CHAPTER 3

MATHEMATICAL FORMULATION OF THE MULTI-OBJECTIVE OPTIMIZATION PROBLEM

The axial flow compressor stage performance is affected by physical phenomena like stalling. The influence of the physical phenomena on the performance of the axial flow compressor stage is studied by mathematical modeling of the physical phenomena as measurable performance parameters. The performance parameters are optimized to improve the axial flow compressor inlet stage performance. In this work, the compressor stage performance influencing parameters considered are stage efficiency, stall margin coefficient, inlet stage specific area and centrifugal stress. These parameters are optimized by formulating them as objective functions and constrained equations in terms of design variables. The design variables chosen are mean diameter of the stage, flow coefficient of air, shaft speed, air inlet angle to rotor and hub-tip radius ratio of the blade. To formulate the objective functions in terms of design variables, the compressor stage design relations and fluid flow correlations are used.

The following assumptions are made in the formulation of the performance influencing parameters of axial flow compressor inlet stage.
The axial velocity component of working fluid is assumed constant throughout the stage. i.e.; $C_{a1} = C_{a2} = C_{a3} = C_{a}$.

The absolute velocity at which the air enters the rotor is equal to the absolute velocity at which the air leaves the stator. i.e.; $C_1 = C_3$. This condition implies that the air angles at inlet to rotor and exit to stator are equal i.e.; $a_1 = a_3$.

The blades of the compressor are assumed symmetrical. Therefore, the blade angle at stator is equal to the air angle at entry to the rotor. Similarly the blade angle at rotor is equal to the air angle at entry to the stator. ($a_1 = \beta_2$ and $a_2 = \beta_1$).

The axial flow compressor stage is assumed as a 50% reaction stage. Degree of reaction is the ratio of rotor’s contribution of static enthalpy rise to the total static enthalpy rise in the stage. A 50% reaction stage ensures uniform diffusion of working fluid and uniform pressure distribution across stator and rotor.

It is assumed that the absolute velocity at which the air enters the rotor is equal to the relative velocity at which the air enters the stator and vice versa. i.e.; $V_1 = C_2$ and $V_2 = C_1$. 
3.1 FORMULATION OF OBJECTIVE FUNCTIONS AND CONSTRAINT

In the formulation of objective functions and constrained equations, the following input data and bounds to the design variables are considered.

3.1.1 Input to the Multi Objective Optimization problem

The input data to the present multi objective optimization problem is taken from A Massardo and A Satta [24].

- Mass flow rate (m) = 4 kg/s
- Inlet temperature to stage (T_{01}) = 300 k
- Exit temperature of stage (T_{03}) = 346 k
- Pressure ratio (P_{03}/P_{01}) = 1.65
- Axial velocity (C_a) = 150 m/s
- Air density (\rho) = 1.165 kg/m^3
- Tip diameter (D_t) = 0.27 m
- Work done or blockage factor (\lambda) = 0.98
- Blade thickness to chord ratio (t/c) = 0.1
- Chord length to mean diameter ratio (c/D) = 0.45
- Specific heat at constant pressure (C_p) = 1.005 KJ/kg k
- Process index for air (\gamma) = 1.4
- Acceleration due to gravity (g) = 9.8 m/s^2
- Mechanical equivalent of heat (J)=4.186 K cal
• Blade material density \( (\rho_b) = 7700 \text{ Kg/m}^3 \)

• Ultimate tensile strength of blade \( (\sigma_u) = 1041 \text{ Mpa} \)
  
  \(4340 \text{ hot rolled carbon alloy steel}\)

• Modulus of Elasticity of blade Material \( (E) \) = 910 Mpa

3.1.2 Lower and Upper bounds to the Design variables

The lower and upper bounds to the design variables i.e.; mean diameter \( (D) \), shaft Speed \( (N) \), flow coefficient \( (\phi) \), air inlet angle to rotor \( (\alpha_1) \), super script “c” denotes angle in radians and hub-tip radius ratio \( (r_r/r_l) \) of the compressor blade are as follows:

- \( 0.3 \leq D \leq 0.4 \)
- \( 0.2 \leq \phi \leq 0.6 \)
- \( 0^c \leq \alpha_1 \leq (\Pi/9)^c \)
- \( 350 \leq N \leq 500 \)
- \( 0.4 \leq (r_r/r_l) \leq 0.9 \)

3.1.3 Formulation of Stage Efficiency as objective function

The stage isentropic efficiency is defined as the ratio of isentropic work output to the actual work absorbed by the compressor rotor. The actual work absorbed by the rotor of the compressor is nothing but the transformation of kinetic energy of working fluid in the rotor to static pressure rise in the stator. The transformation of kinetic energy of
working fluid into static pressure rise results in temperature rise in the stage from $T_{01}$ to $T_{02}$. Therefore, the actual work absorbed by the rotor of the compressor in terms of temperature rise in the stage is:

$$W = m c_p (T_{02} - T_{01}) \quad (3.1)$$

Where “$m$” is the mass flow rate of working fluid, “[subscript](3)cp” is the specific heat at constant pressure. $T_{01}$ and $T_{02}$ are the rotor and stator inlet temperatures. Since no work is absorbed in the stator, there will not be any significant rise in stage temperature at the stator. i.e.; $T_{02} = T_{03}$. Therefore, Eq.3.1 is written as:

$$W = m c_p (T_{03} - T_{01}) \quad (3.2)$$

The ideal isentropic work output from the compressor stage is measured as the isentropic temperature rise in the stage i.e. from $T_{01}$ to $T_{03}$. The stage isentropic temperature rise is shown in Fig.3.1. The isentropic work output in terms of isentropic temperature rise in the stage is:

$$W^i = m c_p (T^i_{03} - T_{01}) \quad (3.3)$$
Fig 3.1: Temperature-Entropy diagram for axial flow compressor stage

From Eq.3.2 and 3.3, the stage isentropic efficiency is:

\[ \eta = \frac{mc_p(T_{03} - T_{01})}{mc_p(T_{03} - T_{01})} = \frac{T_{03} - T_{01}}{T_{03} - T_{01}} \]

\[ = \frac{T_{03} \left( \frac{T_{03}}{T_{01}} - 1 \right)}{(T_{03} - T_{01})} \]

(3.4)

The isentropic temperature ratio \( \frac{T_{03}}{T_{01}} \) of Eq. 3.4 is expressed in
terms of stage pressure ratio by applying the polytropic relations.

The polytropic relation for pressure and volume for an ideal gas existing at states 1 and 2 i.e. before and after expansion is:

\[ P_1 V_1^n = P_2 V_2^n \]  \( (3.5) \)

The ideal gas equation relating pressure, volume and absolute temperature for states 1 and 2 (Before and after expansion) in a polytropic process is:

\[ P_1 V_1 = MRT_1 \& P_2 V_2 = MRT_2 \text{ or } \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \]  \( (3.6) \)

From Eq. 3.5 and 3.6, the relation between pressure and temperature for ideal gas is:

\[ \frac{T_2}{T_1} = \left( \frac{P_2 V_1^{(1/n)}}{P_1 V_2^{(1/n)}} \right) = \left( \frac{P_2^n}{P_1^n} \right)^{(n-1)/(n-1)} = \left( \frac{P_2}{P_1} \right)^{(n-1)/n} \]  \( (3.7) \)

Following the polytropic process, the isentropic temperature ratio between states 01 and 03 of axial flow compressor stage is expressed as:

\[ \left( \frac{T_{03}}{T_{01}} \right) = \left( \frac{P_{03}}{P_{01}} \right)^{(y-1)/\gamma} \]  \( (3.8) \)
Therefore, from Eq. 3.4 and 3.8 the stage efficiency is expressed as:

$$\eta = \frac{T_{01}}{T_{o3} - T_{01}} \left( \frac{P_{03}}{P_{01}} \right)^{\frac{y-1}{y}} - 1$$

(3.9)

The actual work input absorbed by the compressor can also be expressed in terms of the whirl component of velocity of air. As the air passes from rotor to stator, there will be change in angular momentum of air due to whirling. Hence, the work input absorbed by the compressor from Eq. 3.2 is written as:

$$W = mU(C_{w2} - C_{w1})$$

(3.10)

Where $U$ is the peripheral velocity of working fluid, $C_{w2}$ and $C_{w1}$ are whirl component of velocities at stator inlet and rotor inlet respectively. “m” is the mass flow rate of working fluid. The peripheral velocity $U$ is expressed in terms of axial velocity, whirl component of velocity, blade angles and air angles. The stage velocity triangles of Fig. 3.2 are used to express peripheral velocity in terms of axial velocity, whirl component of velocity, blade angles and air angles.
The peripheral velocity of working fluid is expressed as:

\[ U = C_{w1} + C_a \tan \alpha_1 = C_a \tan \alpha_1 + C_a \tan \beta_1 = C_a \tan \alpha_2 + C_a \tan \beta_2 \]  

(3.11)

Since, \((C_{a1}=C_{a2}=C_a)\) Eq.3.11 is expressed as:

\[ \left( \frac{U}{C_a} \right) = \tan \alpha_1 + \tan \beta_1 = \tan \alpha_2 + \tan \beta_2 \]  

(3.12)

Where \(C_{w1}=C_a \tan \alpha_1\) and \(C_{w2}=C_a \tan \alpha_2\)

Therefore, the work input absorbed by the compressor from Eq.3.10 is:
\[
W = mU(C_a\tan \alpha_2 - C_a\tan \alpha_1) \quad (3.13)
\]

From Eq. 3.2 and 3.13, the actual work absorbed by the compressor in terms of stage temperature rise is:

\[
(T_{03} - T_{01}) = \left(\frac{UC_a}{C_p}\right)(\tan \alpha_2 - \tan \alpha_1) \quad (3.14)
\]

The work input absorption capacity of the compressor stage is affected by the formation of boundary layer around the annulus of the compressor. The boundary layer thickens as the flow progresses. This reduces the area available for flow of air. This phenomenon is called blockage. The reduction in work absorption capacity of the stage due to blockage is accounted by a factor called the work done factor or blockage factor denoted by \(\lambda\). The value of work done factor is less than unity and decreases as the number of stages increases. The variation of work done factor or blockage factor against a given no of stages is shown in Fig.3.3.

![Fig. 3.3: Variation of Work done factor against number of Stages](image-url)
The work done or blockage factor $\lambda$, considered for the inlet stage is 0.98.

Considering the effect of blockage, the reduction in the actual work absorption capacity from Eq.3.14 is:

$$(T_{03} - T_{01}) = \left( \frac{\lambda UC_a}{C_p} \right) (\tan \alpha_2 - \tan \alpha_1) \quad (3.15)$$

From Eq.3.9 and 3.15 the inlet stage efficiency is written as:

$$\eta = \frac{T_{01} \left( \frac{P_{03}}{P_{01}} \right)^{\frac{T-1}{T}} - 1}{\left( \frac{\lambda UC_a}{C_p} \right) (\tan \alpha_2 - \tan \alpha_1)} \quad (3.16)$$

The air angle at inlet to stator ($\alpha_2$) shown in Eq. 3.16 is expressed in terms of axial and swirl component velocities of working fluid. The stage velocity triangles of Fig.3.2 are used to express the air inlet angle to stator in terms of axial and swirl component velocities of working fluid.

$$\tan \alpha_2 = \left( \frac{C_{a2}}{C_{a2}} \right) = \left( \frac{U - C_{a2} \tan \beta_2}{C_{a2}} \right) = \left( \frac{U}{C_{a2}} \right) - \tan \beta_2 \quad (3.17)$$

Since, $(C_{a2} = C_{a1} = C_a ; \alpha_2 = \beta_1$ and $\alpha_1 = \beta_2)$ the Eq. 3.17 is expressed as:

$$\tan \alpha_2 = \left( \frac{U}{C_a} \right) - \tan \alpha_1 \quad (3.18)$$

Therefore, from Eq.3.16 and 3.18 the stage efficiency is:
An important parameter which affects the stage efficiency is the flow coefficient. When the compressor stage operates within the design limits of flow coefficient, high value of stage efficiency can be achieved. As flow coefficient reduces, efficiency decreases. The flow coefficient is expressed as:

\[ \phi = \frac{C_a}{U} \text{ or } \phi = \frac{C_a}{\pi DN} \]  

(3.20)

Where \( C_a \) is the axial velocity of flow, \( U \) is the peripheral velocity, \( D \) is the stage diameter and \( N \) is shaft speed.

Therefore from Eq. 3.19 and 3.20 the stage efficiency is expressed as:

\[
\eta = \left( \frac{T_{01}}{1 - \left( \frac{P_{03}}{P_{01}} \right)^{\frac{\gamma-1}{\gamma}} - 1} \right) \left( \frac{\lambda U C_a \left\{ \left( \frac{U}{C_a} \right) - 2 \tan \alpha_1 \right\}}{C_p \left\{ \left( \frac{1}{\phi} \right) - 2 \tan \alpha_1 \right\}} \right)
\]  

(3.21)

Eq.3.21 represents the expression for the objective function stage efficiency in terms of design variables \( D \), \( N \), \( \phi \) and \( \alpha_1 \).
3.1.4 Formulation of Stall Margin Coefficient as objective function

The stall phenomenon in axial flow compressor stage leads to flow reversal and drop in power at the turbine. This situation becomes disastrous in air crafts as the necessary power required for lift is hindered. To optimize the compressor stage against the phenomenon of stalling and to develop a safe design, the physical phenomenon is mathematically modeled.

C C Koch (13), developed a semi empirical model for static enthalpy equivalent pressure rise coefficient or stall margin coefficient. The following equation represents the semi empirical stall margin coefficient expression.

\[
C_h = \left[ J \frac{C_{p_t}}{P_1} \left( \frac{P_3}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] \frac{1}{\text{stage}} - \frac{\left( U_2^2 - U_1^2 \right)_{\text{rotor}}}{2g}
\]

\[
C_h = \left( \frac{V_{\text{rotor}}^2 + V_{\text{stator}}^2}{2g} \right)
\]

The numerator of Eq (3.22) represents the difference between isentropic enthalpy rise due to static pressure rise in the stage and change in enthalpy across the pitch line radius of the rotor. The denominator represents the sum of rotor and stator relative kinetic energies.
In this work, the compressor stage is assumed to be a fifty percent reaction stage. Therefore, the static enthalpy change along the pitch line from stator to rotor is negligible due to uniform pressure distribution. i.e.; $U_2 = U_1$. Therefore Eq.3.22 can be expressed as:

$$
C_h = \frac{2gJC_{p1} t_1 \left( \frac{P_3}{P_1} \right)^{\frac{y-1}{y}} - 1}{(V_{1\text{rotor}}^2 + V_{1\text{stator}}^2)} \text{stage}
$$

(3.23)

The stage inlet temperature $t_1$ of Eq.3.23 is expressed in terms of stage absolute inlet temperature $T_{01}$. The relation between stage inlet temperature and stage absolute inlet temperature in terms of inlet absolute velocity of working fluid ($C_1$) and specific heat ($C_p$) is:

$$
t_1 = T_{01} - \left( \frac{C_1^2}{2C_p} \right)
$$

(3.24)

The Eq.3.24 relating stage inlet temperature in terms of absolute temperature is shown in Fig.3.1.

From the stage velocity triangles Fig.3.2, the absolute velocity of working fluid at inlet to the rotor of the compressor is expressed as:

$$
C_1 = C_{a1} \sec \alpha_1
$$

(3.25)

Therefore from Eq.3.24 and 3.25
\[
\begin{align*}
t_1 &= T_{01} - \left( \frac{C_{a_1} \sec^2 \alpha_1}{2C_p} \right) \\

Similarly \quad
t_2 &= T_{02} - \left( \frac{C_{a_2} \sec^2 \alpha_2}{2C_p} \right) \\

t_3 &= T_{03} - \left( \frac{C_{a_3} \sec^2 \alpha_3}{2C_p} \right)
\end{align*}
\]

Applying polytropic relations between pressure and temperature for the states 1 and 3 of axial flow compressor stage shown in Fig.3.1.

\[
\left( \frac{P_3}{P_1} \right) = \left( \frac{t_3}{t_1} \right)^{\frac{\gamma}{\gamma-1}}
\]

From Eq.3.29, 3.28, 3.26 and 3.23, the stall margin coefficient is expressed as:

\[
C_h = \left[ \frac{2gJC_p \left( T_{01} - \left( \frac{C_{a_1} \sec^2 \alpha_1}{2C_p} \right) \right) \left( T_{03} - \left( \frac{C_{a_3} \sec^2 \alpha_3}{2C_p} \right) \right) - 1}{\left( V_{\text{rotor}}^2 + V_{\text{stator}}^2 \right)} \right]
\]

\[
= \left[ \frac{2JgC_p \{(T_{03} - T_{01})\}}{\left( V_{\text{rotor}}^2 + V_{\text{stator}}^2 \right)} \right]^{\frac{1}{2}}
\]

From Eq.3.15 the stage temperatures rise:
From Eq. 3.18 the air angle at inlet to stator is:

$$\tan \alpha_2 = \left( \frac{U}{C_a} \right) - \tan \alpha_1$$

Therefore, from Eq. 3.18, 3.15 and 3.30, the stall margin coefficient is:

$$C_h = \left[ \left( 2Jg \lambda UC_a \right) \left( \frac{U}{C_a} - 2 \tan \alpha_1 \right) \right]$$

$$(V_{1rotor}^2 + V_{1stator}^2)$$

(3.31)

The denominator of Eq. 3.31 is the sum of stator and rotor relative kinetic energies. This can be expressed in terms of absolute and axial velocity components of working fluid.

From the assumptions, the absolute velocity of air at entry to rotor is equal to the relative velocity at which it enters the stator and vice versa. i.e.; $C_1 = V_2$ and $C_2 = V_1$. Also the relative velocity with respect to a relative frame of reference is equal to the relative velocity of air at entry to the rotor. i.e.; $V_1^1 = V_1$.

Applying the Pythagoras relation on the velocity triangle (Fig. 3.2) at the stator end:

$$V_1^2 + C_2^2 = C_a^2 + (U - C_{w1})^2 + C_a^2 \sec^2 \alpha_2$$

$$= C_a^2 + (U - C_{a1}\tan \alpha_1)^2 + C_{a2}^2 \left( 1 + \tan^2 \alpha_2 \right)$$
\[ C_h = \frac{\left\{ (2Jg\alpha UC_a) \left( \frac{U}{C_a} - 2\Tan\alpha_1 \right) \right\}}{2 \left\{ C_a^2 + (U - C_a\Tan\alpha_1)^2 \right\}} \]

\[ = \frac{\left\{ (Jg\alpha\pi DNC_a) \left( \frac{D}{C_a} - 2\Tan\alpha_1 \right) \right\}}{\left\{ C_a^2 + ((\pi D) - C_a\Tan\alpha_1)^2 \right\}} \] (3.33)

The peripheral velocity \( U \) is expressed as \( U = \pi DN \) where \( D \) is the Mean diameter and \( N \) is the shaft speed. Therefore, Eq.3.33 is expressed as:

\[ C_h = \frac{\left\{ (Jg\alpha\pi DNC_a) \left( \frac{\pi D}{C_a} - 2\Tan\alpha_1 \right) \right\}}{\left\{ C_a^2 + ((\pi D) - C_a\Tan\alpha_1)^2 \right\}} \] (3.34)

The Eq.3.34 represents the expression for stall margin coefficient in terms of the design variables diameter (D), shaft speed (N) and air angle...
at inlet to the rotor \((a_1)\), which is one of the objective functions to the multi objective optimization problem of axial flow compressor inlet stage.

### 3.1.5 Formulation of Inlet Stage Specific Area as objective function

Specific area in reference to the axial flow compressor is defined as the area available for the given mass flow rate of working fluid per unit length in the axial direction. Axial flow compressors are known to absorb high mass flow rate of working fluid, which results in development of high pressure ratios. The increase in specific area increases the mass flow rate and pressure ratios. Therefore the objective of the present work is to increase the inlet stage specific area. The specific area is mathematically expressed as:

\[
\text{Specific Area } A_s = \left( \frac{m}{A} \right) \tag{3.35}
\]

Where \(m\) is the mass flow rate and \(A\) is the net area available for flow of working fluid. The mathematical expression for inlet stage specific area is formulated based on the following assumptions.

- The cross sectional area of the blades remains constant through out the length of the blades.
- The diameter chosen represents the mean diameter of the stage.
• The blades selected are NACA-65 series blades with thickness to chord length ratio of the blade \( t/c \) = 0.1 and the chord length to mean diameter ratio \( c/D \) = 0.45.

• The number of blades per stage \( n = 65 \) and \( h \) represents the height of the blade.

The elemental mass flow rate of air for the given elemental frontal area is:

\[
\delta_m = 2\pi r p C_a \, dr
\]

(3.36)

Where \( r \) is any arbitrary radius from blade root to blade tip, \( C_a \) is the axial velocity component of working fluid and \( p \) is the working fluid density. The total mass flow rate is given by integrating the Eq.3.36 over the mean radius which varies from root of the blade to hub of the blade.

Therefore,

\[
m = 2 \pi p C_a \int_{r_i}^{r_t} r \, dr
\]

\[
= 2 \pi p C_a \left( \frac{r^2}{2} \right)_{r_i}^{r_t}
\]

\[
= 2 \pi p C_a \left( \frac{r_t^2 - r_i^2}{2} \right)
\]

Therefore, the total mass flow rate of working fluid is expressed as:

\[
m = \pi r_i^2 C_a \left( 1 - \left( \frac{r_f}{r_i} \right)^2 \right)
\]

(3.37)
The net area available for the mass flow rate is given by the difference between the total frontal area of the compressor and the area occupied by the blades of the compressor. The area occupied by the blades of the compressor is:

Area occupied by blades $A_2 = nth$ \hspace{1cm} (3.38)

Where $n$ is the number of blades, $t$ is the thickness of each blade and $h$ is the mean height of each blade.

The mean frontal area of the compressor $A_1 = \Pi Dh$ \hspace{1cm} (3.39)

Where $D$ is the mean diameter of the stage and $h$ is the mean height of the blade.

Therefore, from Eq.3.38 and 3.39 the net area available for mass flow of working fluid is:

The net area $A = (\Pi Dh - nth)$ \hspace{1cm} (3.40)

From Eq.3.38 and 3.40 Inlet stage Specific Area is:

$$A_s = \frac{\pi r_t^2 C_a \left\{1 - \left(\frac{r_c}{r_t}\right)^2\right\}}{\left(\Pi Dh - nth\right)}$$

$$= \frac{\pi r_t^2 C_a \left\{1 - \left(\frac{r_c}{r_t}\right)^2\right\}}{D \left(\Pi h - n \left(\frac{t}{D}\right) h\right)}$$ \hspace{1cm} (3.41)
The inlet stage specific area shown in Eq. 3.41 in terms of thickness to chord ratio \((t/c)\) and chord to mean diameter ratio \((c/D)\) is expressed as:

\[
As = \frac{\pi \rho_r^2 C_a \left\{1 - \left(\frac{r_t}{T_r}\right)^2\right\}}{Dh \left(\pi - \frac{t}{c} \left(\frac{c}{D}\right)\right)}
\]

(3.42)

The mean height of the blade \(h\) is expressed as the difference between mean and tip diameters of the stage. Therefore Eq.3.42 is expressed as:

\[
As = \frac{\pi \rho_r^2 C_a \left\{1 - \left(\frac{r_t}{T_r}\right)^2\right\}}{D(D_t-D) \left(\pi - \frac{t}{c} \left(\frac{c}{D}\right)\right)}
\]

(3.43)

Eq.3.43 represents the expression for inlet stage specific area in terms of the design variables hub tip radius ratio of the blade \((r_t/r_h)\) and mean diameter of the stage \(D\), which is one of the objective functions to the multi objective optimization problem of axial flow compressor inlet stage.

### 3.1.6 Formulation of centrifugal stresses as constraint:

The mass flow rate of air increases with the rotational speed of shaft of the rotor. At high rotational speeds of the shaft, the tip speed of the blade increases. This induces centrifugal stresses inside the blade. Hence the
blade material should have properties such as high fatigue strength, high centrifugal stress limit, high bending tensile strength and high temperature, corrosion resistance.

The centrifugal stress acting on the blade surface depends upon the tip diameter, hub-tip radius ratio of the blade, material of the blade and peripheral speed of the shaft of the rotor. Therefore the mathematical expression for the centrifugal stress is modeled as a function of blade material density, blade area, hub-tip radius ratio of the blade and shaft rotational speed as:

\[
\sigma = \left( \frac{\rho_b \omega^2}{a_r} \right) \int_0^1 a_r r dr
\]  (3.44)

Where \( a_r \) is the cross sectional area of the blade at any radius from root to tip of the blade. From the assumptions, the blade cross section is assumed constant from root to tip. Therefore Eq.3.44 is expressed as:

\[
\sigma = \frac{\rho_b}{2} \left( 2\pi N \right)^2 \left( r_i^2 - r_r^2 \right)
\]  

\[
\sigma = \frac{\rho_b}{2} \left( 2\pi N \right)^2 r_i^2 \left[ 1 - \left( \frac{r_r}{r_i} \right)^2 \right]
\]  

\[
\sigma = \frac{\rho_b}{2} \left( \pi D_i N \right)^2 \left[ 1 - \left( \frac{r_r}{r_i} \right)^2 \right]
\]  (3.45)

The centrifugal stress \( \sigma \) in the compressor blade must be within the blade material strength \( \sigma_b \). Therefore Eq. 3.45 is expressed as:
Eq. 3.46 represents the expression for centrifugal stress in terms of the design variables i.e.; hub-tip radius ratio of the blade \((r_r/r_t)\) and rotational speed of the shaft \((N)\).

\[
\sigma = \left( \frac{P_b}{2} \right) (\pi D_t N)^2 \left[ 1 - \left( \frac{r_r}{r_t} \right)^2 \right] \leq \sigma_h
\]  

(3.46)

### 3.1.7 Representation of Multi objective optimization problem

The multi objective optimization problem consisting of the objective functions stage efficiency, stall margin coefficient and inlet stage specific area subject to the centrifugal stress constraint is represented as:

\[ F = \max f( A_s , C_h , \eta) \]

Where \(A_s\) = Objective function (Inlet stage Specific Area)

\(C_h\) = Objective function (Stall Margin Coefficient)

\(\eta\) = Objective function (Stage Efficiency)

For \(x= (D, a_1, N, (r_r/r_t) and \theta)^T\) (Design vector)

Such that \(D^L \leq D \leq D^U\)

\(a_1^L \leq a_1 \leq a_1^U\)

\(N^L \leq N \leq N^U\)  \((L and U are lower and upper bounds)\)
\[ \Phi^L \leq \Phi \leq \Phi^U \]
\[ (r_r/r_t)^L \leq (r_r/r_t) \leq (r_r/r_t)^U \]

Where the design variables:

- \( D \): Mean diameter of Stage
- \( \alpha_1 \): Air angle at inlet to rotor
- \( N \): Shaft rotational speed
- \( \Phi \): Flow coefficient of air
- \( (r_r/r_t) \): Hub-Tip radius ratio of the blade.

Subject to \( \sigma \leq \sigma_b \) (Constraint)

Where \( \sigma \): Centrifugal Stress in the blade at high rotational speeds.

\( \sigma_b \): Blade material strength limit.