WEAK RADIATIVE DECAYS OF BARYONS WITHIN THE
CONSTITUENT QUARK MODEL

5.1 INTRODUCTION

The radiative weak decays of baryons have continually been sparking the interest of theorists for over two decades, as they provide an interesting framework to gain insight into the hadronic weak interactions. Theoretically, though these decays are simpler to handle than the corresponding pionic counterparts, as they present lesser complications due to the strong interaction effects in the final states, yet their low branching ratio \( \approx 10^{-3} \) put limitations on their measurements. For a long time, data on decays, even among the hyperons, other than \( \Sigma^+ \to \rho + \gamma \) was meagre. Though its branching ratio was found to be of expected order of magnitude, yet its large and negative asymmetry \((-0.83 \pm 0.13)\) has caused a constant challenge to the theoreticians. The major conflict lies with the well known Hara’s theorem [1], which forbids \( \Sigma^+ \to \rho + \gamma \) decay in parity-violating (pv) mode and yields null asymmetry for this decay. Recently these decays have come under active experimental investigation [2-4]. Consequently decay parameters of other modes have become available for a detailed analysis. Various physical mechanism and models have been suggested for the description of the weak radiative decays. The earlier theoretical attempts [5], were essentially based on internal symmetries and the pole models, and later the constituent quark-model was incorporated within the framework of the pole- models [6]. Attempts have also been made to include \( 1/2^+ \) baryons [7] and \( K^* \) pole forms [8] to generate the parity-violating decay amplitudes. Although this yielded decay rates of the correct
order of magnitude, the $\Sigma^+ \rightarrow p + \gamma$ decay asymmetry largely remained unexplained.

Under the aegis of the Standard Model new approaches that directly consider quark operators appearing in its lagrangian were developed. At the quark level there exist essentially three modes i.e. the single-quark process (1QP), the two-quark process (2QP) and the three-quark process (3QP) which contribute to these decays. The analysis of Gilman and Wise [9], has shown the inadequacy of the 1QP alone, to explain these decays. In fact in the electroweak gauge theories [10], the $s \rightarrow d + \gamma$ vertex has been shown to be highly suppressed due to the unitarity of the Cabibbo-Kobayashi- Maskawa (CKM) mixing. Subsequent analysis by Shifman et al [11] introducing short distance gluon effects showed some enhancement of the vertex but failed to reproduce the observed branching ratio. In view of the obvious inadequacy of the effective $s \rightarrow d + \gamma$ transition, the quark model analysis has been extended to include the two-quark transitions [12-16], specially the W-exchange bremsstrahlung $s + u \rightarrow u + d + \gamma$. Unlike the meson weak decays, the W-exchange in baryons is neither color nor helicity suppressed since there may exist a spin 0 two-quark system inside the baryon. In fact the contribution from this process has been found to be proportional to $|\psi(0)|^2$ where $\psi(0)$ is the baryon wavefunction at the origin, and is seen to dominate in the baryon weak decays [15-18]. Such quark model results have been found to be consistent with the available data. In particular the $\Sigma^+ \rightarrow p + \gamma$ asymmetry obtained was about -0.60 [13]. This is in apparent conflict with Hara's theorem, since even in the SU(3) limit the pv amplitude for $\Sigma^+ \rightarrow p + \gamma$ does not vanish [15,16]. This astonishing anomaly was explained by Kamal and Riazuddin [19]. They have shown that the effective Hamiltonian for the weak bremsstrahlung operator constitutes an additional term involving a dual electromagnetic field tensor which not only violates
Hara's theorem but in fact dominates over the terms satisfying it. This result received further support from an MIT bag model calculation [14], where the pv amplitude for $\Sigma^+ \to p + \gamma$ arising from W- exchange was of an order of magnitude larger than the pc amplitude. Though now it is commonly believed [20] that the W- exchange 2QP along with the $s \to d + \gamma$ vertex provides the mechanism responsible for these decays, there still remains many a discrepancy [21]. For instance though $\alpha(\Sigma^+ \to p + \gamma)$ is of the right sign, it is lacking in magnitude; further asymmetries of $(\Xi^0 \to \Lambda/\Xi^0 + \gamma)$ got from recent measurements pose problems for their explanations [3,4].

In this chapter, we suggest that these difficulties may be related to the assumed SU(6) symmetry of the baryon wave-functions. This being a low energy phenomenon is not tractable from first principles. In order to study the SU(6) symmetry breaking effects, we employ the constituent quark-diquark model, which has been found adequate for describing various hadronic phenomenon [22,23], including the non-leptonic weak decays [24], besides preserving successes of the SU(6) symmetric quark model results. We obtain $\Sigma^+ \to p + \gamma$ and $\Xi^0 \to \Lambda + \gamma$ asymmetries in better agreement. Though the $\Xi^0 \to \Sigma^0 + \gamma$ asymmetry improves slightly, a new measurement is desired. In section 5.2, the weak Hamiltonian is constructed for the single- and two- quark processes. We discuss the decay rate and asymmetry methodology in Section 5.3. In the following Section 5.4, we use the constituent quark model to estimate the branching ratios and asymmetry parameters for the hyperon radiative decays. Section 5.5 relates to the essence of the SU(6) broken quark-diquark model. Using the diquark model we calculate the parity conserving (pc) and parity violating (pv) decay amplitudes in Section 5.6. Section 5.7 deals with the numerical results and discussions. In Section 5.8 we analyse the weak radiative decays ($\nu_2^+ \to \nu_2^+ + \gamma$) of C = 1 and C = 2 charm baryons in the
Cabibbo enhanced, suppressed and doubly suppressed modes. Section 5.9 describes the Improvised Quark Model developed to account for the large momenta release in these decays. In section 5.10 we estimate weak decay amplitudes and then the decay widths and asymmetry parameters for all these charm changing weak electromagnetic decays including the effect of flavor dependence on the scale. Finally in section 5.11 we discuss the effects of quantum chromo-dynamical (QCD) modifications to these decays. The last Section 5.13 brings summary and conclusion.

5.2 CONSTRUCTION OF THE HAMILTONIAN.

Within the Standard model, the radiative decays arise through the following piece of the electroweak lagrangian, given in Eqn. 2.4 and 2.14,

$$L_{int} = \frac{J_\mu W^\mu}{2\sqrt{2} \sin\theta_W} + h.c. + J^{em}_\mu A^\mu$$  \hspace{1cm} (5.1)

where $J_\mu$ and $J^{em}_\mu$ are the weak and electromagnetic quark currents. In general W-loop and W- exchange processes involving the single-quark, two-quark and three-quark transitions inside the baryon can contribute to these decays. Gluon-exchange effects further modify these processes. In the following we construct the weak effective Hamiltonian for the general weak radiative decays of baryons.

5.2.1 Single Quark Transition Hamiltonian

In hyperons, the 1QP is essentially the $s \rightarrow d + \gamma$ transition involving the W-loop. This transition is allowed in the Standard Model and is not removed with any renormalisation as the photon is emitted at short distances $\sim 1/m_W$ by the internal quarks or the W-boson [10]. Limiting to the 4-quark sector the most general Feynman amplitude can be parameterized [9] as,
\[ M^{1q} = \frac{e G_F \sin \theta_c \cos \theta_c}{\sqrt{2}} \bar{d} \left( a + b \gamma_5 \right) k' s, \]  
(5.2)

where the \( \sqrt{2} \) and the Cabibbo factors are introduced for later convenience.

In the non-relativistic limit, this reduces to,

\[ H^{1q}_{\text{eff}} = \frac{e G_F \sin \theta_c \cos \theta_c}{\sqrt{2}} \bar{d}^+ \left[ i a \vec{d} \cdot (\vec{e} \times \vec{k}) - b \left( \vec{d}' \cdot \vec{e}' \right) \right] s. \]  
(5.3)

In the Glashow-Weinberg-Salam electroweak gauge theory [10] the parameters \( a \) and \( b \) which give the pc and pv contribution respectively, are expressed as,

\[ a = A \left( m_s + m_d \right), \quad b/a = \frac{(m_s - m_d)}{(m_s + m_d)} = \frac{1}{4}. \]  
(5.4)

The GIM cancellation factor \( \frac{m_C^2 - m_U^2}{m_W^2} \) suppresses the scale parameter \( A \) to about \( -8 \times 10^{-5} \). However the QCD correction [11] due to gluonic exchange results in an enhancement (although insufficient) of the scale parameter by about 2 orders of magnitude.

Shifman et al. have shown that in the two loop graphs of \( W \) and a Gluon, the GIM cancellation factor is replaced by quark mass ratio \( m_i^2 / m_j^2 \) where \( i \) and \( j \) are the charge \( 2/3 \) quarks in the loop. In the leading log approximation the QCD modified amplitude becomes,

\[ M_{s \rightarrow d + \gamma}^{\text{QCD}} = -i e G_F \left( \frac{0.02 A}{\pi^2} \right) m_s \bar{d} \left( 1 + \gamma_5 \right) \gamma_{\mu} \gamma_{\nu} k'^\mu e'^\nu \]  
(5.5)

This yields \( a = -0.96 \times 10^{-2} \) GeV and \( b/a = +1 \). A similar enhancement is expected for the \( s \rightarrow d + \gamma + \) gluon vertex [25]. Gilman and Wise made the pioneering attempt in assuming that the weak radiative decays of hyperons proceed via a single quark transition. Without considering the details of the dynamics they performed the calculations using the most
general Feynman amplitude given in Eqn 5.2. Their results are given in the column (i) of Table 5.1. It has now been well established that though the single quark transition is essential to explain decays like $\Xi^- \rightarrow \Sigma^- \gamma$, the $s \rightarrow d \gamma$ mechanism cannot play a dominant role in the weak radiative decays of hyperons, and we extend our analysis to include the two-quark transitions.

5.2.2 Two-Quark Transition Hamiltonian.

The general idea is to expand the effective electroweak Hamiltonian [12] for the weak bremsstrahlung process Fig. 5.1,

$$q_1 (p_1) + q_2 (p_2) \rightarrow q_3 (p_3) + q_4 (p_4) + \gamma (k),$$

(5.6)

with the third quark behaving as a spectator inside the baryon, by carrying out an expansion of the bremsstrahlung amplitude in $k$,

$$H^{2q}_{\gamma} = A_{-1} k + A_0 + A_1 k + \ldots..$$

(5.7)

such that each stage of the expansion is gauge invariant. In the rest frame of the initial baryon, $k$ is fixed, to be

$$k = \frac{(m_B^2 - m_{B'})}{2m_B}$$

(5.8)

so that the expansion in Eqn. 5.7, is in the powers of the flavor symmetry breaking parameter $(m_B - m_{B'})$. Relativistic terms up to the first order are included. The method of calculation and approximation used are best demonstrated through the Feynman expansion of the amplitude for one of the permutations of the photon emission from one of the external quark legs, (say $q_3 (p_3)$):

$$M^{2q}_{\gamma} = \frac{e_3 G_F \text{Cabibbo factors}}{\sqrt{2} 2 k \cdot p_3} \times$$

$$\bar{q_3} (p_3) \left( 2 \epsilon \cdot p_3 + \not{k} \right) \gamma_\mu \left( 1 - \gamma_5 \right) q_1 (p_1) \bar{q_4} (p_4) \gamma^\mu \left( 1 - \gamma_5 \right) q_2 (p_2)$$

(5.9)
where \( q_i \) are Dirac spinors of relevant flavors and all external quarks are assumed to be on their mass shells. We employ the constituent quark model to construct the Hamiltonian for the weak bremsstrahlung process. The calculations are carried out for convenience in momentum space. We work in the Coulomb gauge \( \epsilon_0 = 0, \vec{\epsilon} \cdot \vec{k} = 0 \). The term \( \frac{\epsilon \cdot p_3}{k \cdot p_3} \) vanishes due to the gauge condition, and the effective hamiltonian carries a 'k' dependence in the remaining term through two factors: the propagators \( \sim 1/k \cdot p_3 \) and the current factors. Using the Feynman rules for the electroweak gauge theories, for quark momentum integration (upto first order) and summing over the four photon emission permutations, we get the following effective weak Hamiltonian, for the 2Q parity conserving (pc) and parity violating (pv) process [12,13]:

\[
H^{PC}_{\text{eff}} = \frac{e G_F \text{Cabibbo factors}}{\sqrt{2}} \left| \vec{k} \right| \left[ A q_3^+ i \vec{\sigma} \cdot \vec{\epsilon} x \hat{k} q_1 q_4^+ q_2 + B q_3^+ q_1 q_4^+ i \vec{\sigma} \cdot \vec{\epsilon} x \hat{k} q_2 \right. \\
+ C q_4^+ \vec{\sigma} \cdot \vec{\epsilon} q_1 q_4^+ \vec{\sigma} \cdot \hat{k} q_2 + D q_3^+ \vec{\sigma} \cdot \hat{k} q_1 q_4^+ \vec{\sigma} \cdot \vec{\epsilon} q_2 \\
+ (A + H(k)) q_3^+ i \vec{\sigma} \cdot \vec{\epsilon} x \hat{k} q_1 q_4^+ \vec{\sigma} \cdot \hat{k} q_2 + (B - H(k)) q_3^+ \vec{\sigma} \cdot \hat{k} q_1 q_4^+ i \vec{\sigma} \cdot \vec{\epsilon} x \hat{k} q_2 \\
+ i \vec{\epsilon} \cdot q_3^+ \vec{\sigma} q_1 \cdot q_4^+ \vec{\sigma} q_2 \left. H(k) \right] 
\]

(5.10)

where \( q_i \) indicate the Pauli spinors of relevant flavors, with external quarks assumed to be on-mass shell. Coefficients of the quark operators are,

\[
A = \left[ -H(k) + (Q - G(k) + P + H(k)) \right] 
\]
\[ B = [ +H(k) - (P - G(k) + Q + H(k)) ] \]
\[ C = [ +G(k) - (P + G(k) + Q - H(k)) ] \]
\[ D = [ -G(k) + (Q + G(k) + P - H(k)) ] \]  
\[
(B) = \frac{1}{2} \left[ \frac{e_3}{p_3 \cdot k} + \frac{e_1}{p_1 \cdot k} - \frac{e_4}{p_4 \cdot k} - \frac{e_2}{p_2 \cdot k} \right],
\]
\[
(C) = \frac{1}{2} \left[ -\frac{e_3}{p_3 \cdot k} - \frac{e_1}{p_1 \cdot k} - \frac{e_4}{p_4 \cdot k} + \frac{e_2}{p_2 \cdot k} \right].
\]

\[ G(k) \text{ and } H(k) \text{ are propagator factors defined as,} \]
\[ G(k) = \frac{1}{2} \left[ \frac{e_3}{p_3 \cdot k} + \frac{e_1}{p_1 \cdot k} - \frac{e_4}{p_4 \cdot k} - \frac{e_2}{p_2 \cdot k} \right], \]
\[ H(k) = \frac{1}{2} \left[ -\frac{e_3}{p_3 \cdot k} - \frac{e_1}{p_1 \cdot k} - \frac{e_4}{p_4 \cdot k} + \frac{e_2}{p_2 \cdot k} \right]. \]  

\[ p_i \cdot k = p_{i \mu} k^\mu, \text{ and } e_i \text{ are quark charges in electron units.} \]

The terms with coefficients
\[ P_\tau = \frac{k^\mu}{2 \lambda} \left[ -\frac{5}{m_1^2} + \frac{1}{m_1} \right] \text{ and } Q_\tau = \frac{k^\mu}{2 \lambda} \left[ \frac{-5}{m_3^2} + \frac{1}{m_3} \right]. \]  

where \( m_i \) are the constituent quark masses with \( m_u = m_d = 0.336 \text{ GeV}, \) \( m_s = 0.54 \text{ GeV} \) and \( m_c = 1.5 \text{ GeV}, \) arise through the quark momenta \( p_i \) integration. The propagation factors in Eqn. 5.12 are approximated by using
\[ \frac{1}{p_i \cdot k} = \frac{1}{m_i k}. \]  

In first order, i.e., if terms of the order of \( O(k^2) \) and \( O(\xi^2) \) are dropped, the coefficients of the quark operators are reduced to,
\[ A = [-1 + \xi - X(2 + \xi)] \]
\[ B = [1 - \xi + X(2 + 4 \xi)] \]
\[ C = [\xi + X(3 - 4 \xi)] \]
\[ D = [-\xi - X(3 - \xi)] \]  
with \( X = k_0 / 12m_\mu \) and the SU(3) breaking parameter \( 6 \xi = (1 - m_u / m_s). \)
In the 2QP, the one-loop QCD correction to the W- exchange diagrams [14] show negligible effects on the radiative weak decays. In addition, one may expect contribution from the internal radiative process where the photon is emitted by the W- boson Fig.5.2. This however has been found to be suppressed [19] by a factor $m_u k/m_w^2 \approx 10^{-5}$ as compared to the bremmstrahlung process. Similarly the 3QP involving W- exchange between the two quarks and photon emission by the third quark is also suppressed because of the low probability for adequate kinematic matching.

### 5.3 DECAY RATE AND ASYMMETRY METHODOLOGY.

The gauge invariant form of the radiative weak decay amplitude $B \rightarrow B' + \gamma$ is written as,

$$ M = \frac{e G_F}{\sqrt{2}} B' (F_1 + F_2 \gamma_5) \not k \not i B $$

where $k^\mu$ and $\epsilon^\mu$ are the momentum and polarisation vectors of the emitted photon, $B$ and $B'$ are the Dirac spinors for the initial and final state baryons and $F_1$ and $F_2$ are the pc and pv decay amplitudes. The decay rates are then given by,

$$ \Gamma (B \rightarrow B' \gamma) = \frac{e^2 G_F^2}{2\pi} \left[ |F_1|^2 + |F_2|^2 \right] k^3 $$

and the asymmetry is,

$$ a (B \rightarrow B' \gamma) = \frac{2 \text{Re}(F_1 F_2^*)}{|F_1|^2 + |F_2|^2} . $$

The baryon decay amplitudes, expressed in terms of $F_1$ and $F_2$, are extracted in the spirit of our earlier works [9-11]. Using non-relativistic reduction, eqn.5.16 takes the form
\[ M = \frac{e G_F}{\sqrt{2}} B_i^\dagger \left( F_1 \vec{\sigma} \cdot \vec{r} \cdot \vec{k} + F_2 k \vec{\sigma} \cdot \vec{r} \right) B_i, \] (5.19)

where \( B_i \) and \( B_i^\dagger \) are now Pauli spinors. Considering the two helicity states of the photon one gets

\[ M (\lambda_\gamma = \pm 1) = \frac{e G_F}{\sqrt{2}} k (\pm F_1 - F_2) B_i^\dagger a + B_i. \] (5.20)

Thus

\[ \mathcal{A}^{PC(p\gamma)} = \langle \gamma (\lambda_\gamma = +1), B_i \downarrow \left| H_{PV}^{PC(p\gamma)} \right| B_i \uparrow \rangle \propto k F_1 (-k F_2). \] (5.21)

### 5.4 CONSTITUENT QUARK MODEL

#### 5.4.1 Branching ratios and decay asymmetries

The parity violating and parity conserving decay amplitudes are then obtained by sandwiching the effective weak Hamiltonian given in Eqn.5.10, between the initial and final state constituent quark wave-functions. If we consider the two quark W-exchange process to be dominant, the model immediately predicts \( \Sigma^+ \to p\gamma \) decay asymmetry to be \(-0.30\), which though small in magnitude but is right in sign. This is in conflict with the Hara theorem, since even in the SU(3) limit \( \xi \to 0 \), the \( p\gamma \) decay amplitude for \( \Sigma^+ \to p\gamma \) does not vanish. This astonishing result has been explained by Kamal and Riazzuddin [19]. They have shown that the effective electroweak Hamiltonian can be expressed as

\[ H_{eff}^+ f F^{\mu\nu} J_{\mu\nu} + g \tilde{F}^{\mu\nu} J_{\mu\nu} \] (5.22)

where \( J_{\mu\nu} = [\bar{u} \gamma_\mu (1 - \gamma_5) s \gamma_\nu (1 - \gamma_5) u - (\mu \leftrightarrow \nu)] \) and \( F^{\mu\nu} \) and \( \tilde{F}^{\mu\nu} \) are the electromagnetic field tensor and its dual. The operators generated, to lowest order, by the first term, i.e.,

\[ p\gamma: (u^+ \vec{\sigma} \cdot \vec{r} s d^+ u - u^+ s d^+ \vec{\sigma} \cdot \vec{r} u) \to \langle p\gamma | \Sigma^+ >_{p\gamma} = 0 \]
\[ p_c: (u^+ \vec{u} \cdot \vec{e} s d^+ \vec{d} \cdot \hat{k} u - u^+ \vec{u} \cdot \hat{k} s d^+ \vec{d} \cdot \vec{e} u) \rightarrow < p \gamma | \Sigma^+ >_{p_c} = 0 (5.23) \]

respects Hara theorem, whereas the operators generated by the second term i.e.,

\[ p_v: (i \epsilon \cdot u^+ \vec{d} s \times d^+ \vec{d} u) \rightarrow < p \gamma | \Sigma^+ >_{p_v} \neq 0 \]

\[ p_c: (u^+ i \vec{u} \cdot \vec{e} \times \hat{k} s d^+ u - u^+ s d^+ i \vec{u} \cdot \vec{e} \times \hat{k} u) \rightarrow < p \gamma | \Sigma^+ >_{p_c} \neq 0 (5.24) \]

violate it. It can be seen from the coefficient given in Eqn. 5.11, that the operators which do not satisfy the Hara theorem, in fact, dominate over those which do.

The two quark transitions involve a common scale factor for both the pv and pc modes. Fixing the \( \Sigma^+ \rightarrow p \gamma \) branching ratio, other decay ratios are calculated [13] as given in col. (iii) of Table 5.2. Since the \( \Xi^-\) (dss) and \( \Omega^-\) (sss) hyperons do not contain u quark, their decay cannot occur through the W-exchange diagrams. For other decays, the predicted values are in nice agreement with experiment. Notice that the observed value of \( B(\Xi^- \rightarrow \Sigma^- \gamma) \) is an order of magnitude less than the \( \Delta^- \) decays, thereby indicating the dominance of W-exchange mechanism. Single quark and penguin diagram are the only possible mechanisms for these decays [20]. Eeg [31] and Kamath [32] have observed that the penguin contributions are even smaller than the single-quark transition. So, in order to survive \( \Xi^- \) and \( \Omega^- \) decays, we include the single-quark transition. In view of the uncertain state in the determination of the parameters a and b, we take \( b/a = +1 \) from Eqn. 5.5 and determine the pc parameter a from the \( \Xi^- \rightarrow \Sigma^- \gamma \) branching ratio. Results thus obtained for all the decays are presented in Tables 5.2 and 5.3. The calculated branching ratios are consistent with experimental values. Asymmetry for \( \Sigma^+ \rightarrow p \gamma \) gets doubled to -0.56 and \( \Lambda^+ \rightarrow n \gamma \) asymmetry changes sign in the presence of the single-quark transition. The recently observed \( \Xi^0 \rightarrow \Lambda + \gamma \) decay asymmetry are larger than the experimentally observed values, though of the correct sign.
whereas the $\Xi^0 \rightarrow \Sigma^0 + \gamma$ asymmetry is very badly off from the experimental observation. Either a new measurement for this decay or some new physics is required. Measurement of $\Xi^- \rightarrow \Omega^- + \gamma$ decay asymmetries would provide further clarification. In the next section we consider the effects of strong interaction modifications to the baryon wave-function, which may arise due to the presence of a gluon.

5.5 SU(6) BROKEN QUARK-DIQUARK MODEL AND BARYON WAVEFUNCTIONS

For phenomenological treatment in the low momentum processes, the notion of a diquark representing the quark-quark correlations seems appropriate and has already been widely used. The essential idea of the diquark model is that among the three valence quarks of a baryon, there exists an apparent preferential configuration based on geometric and energetic considerations, such that two of the quarks are closer together in space than in random motion, i.e., in the baryon there seems to exist a more tightly bound two-quark system (the diquark) and a third quark (spectator). A diquark with two identical quarks can appear only in spin 1 state, however, if there are two different quarks then they may exist in spin 0 or spin 1 states. In the limit of exact SU(6) symmetry, the probability amplitudes for a diquark in a baryon to be in spin 1 or spin 0 states are governed by the C.G. coefficients and are seen to have an equal weightage. The possible role of a gluon as a constituent inside the baryon may, however, lead to SU(6) breaking [26]. Thus the quark-gluon interaction may change the probability of finding the diquark in spin 1 and spin 0 states thereby causing SU(6) symmetry breaking in the baryon wavefunctions. For instance, a spin 1 diquark may emit a gluon to become a spin 0 diquark and similarly for a spin 0 diquark to become a spin 1 diquark it must emit a gluon from its quark configuration. In the absence of the knowledge of
strong dynamics, it may be parameterized through the mixing in the follow­
ing manner [16,27];

\[ |ab > 0 = |ab > 0 \cos \varphi_{ab} + |ab > 1 \sin \varphi_{ab} \]

\[ |ab > 1 = |ab > 1 \cos \varphi_{ab} - |ab > 0 \sin \varphi_{ab} \]  \hspace{1cm} (5.25)

where the subscript denotes spin of the diquark made up of 2 different
quarks a and b. The parameter \( \varphi_{ab} \) contains all the dynamics and in the
limit \( \varphi_{ab} \rightarrow 0 \) the SU(6) symmetry is regained. For octet baryons, we have
three diquark pairs ud, us and ds with their corresponding mixing angles
\( \varphi_{ud}, \varphi_{us}, \text{and } \varphi_{ds} \) in the baryon wavefunctions. Invoking the isospin invariance
we get

\[ \varphi_{us} = \varphi_{ds} = \varphi. \]  \hspace{1cm} (5.26)

Further we fix \( \varphi_{ud} = 0 \) to preserve the SU(6) successes in the nucleon sec­
tor. The SU(6) broken baryon wavefunctions thus constructed are given in
the Appendix 5.1.

5.6 DETERMINATION OF DECAY ASYMMETRIES
AND BRANCHING RATIOS

Decay Amplitudes

The decay amplitudes are calculated for both the single- and two-quark
processes. The diquark-diquark transitions occurring under the angular
momentum conservation proceed via the following spin modes:

\[ |1, 1 \rangle \rightarrow |1, 0 \rangle + \gamma \]

\[ |1, 1 \rangle \rightarrow |0, 0 \rangle + \gamma \]

\[ |0, 0 \rangle \rightarrow |1, -1 \rangle + \gamma \]  \hspace{1cm} (5.27)
5.6.1 Single-Quark Contribution

The single-quark transition may occur between diquarks or through the isolated quark $s \rightarrow d + \gamma$. The decay amplitudes are obtained by sandwiching the $H^{1q}_w$ Eq.5.3 between the initial and final state baryons, and summing the contributions from both we get the matrix elements as shown in Table 5.3.

5.6.2 W-Exchange Contribution

The two-quark transition proceeds through the diquark-diquark mode. Matrix-elements at the diquark level for the pc and pv modes are obtained by sandwiching the $H^{2q}_w$ Eqn.5.10, between the diquark-diquark states. The contribution from the different spin modes are given in Table 5.4, which clearly indicates that the dominant contribution occurs through the spin-flip transitions i.e. $(1 \leftrightarrow 0)$. The spin $1 \rightarrow 1$ contribution appear only when symmetry breaking and relativistic corrections are simultaneously imposed.

To evaluate the decay amplitudes at the baryonic level we illustrate for $\Sigma^+ \rightarrow p + \gamma$ decay. Using $\Sigma^+$ and $p$ wavefunctions given in the Appendix 5.1, the two-quark decay amplitude is expressed as,

$$< p \gamma | H_w | \Sigma^+ > = \frac{1}{9 \sqrt{2}} [ < S^2_2 \gamma | H_w | t_2 > (3 \cos \varphi + \sqrt{3} \sin \varphi )$$

$$+ \{ < S^2_2 \gamma | H_w | S^2_2 > -3 < t_1 \gamma | H_w | S^2_1 > \} ( \cos \varphi - \sqrt{3} \sin \varphi ) ] \quad (5.28)$$

Substituting the diquark level transition amplitudes from Table 5.4 we derive the pc and pv amplitudes as,

$$< p \gamma | H_w^{pc} | \Sigma^+ > = \frac{2}{3} \left[ \cos \varphi (\zeta + X (3 - 2 \zeta)) + \frac{\sin \varphi}{\sqrt{3}} (2 - 3 \zeta + X (1 + 6 \zeta)) \right] ,$$

and

156
\begin{equation}
\langle \rho \gamma | H^{\rho \gamma} | \Sigma^+ \rangle = \frac{2}{3} \cos \varphi (1 + \xi - \lambda (2 + 3 \xi)) + \frac{\sin \varphi}{\sqrt{3}} (1 - 3 \xi + \lambda (-4 + 9 \xi))
\end{equation}

Similarly we calculate the decay amplitudes for the other weak decays, which are shown in Table 5.5. The baryon decay amplitudes are extracted in the spirit of the works [12,13,15] after ensuring that there exists necessary kinematic matching between the constituents and baryons. This is required because the baryon B is taken as split into a quark and a diquark, which interact in the intermediate state and emit a photon to give rise to the final state baryon. This requirement introduces scale factors $l_{1q}$ (dimensionless) for the 1QP and $l_{2q}$ (dimension $\text{GeV}^3$) for the 2QP. These scale factors are derived on solving the space part of the matrix elements as given in Appendix of Ref.[12]. The $l_{1q}$ which depends upon the initial and final state wave-function is normalised to unity for $k \to 0$, and is assumed not to depend on the photon momentum $(k)$ significantly [9]. This is justified in view of the successful predictions of $\Delta^+ \to \rho + \gamma$ and $\Sigma^0 \to \Lambda + \gamma$ decays in the constituent quark model framework. However we may remark here that the magnitude of $l_{1q}$ is not explicitly required as it gets absorbed into the free parameters $a$ and $b$. The factor $l_{2q}$ varies very slowly with $k$. We find that the ratio of the scale factors $l_{2q}(k)/l_{1q}(k)$ is independent of $k$.

5.7 NUMERICAL RESULTS AND DISCUSSIONS

We combine the single- and two-quark transition amplitudes as discussed in the previous section after matching their scales appropriately. In view of the short distance QCD corrections due to gluonic exchange we start by
taking the b/a ratio = +1 as given in Eqn 5.5. Further as 
\( \Xi^- \rightarrow \Sigma^- + \gamma \) decay can proceed only via a 1QP we fix the pc parameter 'a' from it, such that \( a = -3.0 \times 10^{-2} \text{GeV} \) indicating significant strong interaction effects on the \( s \rightarrow d + \gamma \) vertex. The scale for the 2QP is fixed from the \( B(\Sigma^+ \rightarrow p + \gamma) \) as \( c \equiv |l_{2q}| / m_u = 1.26 \times 10^{-2} \text{GeV}^2 \). The Branching Ratios and Asymmetries thus obtained are given in Table 5.6.

Here we notice that \( B(\Xi^0 \rightarrow \Sigma^0 + \gamma) \) is in good agreement with the experimental results. \( B(\Lambda \rightarrow n + \gamma) \) is a little below the lower limit and \( B(\Xi^0 \rightarrow \Lambda + \gamma) \) is significantly low. Regarding the asymmetries, though \( \alpha(\Sigma^+ \rightarrow p + \gamma) = -0.63 \) is of correct sign, it is still below the lower limit. \( \alpha(\Xi^0 \rightarrow \Lambda + \gamma) \) is found to be +1 as a consequence of taking \( b/a = +1 \), to be compared with experimentally observed value \( (0.43 \pm 0.44) \). The asymmetry of \( \Xi^0 \rightarrow \Sigma^0 + \gamma \), contrary to the observed small and positive value, is found large and negative. Here we would like to remark that this asymmetry has been calculated to be in the range \((-0.8 \text{ to } -1.0)\) [28] in all the theoretical attempts made so far.

Thus we proceed to introduce SU(6) symmetry violation through our model and study the variation of the branching ratios and asymmetries w.r.t. the b/a ratio and SU(6) violating parameter \( \varphi \). Also as branching ratio, \( B(\Xi^- \rightarrow \Sigma^- + \gamma) \) is a first measurement we include its variation from zero to its earlier upper limit \( 1.2 \times 10^{-3} \) [29] and see its effect on the weak radiative decay parameters. Analysing Fig.5.3, which shows the variation of \( B(\Lambda \rightarrow n + \gamma) \) vs. b/a for different SU(6) breaking angles \( \varphi \), we notice \( B(\Lambda \rightarrow n + \gamma) \) increases as b/a decreases and that for the SU(6) symmetric case \( \varphi = 0 \) agreement with experimental results is obtained for b/a = +0.4. The presence of SU(6) breaking further lends credence to this trend. We may understand this lowering of b/a from +1 as an effective value reached due to the combined short [11]- and long [30]- distance
strong interaction effects. Moreover as all other decays give us consistent results for this choice we henceforth fix $b/a = +0.40$ in our calculations.

$B(\Xi^0 \rightarrow \Lambda + \gamma)$ is seen to increase significantly in the presence of SU(6) breaking, and is more or less independent of $b/a$ variation. Its variation with $\varphi$ and $B(\Xi^- \rightarrow \Sigma^- + \gamma)$ is shown in Fig.5.4. It seems to require $\varphi = 15^\circ$ to match experimental results. However, increase in $B(\Xi^- \rightarrow \Sigma^- + \gamma)$ adversely makes it fall, significantly so at larger angles. Similarly Fig.5.5 shows the variation of $B(\Xi^0 \rightarrow \Sigma^0 + \gamma)$ with $\varphi$ and $B(\Xi^- \rightarrow \Sigma^- + \gamma)$. It shows a tendency to increase with increasing $B(\Xi^- \rightarrow \Sigma^- + \gamma)$. These trends lead us to conclude that a SU(6) breaking angle $>20^\circ$ can be ruled out.

The Asymmetry $\zeta(\Sigma^+ \rightarrow p + \gamma)$ is seen to move in the desired direction with increase in $\varphi$ to a saturating maxima of -1. As $\kappa(\Sigma^+ \rightarrow p + \gamma)$ is insensitive to $b/a$ variation, we show the effects of $\varphi$ and $B(\Xi^- \rightarrow \Sigma^- + \gamma)$ on it for $b/a =0.4$ in Fig.5.6. It can be seen that for SU(6) breaking angle $\varphi = 10^\circ$ it lies in the range -0.89 to -1.0 which is in good agreement with the experimental observations. The variation of $\alpha(\Xi^0 \rightarrow \Lambda + \gamma)$ with $b/a$ is minimal for low $B(\Xi^- \rightarrow \Sigma^- + \gamma)$ but shows a marked decrease as $B(\Xi^- \rightarrow \Sigma^- + \gamma)$ is increased, preferring a $B(\Xi^- \rightarrow \Sigma^- + \gamma)$ around $0.5 \times 10^{-2}$ to match with the experimental result Fig.5.7. Last of all $\alpha(\Xi^0 \rightarrow \Sigma^0 + \gamma)$, which has been the most challenging of them all, is seen to decrease in the desired direction Fig.5.8 with increase in $\varphi$ as well as with increase in $B(\Xi^- \rightarrow \Sigma^- + \gamma)$. However, this fall is insufficient to touch the experimental limits. The small negative or positive $\alpha(\Xi^0 \rightarrow \Sigma^0 + \gamma)$ demands simultaneously a large breaking $\varphi = 30^\circ$ and extremely large $B(\Xi^- \rightarrow \Sigma^- + \gamma)$ nearly equal to $B(\Sigma^+ \rightarrow p + \gamma)$ as seen in Fig. 5.9. Not only are such large values highly
unlikely, they also ruin other results drastically. Therefore a re-measurement of this $\alpha(\Xi^0 \to \Sigma^0 + \gamma)$ is desirable. Results for our choice of parameters SU(6) breaking angle $\varphi = 10^\circ$, b/a ratio of $+0.4$ and $B(\Xi^- \to \Sigma^- + \gamma) = 0.5 \times 10^{-3}$ are presented in Table 5.7.

5.8 Weak Radiative Decays of Charm Baryons

5.8.1 INTRODUCTION

Previous theoretical attempts to study charm baryons are mostly limited to the weak mesonic modes [33]. Strong interaction interference effects between different processes which are prevalent among these modes, casts a shadow on the exact contribution of each process. Final state interactions (FSI) among the hadrons further complicate the situation. Our earlier analysis of charm baryonic nonleptonic weak decays has shown that the pole or $W$-exchange diagrams are certainly important. This dominance was confirmed by the previous section on the hyperon weak radiative decays. As a lot of data is expected in the near future on charm baryon decays [34,35] we have chosen to study the weak radiative modes of these particles. We have extended the analysis to the weak radiative decays of charm baryons, as the $B \to B' \gamma$ weak radiative modes can provide a direct estimate for the $W$-exchange process. In contrast to the strangeness changing hyperon radiative decays, where though the $W$-exchange mechanism is found to be dominant [9-12], single-quark processes can also contribute, for the charm changing $\Delta C = \pm \Delta S \pm 1$ modes, the decay occurs only through a 2Q $W$-exchange process. The mode $\Delta C = -1, \Delta S = 0$ may in addition have some contribution from the single quark process (1QP), but it is expected to be highly suppressed due to the GIM mechanism.
5.9 WEAK RADIATIVE HAMILTONIAN IN THE IMPROVISED QUARK MODEL

The basic features of the charm changing decays are:

**Cabibbo Enhanced**
\[ \Delta C = \Delta S = 1 \quad (c + s \rightarrow d + u + \gamma) \quad \alpha \cos^2 \theta_c \]

**Cabibbo Suppressed**
\[ \Delta C = -1, \Delta S = 0 \quad (c + s \rightarrow s + u + \gamma) \quad (c + d \rightarrow d + u + \gamma) \quad \alpha \cos \theta_c \sin \theta_c \]

**Cabibbo Doubly Suppressed**
\[ \Delta C = -\Delta S = -1 \quad (c + s \rightarrow u + d + \gamma) \quad \alpha = -\sin^2 \theta_c \quad (5.30) \]

Looking at the quarks participating in the various processes, it is obvious that the Cabibbo enhanced decays cannot proceed via the W-loop diagram and hence are free from Penguins and other single quark transitions. The W-exchange Hamiltonian for the respective processes can be directly obtained from the general expression given in Eqn. 5.10, with the appropriate replacement of quarks.

Since the photon momentum in the charm changing baryon decays is large, the use of the naive non-relativistic reduction of G(k) and H(k) through a p/m expansion Eqn.5.11 as done for the hyperon decays, may be incorrect. We therefore follow the "Improvised Quark Model" [36,37], and approximate the propagators G(k) and H(k) using,
\[
\frac{1}{p_i \cdot k} = \frac{1}{E_i k - p_i \cdot k} = \frac{1}{E_i k}.
\]

where we replace the quark energy \( E_i \) by its average value, i.e.,
\[ E_i = (m_i^2 + \langle p_i^2 \rangle)^{1/2} \quad (5.32) \]
This then allows an expansion in the parameter \( p/E \). Physically, one would expect this to be a better approximation than a \( p/m \) expansion, as the average energy \( E_i \) of the quark has a dependence on the momentum of the final baryon, as well as on the cut-off \( \alpha \) provided by the wave-functions. Using the harmonic oscillator (h. o.) wavefunctions for the baryons, we evaluate \( \langle \vec{p}^2 \rangle \) as shown in the Refs. [13,14], and obtain

\[
E_{1,2} = \left( m_{1,2}^2 + \frac{7}{4} \alpha^2 + \frac{k^2}{144} \right)^{\frac{1}{2}},
\]

\[
E_{3,4} = \left( m_{3,4}^2 + \frac{7}{4} \alpha^2 + \frac{25k^2}{144} \right)^{\frac{1}{2}},
\]

where \( \alpha^2 \) is the h. o. parameter.

In addition, one may expect contribution from the internal radiative process where the photon is emitted by the W- boson. This like hyperon decays, have also been found to be suppressed by a factor \( m_u k/m_m^2 \approx 10^{-5} \) as compared to the bremsstrahlung process [19].

5.10 Decay Amplitudes

Among the \( J^P = 1/2^+ \) charmed baryons comprising the \( 20' \) multiplet of SU(4), only the members of the SU(3) submultiplets \( 3^*, 3, \) and \( \Omega_c^0 \) of 6 decay weakly. The remaining decay strongly or radiatively to \( 3^* \). Masses of most of the \( C=1 \) baryons have been experimentally measured [34], and the remaining are taken from the theoretical estimate based on a central two-body potential supplemented by the spin-spin interaction resulting from the Breit-Fermi reduction of the one-gluon-exchange contribution (Table 5.8). We illustrate evaluation of the decay amplitudes at the baryonic level for the \( B(3^*) \to B'(8) + \gamma \), in the Cabibbo enhanced mode. Using
Eqn.5.10 and the quark model wavefunctions [38] we derive the pc and pv amplitudes for the decays of $\Lambda_c^+ \to \Sigma^+ + \gamma$ and $\Xi_c^0 \to \Xi^0 + \gamma$. These up to an overall scale factor $(eG_F/\sqrt{2}) \cos^2 \theta_c$ are

\[
\begin{align*}
\langle \Sigma^+ \gamma | H_{w}^{pc} | \Lambda_c^+ \rangle &= \frac{2}{\sqrt{6}} \left[ H(k) - 2G(k) - X \left\{ (2 + 3\xi_c - 30\xi_s) G(k) + (8 - 6\xi_c) H(k) \right\} \right] \\
\langle \Sigma^+ \gamma | H_{w}^{pv} | \Lambda_c^+ \rangle &= \frac{2}{\sqrt{6}} \left[ 2H(k) - G(k) + X \left\{ (2 + 3\xi_c - 30\xi_s) H(k) + (8 - 6\xi_c) G(k) \right\} \right] \\
\langle \Xi^0 \gamma | H_{w}^{pc} | \Xi_c^0 \rangle &= -\frac{2}{\sqrt{6}} \left[ H(k) - 2G(k) - X \left\{ (2 + 3\xi_c - 30\xi_s) G(k) + (8 - 6\xi_c) H(k) \right\} \right] \\
\langle \Xi^0 \gamma | H_{w}^{pv} | \Xi_c^0 \rangle &= -\frac{2}{\sqrt{6}} \left[ 2H(k) - G(k) + X \left\{ (2 + 3\xi_c - 30\xi_s) H(k) + (8 - 6\xi_c) G(k) \right\} \right] 
\end{align*}
\]

where $X = \frac{k^0}{24m_u}$, and the flavor symmetry breaking parameters are denoted by $6\xi_s = (1 - m_u/m_s)$ and $3\xi_c = (1 - m_u/m_c)$.

In the same manner we calculate the decay amplitudes for the other weak decays, which are shown in Table 5.9. Following the procedure described for the hyperons, to determine the baryon decay amplitudes for the parity conserving and parity violating contributions.

5.11 Determination of Decay Asymmetries and Branching Ratios

To calculate the decay rate, we use the harmonic oscillator wave functions for baryons [12,36] to get,

\[
\Gamma (B \to B' \gamma) = \frac{e^2 G_F^2 \text{Cabibbo factors}}{2\pi} \frac{1}{2} k \left[ \left| A_{pc}^B \right|^2 + \left| A_{pv}^B \right|^2 \right] (l_{2q})^2 \exp \left( -\frac{k^2}{12\pi^2} \right).
\]

(5.35)
Ensuring necessary kinematic matching between the constituents and baryons, introduces a scale factor \( l_{2q} \) (dimension GeV\(^3\)) for the 2QP, and is given by [9,13]

\[
l_{2q} = \delta (\Sigma \vec{p}_1 - \Sigma \vec{p}_f - \vec{k}^*) (2\pi \hbar^2 \alpha)^3.
\] (5.36)

This scale factor \( l_{2q} \), which corresponds to the spatial matrix element \( |\psi(0)|^2 = \langle \psi | \delta^3(r_1 - r_2) | \psi \rangle \) is as yet uncertain for baryons. Its evaluation is complicated because unlike the mesons these are three-body systems. Further the harmonic oscillator potential gives good results only for peripheral processes, but is not realistic for a central quantity like \( |\psi(0)|^2 \). A relative scale can be estimated using the hyperon radiative decays.

For the purely 2QP, the ratio

\[
\frac{\Gamma (\Lambda_c^+ \rightarrow \Sigma^+ \gamma)}{\Gamma (\Sigma^+ \rightarrow p \gamma)} = \frac{\cos^2 \theta_c \times k_{\Lambda_c}^+ \times \frac{|A^0_{\Lambda_c}|^2 + |A^+_{\Lambda_c}|^2}{|A^0_{\Sigma}|^2 + |A^+_{\Sigma}|^2} \times \exp\left( -\frac{k_{\Lambda_c}^2 - k_{\Sigma}^2}{12 \alpha^2} \right)} = 14.9
\]

\[
\frac{\Gamma (\Xi_c^0 \rightarrow \Xi^0 \gamma)}{\Gamma (\Sigma^+ \rightarrow p \gamma)} = \frac{\cos^2 \theta_c \times k_{\Xi_c}^+ \times \frac{|A^0_{\Xi_c}|^2 + |A^+_{\Xi_c}|^2}{|A^0_{\Xi}|^2 + |A^+_{\Xi}|^2} \times \exp\left( -\frac{k_{\Xi_c}^2 - k_{\Xi}^2}{12 \alpha^2} \right)} = 15.1
\]

(5.37)

where \( \alpha^2 = \frac{1}{6} \text{ GeV}^2 \) is fixed from the excitation energy of hyperons [39].

Using \( B (\Sigma^+ \rightarrow p \gamma) = 1.25 \times 10^{-3} \) as an input [34], we obtain,

\[ B (\Lambda_c^+ \rightarrow \Sigma^+ \gamma) = 4.54 \times 10^{-3} \%, \quad \alpha (\Lambda_c^+ \rightarrow \Sigma^+ \gamma) = -0.013 \]

\[ B (\Xi_c^0 \rightarrow \Xi^0 \gamma) = 1.93 \times 10^{-3} \%, \quad \alpha (\Xi_c^0 \rightarrow \Xi^0 \gamma) = -0.042 \]

(5.38)
Naively the branching ratio may have been expected to be larger due to the \( \cos^2 \theta_c \) factor as well as the large photon momentum. These effects are counteracted by the gaussian form-factor appearing through the spatial integral, which amounts to having an extra fine-interaction radius term.

In the above estimate we have considered the spatial overlap to be the same for the strangeness and charm changing modes i.e. \( |\psi(0)|^2 \approx 6.3 \times 10^{-3} \text{GeV}^3 \). However \( |\psi(0)|^2 \) being a dimensional quantity it may be incorrect to ignore its variation with flavor. Evidence to corroborate this is found in quark model [39,40] as well as in lattice calculations [41]. Infact the charm baryons may provide a good and perhaps even dramatic way of testing the flavor dependence of the confinement forces. The absence of a dynamical theory of interactions between quarks limits our evaluation of \( |\psi(0)|^2 \) from first principles. Hence we make a naive estimate for the scale parameter by using the hyperfine splitting (HFS), i.e.

\[
\Delta \text{HFS} = \frac{4\pi \alpha_s}{9 m_1 m_2} |\psi(0)|^2 <a_1 \cdot a_2>
\]  

(5.39)

as explained in Chapter 4. Looking at the uncertainty in \( \Lambda_{QCD} \) (100-300 MeV.) the ratio

\[
\frac{|\psi(0)|^2}{|\psi(0)|^2_c}
\]

(5.41)

may be as high as 2.8, which is still consistent with the estimates of (2 - 3) given by lattice calculations [41].

In addition, the h. o. parameter \( \alpha \) is also expected to have flavor dependence, increasing for heavier quark systems. Following the analysis of Copley, Isgur and Karl [40] for evaluating the excitation energies, we find \( \alpha^2 \approx \frac{1}{3} \) for charm baryons. The net effect of the above scaling is to enhance the ratios in Eqn. (5.37) to yield,
\[ B(\Lambda_c^+ \rightarrow \Sigma^+ \gamma) = 2.91 \times 10^{-2} \%, \quad a(\Lambda_c^+ \rightarrow \Sigma^+ \gamma) = 0.023 \]
\[ B(\Xi_c^0 \rightarrow \Xi^0 \gamma) = 1.26 \times 10^{-2} \%, \quad a(\Xi_c^0 \rightarrow \Xi^0 \gamma) = -0.010 \]  

The asymmetries remain almost unaffected by variation in the scale. The amplitude calculations for these decays indicate the major contribution to be via the parity violating mode. It may be remarked here, that if the propagators \( G(k) \) and \( H(k) \) in Eqn.(5.9) are approximated using a naive nonrelativistic \( p/m \) expansion, as done for the strange baryons in Section 5.2, Eqn 5.14 and earlier in [12,13], the results are interestingly significant, with the asymmetry parameters showing a dramatic change. The asymmetry parameters for both \( a(\Lambda_c^+ \rightarrow \Sigma^+ \gamma) \) and \( a(\Xi_c^0 \rightarrow \Xi^0 \gamma) \) are enhanced by an order of magnitude to -0.27 and -0.30 respectively, with an enhanced parity conserving contribution. An experimental observation of these results will provide a clearer insight into the validity of the improvised quark model scheme as compared to the naive nonrelativistic approach.

Branching Ratios and Asymmetry parameters are similarly calculated for all Cabibbo enhanced, suppressed and doubly suppressed decay modes and are given in Table 5.10.

5.11.1 Inclusion of the Single Quark Transition

The presence of the single quark transitions, though weaker in strength than the W-exchange process, has been well established in the hyperon decays. In the above estimate Eqns.(5.37,5.38) we have fixed the scale for \( \Lambda_c^+ \rightarrow \Sigma^+ \gamma \) using \( \Sigma^+ \rightarrow p \gamma \), arising dominantly from the 2QP, and have neglected the possible single quark contribution to it. While for the Cabibbo enhanced decays there is no explicit single quark transition, yet it can affect them indirectly, through changing the overall scale for W-exchange fixed from \( \Sigma^+ \rightarrow p \gamma \).
In this section we study this effect by varying the ratio $b/a$ in its extreme limits, from -1 to +1, in the single quark weak Hamiltonian Eqn.5.5. Using $B(\Xi^- \to \Sigma^- + \gamma)$ and $B(\Sigma^+ \to p\gamma)$ as inputs we find $a = (-3.25 \pm 4.60) \times 10^{-2}\text{GeV}$ and that $I_{2\mu} \ll 6\text{GeV}^3$, shown in Fig.5.8. This then yields the Cabibbo enhanced modes in the range:

\[ B(\Lambda_c^+ \to \Sigma^+ \gamma) = (2.19 - 3.40) \times 10^{-2}\% \]

\[ B(\Xi^0_c \to \Xi^0 \gamma) = (0.96 - 1.48) \times 10^{-2}\% \]

which were earlier calculated in Eqn. 5.38.

In the charm changing decays, the single quark transition $c \to u$ cannot occur in the $\Delta C = \pm \Delta S$ modes. For the $\Delta C = -1, \Delta S = 0$ mode also, the 1QP is essentially expected to be suppressed by the unitarity of the CKM matrix. In the Standard Model, the Feynman amplitude arising out of the matrix element for $c \to u \gamma$ is,

\[ H_W = \frac{e G_F \sin \theta_c \cos \theta_c}{\sqrt{2}} \bar{u}(a + b\gamma_5)K\sigma c, \]

where the parameters $a$ and $b$ govern the $pc$ and $pv$ contribution respectively.

The single quark contribution in the charm changing decays proceeds via the $c \to u + \gamma$ transition. This process which occurs through SU(3) breaking is expected to be suppressed by at least an order of magnitude than the corresponding $s \to d + \gamma$ transition in the strange baryons which is occurring due to SU(4) breaking. This single quark transition which is present only in the suppressed charm changing decays is neglected for the present analysis. It may be remarked here that the decay $\Xi_{cc}^{++} \to \Sigma_c^{++} \gamma$ is forbidden by the W-exchange process, and can proceed only through a 1QP. The signal for this decay will be able to provide some estimate for the strength of the charm changing single quark transitions.
5.12 QCD MODIFICATIONS

As a consequence of introducing QCD short distance gluon exchange, the 4-Fermi interaction gets modified to

\[ H_{W}^{\text{QCD}} = \frac{e G_F \text{ Cabibbo factors}}{\sqrt{2}} \left[ c_1 \bar{q}_3 \Gamma_\mu q_1 \bar{q}_4 \Gamma^\mu q_2 + c_2 \bar{q}_4 \Gamma_\mu q_1 \bar{q}_3 \Gamma^\mu q_2 \right] \] (5.45)

where \( \Gamma_\mu = \gamma_\mu (1 - \gamma_5) \) and \( c_1, c_2 \) represent combinations of the QCD coefficients \( c_- \) and \( c_+ \) given in Eqn 2.57. In the leading log approximation these are given by Eqn. 2.30,

\[ c_\pm (\mu) = \left[ \frac{\alpha_s (\mu^2)}{\alpha_s (m_W)} \right] \frac{d_\pm}{2b} \] (5.46)

with \( d_- = -2d_+ = 8 \) and \( b = 11 - \frac{2}{3} N_f \), \( N_f \) being the number of flavors, \( \mu \) the mass scale and \( \alpha_s \) is the strong fine structure constant. The precise value of these QCD coefficients is difficult to assign, depending as they do on the mass scale and \( \Lambda_{\text{QCD}} \). Relative to the free field limit \( c_+ = c_- = 1 \), but at the charm mass scale, these quantities are estimated to give substantial enhancements for \( c_- = (1.3 - 2.1) \) and suppressions \( c_+ = (0.6 - 0.9) \) [33].

This then alters the Hamiltonian in Eqn.(5.10), which written in terms of the quark spin operators, and for a fixed photon helicity \( \lambda = +1 \), is given below, up to an overall scale \( \frac{e G_F \text{ Cabibbo factors}}{\sqrt{2}} | \hat{k} | \).

\[ H_{W,\text{QCD}}^{\text{PC}} = \left[ c_1 \left[ A q_3^+ \sigma_- q_1 q_4^+ q_2 + B q_3^+ q_1 q_1^+ \sigma_- q_2 + C q_3^+ \sigma_- q_1 q_4^+ \sigma_- q_2 \right] + D q_3^+ \sigma_3 q_1 q_4^+ \sigma_- q_2 \right] + c_2 \left[ A' q_4^+ \sigma_- q_1 q_3^+ q_2 + B' q_4^+ q_1 q_3^+ \sigma_- q_2 + C' q_4^+ \sigma_- q_1 q_3^+ \sigma_3 q_2 + D' q_4^+ \sigma_3 q_1 q_3^+ \sigma_- q_2 \right] \]
\[ H_{\text{QCD}}^{\mu\nu} = \left[ c_1 \left[ C q_3^+ q_- q_1 q_3^+ q_2 + D q_3^+ q_1 q_3^+ q_2 + A q_3^+ q_- q_1 q_3^+ q_2 + B q_3^+ q_3 q_1 q_3^+ q_2 \right] \right] \\
+ c_2 \left[ C' q_3^+ q_- q_1 q_3^+ q_2 + D' q_3^+ q_1 q_3^+ q_2 + A' q_3^+ q_- q_1 q_3^+ q_2 + B' q_3^+ q_3 q_1 q_3^+ q_2 \right] \] (5.47)

where \( A', B', C' \) and \( D' \) are
\[
A' = \left[ +G(k) - (Q' - H(k)) + P' + G(k) \right] \\
B' = \left[ -G(k) + (P' - H(k)) + Q' + G(k) \right] \\
C' = \left[ -H(k) + (P' + H(k)) + Q' - G(k) \right] \\
D' = \left[ +H(k) - (Q' + H(k)) + P' - G(k) \right] \\ (5.48)
\]

and \( P'_{\pm} = \frac{k_0}{24} \left[ \pm 5 \pm \frac{1}{m_2} \right] \) and \( Q'_{\pm} = \frac{k_0}{24} \left[ \pm 5 \pm \frac{1}{m_1} \right] \).

For the radiative weak decays, effect of these QCD modifications is to alter the decay amplitudes of charm baryons, by an overall scale of \( c_{\pm}(\mu) \). The presence of \( c_1 \) in the overall scale may be understood by noticing that the portion of the Hamiltonian corresponding to \( c_1 \) is symmetric in color indices and hence does not contribute. Here, for a choice of \( c_-(m_s) = 2.80 \) and \( c_+(m_s) = 1.70 \) determined using Eqn. 5.46, the decay rates for the charm changing modes are scaled down by a factor \( \left[ c_-(m_c)/c_-(m_s) \right]^2 = 0.40 \).

### 5.13 SUMMARY AND CONCLUSIONS

In this chapter the weak radiative hyperon decays have been investigated in the framework of the quark-diquark model which provides a natural mode to introduce SU(6) breaking. The contributions from single- and two-quark transition processes via the W-loop and W-exchange diagrams are studied. In addition, the penguin diagram may contribute, however the work of Eeg
[31] and Kamath [32] indicate clearly that this contribution is even less than the 1QP and hence may be neglected. Study at the diquark level with symmetry violations gives us clearer insight into the transition processes (at different levels) through various modes and shows that the main contribution occurs from the spin-flip transitions. The branching ratios and asymmetries are studied in detail with respect to variations of the SU(6) breaking angle $\phi$, the b/a ratio and the $B(\Xi^- \rightarrow \Sigma^- + \gamma)$ which has been used as an input. We find a marked improvement in most of the branching ratios and asymmetries for a SU(6) breaking of the order of $10^0$, b/a ratio +0.4. Further agreement may be obtained by increasing the $B(\Xi^- \rightarrow \Sigma^- + \gamma)$ to 0.50x10^{-5}. The major discrepancy lies with $a(\Sigma^0 \rightarrow \Sigma^0 + \gamma)$ which has been calculated to be large and negative around - 0.9 in all theoretical attempts made so far in contrast to the small positive experimental value 0.20±0.33. In the present scheme, its magnitude decreases as sought with the introduction of SU(6) violation. To obtain a consensus with experimental results, it seems to require a large $f$ and $G(\Xi^- \rightarrow \Sigma^- \gamma)$ nearly equal to that of $B(\Sigma^+ \rightarrow \rho + \gamma)$. As these large values are very unlikely and undesirable keeping other decays into consideration, a new measurement for $a(\Xi^0 \rightarrow \Sigma^0 + \gamma)$ is desired. The conclusive value for b/a can be got directly only if $a(\Xi^- \rightarrow \Sigma^- + \gamma)$ is determined experimentally as due to the quark content constraints it can only decay via the single-quark mode or a possible penguin diagram. Another test can be provided by the experimental determination of $a(\Lambda \rightarrow n + \gamma)$ which, is small and negative in the present scheme.

In the charm sector we have calculated the branching ratios and asymmetry parameters of the weak radiative decays in the Cabibbo enhanced, suppressed and doubly suppressed modes. Noticing the structure of the currents for these decays, shows that the Cabibbo enhanced mode can occur only through the W-exchange process, and there can be no W-loop
or its derivative penguins, since there exists no single quark transition mode. From the experimental viewpoint, the immediate interest lies in the decays of $\Lambda_c^+$ and $\Xi_c$, as with the new high statistical measurements and large event samples, these decays are expected to be observed soon. We find that the branching ratios for these decays are

$$\Lambda_c^+ \to \Sigma^+ \gamma = 0.005\%$$

and

$$\Xi_c^0 \to \Xi^0 \gamma = 0.002\%$$

and they have a null asymmetry. The single quark transitions can affect the $\Delta C = -1, \Delta S = 0$ modes slightly. The observation of the decay mode $\Xi_{cc}^{++} \to \Sigma_c^{++} \gamma$ which can occur through a pure $c \to u + \gamma$ transition, will ascertain the strength of the single quark transitions to the charm baryon decays. Further we find that the effect of flavor dependence can affect the branching ratio predictions upto even an order of magnitude.
References

28 References given in 3.
### Table 5.1 Branching Ratios in Units of $\times 10^{-3}$

<table>
<thead>
<tr>
<th>Decay</th>
<th>Single-Quark Transition $\bar{R}_{q\bar{q}}$</th>
<th>Two-Quark Transition $R_{q\bar{q}}$</th>
<th>Single &amp; Two-Quark Transition $b/a = +1$</th>
<th>Experiment Ref. 2, 3, 24</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma^+ \to \rho\gamma$</td>
<td>1.2*</td>
<td>1.24*</td>
<td>1.24*</td>
<td>1.24 ± 0.08</td>
</tr>
<tr>
<td>$\Lambda \to \pi\gamma$</td>
<td>22.0</td>
<td>1.30</td>
<td>0.49</td>
<td>1.02 ± 0.33</td>
</tr>
<tr>
<td>$\Xi^0 \to \Delta\gamma$</td>
<td>4.0</td>
<td>0.72</td>
<td>0.38</td>
<td>1.06 ± 0.17</td>
</tr>
<tr>
<td>$\Xi^0 \to \Sigma^0\gamma$</td>
<td>9.1</td>
<td>2.62</td>
<td>3.00</td>
<td>3.56 ± 0.43</td>
</tr>
<tr>
<td>$\Xi^- \to \Sigma^-\gamma$</td>
<td>11.0</td>
<td>0</td>
<td>0.23*</td>
<td>0.23 ± 0.10</td>
</tr>
<tr>
<td>$\Omega^- \to \Xi^-\gamma$</td>
<td>41.0</td>
<td>0</td>
<td>0.86</td>
<td>&lt;2.2</td>
</tr>
<tr>
<td>$\Omega^- \to \Xi^*\gamma$</td>
<td>4.5</td>
<td>0</td>
<td>0.09</td>
<td>–</td>
</tr>
</tbody>
</table>

* input

### Table 5.2 Decay Asymmetries

<table>
<thead>
<tr>
<th>Decay</th>
<th>Two-Quark Transition $b/a = +1$</th>
<th>Single &amp; Two Quark Transition $b/a = +1$</th>
<th>Experiment Ref. 2, 3, 24</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma^+ \to \rho\gamma$</td>
<td>0.30</td>
<td>-0.54*</td>
<td>-0.83 ± 0.13</td>
</tr>
<tr>
<td>$\Lambda \to \pi\gamma$</td>
<td>+0.58</td>
<td>-0.36</td>
<td>–</td>
</tr>
<tr>
<td>$\Xi^0 \to \Delta\gamma$</td>
<td>+1.00</td>
<td>+1.00</td>
<td>+0.43 ± 0.44</td>
</tr>
<tr>
<td>$\Xi^0 \to \Sigma^0\gamma$</td>
<td>-0.97</td>
<td>-0.82</td>
<td>+0.20 ± 0.33</td>
</tr>
<tr>
<td>$\Xi^- \to \Sigma^-\gamma$</td>
<td>–</td>
<td>+1.00</td>
<td>–</td>
</tr>
<tr>
<td>$\Omega^- \to \Xi^-\gamma$</td>
<td>–</td>
<td>+1.00</td>
<td>–</td>
</tr>
<tr>
<td>$\Omega^- \to \Xi^*\gamma$</td>
<td>–</td>
<td>+1.00</td>
<td>–</td>
</tr>
</tbody>
</table>
Table 5.3

Single Quark Decay Amplitude with Scale \( e G F \sin \theta_c \cos \theta_c \frac{k_{lq}}{\sqrt{2}} k_{lq} (k) \)

\[ \zeta = \frac{1}{6} \left( 1 - \frac{m_u}{m_s} \right), \quad X = \frac{k_0}{12 m_u}, \quad \varphi = SU(6) \) breaking angle.

For pc amplitude \( A = a \); pv amplitude \( A = b \)

<table>
<thead>
<tr>
<th>Decay</th>
<th>Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Sigma^+ \to \rho + \gamma )</td>
<td>( -\frac{1}{9} (1 + 2 \cos \varphi) A )</td>
</tr>
<tr>
<td>( \Lambda \to n + \gamma )</td>
<td>( \frac{1}{\sqrt{6}} (1 + 2 \cos \varphi) A )</td>
</tr>
<tr>
<td>( \Xi^0 \to \Lambda + \gamma )</td>
<td>( \frac{1}{3 \sqrt{6}} (1 + 2 \cos \varphi) A )</td>
</tr>
<tr>
<td>( \Xi^0 \to \Sigma^0 + \gamma )</td>
<td>( \frac{1}{9 \sqrt{2}} (3 - 10 \cos \varphi - 8 \cos^2 \varphi) A )</td>
</tr>
<tr>
<td>( \Xi^- \to \Sigma^- + \gamma )</td>
<td>( -\frac{1}{9} (3 - 10 \cos \varphi - 8 \cos^2 \varphi) A )</td>
</tr>
<tr>
<td>( \Omega^- \to \Xi^- + \gamma )</td>
<td>( -\frac{2}{3 \sqrt{6}} (1 + 2 \cos \varphi) A )</td>
</tr>
</tbody>
</table>

Table 5.4

Decay Amplitudes at Diquark transition level for the 2-Quark Processes.

\[ \zeta = \frac{1}{6} \left( 1 - \frac{m_u}{m_s} \right), \quad X = \frac{k_0}{12 m_u} \]

<table>
<thead>
<tr>
<th>Process</th>
<th>Parity conserving</th>
<th>Parity violating</th>
</tr>
</thead>
<tbody>
<tr>
<td>( &lt; S^+_2 \gamma</td>
<td>H_w</td>
<td>S^+_4 &gt; )</td>
</tr>
<tr>
<td>( &lt; S^+_2 \gamma</td>
<td>H_w</td>
<td>S^+_4 &gt; )</td>
</tr>
<tr>
<td>( &lt; S^-_2 \gamma</td>
<td>H_w</td>
<td>t_2 &gt; )</td>
</tr>
<tr>
<td>( &lt; t_0 \gamma</td>
<td>H_w</td>
<td>S^+_4 &gt; )</td>
</tr>
<tr>
<td>Decay</td>
<td>PC Amplitude</td>
<td>PV amplitude</td>
</tr>
<tr>
<td>-------------</td>
<td>------------------------------------------------------------------------------</td>
<td>------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>( \Sigma^- \rightarrow p + \gamma )</td>
<td>( \frac{2}{3} \left[ \cos \varphi \left( \frac{1}{2} + X \left( 3 - 2 \zeta \right) \right) \right] + \frac{\sin \varphi}{\sqrt{3}} \left( 2 - 3 \zeta + X \left( 1 + 6 \zeta \right) \right) )</td>
<td>( \frac{2}{3} \left[ \cos \varphi \left( -1 + \zeta - X \left( 2 - 3 \zeta \right) \right) \right] + \frac{\sin \varphi}{\sqrt{3}} \left( 1 - 3 \zeta + X \left( -4 + 9 \zeta \right) \right) )</td>
</tr>
<tr>
<td>( \Lambda \rightarrow n + \gamma )</td>
<td>( \frac{2}{3\sqrt{6}} \left[ \cos \varphi \left( 1 - 3 \zeta + X \left( -4 + 9 \zeta \right) \right) - \sqrt{3} \sin \varphi \left( -1 + \zeta - X \left( 2 + 3 \zeta \right) \right) \right] )</td>
<td>( \frac{2}{3\sqrt{6}} \left[ \cos \varphi \left( 2 - 3 \zeta - X \left( 1 + 6 \zeta \right) \right) - \sqrt{3} \sin \varphi \left( 1 + X \left( 3 - 2 \zeta \right) \right) \right] )</td>
</tr>
<tr>
<td>( \Xi^- \rightarrow \Lambda + \gamma )</td>
<td>( \frac{2}{3\sqrt{6}} \left[ \cos \varphi + \sqrt{3} \sin \varphi \right] \left( 1 - 2 \zeta + X \left( -1 + 5 \zeta \right) \right) )</td>
<td>( \frac{2}{3\sqrt{6}} \left[ \cos \varphi + \sqrt{3} \sin \varphi \right] \left( 1 - 2 \zeta + X \left( -1 + 5 \zeta \right) \right) )</td>
</tr>
<tr>
<td>( \Xi^0 \rightarrow \Sigma^0 + \gamma )</td>
<td>( \frac{\sqrt{2}}{3} \left[ \cos \varphi \left( 1 + X \left( 5 - \zeta \right) \right) \right] - \frac{\sin \varphi}{\sqrt{3}} \left( 1 - X \left( 5 + 3 \zeta \right) \right) )</td>
<td>( \frac{\sqrt{2}}{3} \left[ \cos \varphi \left( 1 + X \left( 5 - \zeta \right) \right) \right] - \frac{\sin \varphi}{\sqrt{3}} \left( 1 + X \left( 5 + 3 \zeta \right) \right) )</td>
</tr>
</tbody>
</table>
Table 5.6
Branching ratios and asymmetries for SU(6) symmetric case:
Parameters: $\phi = 0^\circ$, $b/a = +1$, $B (\Xi^- \to \Sigma^- + \gamma) = 0.227 \times 10^{-3}$

<table>
<thead>
<tr>
<th>Decay</th>
<th>Branching ($\times 10^{-3}$)</th>
<th>Asymmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Theory</td>
<td>Experiment</td>
</tr>
<tr>
<td>$\Sigma^+ \to p + \gamma$</td>
<td>1.24 *</td>
<td>1.24 ± 0.08</td>
</tr>
<tr>
<td>$\Lambda \to n + \gamma$</td>
<td>0.63</td>
<td>1.02 ± 0.33</td>
</tr>
<tr>
<td>$\Xi^0 \to \Lambda + \gamma$</td>
<td>0.39</td>
<td>1.06 ± 0.17</td>
</tr>
<tr>
<td>$\Xi^- \to \Xi^- + \gamma$</td>
<td>3.22</td>
<td>3.56 ± 0.43</td>
</tr>
<tr>
<td>$\Xi^- \to \Sigma^- + \gamma$</td>
<td>0.23 *</td>
<td>0.227 ± 0.100</td>
</tr>
<tr>
<td>$\Omega^- \to \Xi^- + \gamma$</td>
<td>0.86</td>
<td>&lt; 2.2</td>
</tr>
</tbody>
</table>

* Input

Table 5.7
Branchings and asymmetries for SU(6) $b_\omega\Lambda K\Xi$ case:
Parameters: $\phi = 10^\circ$, $b/a = +0.4$, $B (\Xi^- \to \Sigma^- + \gamma) = 0.50 \times 10^{-3}$

<table>
<thead>
<tr>
<th>Decay</th>
<th>Branching ($\times 10^{-3}$)</th>
<th>Asymmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Theory</td>
<td>Experiment</td>
</tr>
<tr>
<td>$\Sigma^+ \to p + \gamma$</td>
<td>1.24 *</td>
<td>1.24 ± 0.08</td>
</tr>
<tr>
<td>$\Lambda \to n + \gamma$</td>
<td>0.69</td>
<td>1.02 ± 0.33</td>
</tr>
<tr>
<td>$\Xi^0 \to \Lambda + \gamma$</td>
<td>0.49</td>
<td>1.06 ± 0.17</td>
</tr>
<tr>
<td>$\Xi^- \to \Xi^- + \gamma$</td>
<td>3.14</td>
<td>3.56 ± 0.43</td>
</tr>
<tr>
<td>$\Xi^- \to \Sigma^- + \gamma$</td>
<td>0.50 *</td>
<td>0.227 ± 0.100</td>
</tr>
<tr>
<td>$\Omega^- \to \Xi^- + \gamma$</td>
<td>1.8</td>
<td>&lt; 2.2</td>
</tr>
</tbody>
</table>

* Input
### Masses of Charmed Baryons (20' of SU(4))

<table>
<thead>
<tr>
<th>SU(3)</th>
<th>PARTICLE</th>
<th>MASS</th>
<th>LIFETIME (10^{-12} sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3*</td>
<td>Λ_c^+</td>
<td>2285.0 ± 0.6</td>
<td>0.196 ± 0.016</td>
</tr>
<tr>
<td></td>
<td>Ξ_c^+</td>
<td>2466.2 ± 2.2</td>
<td>0.57 ± 0.14</td>
</tr>
<tr>
<td></td>
<td>Ξ_c^0</td>
<td>2472.8 ± 1.7</td>
<td>0.082 ± 0.06</td>
</tr>
<tr>
<td>6</td>
<td>Σ_c</td>
<td>2453</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Σ'_c</td>
<td>2561</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ω_c^0</td>
<td>2740</td>
<td>0.79 ± 0.34</td>
</tr>
<tr>
<td>3</td>
<td>Ξ_cc</td>
<td>3616</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ω_{cc}</td>
<td>3706</td>
<td></td>
</tr>
</tbody>
</table>
Table 5.9
Two Quark Decay Amplitude with Scale $aG_F$ Cabibbo Factors $f_{2q}(k)$

\[ 
\begin{align*}
\tilde{\gamma}_s & = \frac{1}{6} \left( 1 - \frac{m_u}{m_s} \right), \quad \tilde{\gamma}_c = \frac{1}{3} \left( 1 - \frac{m_u}{m_c} \right) \quad X = \frac{k^0}{24 m_u} \\
\end{align*} \]

<table>
<thead>
<tr>
<th>Decay</th>
<th>PC Amplitude ($A^{PC}$)</th>
<th>PV amplitude ($A^{PV}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta C = \Delta S = -1$</td>
<td>$c + d \to s + u + \gamma$</td>
<td>$\frac{2}{\sqrt{6}} \left[ (H(k) - 2 G(k)) - X \left( (2 + 3 \zeta_c - 30 \zeta_s) G(k) + (8 - 6 \zeta_c) H(k) \right) \right] ]</td>
</tr>
<tr>
<td>$\Lambda_c^+ \to \Sigma^+ + \gamma$</td>
<td>$\frac{2}{\sqrt{6}} \left[ (H(k) - 2 G(k)) - X \left( (2 + 3 \zeta_c - 30 \zeta_s) G(k) + (8 - 6 \zeta_c) H(k) \right) \right] ]</td>
<td>$\frac{2}{\sqrt{6}} \left[ (2 H(k) - G(k)) + X \left( (2 + 3 \zeta_c - 30 \zeta_s) H(k) + (8 - 6 \zeta_c) G(k) \right) \right] ]</td>
</tr>
<tr>
<td>$\Sigma^0 \to \Sigma^0 + \gamma$</td>
<td>$\frac{2}{\sqrt{6}} \left[ (H(k) - 2 G(k)) - X \left( (2 + 3 \zeta_c - 30 \zeta_s) G(k) + (8 - 6 \zeta_c) H(k) \right) \right] ]</td>
<td>$\frac{2}{\sqrt{6}} \left[ (2 H(k) - G(k)) + X \left( (2 + 3 \zeta_c - 30 \zeta_s) H(k) + (8 - 6 \zeta_c) G(k) \right) \right] ]</td>
</tr>
<tr>
<td>$\Sigma_c^+ \to \Xi_c^+ + \gamma$</td>
<td>$\sqrt{2} \left[ (H(k) + G(k)) \left( 1 + X \left( 10 - 2 \zeta_c - 20 \zeta_s \right) \right) - 20 X \zeta_s G(k) \right] ]</td>
<td>$\sqrt{2} \left[ (H(k) + G(k)) \left( 1 + X \left( 10 - 2 \zeta_c - 20 \zeta_s \right) \right) - 20 X \zeta_s H(k) \right] ]</td>
</tr>
<tr>
<td>$\Sigma^{++} \to \Xi^{++} + \gamma$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>Equation</td>
<td>Value</td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>-------</td>
<td></td>
</tr>
<tr>
<td>((\text{H}) \cdot \text{Z} = (\text{E}))</td>
<td>((\text{H}) \cdot \text{Z} = (\text{E}))</td>
<td></td>
</tr>
<tr>
<td>((\text{H}) \cdot \text{Z} = (\text{E}))</td>
<td>((\text{H}) \cdot \text{Z} = (\text{E}))</td>
<td></td>
</tr>
<tr>
<td>((\text{H}) \cdot \text{Z} = (\text{E}))</td>
<td>((\text{H}) \cdot \text{Z} = (\text{E}))</td>
<td></td>
</tr>
<tr>
<td>((\text{H}) \cdot \text{Z} = (\text{E}))</td>
<td>((\text{H}) \cdot \text{Z} = (\text{E}))</td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Continued.
Table 5.10
Decay Rates and Asymmetry Parameters for Charm Baryon Weak Radiative Decays.

<table>
<thead>
<tr>
<th>Process</th>
<th>Decay Width x 10^9 (s^{-1})</th>
<th>Asymmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Lambda C = \Delta S = -1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Lambda_c^+ \to \Sigma^+ + \gamma)</td>
<td>1.48</td>
<td>0.02</td>
</tr>
<tr>
<td>(\Xi_c^0 \to \Xi^0 + \gamma)</td>
<td>1.54</td>
<td>-0.01</td>
</tr>
<tr>
<td>(\Xi_c^+ \to \Xi^+_c + \gamma)</td>
<td>0.41</td>
<td>1.00</td>
</tr>
<tr>
<td>(\Xi^{++}<em>{cc} \to \Xi^{++}</em>{c} + \gamma)</td>
<td>10.15</td>
<td>-0.99</td>
</tr>
<tr>
<td>(\Lambda C = -1, \Delta S = 0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Lambda_c^+ \to \rho + \gamma)</td>
<td>0.11</td>
<td>-0.25</td>
</tr>
<tr>
<td>(\Xi^+_c \to \Sigma^+ + \gamma)</td>
<td>0.09</td>
<td>-0.07</td>
</tr>
<tr>
<td>(\Xi^0_c \to \Sigma^0 + \gamma)</td>
<td>0.06</td>
<td>-0.48</td>
</tr>
<tr>
<td>(\Xi^0_c \to \Lambda + \gamma)</td>
<td>0.14</td>
<td>-0.01</td>
</tr>
<tr>
<td>(\Xi^{++}_{cc} \to \Lambda^{++}_c + \gamma)</td>
<td>0.02</td>
<td>1.00</td>
</tr>
<tr>
<td>(\Xi^{++}_{cc} \to \Xi^{++}_c + \gamma)</td>
<td>0.88</td>
<td>-0.99</td>
</tr>
<tr>
<td>(\Xi^0_{cc} \to \Xi^0_{c} + \gamma)</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>(\Omega^0_{cc} \to \Xi^0_{c} + \gamma)</td>
<td>0.38</td>
<td>-0.79</td>
</tr>
<tr>
<td>(\Omega^{+}<em>{cc} \to \Xi^+</em>{c} + \gamma)</td>
<td>0.02</td>
<td>1.00</td>
</tr>
<tr>
<td>(\Omega^{++}<em>{cc} \to \Xi^{++}</em>{c} + \gamma)</td>
<td>0.59</td>
<td>-0.99</td>
</tr>
<tr>
<td>(\Lambda C = - \Delta S = -1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Xi^+_c \to \rho + \gamma)</td>
<td>2.38 \times \tan^4 \theta_c</td>
<td>-0.33</td>
</tr>
<tr>
<td>(\Xi^0_c \to n + \gamma)</td>
<td>2.26 \times \tan^4 \theta_c</td>
<td>0.41</td>
</tr>
<tr>
<td>(\Omega^0 \to \Lambda + \gamma)</td>
<td>0.32 \times \tan^4 \theta_c</td>
<td>1.00</td>
</tr>
<tr>
<td>(\Omega^0 \to \Sigma^0 + \gamma)</td>
<td>19.01 \times \tan^4 \theta_c</td>
<td>-0.99</td>
</tr>
<tr>
<td>(\Omega^{+}_{cc} \to \Lambda^{+}_c + \gamma)</td>
<td>0.32 \times \tan^4 \theta_c</td>
<td>1</td>
</tr>
<tr>
<td>(\Omega^{++}_{cc} \to \Sigma^{++}_c + \gamma)</td>
<td>18.16 \times \tan^4 \theta_c</td>
<td>-0.99</td>
</tr>
</tbody>
</table>
Diquartepin = 0 : f1 (ad), f2 (as), f3 (ds)

with the following diquark content:

\[
\begin{align*}
\text{(sp)} f_1 & = \frac{1}{\sqrt{2}} (\frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta) - \frac{1}{\sqrt{2}} (\frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta), \\
\text{(sp)} f_2 & = \frac{1}{\sqrt{2}} (\frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta) + \frac{1}{\sqrt{2}} (\frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta), \\
\text{(sp)} f_3 & = \frac{1}{\sqrt{2}} (\frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta) - \frac{1}{\sqrt{2}} (\frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta).
\end{align*}
\]

Appendix 5.1
Fig. 5.1

Fig. 5.2
Fig. 5.3 Variation of $B(\Lambda \rightarrow n \pi)$ vs $b/a$ for different $SU(6)$ breaking angle ($\phi$).
Fig. 5. Variation of $B(\equiv \lambda \chi \propto 10^3)$ vs $\phi$ for different $B(\equiv \Sigma \gamma)$. 

(18)
Fig. 5.5 Variation of $B(e^0 \rightarrow \Sigma^0 \bar{\nu}) \times 10^3$ vs $\phi$ for different $B(e^0 \rightarrow \Sigma^- \bar{\nu})$. 

(11)
Fig. 5.6 Variation of $\alpha_{\Sigma^+} - \rho_Y$ vs $B(\Xi^- \rightarrow \Sigma^+\pi)$ for different $\phi$. 
Fig. 5.7 Variation of $\phi - \Lambda\gamma$ vs $B(\Xi^+ \to \Sigma^-\gamma)$ for different $\phi$. 
Fig. 5.8 Variation of $\rho$ with $\phi$ for different $\phi$. 

$\phi = 20^\circ$

$\phi = 10^\circ$

$\phi = 0^\circ$
Fig. 5. Variation of $\phi = 30$ vs $b/a$.  

(23)
Fig 5.10

$b/a$ ratio

$a \left(10^{-2} \text{GeV}^2\right)$

$1_{2q} \left(10^{-3} \text{GeV}^3\right)$