Subspace analysis is one of popular multivariate data analysis methods widely used in pattern recognition. PCA, ICA, and LDA are well-known approaches of biometric recognition that use feature subspaces. These methods find a mapping between the original feature spaces to a lower dimensional feature space. In this chapter we have implemented two traditional subspace analysis methods namely PCA, and LDA or FLD to extract various features from imprint images used for personnel identification and system authentication. A brief review of all these is covered in the subsequent sections.

5.1 Principal Component Analysis

Principle Component Analysis (PCA) is also called as the “Hotteling Transform.” The main use of PCA is to reduce the dimensionality of the data set while retaining as much information as is possible. It computed a compact and optimal description of the facial data set. PCA is a statistical procedure which rotates the data such that maximum variability is projected onto the axes (see Figure 5.1).

Essentially, a set of correlated variables are transformed into a set of uncorrelated variables which are ordered by reducing variability. Un correlated variables are linear combinations of the original variables and the last of these variables can be removed with minimum loss of real data. Hence, PCA has been defined as, finding an orthogonal coordinate system such that the correlation between different axes is minimized.
An image may be viewed as a vector of pixels where the value of each entry in the vector is the grayscale value of the corresponding pixel. For example, an $8 \times 8$ image may be unwrapped and treated as a vector of length 64. Image is said to sit in $N$-dimensional space, where $N$ is the number of pixels (and length of the vector). This vector representation of the image is considered to be the original space of the image. The original space of an image is just one of infinitely many spaces in which the image can be examined. The subspace created by the eigenvectors of the covariance matrix of the training data and the basis vectors calculated by Fisher discriminant are two specific subspaces. The majority of subspaces, including Eigen space, do not optimize discrimination characteristics. Variance among the images is optimized using Eigen space.

There is a basic algorithm for identifying images by projecting them into a subspace. The first one selects a subspace on which to project the images. After selecting this subspace, all training images are projected into this subspace. Each test image is then projected into this subspace. Each test image is compared to all the training images by a similarity or distance measure, the training image found to be most similar or closest to the test image is used to identify the test image.

Projecting images into subspaces has been studied for many years as discussed in the previous section. The research into these subspaces has helped to revolutionize image recognition algorithms. When studying these subspaces an interesting question arises: under what conditions does projecting an image into a subspace improve performance. Answer to this question is not an easy one. What specific subspace (if any at all) improves performance depends on the specific problem. Furthermore,
variations within the subspace also affect performance. The selection of vectors to create the subspace and measures to decide which images are a closest match both affect the performance. Basically PCA projects images into a subspace such that the first orthogonal dimension of this subspace captures the greatest amount of variance among the images and the last dimension of this subspace captures the least amount of variance among the images. A similarity measure is used to decide which images are closest matches, once images are projected into one of these spaces. PCA is a standard de-correlation technique and following its application one derives an orthogonal projection basis that directly leads to dimensionality reduction, and possibly to feature selection. Principal components (PCs) are a set of orthonormal basis vectors generated by PCA that maximize the scatter of all the projected samples. If we draw the magnitude plot of Eigen values after sorting the Eigen values in decreasing order, then it is observed that the first few leading eigenvectors define the matrix. One can see from Figure 5.2 that the first 15 Eigen values capture most of the energy and that the Eigen values whose index is greater than 30 are fairly small and most likely capture noise.

Figure 5.2 Magnitude plot of Eigen values in descending order.
PCA (or Karhunen-Loeve expansion) identifies variability between human palmprints. PCA does not attempt to categorize palmprints using familiar geometrical differences instead a set of human palmprints is analyzed using PCA to determine which 'variables' account for the variance of palmprints. In palmprint recognition, these variables are called eigen palms. Any grey scale palm image \( I(x, y) \), is a two dimensional \( N \times N \) array of intensity values (usually 8 bit gray scale) that may be considered a vector of dimension \( N^2 \) so that an image of size \( 256 \times 256 \) becomes a vector of dimension \( 65,536 \) or a point in \( 65,536 \)-dimensional space. The ensemble of images then maps to a collection of points in this huge space. Central idea is to find a small set of palmprints (the eigenpalms) that can approximately represent any point in the palm space as a linear combination. Each of the eigenpalms is of dimension \( N \times N \), and can be interpreted as an image [126].

The eigen space is a subspace of the image space spanned up by a set of eigenvectors of the covariance matrix of the trained images. These eigenvectors are also called eigenpalms because of their palm-like appearance. The covariance matrix is constructed by performing PCA which means rotating the dataset so that its primary axes, the eigenvectors with the highest modes of variation, lie along the axes of the coordinate space and move it so that its centre of mass corresponds with the origin (see Figure 5.3).

![PCA Diagram](image)

Figure 5.3 The covariance matrix construction using PCA.
5.2 Linear Discriminant Analysis (LDA)

Fisher’s Linear Discriminant (FLD) known as Linear Discriminant Analysis (LDA) is a popular discriminant criterion that measures the between-class scatter normalized by the within-class scatter.

Let \( \omega_1, \omega_2, \ldots, \omega_L \) denote the number of classes and \( N_1, N_2, \ldots, N_L \) are the number of images within each class. Let \( M_1, M_2, \ldots, M_L \) are the means of the classes and \( M \) be the grand mean. The within-class and between-class scatter matrices \( \sum \omega \) and \( \sum b \) are defined as,

\[
\sum \omega = \sum_{i=1}^{L} p(\omega_i) \varepsilon \{ (Y^{(\omega_i)} - M_i)(Y^{(\omega_i)} - M_i)' \} \\
\sum b = \sum_{i=1}^{L} p(\omega_i) \varepsilon (M_i - M)(M_i - M)'
\]

(5.1)

Where,

\( p(\omega_i) \) is a priori probability,

\( \sum \omega, \sum b \in mR \), and

\( L \) denotes the number of classes.

FLD derives projection matrix that maximizes the ratio \( |P' \sum bP|/|P' \sum \omega P| \). This ratio is maximized when \( P \) consists of the eigenvectors of the covariance matrix \( A \) i.e.

\[
\sum^{-1} \omega \sum b \psi = \psi \Delta
\]

(5.2)

where,

\( \psi, \Delta \in R^{m \times m} \) are Eigen vector and Eigen value matrices of \( \sum^{-1} \omega \sum b \) respectively.
FLD known as LDA finds a small number of features that differentiates individual palms but recognizes palms of the same individual. Small number of features are found by maximizing the Fisher Discriminant Criterion [127] which is achieved by maximizing the grouping of individual palms whilst minimizing the grouping of different individual palms. Hence, by grouping palms of the same individual these features can be used to determine the identity of individuals. The between scatter class $S_B$ and within scatter class $S_W$ define LDA. The between scatter class $S_B$ are palms of different individuals while the within scatter class $S_W$ are palms of the same individuals. The between scatter class $S_B$ specifically represents the scatter of features around the mean of each palm class whilst the within scatter class $S_W$ represents the scatter of features around the overall mean for all palm classes. Figure 5.4 is a comparison of PCA and FLD for a two-class problem in which the samples from each class are randomly perturbed in a direction perpendicular to a linear subspace. Both

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**Figure 5.4** A comparison of PCA and FLD for a two class problem.
the PCA and FLD reduce the dimension by projecting the points from 2-D to 1-D. The comparison of the projections in the Figure indicates PCA smears the class together so that the samples in the projected space are no longer linearly separable. FLD achieves greater between-class scatter compared to PCA that achieves larger total scatter and thus the classification is simplified. The number of classes, the number of samples and the original space dimensionality strongly decide LDA transformation.

5.3 Feature Extraction using Subspace analysis Techniques

Let a palm image \(I(x, y)\) be a two dimensional array of intensity values, or a vector of dimension \(N\). Let the training set of images be \(I_1, I_2, \ldots, I_N\). The average palm image of the set is defined by \(m = \frac{1}{N} \sum_{i=1}^{N} I_i\). Each palm differs from the average by the vector \(\vec{I}_i = I_i - m\). This set of very large vectors is subject to principal component analysis which seeks a set of \(K\) orthonormal vectors \(v_k\) for \(k = 1, \ldots, K\) and their associated Eigen values \(\lambda_k\) which best describe the distribution of data.

![Eigen vector images](image)

Figure 5.5 Different Eigen vector images of palmprints.

The covariance matrix \(C\) is computed as,
\[ C = \frac{1}{N} \sum_{i=1}^{N} \bar{I}_i \bar{I}_i^T = AA^T \]  \hspace{1cm} (5.3)

where,

the matrix \( A = [\bar{I}_1, \bar{I}_2, ..., \bar{I}_t] \).

Eigen values \( \lambda_k \) and related eigenvectors \( v_k \) are computed for the covariance matrix.

\[ C v_k = \lambda_k v_k \]  \hspace{1cm} (5.4)

The space spanned by the eigenvectors \( v_k, k = 1, ..., K \) corresponding to the largest \( K \) eigenvalues of the covariance matrix \( C \), is called the palm space. The eigenvectors of matrix \( C \) which are called eigenpalms form a basis set for the palm images. A new palm image \( G \) is transformed into its eigenpalm components (projected onto the palm space) by,

\[ w_k = \langle v_k, (G - \bar{I}_t) \rangle = v_k^T (G - \bar{I}_t) \]  \hspace{1cm} (5.5)

for \( k = 1, ..., K \).

The projections \( w_k \) form the feature vector \( w = [w_1, w_2, ..., w_k] \) that describes the contribution of each of each eigenpalm in representing the input image. Figure 5.5 shows different Eigen vectors of palms as independent vectors. Projection data with low dimensionality (see Figure 5.6) and different Eigenpalms (see Figure 5.7) from each vector has been obtained using all the vector matrices.

Figure 5.6 Mean palm image.
Feature Extraction of Handbased Biometric using Subspace Analysis

We have implemented PCA by selecting the $k$ eigenvectors with the largest Eigen values as the basis i.e. we have selected the dimensions which can express the greatest variance in the palmprint images. It is found that using this co-ordinate system, a palm can be reasonably reconstructed with as few as 6 co-ordinates and hence selected only six Eigen values for our experimental work. It means that a $128 \times 128$ pixel palm, which previously took 16,384 bytes to represent in image space will require only 6 bytes. Figure 5.7 shows some examples of the eigenpalms sorted with respect to decreasing Eigen values. This reduction in dimensionality makes the problem of palm recognition much simpler since we concern ourselves only with the relevant and most discriminatory attributes of the palmprint. Similar experimentations were carried out for fingerprint and FKP images. Figure 5.8 and Figure 5.9 shows eigenfingers and eigenFKPs sorted with respect to decreasing Eigen values.
Determining alike images is the next task, once images are projected into a subspace [128]. There are two ways in general to determine how alike images are. One is to measure the distance between the images in $N$-dimensional space. The second way is to measure the distance between the images in $N$-dimensional space. Second way is to measure how similar two images are. The similarity measurement requires maximization of similarity such that two like images produce a high similarity value. $L_1$ norm, $L_2$ norm, covariance, Mahalanobis distance and correlation are some of the possible similarity and distance measures.

### 5.4 Results and Discussions

Both the subspace analysis algorithms were implemented and tested on Pentium-IV processor with 2.6 GHz, 512 MB RAM under MATLAB environment. The performance of the imprint identification system using subspace analysis techniques has been tested using the standard databases FVC2000 and PolyU available on the website to compute recognition rate. The recognition rate for both the proposed algorithms is carried out in two parts as in-database and out-database. Training images and testing images are same for in-database whereas for out-database training images and testing images are different. The result obtained are listed in Table 5.1.

<table>
<thead>
<tr>
<th>Imprints</th>
<th>Testing database</th>
<th>Recognition rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>PCA</td>
</tr>
<tr>
<td>Fingerprint</td>
<td>In-database</td>
<td>84.74%</td>
</tr>
<tr>
<td></td>
<td>Out-database</td>
<td>69.35%</td>
</tr>
<tr>
<td>Palmprint</td>
<td>In-database</td>
<td>82.25%</td>
</tr>
<tr>
<td></td>
<td>Out-database</td>
<td>64.67%</td>
</tr>
<tr>
<td>FKP</td>
<td>In-database</td>
<td>79.98%</td>
</tr>
<tr>
<td></td>
<td>Out-database</td>
<td>61.33%</td>
</tr>
</tbody>
</table>

The results obtained for in-database of FLD have achieved 98%, 95% and 93% identification rates for fingerprint, palmprint and FKP images respectively. In contrast, PCA has only 84.74%, 82.25% and 79.98% for fingerprint, palmprint and FKP images respectively. FLD maximizes the distance of each subject hence, these
Projected coefficients of different subject could be distributed in the same area and FLD system cannot identify these imprints of palms. The results obtained for out-database degrades slightly because the imprint of palm images used for recognition have not been presented to the algorithm during training.

Also we have performed another experiment on PCA to find the minimum number of principal components required for correct recognition of the palmprints. We evaluated the performance of the system by varying principal components from 2 to 100. Figure 5.8 shows plot of number of principal components versus recognition accuracy. Principal component beyond 40, consistent accuracy of 82.25% was obtained in case of PCA.

![Figure 5.10 Recognition accuracy with PCA.](image)

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