Chapter 7
Mass dependence of the production of light fragments in heavy-ion collisions

7.1 Introduction

Does the mass of the system affect the dynamics? This question has always captured the central place in present day nuclear research. Ranging from the problems of nuclear structure to the decay of (excited) compound nucleus at low incident energies as well as the particle emission and its production at intermediate and high energies, the mass of the system is expected to play a dominant role. One has always tried to understand the system size effects in terms of scaling factors. The low energy heavy ion physics (where the typical phenomena like the fusion, fission as well as radioactive decay of the cold or excited compound system formed in heavy ion collision occur) is reported to be affected by the size of the system [1, 2]. Due to the hindrance from the Pauli-principle at low incident energies, the imaginary part of the potential does not contribute, therefore, the low energy dynamics is governed by the real part of the potential. Naturally, at low incident energies, the Coulomb force also contributes significantly. The interplay between the attractive nuclear potential and repulsive Coulomb force depends on the size of colliding nuclei [1, 2]. One has tried to understand the mass- dependence, for instance, in the fusion process [1, 2] where one concluded that the Coulomb force (∝ \( \frac{Z_1 Z_2}{r} \)) contributes significantly towards the barrier which can be parametrized in terms of the masses (and charges) of the colliding nuclei [1, 2].
Similar efforts are also made at intermediate energies to pin down the system-size dependence in various phenomena. This includes the temperature as well as the density, nuclear flow of nucleons/fragments, disappearance of flow, particle-production, multifragmentation, etc. The study of the mass-dependence in the evolution of the density and temperature (involving different nuclei) reveals that the maximum temperature is insensitive towards the mass of the system. However, the maximum density scales with the size of the interacting system \([3, 4, 5]\). The hot and dense nuclear matter (formed in a heavy ion collision) lasts longer in heavier colliding nuclei compared to lighter colliding nuclei \([3, 4]\). Interestingly, the equation of state also depends on the size of the system. Due to less compression in lighter colliding nuclei, the different equations of state (i.e., different compressibilities) do not yield different results. On the other hand, a clear difference can be seen with heavier colliding nuclei \([3, 5, 6]\). As noted in Refs. \([6, 7]\), the reaction volume is much larger in heavier systems which leads to significant higher average baryonic density. Note that smaller densities enhance the rate of the emission of fragments \([6, 19]\).

Another interesting study (of the system size effect) was made for the particle production by Hartnack et al. \([8]\) who found that the probabilities of the kaon production scale with the size of the system which can be parametrized in terms of a power law \(A_{\text{tot}}^\tau\); \(A_{\text{tot}}\) is the mass of the composite system \([8]\). The value of the parameter \(\tau\) (in the power law \(A_{\text{tot}}^\tau\)) was found to depend on the incident energy as well as on the different equations of state and their momentum dependence \([8]\). On the other hand, the kaon production probability per participant was independent of the projectile-target combination \([8]\). In a recent experiment \([7]\), the KAOS group reported the \(K^+\) production per nucleon which increases with the size of the system. Note that the pion multiplicity per nucleon decreases in the same measurements. The interesting observation was that the multiplicity of the high energy pions was nearly independent of the size of the interacting system and beam energy \([7]\).

In another experimental analysis, the entropy of the system (which is closely re-
lated with the production of the light charged particles and with the signatures of the early high density and temperature phase of the relativistic heavy ion collisions) was found to depend weakly on the size of the system [9]. The other signature of the compressional effects (predicted by the equation of state) is the collective flow. This most sensitive observable has been a topic of major nuclear research. Its dependence on the mass of the colliding nuclei has been investigated extensively during last few years [5, 10]. The disappearance of the collective flow (which occurs due to the balancing between the attractive and repulsive scatterings) depends strongly on the (composite) mass of the system [11]. It is now clear that the balance energy (at which the flow disappears) scales as $A_{tot}^{-1/3}$ [11]. The mass-dependence of the rapidity distributions measured in the plastic ball experiments was discussed in Ref. [12].

In contrary, fewer attempts exist in the literature which deals with the systematic study of the mass dependence of the multifragmentation. [13, 14, 15, 16, 17]. Most of the reported studies involve the asymmetric colliding nuclei at a fixed relative velocity [15, 17]. The asymmetry of a reaction can be defined by the asymmetry parameter $\eta = (A_T - A_p)/(A_T + A_p)$; $A_T$ and $A_p$, is respectively the mass of target and projectile. The recent reports from the FOPI experiments [13] depict the dependence of the multiplicity of heavy fragments on the size of the interacting system. This was carried out for symmetric nuclei like Ni+Ni, Ru+Ru, Xe+CsI, and Au+Au. In other words, the center-of-mass velocity is kept fixed in all these cases. The above study was based on the participant-spectator model where yields are analyzed separately for the participant and spectator fragments [13]. The universality in the production of the spectator fragments was achieved in the above study which confirms the results of ALADiN experiments [17]. However, a strong mass dependence was seen for the participant fragments [13]. Until recently, no systematic theoretical attempt was made to study the role of the masses of colliding nuclei in multifragmentation [18].

Note that the dynamics in light colliding nuclei can be quite different compared to heavy colliding nuclei. The surface contribution in light nuclei (like C, O, Ne etc.) is much
larger than for the heavy nuclei (like Pb, U etc). In other words, the surface to volume ratio (which depends on the size of the system) can play a vital role [5, 6, 12, 13, 14, 15]. The binding energy of a nucleus, for instance, is modified heavily by the surface as well as by the Coulomb and asymmetric terms. These effects contribute to \( \approx 50 \% \). Further, as noted by several authors, the heavy nuclei can be compressed strongly which may lead to faster expansion of the compressed matter [5, 6]. This also points toward the relation between the production of intermediate mass fragments and the amount of collective flow built during the compression [19]. The light and medium mass fragments \((A^{frag} \leq 10)\) which originate from the participant zone are formed at a very early stage of the reaction, therefore, one does not need to divide the matter into participant and spectator zones.

It was noted in Ref. [15] that the initial radial kinetic energy of the composite system depends strongly on the initial compression-decompression dynamics which varies with the mass ratio of the target/projectile. For asymmetric systems \((A_P \ll A_T)\), the compressional effects and radial expansion velocities will be small. Whereas for symmetric or nearly symmetric cases \((A_P \approx A_T)\), a large initial compression will be followed by the rapid expansion. Motivated by these findings, we present here a complete study of the mass dependence of the production of light and medium mass fragments. We here model the symmetric reactions involving the nuclei with masses between 40 and 238 [18].

In this chapter, we shall first concentrate on the time evolution of fragments and entropy production in the reaction of symmetric nuclei and then shall present the universal mass dependence in multifragmentation.

### 7.2 Results and discussion

Here we simulate several thousand events involving the symmetric reactions like \(^{40}\text{Ca} + ^{40}\text{Ca}, ^{58}\text{Ni} + ^{58}\text{Ni}, ^{93}\text{Nb} + ^{93}\text{Nb}, ^{131}\text{Xe} + ^{131}\text{Xe}, ^{168}\text{Er} + ^{168}\text{Er}, ^{197}\text{Au} + ^{197}\text{Au}, \text{and} ^{238}\text{U} + ^{238}\text{U}\) at incident energies between 50 MeV/nucleon and 1 GeV/nucleon and at different impact parameters \(b = b/b_{\text{max}}\); \(b_{\text{max}} = R_1 + R_2\); \(R_i\) is the radius of either projectile or target. The use of the symmetric nuclei \((\eta = 0)\) simplifies the theoretical consideration
and rescaled impact parameter \( \hat{b} \) assures the same geometrical overlap in all cases. By using the symmetric (colliding) nuclei, the system size effects can be analyzed without varying the asymmetry \( \eta \) and excitation energy of the system. It is worth mentioning that the experimental studies by the MSU miniball and ALADiN [15, 17] groups vary the asymmetry \( \eta \) of the reaction whereas the plastic ball [12] and FOPI experiments [13] are performed for symmetric reactions. In the following paragraphs, we first discuss the time evolution of different reactions and then, shall discuss the different light particle ratios and the production of entropy in the heavy-ion collisions.

### 7.2.1 Time evolution of density and nucleon-nucleon collision

We display in Fig. 7.1, the average density of the reaction which is defined in eq. (3.4). In Fig. 7.1, we show the evolution of the density \( \langle \rho_n \rangle \) at two typical incident energies 50 and 400 MeV/nucleon and at two impact parameters \( \hat{b} = 0 \) and 0.6. The central collisions (at low incident energies) as well as the peripheral collisions (at all incident energies) lack the frequent nucleon-nucleon collisions and, therefore, most of the initial memories of the nucleons (and the correlations among them) are preserved. In contrary, the frequent occurrence of the nucleon-nucleon collisions at central higher incident energies destroys the most of the correlation among them. If one goes beyond 400 MeV/nucleon, a little change in the fragmentation yield was reported in the experiments [13, 14, 17, 20]. From Fig. 7.1, we see that the heavier colliding nuclei are more compressed compared to lighter one. In addition, the dense (and hot) matter exists longer in heavier colliding nuclei compared to lighter nuclei. It is worth mentioning that the maximum temperature is unaffected by the size of the system whereas the density (both the maximum and average) in central region depends on the size of the system [3, 4, 15]. The maximum temperature can be correlated with the excitation energy of the system. After the compression, the matter expands and breaks into fragments (consisting of the entities of all sizes). As the higher compression exists longer for the heavy nuclei, one would expect a delayed triggering of the multifragmentation in these reactions. Note that the higher density \( \langle \rho_n \rangle \geq 1 \) at 50 MeV/nucleon remains till about 75 fm/c for U+U reaction compared to 40 fm/c for
Figure 7.1: The average density \( \langle \rho/\rho_0 \rangle \) as a function of the time. The top panel is at 50 MeV/nucleon whereas the bottom panel represents the reaction at 400 MeV/nucleon. The left and right hand sides represent, respectively, the central collision \( \hat{b} = 0 \) and peripheral collision \( \hat{b} = 0.6 \). All the reactions are for symmetric colliding nuclei \( X + X \), where \( X \) represents the Ca (filled triangle), Ni (solid line), Nb (dashed line), Xe (dotted line), Er (dashed-dotted line), Au (solid circle and dashed-double-dotted line) and U (inverted triangle)
Figure 7.2: Same as Fig. 7.1, but for the rate of collision $\frac{dN_{\text{coll}}}{dt}$.
Ca+Ca reaction. In other words, the excited heaviest fragment $A_{\text{max}}$ (detected in the MST method) will remain for longer time. A large freeze out density (at 200 fm/c) for heavier masses indicates the existence of the heavier fragments. On the other hand, one should expect universality beyond 400 MeV/nucleon.

The preservation of the initial nucleon-nucleon correlations can be linked with the collision rate which is displayed in Fig. 7.2. Naturally, the peripheral collisions have lesser overlap and hence lesser collision rate. Due to (available) free phase space at higher incident energies, the collision rate is very large. We also notice that the maximum collision rate for U+U system (at 400 MeV/nucleon) is about 80, which is less than 6 at 50 MeV/nucleon. Similar evolution can also be seen at peripheral geometry. The trends of the collision rate and density are quite similar. The maximal collision rate, which lasts longer in heavier colliding nuclei, will not allow the fragment distribution to saturate for long time. In other words, the saturation time of the fragmentation yield will be shorter in lighter systems compared to heavy systems. The finite collision rate at peripheral reactions at freeze-out times points towards the compactness of the nuclear matter. It is worth mentioning that the collision rate per nucleon (i.e., \( \frac{1}{A_{T}+A_{P}} \frac{dN_{\text{coll}}}{dt} \)) is nearly independent of the mass at 50 MeV/nucleon. Whereas it depends on the mass of the system at 400 MeV/nucleon.

7.2.2 Time evolution of various fragments

In Figs. 7.3-7.6, we show the time evolution of different fragments. The time evolution of the heaviest fragment $A_{\text{max}}$, emitted nucleons, light mass fragments (2 $\leq A \leq 4$), and medium mass fragments MMF’s (5 $\leq A \leq 9$) is displayed, respectively, in Figs. 7.3, 7.4, 7.5 and 7.6. The top panel in all figures is at 50 MeV/nucleon whereas the bottom 400 MeV/nucleon. As expected from Fig. 7.1 (where the evolution of the density was shown), the $A_{\text{max}}$ last longer in heavier systems compared to lighter systems. The excited $A_{\text{max}}$ in heavier systems continues to emit the nucleons till the end of the reaction whereas it saturates around 100 fm/c in light systems indicating the cold and separated matter.
Figure 7.3: Same as Fig. 7.1, but for the time evolution of the heaviest fragment $A_{\text{max}}$ as a function of the time.
Figure 7.4: Same as Fig. 7.1, but for the time evolution of the multiplicity of free particles.
Figure 7.5: Same as Fig. 7.1, but for the time evolution of the light mass fragment ($2 \leq A \leq 4$).
Figure 7.6: Same as Fig. 7.1, but for the time evolution of the medium mass fragment MMF's ($5 \leq A \leq 9$).
The emission of the nucleons (shown in Fig. 7.4) reflects the same trend. Due to finite collision rate (in heavier colliding nuclei), the emission of the nucleons and light charged particles continues till the end of reaction. Note that the saturation occurs around 80 fm/c in Ca+Ca reaction whereas it takes 250 fm/c for U+U system. The saturation of the free nucleons occurs earlier at higher incident energies which indicates a faster disintegration of the matter at these energies. The time evolution of the light charged particles \((2 \leq A \leq 4)\) (not shown here) also follows similar trend.

On the other hand, the formation of the MMF’s (Fig. 7.6) has different evolution. While the MMF’s at 50 MeV/nucleon are stable and saturate around 120-200 fm/c, the MMF’s in central 400 MeV/nucleon are very excited and unstable which continuously emit the light fragments/particles. Again the saturation time is much shorter for lighter system. These results are in agreement with earlier calculations [19]. It is worth mentioning that the MST method which is based on the spatial correlation, produces excited fragments. In this situation, other modified methods (like the MST method with momentum or binding energy cut [23]/or energy minimization method like the simulated annealing clusterization algorithm [22]) avoid the creation of weakly bound fragments. One also notices that the triggering of the fragmentation is delayed in heavier colliding nuclei compared to lighter nuclei. If one plots the final state multiplicity of the MMF’s as a function of the impact parameter, one will observe the well known rise and fall of the multiplicity [19].

The above findings show that the light mass fragments are formed at a very early stage of the reaction. The light charged particles \((2 \leq A \leq 4)\) do not decay and seem to be originating from the surface of the confined system at higher incident energies. As seen from Figs. 7.3-7.6 and also reported in the literature, these fragments measure the violence of the reaction therefore, depend on the impact parameter. We have also checked the binding energy of different fragments produced in the above reactions and find that they are properly bound at the end of the reaction.
7.2.3 Entropy production

The yield of the light charged particles is closely related with production of entropy in heavy ion reactions. It has been proposed in Ref. [24] that the entropy in heavy ion collisions can be determined by calculating the ratio of deuterons to protons. The entropy per particle $S/A$ is connected to the ratio of deuterons to protons $R_{dp}$ as

$$S/A = 3.945 - \ln R_{dp}.$$  \hfill (7.1)

The heavier particles can also be included in $R_{dp}$ by using more general formula [25]:

$$R_{dp} = \frac{d + \frac{3}{2}(t + 3\,^{3}He) + 3^{4}He}{p + d + t + 2(3^{3}He + ^{4}He)}.$$  \hfill (7.2)

Since no isospin identification is done in present QMD model, we cannot distinguish between different isobars. For example, a fragment with mass = 3 represents both the tritium and $^{3}He$. This problem can be overcome by using following alternate prescription for $\tilde{R}_{dp}$

$$\tilde{R}_{dp} = \frac{Y(A = 2) + \frac{3}{2}Y(A = 3) + 3Y(A = 4)}{N_{p}},$$  \hfill (7.3)

where $Y(A = n)$ stands for the number of fragments with mass $n$ in one event. In analogy to the experimental results, we define the total participant proton multiplicity $N_{p}$ in the following way [9, 21]

$$N_{p} = \frac{Z_{p} + Z_{T}}{A_{p} + A_{T}}[Y(A = 1) + 2Y(A = 2) + 3Y(A = 3) + 4(A = 4)].$$  \hfill (7.4)

It would be of interest to look for the mass dependence of proton multiplicities $N_{p}$ as a function of reduced impact parameter. Fig. 7.7, shows the relation between $\tilde{N}_{p}$ and reduced impact parameter for the reactions of Ca+Ca, Nb+Nb and Au+Au at 400 MeV/nucleon. Two different behavior can be seen: (i) The first one is for $\tilde{b} < 0.2$ where $\tilde{N}_{p}$ is nearly constant. (ii) The second one for $\tilde{b} \geq 0.2$. One sees that the proton multiplicity decreases almost linearly with increase in the impact parameter. One can, therefore, conclude that the reaction $\tilde{b} < 0.2$ cannot be distinguished experimentally and represent most central collisions.
The relation between reduced impact parameter $\hat{b}$ and the rescaled multiplicity of participant protons $N_p$ for the reactions of Ca+Ca, Nb+Nb and Au+Au. Here a soft equation of state is used.

In Fig. 7.8, the ratio of different isotopes to proton ($X/P$, $X$ denotes an isotope) is presented for the reactions of Ca+Ca, Nb+Nb and Au+Au. Here $X$ stands for a fragment with mass = 2, 3 and 4. Interestingly, various ratios first increase with $N_p$ and then saturate for very high value of $N_p$. In figure 7.9(a), we show the $R_{dp}$ (defined in eq. (7.3)) versus $N_p$. The entropy production is displayed in Fig. 7.9(b). Here we observe a moderate mass dependence in the production of $R_{dp}$ ratio and entropy.

### 7.2.4 Mass dependence and power law behavior

We display, in Figs. 7.10-7.14, the reduced multiplicity (multiplicity per nucleon) of the free nucleons as well as of the fragments with mass $A = 2$, LMF’s and MMF’s and MMF’s*. Note that in contrary to the FOPI and ALADiN experiments, we do not divide the matter into participant and spectator zones. The top panel in each figure displays the multiplicities at 50 and 100 MeV/nucleon whereas the bottom panel is at 600 MeV/nucleon and 1000 MeV/nucleon. The middle panel represents the 200 and 400
Figure 7.8: The $X/p$ ratios for the reactions of Ca+Ca, Nb+Nb and Au+Au at $E = 400$ MeV/nucleon as a function of rescaled multiplicity of participant protons $N_p$. Here $X$ stands for fragments with mass 2, 3 and 4.
Figure 7.9: Same as Fig. 7.7, but for $R_{dp}$ (upper part) and entropy $S/A$ (lower part).
MeV/nucleon. The windows in each panel contain four different curves which correspond, respectively, to the scaled impact parameter $\hat{b} = 0.0, 0.3, 0.6$ and 0.9. First of all, the wide range of the incident energy between 50 MeV/nucleon and 1000 MeV/nucleon and impact parameter between 0 and $b_{\text{max}}$ deals with different dynamics emerging at low, intermediate and high energies. The nature of the dynamics at low energy is more of a fusion-fission whereas the multifragmentation dominates the scenario at medium energies. At higher incident energies, one has complete disassembly of the nuclear matter which makes the multifragmentation a rare process. The central collisions lead to the participant matter dynamics whereas the dynamics at peripheral geometries is more of a spectator physics. In view of this, the medium incident energies provide unique possibility to study the fragmentation. The Fermi-spheres of the projectile and target become separated at incident energies of about 50-100 MeV/nucleon and hence, one may expect that it is the beginning of the transition regime between the low energy heavy-ion reactions, demanded by the compound nucleus formation and the high energy heavy-ion reaction where clear participant-spectator picture emerges.

As discussed above, the general behavior of all light mass fragments follows the well known trends. In peripheral collisions, the geometry is dominated by the spectator physics. The free nucleons as well as the light charged particles scale with the size of the participant matter. Their multiplicity is maximum for the central collisions which decreases with increase in the impact parameter. One also sees that the number of the emitted nucleons and light mass fragments ($A \leq 4$) increases with the incident energy. At higher incident energies, most of the initial nucleon-nucleon correlations are destroyed in participant matter and, therefore, only light particles survive from the reaction zone.

In contrast, due to large Pauli-blocking at low incident energies, many nucleons in the reaction zone survive the reaction without suffering the collisions with large momentum transfer. The energy received by the target in peripheral collisions is not enough to excite the matter far above the Fermi-level resulting in fewer light fragments. In other words, the emission of the heavier fragments becomes more and more a phenomena of peripheral collision with increase in the incident energy. To deal with this situation, the FOPI [13]
Figure 7.10: The final state multiplicity (calculated at 300 fm/c) of the free particles per nucleon as a function of the composite mass of the system $A_{tot} (= A_T + A_P$; $A_T$ and $A_P$ are, respectively, the mass of the target and projectile). The left hand side of the top, middle and bottom panels represent, respectively, the reaction at 50, 200 and 600 MeV/nucleon. The right hand side of the top, middle and bottom panels represent, respectively, the reaction at 100, 400, 1000 MeV/nucleon. In each window, four symbols i. e., the inverted triangle, triangle, solid circle and diamond represent, respectively, the reaction at $\tilde{b} = 0, 0.3, 0.6$ and 0.9. All curves are using $y = c. A_{tot}^\tau$. 

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Figure 7.11: Same as Fig. 7.10, but for the final state multiplicity of the fragments with mass = 2.
Figure 7.12: Same as Fig. 7.10, but for the final state multiplicity of the fragments with mass $2 < A < 4$. 

$\hat{b} = 0$ 
$\hat{b} = 0.3$ 
$\hat{b} = 0.6$ 
$\hat{b} = 0.9$ 

$E = 50$ A MeV 
$E = 100$ A MeV 
$E = 200$ A MeV 
$E = 400$ A MeV 
$E = 600$ A MeV 
$E = 1000$ A MeV 

$A_{tot}$
and ALADiN groups [17] divided the nuclear matter into spectator and participant zone. Our present interest lies in the light mass fragments (with mass less than 10), therefore, we do not divide the matter into participant and spectator zones.

Remarkably, independent of the mass of the fragments as well as incident energy and impact parameter, the multiplicity of any kind of fragment (i.e., of free nucleons, fragments with masses \( A = 2, 2 \leq A \leq 4 \) and \( 5 \leq A \leq 9 \)) scales with the size of the system which can be parametrized by a power law of the form \( cA_{\text{tot}}^{\tau} \); \( A_{\text{tot}} \) is the composite mass of the system. The values of the constants \( c \) and \( \tau \) depend on the size of the fragments as well as on the incident energy and impact parameter [18]. This dependence of \( \tau \) will be discussed in the following paragraphs. We have also tried a functional form \( c.e^{-\tau A} \). The fits were worse than the one obtained with power law.

A word of caution should be added here: It has been shown and discussed extensively in the literature that the mass yield curve approximately obeys a power law behavior \( \propto A_{\text{frag}}^{\tau} \) [26]. It has been conjectured (though controversial) that this behavior (which has also been termed as "accidental" [21]) is an indication of the phase transition between a gaseous and liquid phase of the nuclear matter. Note that the said power law behavior of the mass (or charge) distribution is for a "given system" [26]. The above power law dependence, which we are discussing, is something very different. The above power law function is for the multiplicity of a "given fragment" which scales with the size of the system. The existence of the above power law dependence at all impact parameters and incident energies indicates the universality of the power law behavior for the system size effect in the production of light mass fragments. A similar power law behavior was reported by us for the production of heaviest fragment as well as for the production of the intermediate mass fragments [18].

From Fig. 7.10, we also notice that the percentage of the free particles increases drastically with the incident energy which can be as high as 80\% for central collision. If we label the reaction above 60\% or more free-particle as total disassembly, we see a clear
disassembly of the matter in central collision above 400 MeV/nucleon.

From Figs. 7.10-7.14, we see maximum effect (of system size) at low incident energies which decreases with incident energy. The emission of the light charged particles exhibits linear dependence at higher incident energies. One of the possible causes of this sharp dependence is the late saturation in heavier colliding nuclei (i.e., Fig. 7.4). From Fig. 7.4, we see that the multiplicity of the free nucleons and light charged particles in lighter colliding nuclei saturates around 200 fm/c whereas it takes much longer time for heavier nuclei. As our nuclei are stable for a typical time span of 300 fm/c, we cannot follow the reaction beyond this time. If we analyze the mass dependence at later stage, the multiplicity is likely to be changed for heavy systems. To demonstrate this, we show the outcome of various fragments at 800 fm/c (dashed lines). We see that the particle emission in heavier nuclei changes drastically beyond 300 fm/c at low incident energies whereas nearly no effect exists at higher incident energies. The multiplicity of light charged particle does not change.

Our results at 400 MeV/nucleon are in agreement with Ref. [12] where the fraction of the observed charge was reported to decrease from Ca+Ca to Au+Au. In contrary to the light charged particles, the multiplicity of the medium mass fragments (MMF’s) (Fig. 7.13) has a sharp dependence on the size of the system in some cases. If we look at Fig. 7.3 (where the size of heaviest fragment is displayed), we find that the size of $A_{\text{max}}$ in many cases, is quite close to the range of the MMF’s ($5 \leq A \leq 9$). In order to strengthen our argument, we show, in Fig. 7.14, the multiplicity of MMF’s excluding the $A_{\text{max}}$ (marked by $MMF’s^*$). Once the $A_{\text{max}}$ is excluded, the sharp dependence of the MMF’s on the size of system washes away in most of the cases. This happens because the $A_{\text{max}}$ [which is close to the extreme limits of the MMF’s (either 5 or 9)] will be included sometime whereas may be excluded other times which makes a sharp system size dependence.

It is worth mentioning that if one plots the reported results of the FOPI experiments [13] as a function of the size of the system, a similar power law fit can also be
Figure 7.13: Same as Fig. 7.10, but for the MMF's.
Figure 7.14: Same as Fig. 7.13, but for the MMF’s excluding the heaviest fragment $A_{\text{max}}$ (denoted by MMF’s*).
obtained. Note that the analysis of the FOPI experiments [13] has been done for the participant zone only. Our present calculations include both the participant and spectator zones. As discussed above, similar power law dependencies have also been reported for other observables. For example: the probability of the kaon production was reported to depend on the size of the system and was parametrized in terms of a power law function [8]. A similar power law dependence was also obtained for the collective flow. The different slopes of the power law at low and higher incident energies can be coupled with the collective flow that depends on the incident energy as well as on the mass of the system.

7.2.5 Energy and impact parameter dependence of parameter $\tau$

The $\tau$ dependence is displayed in Fig. 7.15 where it is plotted as a function of the incident energy. Different symbols in the figure represent different impact parameters. We do not see any unique value of the $\tau$. For the central collisions, the value of the parameter $\tau$ is close to $1/3$ at 50 MeV/nucleon which first increases with incident energy and then finally saturates at very high incident energy. In other words, the total multiplicity of the fragments will be $A/A^{-1/3} = A^{2/3}$ which is like a surface of the colliding nuclei representing the mean field. Therefore, it seems that the mass dependence at low incident energies is like that of a mean field. With the increase in the incident energy, the value of $\tau$ tends to approach 0 (the unscaled value will be $\approx 1$). This corresponds to a linear dependence. It has been stated by a number of authors that the repulsive nucleon-nucleon interactions at high energies scale like $A$ [11]. Unlike the disappearance of flow (which rescales as $A^{-1/3}$ [11]), the present $\tau$ dependence is not unique. It is worth mentioning that the power law factor $\tau$ in the parameterization of the kaon production also depends on the incident energy as well as on the equation of state one is using. As reported by Hartnack et al. [8] no unique dependence of $\tau$ could be obtained for the kaon production.

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Figure 7.15: The parameter $\tau$ (appearing in the power law function $A_{\text{tot}}$) as a function of the incident energy. The top panel displays the values of $\tau$ for free nucleon (left part) and fragments with mass $A=2$ (right part). The bottom panel represents the values of $\tau$ for LMF’s (left panel) and MMF’s (right panel). The different symbols namely, the inverted triangle, triangle, solid circle and square, represent the results at impact parameter $b = 0, 0.3, 0.6$ and 0.9.
7.2.6 Mass yield and power law behavior

In order to see the dependence of the mass yield on the incident energy as well as on the impact parameter and size of the system, we plot in Fig. 7.16, the mass yield \((dN/dA)\) as a function of the mass of the fragments. The top panel is at 50 MeV/nucleon whereas the middle and bottom panels are, respectively at 200 and 400 MeV/nucleon. We see well known trends in the mass yield. It decreases with the increase in the mass of the fragment. The slope of the mass curve becomes steeper with incident energy whereas it decreases with impact parameter. One also notices that the heavier colliding nuclei emit more light charged particles at higher incident energies which is in agreement with the results of FOPI experiment [13]. The difference between the mass yield of various reactions decreases with decrease in the incident energy, which is again in agreement with FOPI experiment [13]. All the mass yields can be parametrized in terms of well known power law \(\propto A_{frag}^{7\gamma}\). The interesting feature is that if we rescale the mass yield with the size of the system (Fig. 7.17), nearly a single mass yield curve (\(\propto\) power law) can be obtained.

7.2.7 Rapidity distribution and mass dependence

To look for the (possible) equilibrium in heavy ion collisions and its dependence on the size of system, we also calculate the rapidity distribution. The rapidity distribution of \(i^{th}\) particle is calculated as [3, 6, 8, 19]:

\[
\frac{1}{2} \ln \frac{E(i) + p_z(i)c}{E(i) - p_z(i)c}
\]  

(7.5)

Here \(E(i)\) and \(p_z(i)\) denotes, respectively, the total energy and longitudinal momentum of the \(i^{th}\) particle. Naturally, for a complete equilibrium, one expects a single Gaussian shape in the rapidity distribution. In Fig. 7.18, we display the rapidity distribution for different colliding nuclei at two typical incident energies of \(E = 50\) and 400 MeV/nucleon. Here we display the rapidity of free nucleons as well as of medium mass fragments. We notice that the free nucleons exhibit nearly a Gaussian shape distribution for heavy colliding nuclei whereas the sharp peak flattens for lighter colliding nuclei. Similarly, the
Figure 7.16: The mass yield $dN/dA$ as a function of the mass of the fragment $A_f$. The left and right hand sides of the top, middle and bottom panel represent, respectively, the reaction at 50 MeV/nucleon, 200 MeV/nucleon and 400 MeV/nucleon and at impact parameter $b = 0$ and 0.6. The symbols open circle, solid circle, open triangle, solid triangle, open square, solid square and solid diamond represent, respectively, the reaction of Ca+Ca, Ni+Ni, Nb+Nb, Xe+Xe, Er+Er, Au+Au and U+U.
Figure 7.17: Same as Fig. 7.16, but for the rescaled mass yield (\( \frac{dN/dA}{\Sigma dN/dA} \)) as a function of the mass of the fragment.
Figure 7.18: The rapidity distribution $\frac{dN}{dy}$ as a function of the scaled rapidity $Y/Y_{beam}$. The left hand side is at 50 MeV/nucleon whereas the right hand side is at 400 MeV/nucleon. The upper and bottom panels represent, respectively, the free nucleons and MMF’s.
rapidity distribution of the medium mass fragments (MMF’s) is very sensitive to the size of the system. We see that at both incident energies, the lighter nuclei have wider distribution compared to heavier colliding nuclei. Interestingly, very light nuclei (like Ca+Ca) exhibits totally different shape with very flat plateau. In some lighter cases, one even finds a peak at target/projectile rapidity indicating a non-equilibrium situation. These findings are again in agreement with the other theoretical investigations and experimental data [8, 13, 14]. We have also checked the directed transverse momentum and $\frac{dN}{d\eta}$ (as a function of $p_t$, where $p_t$ is the transverse energy). Both quantities scales with the size of the system.

7.3 Summary

Using quantum molecular dynamics (QMD) model coupled with the minimum spanning tree (MST) method, we investigated in detail the formation of various light mass fragments and entropy production as well as their dependence on the size of the system [18]. For detailed analysis, we studied the reactions at incident energies between 50 and 1 GeV/nucleon and over full geometrical overlap using symmetric colliding nuclei with mass between 40 and 238. As we know, the ratio of the surface to volume decreases with the size of the system whereas the compressional effects increase. The lighter colliding nuclei generate less density whereas higher density is achieved with heavy nuclei which gives ample space for compression-decompression as well as radial expansion.

We also find that the light mass fragments are formed at a very early stage of the reaction. The light charged particles ($2 \leq A \leq 4$) do not decay and seem to be originating from the surface of the confined system at higher incident energies. As shown above and also reported in the literature, these fragments measure the violence of the reaction and therefore, depend on the impact parameter. The relative yields of the light fragments ($d$, $t$, $\alpha$) show a strong impact parameter and bombarding energy dependence. The entropy production, which is connected with the production of light particles also exhibits similar picture.
The system-size effects depend on the reaction inputs as well as on the colliding geometry. The multiplicity of any kind of fragment can be parametrized in terms of a power law \( \propto A_{tot}^{\tau} \) where \( A_{tot} \) is the total mass of the composite system. This is true for a wide range of impact parameter and incident energy considered here. However, the parameter \( \tau \) does not have a unique value. Rather, it seems that the parameter \( \tau \) is close to 2/3 at lower energies suggesting the dominance of the mean field that scales as \( A^{2/3} \). In contrast, we obtain a nearly linear dependence at higher incident energies suggesting the dominance of the repulsive scattering at higher incident energies. The mass yield also follows similar trends. The rapidity distribution suggests that the shape of the distribution is very sensitive to the size of the system. It is nearly Gaussian (with sharp peak) for heavier colliding nuclei whereas a flat plateau is obtained for lighter colliding nuclei. We even obtain small peaks at target/projectile rapidities for lighter colliding nuclei. Similar system-size power law dependence has also been reported in other observables like in the disappearance of flow [11] as well as in the production of kaon/pion [8] and in low incident energy phenomena like the fusion etc [1, 2]. Such trends can also be seen in the preliminary experimental results of FOPI group which has measured the intermediate mass fragment yields [13].

It is worth mentioning that the results of the multifragmentation are found to be sensitive towards the different model ingredients like the equation of state (with/without momentum dependent interactions [27] and nucleon-nucleon cross section [28]) as well as towards the clusterization method one is using [23]. As has been reported in the literature, the MST method does not yield proper results at higher incident energies whereas more sophisticated model can yield better results [22]. In view of these points, we would like to add that the value of parameter \( \tau \) should depend on the model ingredients one is using. Its value is likely to change with the method/input, but the power law dependence of the system-size effect should exhibit.
Bibliography


