CHAPTER V: Rotational Bands in Fe and Kr

The absence of well developed rotational spectra in the medium weight region $40 \leq A \leq 150$ was noted several years ago. The nuclei in this region are assumed to be spherical in shape and, away from closed shells, are expected to exhibit oscillations about their equilibrium shape. The excitations involving these shape oscillations are explained on the vibrational or phonon excitation model. On the other hand Coulomb excitation experiments$^{86)}$ had established that essentially all even even transitions in medium weight non-magic nuclei (roughly from Ti to Te) are from 10 to 30 times faster than single particle transitions and must therefore involve some degree of collective participation. The purpose of the present study is to look for the occurrence of rotational spectra in this region of medium weight nuclei.

It has been recently pointed out$^{150)}$ that a fairly abrupt change occurs between the shell model region below the neutron number $N = 30$ and the vibrational region above $N = 32$. Whereas the nuclei around mass number 60 can be termed as vibrational whose spectra is explained in terms of phonon excitation$^{151)}$, the nuclei in the transition region, e.g. chromium and iron isotopes, cannot be satisfactorily treated on either of these models$^{152)}$. Earlier experimental$^{132)}$ and theoretical$^{133)}$ studies at the Lawrence Radiation Laboratory have also pointed out certain extended regions of deformations for relatively neutron deficit nuclei in the regions where both proton and neutron numbers go from 50 to 82 and 82 to 128. In particular it has been found that even-even Ba isotopes are deformed and their
deformations are comparable with nuclei from other regions of
deformation. Deformations are also expected in certain other
limited regions e.g. Fe isotopes other than Fe$^{54}$ and As and Se
isotopes. Kumar and Baranger$^{153}$ have calculated the nuclear
shapes in this region and find these nuclei to have large
negative intrinsic quadrupole moments i.e. oblate or disc shaped,
in contrast to the prolate shapes (i.e. positive quadrupole
moments) for all the strongly deformed nuclei known so far.
Certain other recent nuclear reaction studies$^{152, 154}$ have
indicated that Fe$^{56}$ may possess deformed shape and show rotational
characteristics. In the following we discuss$^{155, 156}$ the level
scheme of this nucleus based on rotational model.

Normally one expects that nuclear spectra in this
neighbourhood can be explained on the vibrational model. The
first $4^+$ excited state is then a member of the two-phonon
excitation triplet. The differential scattering cross-section
for the $0^+ \rightarrow 4^+$ transition in Fe$^{56}$ was studied by Matsuda$^{157}$
and a comparison of the experimental results with the theoretical
predictions for two-phonon excitation showed marked disagreement.
On the other hand, the relative excitation of the $2^+$ and $4^+$
levels of Fe$^{56}$ in proton reactions$^{152, 154}$ suggests that they
are members of the ground state rotational band.

The collective effects in this nucleus are apparent from
the Coulomb excitation studies$^{158}$ and lifetime measurements$^{159}$
of the 0.845 MeV first excited level. These experiments give
an E2 enhancement factor of 16 over the single particle estimate.

The stable equilibrium deformation and rotational spectra
may occur for even even nuclei if they obey the approximate
criterion suggested by Alder et al.$^8$ that $E_{2^+}$, the energy of the
first excited $2^+$ state, is smaller than a critical value

$$(E_2)_{\text{crit.}} \approx 13 \frac{k^2}{J_{\text{rigid}}}$$  \hspace{1cm} (1.13)

where $J_{\text{rigid}}$, the moment of inertia for rigid rotations, is a function of the deformation parameter $\beta$, which in turn is related to the intrinsic quadrupole moment $Q_0$ (section 1.3). Taking $Q_0 = 1.0$ barn from the Coulomb excitation experiments\textsuperscript{158} we obtain a deformation parameter $\beta \approx 0.20$ and the $E_2$ critical value $\approx 0.21$ MeV. The observed energy $0.845$ MeV for the first excited $2^+$ state is smaller than the critical value and hence the rotational model interpretation can be expected to hold for this nucleus.*

The energy levels in this model are given by the formula

$$E_I = AI(I+1) - BI^2 (I+1)^2$$  \hspace{1cm} (1.3)

where $A$ is related to the moment of inertia of the deformed nucleus ($A = \frac{k^2}{2J}$) and $B$ is the rotation-vibration coupling constant. These constants $A$ and $B$ are evaluated by fitting the $0.845$ MeV, $2^+$ level and $2.03$ MeV, $4^+$ level in Fe\textsuperscript{56}. With the values of $A$ and $B$ thus obtained two other $K = 2$ bands are constructed on the $2^+$ levels at $2.66$ MeV and $2.95$ MeV respectively. The resulting rotational spectrum is shown in fig. 5.1(b) to be compared with the experimental level scheme\textsuperscript{161,162} shown in fig. 5.1(a). It is seen that the agreement between the theory and experiment is quite good. An excited $0^+$ level has also been suggested by Matsuda\textsuperscript{157} from the angular distribution studies of $3.61$ MeV level in Fe\textsuperscript{56} by the inelastic scattering of $14.69$ MeV.

*The characterization of the even even Ba isotopes as deformed\textsuperscript{132} was based on the regularities shown by their first excited state energy in three different ways: (i) a direct comparison with $E_2$ critical value; (ii) a determination of an empirical deformation $\beta_{\text{emp}}$ with a relationship\textsuperscript{115} between $J_{\text{rigid}}$ and $\beta$ ; and (iii) a comparison of $\beta_{\text{Mig}}$ from the relationship of Migdal\textsuperscript{180}. For Fe\textsuperscript{56}, the magnitudes of $\beta_{\text{emp}}$ ($\sim$ of the order of $\beta_{\text{expt}}$) and $\beta_{\text{Mig}}$ ($\sim 0.156$) are found to be comparable with the values for other deformed nuclei having similar deformations.
<table>
<thead>
<tr>
<th>MeV</th>
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<th>MeV</th>
<th>$I^\pi$</th>
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</thead>
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<td>$4^+\ 0$</td>
<td>5.3</td>
<td>$4^+\ 0$</td>
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<tr>
<td>5.1</td>
<td>$(4^+)$</td>
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<td>$4^+\ 2^-$</td>
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</tr>
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<td>$4^+\ 2$</td>
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</tr>
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<td>3.70</td>
<td>$3^+\ 2^-$</td>
<td></td>
</tr>
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<td>$0^+\ 0$</td>
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<td>3.32</td>
<td>$3^+\ 2$</td>
<td>3.37</td>
<td>$3^+\ 2$</td>
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<td>$2^+\ 0$</td>
<td>2.95</td>
<td>$2^+\ 2$</td>
<td></td>
</tr>
<tr>
<td>2.66</td>
<td>$2^+\ 2$</td>
<td>2.66</td>
<td>$2^+\ 2$</td>
<td></td>
</tr>
<tr>
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<td>$4^+\ 0$</td>
<td></td>
</tr>
<tr>
<td>0.845</td>
<td>$2^+\ 0$</td>
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<td>$2^+\ 0$</td>
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<tr>
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<td>$0^+\ 0$</td>
<td>$0^+\ 0$</td>
<td>$0^+\ 0$</td>
<td></td>
</tr>
</tbody>
</table>

**EXPT.**

(a)

**PREDICTED**

(b) < « >

(c)

Fig. 5.1
protons. Assuming this as a $\beta$-vibrational band head, a
K = 0, $\beta$-vibrational band has been formulated. The 2$^+$ and
4$^+$ levels are predicted at 4.43 MeV and 5.69 MeV respectively.
A 4$^+$ level at 5.1 MeV has been indicated in the study of the
inelastic scattering of 150 MeV electrons\(^{163}\), in addition to
many other levels in its neighbourhood with no assignments\(^{164}\).
Definite spin-parity assignments for these levels would check
these predictions. Thus, for this nucleus, in addition to the
well developed ground state K = 0 rotational band, two other
K = 2 bands and the possible existence of a $\beta$-vibrational K = 0
band are indicated. However, when one looks at the predictions
for relative gamma-ray intensities, it is found that the agreement
is not so good. This, of course, is not surprising since we know
that even in rare earth and heavy deformed nuclei the transition
probabilities cannot be predicted on this simple model; one has
to invoke band mixing or introduce other parameter\(^{165}\). (A brief
discussion on these effects is given in Appendix II).

As a further check on this approach we compare the
rotational constants for this case with the known cases of
deformed nuclei. This is done in table 5.1. The rotational
constant $A$ progressively decreases from about 540 KeV for lp
shell nuclei to about 7 KeV in the actinide region. On the
other hand, the rotation-vibration coupling constant $B$ is
less than two per cent of $A$ in all regions of the periodic table.
The values of $A$ (156.8 KeV) and $B$(2.63 KeV) for Fe$^{56}$ fit quite
nicely in the general picture.

This observation, coupled with the approximate criterion
applied above, the agreement of the theoretical and experimental
level schemes, and the evidence from nuclear reactions\(^{152,154}\)
Table 5.1

The rotational constants $A$ and $B$ in KeV obtained by fitting the $2^+$ and $4^+$ states of the ground state rotational band in various nuclei.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Ne$^{20}$</th>
<th>Mg$^{24}$</th>
<th>Si$^{28}$</th>
<th>Fe$^{56}$</th>
<th>Sm$^{152}$</th>
<th>Th$^{228}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>298</td>
<td>237</td>
<td>323</td>
<td>157</td>
<td>21</td>
<td>10</td>
</tr>
<tr>
<td>B</td>
<td>4.3</td>
<td>1.6</td>
<td>4.6</td>
<td>2.6</td>
<td>0.14</td>
<td>0.02</td>
</tr>
</tbody>
</table>
strongly suggest that the rotational band structure for Fe\textsuperscript{56} is meaningful.

On the basis of rotational model studies of other deformed nuclei (Chapter III) it may be argued that the relatively large value of the coefficient $B$ of the correction term necessitates the inclusion of the higher order corrections. But, as already mentioned in section 3.1, in the microscopic theory of collective motions in nuclei\textsuperscript{100,134} the rotational sequence based on the ground state effectively terminates for a critical value $I_c$ of the nuclear spin due to complicated interplay of rotational motion, Coriolis forces and pairing correlations; the states with $I > I_c$ lie above the energy gap and are thus intrinsic in character. The energy gap for this nucleus\textsuperscript{166} is approximately 3 MeV (2.66 MeV by considering the neutron pair and 3.66 MeV for the proton pair). As such $6^+$ and higher spin states would lie well above the energy gap. Consequently the ground state band comprising of $0^+ - 2^+ - 4^+$ sequence is essentially complete in the above sense and any higher order corrections to the expression for energy cannot be of any practical interest.

Next we discuss the excited bands in Fe\textsuperscript{56} in the light of the formalism developed in the previous chapter. It has been suggested that it may be more appropriate to use the rotation-vibration coupling constants derived from the band head energy rather than from the ground state band. This suggestion to use distinct constants for the $\gamma$- and $\beta$- vibrations separately leads to the following equations:

\begin{align*}
B_\gamma &= 4A^3/(\kappa \omega_\gamma)^2 \\
B_\beta &= 12A^3/(\kappa \omega_\beta)^2
\end{align*}

\textsuperscript{166} (4.1).

The energies for levels within an excited band are then given by
In Fe$^{55}$ two excited $\gamma$-bands ($K = 2$ and $K = 2'$) with band heads at 2.66 and 2.95 MeV respectively have been constructed. Using eq. (4.1) we obtain $B_{\gamma} = 2.17$ KeV and $B_{\gamma'} = 1.76$ KeV respectively for the two $K = 2$ and $K = 2'$ bands as compared to $B_0 = 2.53$ KeV for the ground state band. The constant $A$, which is connected with the inertial parameter $J$, the effective moment of inertia, is taken to be same for all bands. The predicted level scheme based on eq. (4.2) is compared in fig. 5.1(c). The overall agreement of the predicted energies with the experiment may be termed as quite satisfactory, considering the fact that band mixing effects, mutual repulsion of the levels with same spin and parity, etc., have been neglected. The better agreement obtained by using $B_{\gamma}$ instead of $B_0$ may be noted by pointing out that the energy of the $4^+$ state in $K = 2$ band is 3.89 MeV for $A_0$, $B_0$ and 4.06 MeV for $A_0$, $B_{\gamma}$, to be compared with the experimental value of 4.10 MeV.

For $\beta$-vibrational $K = 0$ band, accepting 3.61 MeV as the $\beta$-vibrational state, the calculated value of $B_\beta = 3.54$ KeV. This gives

$$b = \frac{B_{\text{theo}}}{B_0} = \frac{B_{\gamma} + B_\beta}{B_0} = 2.17$$

which fits in quite well with the general systematics$^{9,93}$. Using this value of $B_\beta$ the energies for the $2^+$ and $4^+$ states of the $\beta$-vibrational band are calculated, by using eq. (4.2), as 4.4 MeV and 5.3 MeV respectively. This is also shown in fig. 5.1(c). The improved agreement for the $4^+$ state, given in this formalism, over the calculations for the ground state parameters, when compared with the $4^+$ level at 5.1 MeV (suggested in the study of the inelastic scattering of 150 MeV electrons) is to be noted.
Another instance, of medium weight nuclei showing rotational spectrum, has been noticed in the case of Kr$^{82}$. The reduced E2 transition probability is $0.13 \times 10^{-48} \text{e}^2 \text{cm}^4$ from the Coulomb excitation of the first $2^+$ excited state shows an enhancement of about 17 times over the single particle estimate. For this value of the reduced E2 transition probability, the intrinsic quadrupole moment $Q_0 = 1.35$ barn and the deformation parameter $\beta \approx 0.13$. These values of the enhancement factor and deformation parameter $\beta$ are comparable with the values for other deformed nuclei with well developed rotational spectra.

The rotational constants $A$ and $B$ for Kr$^{82}$, calculated by using the experimental energies of the first $2^+$ (777 KeV) and $4^+$ (1231 KeV) excited states, are 146 KeV and 2.75 KeV respectively. These values of $A$ and $B$ for Kr$^{82}$, when compared with the known cases of deformed nuclei given in table 5.1, fit quite nicely in the general picture.

Using the values of $A$ and $B$ thus obtained, we calculate the energy separations in the excited bands. Taking $2^+$, 1475 KeV level as the band head for $K = 2$, $\gamma$-vibrational band, a $\gamma$-band is constructed. The calculated level scheme is shown in fig. 5.2(b) to be compared with the experimental level scheme shown in fig. 5.2(a). The agreement for the $3^+ 2 (1^+ \pi \pi K)$ level is very good. The $4^+ 2$ level is predicted at 2519 KeV. This may correspond to an experimentally observed level at 2426 KeV whose spin-parity assignment has yet to be determined. Thus in addition to the ground state $K = 0$ rotational band, a $K = 2$ $\gamma$-vibrational band is indicated for this nucleus.

Finally, it remains to be seen whether these nuclei showing rotational bands are just random cases in this region.
<table>
<thead>
<tr>
<th>KeV</th>
<th>$I^\pi$</th>
<th>KeV</th>
<th>$I^\pi$</th>
<th>K</th>
</tr>
</thead>
<tbody>
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<td>4^+</td>
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<td>777</td>
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</table>

Expt.        Predicted
(a)          (b)

Fig 5.2
or define some new regions of deformation. In this connection we refer to an observation made by us\textsuperscript{169} (to be discussed in the next chapter) while discussing certain regularities in the spectra of even even nuclei in 1d-2s shell. We had observed that there may not be a continuous region of deformation in light nuclei since 4n nuclei appear to possess rotational structure whereas (4n + 2) nuclei within the region are better explained on weak coupling models. It may happen that a similar situation exists here and we may find certain groups (rather than a continuous set) of deformed nuclei in medium weight region.