CHAPTER II: Nuclear Systematics in Deformed Even-Even Nuclei.

The regions of validity of different nuclear models have, generally, been marked out on the basis of certain regularities in spin sequence, level spacings, deformation parameter $\beta$, and $E2$ enhancement factor $F$, etc. We here re-examine some of these systematics of nuclear properties observed in the even-even nuclei for two regions of stable deformations (rare-earth and actinide regions). Such regularities allow us to revise certain definitions for the demarcation of these regions. More quantitative studies to this effect will be made in the next chapter. The necessity of such a study arises at the present time primarily because of the following considerations.

The regions of deformation have been given differently by different authors based on neutron number $N$, proton number $Z$ or the mass number $A$, and have been generally mentioned as $150 < A < 190$ and $A > 224$. In rare earth region a sudden transition from the spherical to the deformed region in going from $N = 88$ to $N = 90$ had been considered to be very striking feature of nuclear systematics in the medium weight nuclei, such that for $N > 90$ the nuclear spectra was believed to show very little deviations from that for the pure rotations, assuming that these deviations can be taken into account by considering only first order correction as in eq. (1.3). However, recent evidence shows that the deformation parameter exhibits a smooth behaviour in going from $N = 82$ to $N = 92$. Large deviations from the energy ratio (1.2) and hence higher order correction terms can be expected to be significant for nuclei in the transitional region. Furthermore, recently in the middle of this region some new isotopes, far removed from the line of $\beta$-stability (neutron
deficit) have been observed following the heavy-ion nuclear reactions\textsuperscript{109,125}. The ground state band has been shown to be populated up to 16\textsuperscript{+} spin on the average. In the actinide region, the question of whether a nucleus is spherical or deformed has been shown\textsuperscript{110} to be decided primarily by the proton number \(Z\); the change from spherical to deformed occurring between \(Z = 86\) and \(Z = 92\). In the following systematics we investigate some of these aspects.

Fig. 2.1(a) shows a plot of the deformation parameter \(\beta\) as a function of the mass number \(A\) in the region 50-250. Fig. 2.1(b) shows a similar plot for \(E2\) enhancement factor \(F\). The data has been taken from the various sources available and also calculated for some of the cases. A general reference can be made to a recent\textsuperscript{126} compilation of the reduced transition probabilities \(B(E2)\) and the half-life times of the first excited 2\textsuperscript{+} states for all the regions of periodic table. From the figure we see, as expected, the variation of \(\beta\) and \(F\) with \(A\) is quite similar. We have also plotted these quantities as functions of neutron number \(N\) and proton number \(Z\), and no distinct features are exhibited in these plots. In this figure we see that the deformation parameter \(\beta\) assumes comparable values in all the regions of periodic table in between the major shells — as such it does not provide us with a criterion for separating out the deformed and spherical regions. On the other hand, such a separation is quite striking in the variation of \(F\). Whereas for spherical nuclei the factor \(F\) does not exceed 70, nearly in all cases of deformed nuclei its value exceeds 100; the peaks in each case appearing in the regions farthest removed from the closed shells.
Fig. 2.1
We have not included light nuclei in the present survey and have also left out the nuclei in the immediate neighbourhood of major closed shells for the obvious reasons. However, for light nuclei the deformation parameter $\beta$ is very large; even more than double the value for a most deformed nuclei; whereas the enhancement factor is only of the order of 30 or so. For example, $^{127,128}$ Ne$^{20}$ has $\beta = 0.69$ and $F = 30$. The light nuclei will be further discussed in Chapter VI.

Fig. 2.2 shows a plot of the energy ratio $E_4/E_2$ as a function of the neutron number $N$. The recently$^{109,125}$ observed neutron deficit nuclei in the rare-earth region have been shown by cross mark (x); their energy ratio varying from 2.97 to 3.22. Again the separation between the spherical and non-spherical regions is distinct. An abrupt change is noticed in going from $N = 88$ to $N = 90$. The very sharp distinction between the two kinds of states at $N = 90$ is clearly demonstrated$^{129}$ by the decay of Eu$^{152}$. In this nucleus both the ground state and the isomeric state develop rotational levels in Sm$^{152}$ ($N = 90$) by $\beta^+$ emission with $E_4/E_2 = 3.0$ (rotational character) while for Gd$^{152}$ ($N = 88$), which is obtained by $\beta^-$ emission, $E_4/E_2 = 2.2$ (vibrational character). From the figure, the nuclei seem to divide themselves in three groups according to the order of correction expected to be necessary to explain the deviations from pure rotations. The nuclei with the energy ratio $E_4/E_2 > 3.25$ may be termed as strongly deformed whose spectra can be well explained in terms of the rotational model with only small deviations to be explained in terms of the rotation-vibration perturbation effects in the first order. The nuclei with ratio in the range 3.05 - 3.25 may be explained on the rotational model with the inclusion of large second order corrections. Then
there appears a set of nuclei — particularly those with $N = 90$ — for which the level sequence is apparently of the rotational type but their interpretation in terms of the usual BM-rotational model breaks down as the correction terms dominate. These so-called transitional nuclei for which presently no theory exists, will be discussed in the next chapter. The characteristic properties of $N = 90$ nuclei have also been pointed out in the studies of various other authors\textsuperscript{105,111,130,131}. Bjerregard et al.\textsuperscript{105} studied the variation of $^3_\text{rigid}$ with the deformation parameter $\beta$ and find that all the $N = 90$ nuclei fall definitely outside the normal trend. All the four $N = 90$ nuclei have almost the same $\beta \approx 0.28$ which seems to be the smallest deformation possible in this part of the periodic table. Theoretical calculations of B"es and Szymanski\textsuperscript{111} for the deformation parameters also show that near $N = 90$ the calculations show some Coulomb effects. Trainor\textsuperscript{130} has put forward a hypothesis that strong nuclear deformation in the region $N = 90$ is associated with strong neutron-proton correlation in the $1h$ shells. Barber et al.\textsuperscript{131} have shown that the mass effects associated with the nuclear deformations show a striking change in going from $N = 88$ to $N = 90$ which corresponds to the onset of nuclear deformation. However, such dislocations corresponding to the onset of collective motion extend for the range $N = 88-92$, after which essentially normal behaviour resumes; thus confining the mass effects only to the region $88 < N < 92$.

There remains an additional set of nuclei, the so-called neutron deficit nuclei, shown by crosses, for which the studies of the observed ground state rotational bands show that the BM-rotational model does not predict the energy levels with any
order of correction (Chapter III), although the energy ratio $E_a/E_2$ is even as large as 3.2 (not far from 3.33). The existence of such a situation in the middle of a region showing well developed rotational bands is very striking. This evidently points out that BM model is applicable only in the nuclei close to the line of $\beta$-stability and one has to define $N/Z$ ratio in addition to neutron-, proton- or mass-number, so as to define the regions of stable nuclear deformations. Nathan and Nilsson$^{87}$ have also observed that the neutron-rich or neutron-deficit nuclei far away from the bottom of the nuclear mass valley are expected to show new regions of deformations.* It may be relevant to note that certain experimental$^{132}$ and theoretical$^{133}$ studies at the Lawrence Radiation Laboratory have pointed out certain extended regions of deformations for relatively nucleon deficit nuclei in the regions where both proton- and neutron-numbers go from 50 to 82 and 82 to 126. In particular the even-even Ba-isotopes are found to have deformations quite comparable with nuclei in other regions of deformation. Deformations are also expected in certain other limited regions e.g. Fe isotopes other than Fe$^{54}$, As and Se and Cr isotopes. Studies to this effect are made further in Chapter V.

At the other end of the rare-earth region (fig. 2.2) we see that $A = 188$ (and not $N = 112$) is well within the deformed regions whose spectra are expected to be explained with the inclusion of second order corrections whereas Cs$^{190}$ has a spectrum similar to $N = 90$ nuclei. This points out that at this boundary $A$ is better parameter than $N$ or $Z$. Similarly for the actinide region we notice that the spectrum of $A = 224$ (and not $N = 136$) again corresponds to $N = 90$ nuclei. Since no Rn$^{224}$ and Th$^{224}$ isotopes

*This was noticed after the present studies have been already completed.
are observed, the present situation points out a clear dependence of the proton number \( Z \) along with mass number \( A \) for defining the transition from spherical to deformed nuclei in actinide region. The transition occurs in going from \( Z = 86 (\text{Rn}) \) to \( Z = 88 (\text{Ra}) \) with \( A = 224 \). Combining all the results we find that for the valid description of BM-rotational model the boundaries of the rare earth region are defined for \( N \geq 92 \) and \( A \leq 188 \) and that for actinide region \( Z \geq 88 \) and \( A \geq 226 \). Also defined are the nuclei with \( N = 90, \text{Os}^{190} \) and \( \text{Ra}^{224} \) in addition to the neutron-deficit nuclei (which points out the importance of \( N : Z \) ratio) where rotational model interpretation breaks, although their spectra are clearly of rotational type. These results are also clearly reflected for a plot of \( E_2/E_2^* \) vs. \( A \).

We have also plotted this ratio as a function of \( E_2^* \), the energy of the first excited \( 2^+ \) state, and find no points in the \( 2^+ \) energy range 210–310 KeV. This can also be taken as another criterion to separate the deformed- from the spherical-region.

Fig. 2.3(a) shows a plot of \( E_2^* \) vs. mass number \( A \) for rare-earth region. The dotted line shows the \((E_2^*)_{\text{crit.}}\) value. For the rotational description to be valid \( E_2 \) should be less than \((E_2^*)_{\text{crit.}}\) as is shown to be the case. This, however, also includes all the neutron-deficit nuclei for which the rotational model description is found to be invalid. Such a situation removes our faith in the applicability of this criterion. The approximate nature of the criterion is shown by the slope of the \((E_2^*)_{\text{crit.}}\) curve. For nuclei showing the rotational spectra including the various orders of corrections we notice from the figure that the \( E_2 \) value increases from 82 KeV (corresponds to \((E_2^*)_{\text{crit.}} \approx 199 \text{ KeV}\)) to 165 KeV (nearly equal
\[ E_{2+} \text{(in KeV)} \]
to \((E^*_2)_{\text{crit. value}}\) in going from \(\text{Sm}^{154}\) to \(\text{Os}^{192}\). This gives a slope reverse to that of the \((E^*_2)_{\text{crit.}}\) graph. For actinide region, however, the present experimental situation favours the validity of this criterion (shown in fig. 2.3(b)).

Finally in fig. 2.4(a) we plot the experimental energies for the various levels in the ground state rotational bands and the ratios of the energies of the excited states to the energy of the first excited \(2^+\) state for nuclei in transitional regions i.e. \(\text{Sm}^{146} = \text{Sm}^{154} (N = 82-92), \text{Gd}^{154} = \text{Gd}^{160} (N = 90-96)\) and \(\text{Dy}^{156} = \text{Dy}^{164} (N = 90-98)\). The choice of the transitional nuclei is made since we are dealing with the corrections to rotational spectra and marked deviations from the pure rotational pattern are expected in these cases. The dotted horizontal lines correspond to the energy ratios given by eq. (1.2) for pure \(\pi\) rotations. The plots for \(\text{Sm}\) show a sharp break in going from \(N = 88\) to \(90\) whereas that of \(\text{Gd}\) and \(\text{Dy}\) demonstrate that the onset of deformation is not an abrupt process; the energy ratios rise gradually and smoothly in going from \(N = 90\) to \(N = 98\) to meet the predicted values for pure rotations. This behaviour is very neatly reflected in the corrections needed to describe the spectra satisfactorily (Chapter III). Fig. 2.4(b) shows similar plots for \(\text{Os}\)-isotopes. Again smooth transition into the vibrational region is indicated. The systematics also suggest the spin-parity assignment \(6^+\) to the 1065 KeV level in \(\text{Os}^{192}\). Similar situation is reflected in the plots shown in fig. 2.4(c) for the actinide region.

Summarizing the results we find that \(E2\) enhancement factor over the single particle estimate (rather than \(E1\) the
Fig. 2.4(b)
Fig. 2.4 (c)
deformation parameter $\beta$ is a better criterion in demarcating the spherical and non-spherical regions. Further, there is quite an abrupt change in the nuclear properties in going from $N = 93$ to $90$; nuclei with $N = 90$ are deformed but not rotational in character in the usual sense; the transition to the pure rotational pattern is gradual and smooth. For the other boundaries we find that around $A \sim 190$, mass number is important and around $A \sim 224$ both proton- and mass-numbers are important parameters for defining the separation boundaries. In addition one has to define $N : Z$ ratio for defining the regions of deformation. This result in a way points out that rotational model interpretation is applicable only in the regions of approximately half-filled neutrons and protons shells i.e. nuclei not too far removed from the line of $\beta$-stability. Such a result, evidently, shows the possible invalidity of the $(E_2)^\text{crit.}$ criterion. We shall discuss some of these results more quantitatively in the next Chapter.