Appendix II
GAMMA RAY INTENSITIES IN Dy$^{160}$ AND ROTATIONAL MODEL.

The level scheme for Dy$^{160}$ is evidently rotational in character and therefore quite naturally one attempts to explain the transition probabilities on the basis of the various bands in the rotational spectrum. The level structure in Dy$^{160}$ has been the subject of various investigations$^{1-4}$ recently. However, in spite of various lines of approaches and careful experimentation, the results obtained by the different investigators are widely different — sometimes differing by a factor of two or more.

The aim of the present study is to see whether a check of the level energies derived from the experimental gamma ray intensities can lead to some selective choice out of these differing experimental results. Further, the role of mixing between the neighbouring rotational bands in giving a consistent picture of the level spectrum as well as the transition probabilities is examined.

The relation between the experimentally observed gamma ray intensities and the reduced $E^2$ transition probabilities is given as$^{5}$

$$\frac{T(\gamma_1)}{T(\gamma_2)} \left( \frac{E_{\gamma_2}}{E_{\gamma_1}} \right)^5 = \frac{B(E^2; I_1 \to I_f, (\gamma_1))}{B(E^2; I_1 \to I_f, (\gamma_2))}$$

where $E_{\gamma_1}$ and $E_{\gamma_2}$ are the energies of the gamma rays involved in the transitions. The quantity on the right can be calculated by assuming a definite model. Based on the rotational model and assuming the purity of various rotational bands it is given$^{6}$ as the ratio of certain Clebsch-Gordan coefficients. However, it has been found that the rotational model, for the axially symmetric case of Bohr and Mottelson$^{7}$ as well as for asymmetric case of
Davydov and Fillippov\textsuperscript{2)}, has not proved to be as successful in explaining the transition probabilities as it proved to explain the level schemes. As initiated by Bohr and Mottelson\textsuperscript{9)} and later on worked out by various authors\textsuperscript{3,9–13)}, the agreement for the transition probabilities can be improved by assuming that there is some mixing in the various rotational bands and even a mixing of a few percent, although affecting the level scheme insignificantly, can alter the transition probabilities appreciably. The effect of band mixing is taken into account by writing

\[ B(E2; I_1 \rightarrow I_\pi (\gamma)) = B(E2; I_1 \rightarrow I_\pi (\gamma))_{\text{pure band}} + f(Z_k, I_1, I_\pi) \]

where \( Z_k \) is known as the band mixing parameter which essentially is the \( E2 \) transition strength of the admixed components. The correction factor \( f(Z_\gamma, I_1, I_\pi) \), as calculated by Mottelson, for mixing between the ground state band \( K = 0 \) and the \( \gamma \)-vibration band \( K = 2 \) is tabulated in reference 9. We are concerned here only with these two bands as no \( \beta \)-vibrational band is observed in Dy\textsuperscript{160} so far.

The determination of \( Z_k \) has been a subject of considerable interest: firstly to find out a single value of \( Z_\gamma \) which could explain the intensities of all observed transitions between the \( K = 0 \) ground state and \( K = 2 \) bands\textsuperscript{3,12–14)}, and secondly to correlate the magnitude of \( Z_k \) with other effects of band mixing\textsuperscript{10,13,15,16)} such as the occurrence of M1 admixtures in the predominant \( E2 \) radiation and the deviation of the rotational spectra from the simple \( I(I + 1) \) law. Our procedure is to take the known first excited state energy \( E_{2+} \) and the various experimentally determined gamma ray intensities connecting this
level with other levels, and then use equations (II.1) and (II.2) to deduce the relative position of these levels as a function of the band mixing parameter $Z_{\gamma}$. This enables us to choose a value of $Z_{\gamma}$ that gives the best fit simultaneously for the level scheme as well as the transition probabilities.

For the present analysis on Dy$^{160}$ we take the results from the two recent experimental studies by Ewan et al.\(^3\) and by Michaelis\(^4\). The level schemes are shown as a function of $Z_{\gamma}$ in fig. II.1(a) — the broken line corresponds to the results of Michaelis and the solid line that for Ewan et al. Strangely enough it is found that the $3^+_2 (I^0_1 K)$ level based on the results of Michaelis always lies lower than $2^+_2$ level for pure band ($Z_{\gamma} = 0$) and for any band mixing. This is evidently incompatible with the basic requirement of the rotational spectrum which requires that the $\gamma$-vibrational, $K = 2$ band is built on $2^+$ as the band head with level sequence $2^+, 3^+, 4^+, ...$. Hence the accuracy of Michaelis results and the validity of rotational model interpretation of this nucleus cannot be reconciled. On the other hand we find that the results of Ewan et al. for the three transition probabilities

$B(EE; 2^+_2 \rightarrow 0^+_0) / B(EE; 2^+_2 \rightarrow 2^+_0)$, $B(EE; 2^+_2 \rightarrow 2^+_0) / B(EE; 2^+_2 \rightarrow 4^+_0)$ and $B(EE; 3^+_2 \rightarrow 4^+_0) / B(EE; 3^+_2 \rightarrow 2^+_0)$ can give complete agreement with the above theory corresponding to the respective values of 0.048, 0.061 and 0.057 for the band mixing parameter $Z_{\gamma}$. We may represent this as

$Z_{\gamma} = 0.055 \pm 0.007$.

If we compare the solid line curves in fig. II.1(a), due to the results of Ewan et al., with the experimental level scheme shown in fig. II.1(b), the best possible agreement is obtained for $Z_{\gamma} = 0.048$. 
The level spectrum for this choice of $Z_\gamma$, calculated by using the above formalism, is shown separately in fig. II.1(c) and the transition probabilities are given in table II.1. The agreement for level spectrum can, however, be improved by going to the fourth decimal place for the value of $Z_\gamma$. The level scheme is also shown in fig. II.1(d) for $Z_\gamma$ having mean value 0.055 (which agrees with the value adopted by Ewan et.al.). The transition probabilities for this value of $Z_\gamma$ are also given in the table II.1. From the comparison we see that an improved agreement to give a consistent picture for both the level spectrum as well as transition probabilities is obtained for the band mixing parameter $Z_\gamma = 0.048$.

Thus we see that our analysis points to the fact that the results of Michaelis need be carefully checked again to conform to the rotational pattern. The results of Ewan et.al. appear to be in definitely better agreement. It is satisfying that nearly exact agreement is obtained for both the energy levels and branching ratios by introducing the band mixing of about 5 per cent only which also is consistent with the band mixing estimates in neighbouring nuclei. It may, however, be mentioned that the present method of analysis is significant because of the existence of well separated ground state and $\gamma$-vibrational bands in Dy$^{160}$. 
Table II.1

Reduced E2 transition probabilities for levels in ground state $K = 0$ and $Y$-vibrational $K = 2$ bands in Dy$^{160}$

<table>
<thead>
<tr>
<th>Reduced E2 transition probabilities</th>
<th>Experimental</th>
<th>Pure $K = 2$ with $K=0$ admixture</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ewan et. al. Michaelis</td>
<td>$Z_\gamma=0.048$</td>
</tr>
<tr>
<td>$2^+_2 - 0^+_0$</td>
<td>$0.53 \pm 0.05$</td>
<td>0.7</td>
</tr>
<tr>
<td>$2^+_2 - 2^+_0$</td>
<td>$10.6 \pm 3$</td>
<td>20</td>
</tr>
<tr>
<td>$2^+_2 - 4^+_0$</td>
<td>$0.81 \pm 0.08$</td>
<td>0.4</td>
</tr>
<tr>
<td>$3^+_2 - 2^+_0$</td>
<td>$0.35 \pm 0.15$</td>
<td>0.4</td>
</tr>
</tbody>
</table>
References.


