ni$^{57}$. Many more, as yet unobserved, levels are predicted in the energy range of 2.5-3.2 MeV and it is pointed out that their exact location would be very helpful in understanding the detailed nature of the two-body forces. Finally the extension of the above analysis to the three particle configuration suggests a $3/2^-$ ground state for Ni$^{59}$.

§ 1. Introduction

It has been suggested by Moszkowski$^1$ that the study of $s$-state interactions would be able to give us a qualitative idea about the level structure of a shell model nucleus. The calculations can further be simplified by considering the interactions to be spin-independent. As far as configurations such as $(jj)$ are concerned, this would obviously give a non-degenerate set of energy eigen states. But for the configurations of $(j_1j_2)$-type, one would obtain the odd states as degenerate if triplet forces are neglected. In many cases, this gives a convenient way of handling the situation.

With the calculations of the low lying energy levels in terms of relative coordinates on the basis of the method developed by Lawson and Mayer,$^2$ Arima and Terasawa,$^3$ Moshinsky,$^4$ and Mitra and Pandya,$^5$ one could also set up a sort of non-locality if different strengths are assumed for the various $l$-states. Calculations on these lines have recently been done by Shah and Pandya,$^6$ who find on the analysis of various configurations that the triplet forces are negligible. These authors have also determined the strengths of the $s$- and $d$-states along with their range, which was assumed to be the same for both these states.

$^1$ Supported in part by the U. S. Atomic Energy Commission.
In a similar way, we have in this paper tried to reproduce the energy levels of \( \text{Ni}^{58} \) by assuming only the \( s \)-state forces and neglecting all the triplet potentials. The choice of this nucleus is obvious. It lies in the magic-region with two neutrons outside the \( j_{1}/2 \)-shell, i.e. in the \( p_{1/2} \) shell. Moreover the single particle energies of \( \text{Ni}^{57} \) are as yet not well established. Some calculations\(^7,8\) in this direction are previously reported and we discuss them in the text.

\section*{2. Calculations}

The method of calculations is straightforward and is well illustrated in the paper of Shah and Pandya.\(^9\) As has been remarked in the introduction we would be interested in the calculations based on the relative coordinates rather than in that developed by Talmi\(^9\) and Thieberger.\(^10\) In many ways these methods illustrate different properties of the effective two-body interaction. For example, in the method of Talmi, the multipoles occurring in the Slater integrals \( F^{(k)} \) mask the configuration dependence of the two-body Hamiltonian. However, the calculations show that the configuration dependence enters through the parameter \( \lambda = (r_0/r_1) \) rather than the \( k \)-multipoles; \( r_1 \) is the range of the Gaussian potential and \( r_0 \) that of the nucleon wave function. It is also difficult to get an idea of the contributions due to the states of different angular momenta.

The Hamiltonian Matrix is written as

\[ \langle j_1 j_1 \rangle : JM | H_{\mu} | j_1' j_1' : JM \rangle. \]  

Expanding the two-particle wave functions by means of the Moshinsky brackets, we have

\[ | j_1 j_1 \rangle : JM = \sum_{ LS \text{ij}} A \left( \begin{array}{ccc} l_1 & s_1 & j_1 \\ l_2 & s_2 & j_1 \\ L & S & J \end{array} \right) B_{N_{15/2}^{(J)}}^{(J)}(L) \]

\[ \times | n_{15/2} l_{15/2} : LS \rangle, \]  

where \( A \)'s are the \( LS \)-\( jj \) transformation coefficients and \( B \)'s are the Moshinsky brackets. Substituting this expression in (1) we obtain

\[ \langle j_1 j_1 | H_{\mu} | j_1' j_1' \rangle : JM \rangle = \sum_{ LS \text{ij}} A \left( \begin{array}{ccc} l_1 & s_1 & j_1 \\ l_2 & s_2 & j_1 \\ L & S & J \end{array} \right) A \left( \begin{array}{ccc} l_1' & s_1' & j_1' \\ l_2' & s_2' & j_1' \\ L' & S' & J' \end{array} \right) B_{N_{15/2}^{(J)}}^{(J)}(L') B_{N_{15/2}^{(J)}}^{(J)}(L') \]

\[ \times \delta_{LL'} \langle n_l, S | H_{\mu} | n_{l'}, S \rangle \]  

for the central forces. For singlet forces \( (S=0) \) the \( A \)-coefficients give terms of the type \( \delta_{JJ} \) and \( \delta_{LL} \). Still further simplification to only the \( s \)-state \( (l=0) \) interactions, which we adopt throughout this paper, would give the term \( \delta_{JJ} \) as is easily seen from expression (3).
The values of the matrix elements $I_{\omega} = \langle n \ell | V_{\omega} | n \ell \rangle$ for the Gaussian potential $\exp(-r^2/\sigma^2)$ are given in Table I. For completeness of this table which is extremely useful for many further calculations of odd state forces as well as even state forces, we tabulate all the integrals that we normally come across in these calculations.

§ 3. Results and Discussion

Calculations of energy levels of Ni$^{58}$ have been carried out recently by Hamamoto and Arima. These authors have assumed $p\pi$-force plus a $\delta$-interaction for the exchange part and a Gaussian dependence for the radial part of the two-body Hamiltonian. The calculations are made in the framework of Talmi and Thieberger's method. However, as remarked earlier we carry out the calculations by the method of relative coordinates with pure singlet $s$-state forces. The attempt is to see in a simple way, as suggested by Moszkowski, how far the energy level data can be qualitatively reproduced by the consideration of such forces only. Further simplification could be introduced, as has been suggested by some authors, by neglecting even the configuration mixing. However,
in the present paper we do investigate the effect of such admixtures, as we do not see any reason for neglecting them.

Although the correct way of making calculations of the energy levels is to assume the single-particle energies for the excited state configurations from the neighbouring odd-$\Lambda$ nucleus with only one nucleon inside a shell, but there is no satisfactory information regarding the single particle levels in Ni$^{58}$ required for the present analysis. For this reason, the spacing of $f_{5/2}$ and $p_{3/2}$ (ground state), namely

$$J = E(f_{5/2}) - E(p_{3/2})$$

has been taken as a free parameter for our study. One can then find out a suitable combination of $\lambda$ and $J$ to reproduce the best possible fit. Such calculations have also been done by Bouten and Van Leuven.\(^7\) The values of the parameters in their calculations are varied from 1.5 to 1.8 MeV for $E(p_{3/2}) - E(p_{1/2})$ splitting and from 1.25 to 1.40 MeV for the $E(f_{5/2}) - E(p_{3/2})$ separation. However, in our calculations we vary the parameter $J$ from 0.8 to 1.8 MeV, since there is some evidence that in Ni$^{58}$, two levels at approximately 0.85 MeV and 1.15 MeV are observed, although no definite spin and parity assignment has been made.\(^11\)

### Table II. Variation of $v_0$ with $\lambda$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>0.5</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-v_0$ (in MeV)</td>
<td>68.6</td>
<td>38.2</td>
<td>30.9</td>
<td>25.9</td>
<td>22.4</td>
</tr>
</tbody>
</table>

![Graph](#)

**Fig. 1.** Energy levels of Ni$^{58}$. The calculated energy levels are shown in (A), (B) and (C) for different values of $\lambda$. The experimental level spectrum is shown in (D).

Figure 1 of $J(f_{5/2} - p_{3/2}$ spacing) vs. $E$(MeV) shows the calculated energy levels of Ni$^{58}$ for various values of $\lambda$. These include the effect of configuration...
mixing. It is observed that the ground state is much affected due to the presence of the excited 0+ level from \((f_5/2)^2\) configuration. This shifts the first 2+ excited state upwards and consequently the other low-lying levels. The potential-strength parameter \(V_0\) was calculated from comparison with the observed splitting of the 2+ and 0+ levels of the ground state configuration. The variation of \(V_0\) with \(\lambda\) is shown in Table II. It is then obvious that the calculated positions of all the levels including the observed 0+ and 2+ levels will depend upon the suitable choice of \(\lambda\) and \(J\). To fix such a suitable combination, we proceed as follows:

From Fig. 1 it is evident that the value of \(\lambda\) is approximately 1.0. For \(\lambda<1.0\) the 0+ excited level comes below 4+ for any reasonably acceptable value of the latter. The 4+ level is observed\(^\text{150}\) to be at 2.46 MeV. This can be made to agree with the theoretical one for \(J=0.9\) MeV as can be seen from Fig. 1C. However, for this choice of \(J\) the first 2+ level is raised up to approximately 1.75 MeV, i.e. off by about 0.3 MeV. It is also clear from all the three curves in Fig. 1 that the observed levels (i.e. 2+ and 4+) are not sensitive to the value of \(\lambda\). On the other hand, the choice of \(\lambda\) as 1.0 is made from the first excited (unobserved) 0+ level. It is thus clear that by any means, for example, reducing the strength of the admixture of the 0+ levels (raising the ground state by \(~0.3\text{ MeV}\)), or in other words, treating the two 0+ levels as almost pure, the agreement can be made in a reasonable way. This situation seems to be similar to that treated by Thankappan, Waghmare and Pandya\(^\text{14}\) wherein the effect has been attributed to the configuration dependence of the effective two-body nuclear interaction. In any case, our agreement even with the present consideration and neglecting the configuration dependence seems to be fairly satisfactory for the values of the parameters

\[
\lambda=1.0 \text{ and } J=0.8-0.9 \text{ MeV.}
\]

This would mean that between the energy range of 2.5 to 3.2 MeV there are as many as five levels, namely, 4+, 0+, 1+, 3+ and 2+. It would thus indeed be very much interesting to observe the exact positions of these levels experimentally, if possible. This would incidentally give a clearer picture of the application of this method. It should be mentioned that at \(~2.90\text{ MeV}\) a close doublet of 1+ and 2+ levels is already suspected.

These results can now be compared with the calculations made by using a Serber force, i.e. all even \(l\) states. In this case the potential strength \(v_0\) has the following values:

<table>
<thead>
<tr>
<th>(\lambda)</th>
<th>1.0</th>
<th>0.9</th>
<th>0.8</th>
<th>0.7</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-v_0)</td>
<td>18.95</td>
<td>22.35</td>
<td>27.41</td>
<td>34.94</td>
<td>66.21</td>
</tr>
</tbody>
</table>

Table III. Variation of \(\lambda\) with \(v_0\) for a Serber potential.
It is clear that these values do not differ much from those given in Table II. With these values of the singlet force (calculated once again from the observed $2^\text{+} - 0^\text{+} (G-S')$ separation) we obtain the energy levels of Ni$^{58}$ as given in Table IV.

Table IV. Energy levels of Ni$^{58}$ by using a Serber force.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$J$</th>
<th>1.0</th>
<th>0.8</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(p_{3/2})^2$</td>
<td>0</td>
<td>G.S</td>
<td>G.S</td>
<td>G.S</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.49</td>
<td>1.55</td>
<td>1.70</td>
</tr>
<tr>
<td>$(f_{5/2}p_{3/2})$</td>
<td>2</td>
<td>2.99</td>
<td>2.92</td>
<td>3.11</td>
</tr>
<tr>
<td></td>
<td>1.3</td>
<td>3.16</td>
<td>3.13</td>
<td>3.16</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>2.58</td>
<td>2.63</td>
<td>2.80</td>
</tr>
<tr>
<td>$(f_{5/2})^2$</td>
<td>0</td>
<td>2.63</td>
<td>2.53</td>
<td>2.38</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3.23</td>
<td>3.32</td>
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<td></td>
<td>4</td>
<td>3.70</td>
<td>3.74</td>
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</tbody>
</table>

It is thus evident that the agreement of the calculations with the Serber force and the experimental results is satisfactory for $J=1.0$. However, in these calculations the value of the single particle spacing $J$ is assumed to be 0.85 MeV. It is also clear that the general characteristics of the levels remain similar to those calculated with pure $s$-states. Thus, the suggestion of Moszkowski$^7$ to neglect higher $l$ contributions seems to be justified at least in this case.

In the above calculations, we have not taken into account the effect of the $(p_{3/2} p_{1/2})$ excited configurations. The levels from these configurations would depress to some extent some of the low-lying levels. However, the preliminary calculations show that the off-diagonal matrix elements are not strong enough to depress these states to the required energies, and it would not disturb the ground state by a considerable amount.

We now remark on the odd-state forces. There are many levels observed in the region of 2.50 to 3.50 MeV and 1$^+$ is one of them. From general considerations of the shell model, the other odd-$j$ level can be suspected to be not much far; rather it may come very close to the first 4$^+$ excited level. It would be thus interesting to know the experimental separation of the 3$^+$ and 1$^+$ levels, which would give us obvious information regarding the strength of the triplet forces.

Finally, extending the two-particle nuclear spectroscopy to the three particle configuration by the use of fractional parentage coefficients, one could discuss the level structure of Ni$^{58}$. Since nothing is known as yet about the spin and parity of the levels for this nucleus, it is not possible to discuss it in detail here. The levels of $(j)^3$ configuration can be derived directly from the known levels of a $(j)^2$ configuration by the relation

$$E_i[(j)^3] = 3\sum\langle (j)^3 : J | (j)^3 : X \rangle E_i[(j)^3]$$
where \( \langle j \rangle^3 : J \langle j \rangle^2 : X \rangle \) are the c.f.p.s. and are tabulated for \( j = 5/2 \) by Rose. The calculations then predict the ground state to be \( 3/2^- \), which has also been suspected experimentally. Also it is observed that at about 3.50 MeV three levels, namely, \( 1/2^- \), \( 5/2^- \) and \( 7/2^- \) can be suspected to be very close together (degenerate in our calculations). The explicit calculations for this nucleus are in progress and will be reported in due course. From the above analysis, it is thus clear that the agreement of the theoretical and experimental results can be well obtained, qualitatively, on the simple considerations of pure singlet \( s \)-state forces alone.

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References

Rotational Bands in Iron-56

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The absence of well developed rotational spectra in the medium weight region 40 ≤ A ≤ 50 was noted several years ago. The nuclei in this region are assumed to be spherical in shape and, away from closed shells, are expected to exhibit oscillations about their equilibrium shape. It has been recently pointed out that a fairly abrupt change occurs between the shell model region below the neutron number N=30 and the vibrational region above N=32. Whereas the nuclei around mass number 60 can be termed as vibrational whose spectra is explained in terms of phonon excitations, the nuclei in the transition region, e.g. chromium and iron isotopes, cannot be satisfactorily treated on either of these models. Certain recent nuclear reaction studies have indicated that Fe-56 may possess deformed shape and show rotational characteristics. In the present note we discuss the level scheme of this nucleus based on rotational model.

The collective effects in this nucleus are apparent from the Coulomb excitation studies and lifetime measurements of the 0.845 MeV first excited level. These experiments give an enhancement factor of 16 over the single particle estimate. The stable equilibrium deformation and rotational spectra may occur for even-even nuclei if they obey the approximate criterion suggested by Alder et al. That E2, the energy of the first excited 2+ state, is smaller than a critical value

\[ E2_{\text{crit}} = \frac{138}{B_{\text{rigid}}} \]  

where \( B_{\text{rigid}} \) is the moment of inertia for rigid rotations, is a function of the deformation parameter \( \delta \), which in turn is related to the intrinsic quadrupole moment \( Q_\ell \). Taking \( Q_\ell = 1.0 \) barn from the Coulomb excitation experiments we obtain a deformation parameter \( \delta = 0.20 \) and the \( E2 \) critical value = 0.91 MeV. The observed energy 0.845 MeV for the first excited 2+ state is smaller than the critical value and hence the rotational model interpretation can be expected to hold for this nucleus.

The energy levels in this model are given by the formula

\[ E_\ell = A\ell(\ell+1) - B\ell(\ell+1)^2 \]  

where \( A \) is related to the moment of inertia of the deformed nucleus \( (A = \frac{B_{\text{rigid}}}{2\delta}) \) and \( B \) is the rotation-vibration coupling constant. These constants \( A \) and \( B \) are evaluated by fitting the 0.845 MeV 2+ level and 2.08 MeV 4+ level in Fe-56. With the values of \( A \) and \( B \) thus obtained two other \( K=2 \) bands are constructed on the 2+ levels at 2.66 MeV and 2.95 MeV respectively. The resulting rotational spectrum is shown in Fig. 1(b) to be compared with the experimental level scheme shown in Fig. 1(a). It is seen that the agreement between the theory and the experimental results is quite good. Thus in addition to the well developed ground state \( K=0 \) rotational band, two other \( K=2 \) bands are indicated for this nucleus. However, when one looks at the predictions for relative gamma-ray intensities, it is found that the agreement is not so good. This, of course, is not surprising since we know that even in rare earth and heavy deformed nuclei the transition probabilities cannot be predicted on this simple model; one has to invoke band mixing or introduce other parameters. We shall deal with this aspect in a later communication.

As a further check on this approach we compare the rotational constants for this case with the known cases of deformed nuclei. This is done in Table I. The rotational constant \( A \) progressively decreases from about 540 KeV for 1p-shell...
nuclei to about 7 keV in the actinide region. On the other hand the rotation-vibration coupling constant $B$ is less than two percent of $A$ in all regions of the periodic table. The values of $A$ (156.6 keV) and $B$ (2.63 keV) for Fe$^{56}$ fit quite nicely in the general picture.

Table I. The rotational constants $A$ and $B$ in keV obtained by fitting the 2$^+$ and 4$^+$ states of the ground state rotational band in various nuclei.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Ne$^{56}$</th>
<th>Mg$^{56}$</th>
<th>Si$^{56}$</th>
<th>Fe$^{56}$</th>
<th>Sm$^{152}$</th>
<th>Tb$^{158}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>496</td>
<td>383</td>
<td>323</td>
<td>157</td>
<td>21</td>
<td>10</td>
</tr>
<tr>
<td>$B$</td>
<td>4.3</td>
<td>1.6</td>
<td>4.6</td>
<td>2.6</td>
<td>0.14</td>
<td>0.02</td>
</tr>
</tbody>
</table>

This observation, coupled with the approximate criterion applied above, the agreement of the theoretical and experimental level schemes, and the evidence from nuclear reaction, strongly suggests that the rotational band structure for Fe$^{56}$ is meaningful. Detailed calculations for this nucleus and other similar cases in this region are in progress.

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7) K. Alder, A. Bohr, T. Huus, B. Mottelson and A. Winther, Rev. Mod. Phys. 28 (1956), 432.
Nuclear Energy Levels in Fe\(^{56}\) &
the Rotational Model

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The interpretation of nuclear energy levels of Fe\(^{56}\) on the rotational model is discussed. The energies of the levels in excited rotational bands based on the gamma-vibrational states are calculated using the rotation-vibration coupling constant derived from the band head energy. The agreement between the predicted values and the experimental results is good. The possible existence of a beta-vibrational excited band is discussed.

Some recent nuclear reaction studies\(^1\)\(^-\)\(^5\) have indicated that some iron and chromium isotopes may possess permanently deformed shapes. In this note a discussion of the energy levels of Fe\(^{56}\) based on the rotational model is presented, in continuation of our earlier communication\(^4\) on the subject.

On the vibrational model the first excited 4\(^+\) level is a member of the two-phonon excitation triplet. The differential scattering cross-section for the 0\(^+\)–4\(^+\) transition in Fe\(^{56}\) was studied by Matsuda\(^5\) and a comparison of the experimental results with the theoretical predictions showed marked disagreement. On the other hand, the relative excitation of the 2\(^+\) and 4\(^+\) levels in this nucleus (Fe\(^{56}\)) in proton reactions\(^1\)\(^-\)\(^3\) suggests that they are members of the ground state rotational band.

The 0\(^-\)–2\(^-\)–4\(^-\) sequence of the ground state band gives the values \(A = 156-6\) keV. and \(B_\gamma = 2-63\) keV. for the rotational constants in conformity with the general trend of their values for other nuclei. In the microscopic theory of collective motions in nuclei\(^6\)\(^-\)\(^7\) the rotational sequence based on the ground state effectively terminates for a critical value \(I_c\) of the nuclear spin due to complicated interplay of rotational motion, Coriolis forces and pairing correlations; the states with \(I > I_c\) lie above the energy gap and are thus intrinsic in character. The energy gap for this nucleus\(^8\) is approximately 3 MeV. (2-66 MeV. considering the neutron pair and 3-36 MeV. for the proton pair). As such 6\(^+\) and higher spin states lie well above the energy gap. Consequently, the ground state band is essentially complete in the above sense and any higher order corrections to the expression for energy cannot be of any practical interest.

On the Bohr-Mottelson model the \(\beta-(K = 0)\) and \(\gamma-(K = 2)\) vibrational frequencies are related to the rotation-vibration coupling constant \(B\) through the relation

\[
B_{\text{theo}} = 4A^3/(\hbar\omega_0)^2 + 12A^3/(\hbar^2)^2 \ldots \ldots (1)
\]

However, marked deviations from this relation are well known. The suggestion\(^9\) to use distinct constants for the \(\beta\) and \(\gamma\) vibrations separately leads to the following equations:

\[
\begin{align*}
B_{\text{theo}} & = B_\beta + B_\gamma \\
B_\beta & = 4A^3/(\hbar\omega_0)^2 \ldots \ldots (2) \\
B_\gamma & = 12A^3/(\hbar^2)^2
\end{align*}
\]

In view of the authors’ earlier success in interpreting the energy level schemes in elements falling in the rare earth region with this approach\(^10\), the same procedure has been adopted in the present study also. Thus the energies for levels within an excited band are given by

\[
E(I) = E_0 + AI(I+1) - B_{\text{band}} P(I+1)^2 \ldots \ldots (3)
\]

In Fe\(^{56}\) two excited \(\gamma\) bands \((K = 2\) and \(K = 2')\) with band heads at 2-66 and 2-95 MeV. respectively can be constructed. Using Eq. (2) we obtain \(B_\beta = 2-17\) keV. and \(B_\gamma = 1-76\) keV. respectively for the two bands. The predicted level scheme based on Eq. (3) is compared with the experimental results\(^11\)\(^-\)\(^12\) in Fig. 1. The overall agreement of the predicted energies with the experiment may be termed as quite satisfactory, considering the fact that band-mixing effects, mutual repulsion of levels with same spin and parity, etc., have been neglected.

The possible existence of a \(\beta\) band is discussed in the following: The angular distribution of 3-61 MeV. level, studied by the inelastic scattering of 14-65 MeV. protons\(^5\), suggests that it may be the 0\(^+\) level. Assuming this as the \(\beta\)-vibrational state the calculated value of \(B_\beta = 3-54\) keV. This gives

\[
b = B_{\text{theo}}/B_\beta = (B_\beta + B_\gamma)/B_\beta = 2-17 \ldots \ldots (4)
\]

which fits in quite well with the general systematics. The energies for the 2\(^+\) and 4\(^+\) states of the \(\beta\)-vibrational band calculated using Eq. (3) are 4-4 and 5-3 MeV. respectively. A 4\(^+\) level at 5-1 MeV. has
been indicated in the study of the inelastic scattering of 150 MeV. electrons. There are many other levels in the neighbourhood with no assignments. Definite spin parity assignments for these levels should be looked for to check these predictions.

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References