Chapter 2

Methodology

2.1 Introduction

Many theoretical attempts have been made to understand the formation and decay of super-heavy elements (SHEs). Theoretical approaches of finite potentials [1]-[6] using the Strutinsky macro-microscopic method, the (non-relativistic) self consistent Skyrme (or Gogny) Hartree-Fock [7]-[10] and the relativistic mean field theories [11]-[13] have been successful in the study of SHEs. Among these, the calculations based on macro-microscopic method have been particularly successful. The model used in this thesis for the study of synthesis of super-heavy elements is the Quantum Mechanical Fragmentation Theory (QMFT) [14]-[27] where the potential is calculated in macro-microscopic method. This theory, based on two center shell model, using as an average two body potential in Strutinsky macro-microscopic method [28], has been successful in explaining both the cold and hot fusion reactions for the synthesis of super-heavy elements. Infact, the QMFT predicted the phenomenon of cold fusion reactions which is successfully used for the synthesis of new super-heavy elements, thereby signalling a complete success of the quantum mechanical concept of probability.

In this thesis, we have calculated the fusion cross-sections for SHEs, using the dynamical cluster-decay model (DCM) of Gupta and collaborators [29]-[45], based on the QMFT which, in binary fragmentation, uses a collective mass transfer pro-
cess. This model is a two step model, where the first step is the quantum mechanical preformation probability of the decay products or clusters formed in the mother nucleus and the second step is the penetration of the so formed clusters through the interaction barrier. In this model, the preformation probability of all possible clusters within the mother nucleus are to be calculated. It is important to note that deformations and orientation effects of the reaction partners and decay products are explicitly included in this model together with temperature, and the angular momentum contributions.

The details of Quantum Mechanical fragmentation theory are presented in Section 2.2, and the theory of DCM is discussed in Section 2.3.

2.2 Quantum Mechanical Fragmentation Theory

The QMFT [14]-[27] is a unified description of two body channels in both fusion and fission processes. Here the quantum mechanical concept of probability is utilized to investigate the role of shell effects in the fusion, fission and cluster radioactivity. In QMFT, the dynamics of process is determined by the mass parameters, defining the kinetic energy of the system while the static properties of nuclear system are contained in the potential energy surfaces. The QMFT is worked out in terms of the following collective variables:

(i) relative separation coordinate $R$ between the two nuclei or, in general, two fragments (or, equivalently, the length parameter $\lambda = L/2R_0$, with $L$ as the length of the nucleus and $R_0$ as the radius of an equivalent spherical nucleus).

(ii) The deformation co-ordinates $\beta_i (\lambda=2,3,4..., \text{and } i=1,2)$ of the colliding nuclei.

(iii) The orientation degrees of freedom $\theta_i (i=1,2)$ of the deformed nuclei (see Figure 2.1).

(iv) Azimuthal angle $\phi$ between the principal planes of the two colliding nuclei.

(v) Neck parameter $\varepsilon$, defined by the ratio $\varepsilon = E_0/E'$ for the interaction region ($R < R_1 + R_2$, $R_i$ as the radius of the two nuclei);

where $E_0$ is the actual height of the barrier and $E'$ is the fixed barrier of the two
center oscillator. $\epsilon = 0$ represents a broad or no neck formation, whereas $\epsilon = 1$
gives that the neck is fully squeezed in corresponding to the asymptotic region
($R > R_1 + R_2$).

vi) Mass and charge fragmentation co-ordinates [14, 15, 26].

For two body channels, the mass and charge fragmentations for separated nu-
clei/fragments are defined by the mass and charge-asymmetry coordinates as

$$\eta = \frac{A_1 - A_2}{A}, \quad \eta_Z = \frac{Z_1 - Z_2}{Z}$$  \hspace{1cm} (2.1)

Similarly, the neutron asymmetry coordinate [15]

$$\eta_N = \frac{N_1 - N_2}{N}$$  \hspace{1cm} (2.2)

can also be used, but it is sufficient to treat only two of them as dynamical co-
ordinates since they are related as

$$\eta = \frac{Z}{A} \eta_Z + \frac{N}{A} \eta_N$$  \hspace{1cm} (2.3)

Here $A = A_1 + A_2$, $Z = Z_1 + Z_2$ and $N = N_1 + N_2$. $A_i$, $Z_i$ and $N_i$ ($i = 1, 2$) are
respectively the mass number, the charge number and the neutron number of
two fragments. $A$, $Z$ and $N$ are respectively the mass number, charge number
and neutron number of the compound system. The limiting values of $\eta$ are $0 \leq |\eta| \leq 1$
and thus allows a unified description of a few-nucleon or multi-nucleon
(a cluster) transfer, a large-mass transfer, the complete fusion ($|\eta| = 1$) of nuclei
and the symmetric ($\eta = 0$), asymmetric and super-asymmetric fission of a nucleus
or compound nucleus. The $\eta_Z$ coordinate gives the associated charge distribution
effects. In terms of these collective coordinates and their velocities, the collective
Hamiltonian can be written as (taking $\beta$ to stand for $\beta_{31}$ and $\beta_{32}$ ($\lambda=2,3,4,...$)).

$$H = K(R, \beta, \epsilon, \eta, \eta_Z; \dot{R}, \dot{\beta}, \dot{\epsilon}, \dot{\eta}, \dot{\eta}_Z) + V(R, \beta, \epsilon, \eta, \eta_Z).$$  \hspace{1cm} (2.4)
For the compound nucleus formation, the neck parameter $\epsilon = 0$ is assumed, since once the neck formation starts between the two colliding nuclei, then fission phenomenon takes place, i.e. excited compound nucleus will proceed towards the disintegration process.

For the potential $V(\eta, \eta_Z, R)$, minimized in the $\eta_Z$ co-ordinate, Schrödinger wave equation, in terms of mass asymmetry $\eta$ and relative separation $R$ co-ordinates, can be written as:

$$H(\eta, R)|\psi(\eta, R) = E(\eta, R)|\psi(\eta, R)$$

(2.5)

with the Hamiltonian,

$$H(\eta, R) = K(\eta) + K(R) + K(\eta, R) + V(\eta) + V(R) + V(\eta, R)$$

(2.6)

Here, $K$ refers to the kinetic energy and $V$ to the collective potential energy. The mass parameters $B_{ij}$, defining the kinetic energy term $K$ in the above Eqs. (2.4) and (2.6) are either the consistently calculated cranking masses using the Asymmetric Two-Center Shell Model (ATCSM) or the classical hydrodynamical masses, which are shown to have good agreement with microscopic cranking calculations. The coupling term of the kinetic energy $K(\eta, R)$, proportional to $\frac{\partial^2}{\partial \eta \partial R}$, is neglected here, since the coupled cranking masses are very small [14,15] ($B_{\eta} \ll (B_{RR} B_{\eta})^{1/2}$ and $B_{\eta Z} \ll (B_{RR} B_{\eta Z})^{1/2}$). Same is true for the coupling term of potential energy $V(\eta, R)$.

Therefore, in a decoupled approximation [27], the Schrödinger equation (2.5) can be solved for which the Hamiltonian takes the form:

$$H = -\frac{\hbar^2}{2\sqrt{B_{\eta}} \sqrt{B_{\eta}} \sqrt{B_{\eta Z}}} \frac{\partial}{\partial \eta} \frac{1}{\sqrt{B_{\eta}}} \frac{\partial}{\partial \eta} - \frac{\hbar^2}{2\sqrt{B_{RR}} \sqrt{B_{RR}} \sqrt{B_{RR}} \sqrt{B_{RR}}} \frac{\partial}{\partial R} \frac{1}{\sqrt{B_{RR}}} \frac{\partial}{\partial R} + V(\eta) + V(R).$$

(2.7)
Figure 2.1: Schematic configurations of two (equal/unequal) axially symmetric deformed, oriented nuclei, lying in the same plane and for various $\theta_1$ and $\theta_2$ values in the range 0° to 180°. The $\theta$'s are measured in anti-clockwise from the colliding axis and the angle $\alpha$'s in clockwise from the symmetry axis.
For decoupled Hamiltonian (2.7), Schrödinger wave equation (2.5) can be separated for the two co-ordinates \( \eta \) and \( R \) as follows,

\[
\left[ -\frac{\hbar^2}{2\sqrt{B_{\eta\eta}}} \frac{\partial}{\partial \eta} \sqrt{B_{\eta\eta}} \frac{\partial}{\partial \eta} + V(\eta) \right] \psi^\nu(\eta) = E^\nu_\eta \psi^\nu(\eta) \tag{2.8}
\]

and

\[
\left[ -\frac{\hbar^2}{2\sqrt{B_{RR}}} \frac{\partial}{\partial R} \sqrt{B_{RR}} \frac{\partial}{\partial R} + V(R) \right] \psi^\nu(R) = E^\nu_R \psi^\nu(R) \tag{2.9}
\]

with

\[
\psi(\eta, R) = \psi(\eta)\psi(R) \tag{2.10}
\]

and

\[
E = E_\eta + E_R \tag{2.11}
\]

The states \( \psi^\nu(\eta) \) are the vibrational states in the potential \( V(\eta) \) and are labelled by the quantum numbers \( \nu = 0, 1, 2, \ldots \).

In the following subsections, we first discuss the various terms of Schrödinger wave equations (2.8) and (2.9) and then give the solution of Eq. (2.8) for the determination of preformation probability \( P_0 \propto |\psi^0(\eta)|^2 \).

### 2.2.1 The Scattering Potential \( V(R) \)

For a fixed \( \eta \), i.e. for a given target projectile \((A_1, A_2)\) combination, the scattering potential \( V(R) \) in Eq. (2.9) is defined as the sum of the deformations, orientations dependent Coulomb potential, proximity potential and angular momentum dependent potential, i.e.,

\[
V(R, \ell, T) = V_c(R, Z_1, \beta_{\lambda_1}, \theta_1, \phi, T) + V_p(R, A_1, \beta_{\lambda_1}, \theta_1, \phi, T) + V_\ell(R, A_1, \beta_{\lambda_1}, \theta_1, \phi, T) \tag{2.12}
\]

For co-planar nuclei, \( \phi = 0^\circ \), and for spherical-plus-deformed nuclear collisions, only one orientation angle \( \theta \) is enough, referring to the rotationally-symmetric deformed nucleus.
2.2.2 The Fragmentation potential \( V(\eta) \)

The collective potential energy or the fragmentation potential \( V(\eta, R) \), appearing in equation (2.8), is calculated as,

\[
V(\eta, R, \ell, T) = - \sum_{i=1}^{2} B_i(A_i, Z_i, \beta_{Si}, T) + V_c(R, Z_i, \beta_{Si}, \theta_i, \phi, T) \\
+ \delta U(R, A_i, \beta_{Si}, \theta_i, \phi, T) + V_t(R, A_i, \beta_{Si}, \theta_i, \phi, T) 
\] (2.13)

Here \( B_i \) \((i=1,2)\) are the binding energies of the two nuclei, available at \( T=0 \) from the experimental data of Audi-Wapstra [54]. Wherever the experimental \( B \)'s are not available, or we wish to add temperature dependence, the theoretical binding energies of Möller et al. [55] are used. Writing Eq. 2.13 at fixed \( R \) and at \( T\neq0 \) as,

\[
V(\eta, T) = \sum_{i=1}^{2} V_{LDM}(A_i, Z_i, T) + \sum_{i=1}^{2} \delta U \exp\left(-\frac{T^2}{T_0^2}\right) + V_c(R, Z_i, \beta_{Si}, \theta_i, \phi, T) \\
+ \delta U(R, A_i, \beta_{Si}, \theta_i, \phi, T) + V_t(R, A_i, \beta_{Si}, \theta_i, \phi, T) 
\] (2.14)

Here, \( V_{LDM}(A_i, Z_i, T) \) is the liquid drop part of the binding energy and \( \delta U \), the shell corrections. Note that the experimental binding energies contain both the macroscopic (liquid drop part) and the microscopic (shell correction) parts.

The fragmentation potential \( V(\eta) \) is calculated at a fixed distance \( R = R_1 + R_2 + \Delta R \) or \( R = C_1 + C_2 + \Delta C \) fm, with \( C_i \) \((i=1,2)\) as the Süßmann central radii related to the radius vector \( R_i \) as \( C_i = R_i(1 - \frac{b_i^2}{R_i^2}) \), with

\[
R_i = R_{0i}[1 + \sum_{\lambda} \beta_{Si} Y_{\lambda}^{(0)}(\alpha_i)], 
\] (2.15)

and

\[
R_{0i}(T) = [1.28A_i^{1/3} - 0.76 + 0.8A_i^{1/3}][1 + 0.0007T^2]. 
\] (2.16)

The diffuseness of the nuclear surface (i.e. the surface thickness) \( b=0.99 \) fm at \( T=0 \).

For \( T\neq0 \), \( b \) is defined later. The charges \( Z_i \) are fixed by minimizing the potential \( V(\eta) \) in the \( \eta_z \) coordinate at each \( \eta \) value.

The shell corrections \( \delta U \) are considered to vanish exponentially with \( T \) for \( T_0=1.5 \).
MeV [56]. At higher excitation energies the shell corrections vanish completely and only the liquid drop part of energy is present.

The shell corrections play an important role in determining or empirical fitting of nuclear masses, because the nuclear masses calculated by using the smooth liquid drop formula show large deviations with respect to the experimental masses. It means that in the experimental masses there exist deep minima at specific neutron and/or proton numbers indicating the presence of shell structure, the so-called magic numbers in nuclei. This characteristic behavior cannot be reproduced by the liquid drop part alone, which means that the introduction of microscopic shell correction in the mass formula is essential. Thus, shell corrections accounts for the removal of deviation from the liquid drop calculations (uniform distribution of nucleons), and are defined, within Strutinsky [28] method as

\[ \delta U = U - \bar{U} \]  

(2.17)

where, \( U = \sum_{\nu} E_{\nu} 2n_{\nu} \) is the sum over all occupied single particle states and \( \bar{U} = 2 \int_{-\infty}^{\lambda} E\tilde{g}(E)dE \) 

(2.18)

is the average energy for uniform distribution. In general, the microscopic shell correction, together with the liquid drop part, gives a proper description of the binding energy of the nucleus. This method, however, does not give a proper description of light mass nuclei. The difficulty is the inadequacy of shell model for very light nuclei. For this reason, the macro-microscopic calculations of Möller et al. [55] are tabulated for \( Z \geq 8 \) only. For \( Z \leq 8 \), one could alternatively use the empirical shell correction method of Myers-Swiatecki [57] which again is not very satisfactory for light nuclei \( (Z \leq 16) \). Gupta and collaborators have modified this empirical method and obtained a better description of the shell corrections for the light as well as heavy mass region, i.e, \( 1 \leq Z \leq 118 \) [33].

The other \( T \)-dependent terms are, respectively, the Coulomb, proximity and...
centrifugal potentials for deformed and oriented nuclei.

The $T$-dependent liquid drop energy $V_{ldm}(A, Z, T)$ of Davidson et al. [58] is

$$V_{ldm}(T) = \alpha(T)A + \beta(T)A^{\frac{2}{3}} + \left(\gamma(T) - \frac{\eta(T)}{A^{\frac{2}{3}}}\right)\left(I_a^2 + 2|I_a|\right) + \frac{Z^2}{r_0(T)A^{\frac{2}{3}}}\left(1 - \frac{0.7636}{Z^{\frac{2}{3}}} - \frac{2.29}{[r_0(T)A^{\frac{1}{3}}]^2}\right) + \delta(T)f(Z, A)A^{\frac{4}{3}},$$

(2.19)

where

$$I_a = a_a(Z - N), \quad a_a = 1,$$

and, respectively, for even-even, even-odd, and odd-odd nuclei,

$$f(Z, A) = (-1, 0, 1).$$

For $T=0$, Seeger [59] obtained the constants, by fitting all even-even nuclei and 488 odd-$A$ nuclei available at that time (in 1961), as

$$\alpha(0) = -16.11\text{MeV}, \quad \beta(0) = 20.21\text{MeV},$$

$$\gamma(0) = 20.65\text{MeV}, \quad \eta(0) = 48.00\text{MeV},$$

with the pairing energy term from Ref. [60],

$$\delta(0) = 33.0\text{MeV}.$$

For the large amount of data available now on ground-state binding energies, these constants of $V_{ldm}(T = 0)$ needed re-fitting, which was done by some of us [32, 33] to get the experimental binding energies [54] with shell corrections determined from Myers and Swiatecki [57]. Wherever the experimental data were not available, the theoretically estimated binding energies of Möller et al. [55] were used. The magic numbers used for the superheavy region, both in fitted constants of $V_{LDM}(T = 0)$ [32, 33] and shell corrections [57], were Z=126, N=184. Evidently, the same job of fitting the constants of $V_{ldm}(T = 0)$ to give the experimental binding energy $B$
for shell corrections with new proton magic numbers at \(Z=120\) or \(114\), respectively, need be re-done if the magicity at \(Z=126\) is to be changed to that of the new ones at \(Z=120\) or \(114\).

The shell corrections according to the “empirical” formula of Myers and Swiatecki [57], for spherical shapes, are

\[
\delta U = C \left[ \frac{F(N) + F(Z)}{(A/2)^{2/3}} - cA^{1/3} \right]
\]

where

\[
F(X) = \frac{3}{5} \left( \frac{M_i - M_{i-1}}{M_i - M_{i-1}} \right) (X - M_{i-1}) - \frac{3}{5} \left( X^{2/3} - M_{i-1}^{2/3} \right)
\]

with \(X = N\) or \(Z\), \(M_{i-1} < X < M_i\), and \(M_i\) as the magic numbers 2, 8, 14 (or 20), 28, 50, 82, 126 and 184 for both neutrons and protons. The constants \(C=5.8\) MeV and \(c=0.26\) MeV. In the following, we use the more often used \(Z=N=14\), instead of 20, as the light magic numbers, and change \(Z=126\) to \(Z=120\) or \(114\), respectively, for the re-fitting of constants of \(V_{LDM}(T=0)\).

Note that the calculation of fragmentation potential involves all the possible decay channels and the number of all such possible decay channels becomes more and more with the increasing mass of the mother nucleus. The nuclear temperature \(T\) (in MeV) is related to the excitation energy \(E_{CN}^*\) of the compound nucleus, through a semi-empirical statistical relation as:

\[
E_{CN}^* = \frac{1}{10} AT^2 - T \quad (MeV).
\]

2.2.3 The proximity potential for deformed, oriented and co-planar nuclei

When two surfaces approach each other within a small distance of less than \(\sim 2fm\), comparable with the surface thickness of interacting nuclei, or when a nucleus is at the verge of dividing into two fragments, then the two surfaces actually face each other across a small gap or crevice. In both cases, the surface energy term alone
could not give rise to the strong attraction that is observed when the two surfaces are brought in close proximity. Such additional attractive forces are called proximity forces and the additional potential due to these forces is called the nuclear proximity potential. Blocki et al. [61] have reanalyzed and extended a theorem, originally due to Deryagin [62], according to which the force between two gently curved surfaces in close proximity is proportional to the interaction potential per unit area between the two flat surfaces. The proximity potential [63] for hot deformed nuclei is given as

\[ V_p(A_i, \beta_i, \theta_i, T) = 4\pi R(T)\gamma b(T)\Phi(s_0(T)). \]  

(2.23)

\( \Phi(s_0) \) is the universal function, independent of the shapes of nuclei or the geometry of nuclear system, but depends on the minimum separation distance \( s_0 \),

\[
\Phi(s_0) = \begin{cases} 
-\frac{1}{3}(s_0 - 2.54)^2 - 0.0852(s_0 - 2.54)^3 \\
-3.437\exp\left(-\frac{s_0}{0.75}\right)
\end{cases}
\]  

(2.24)

respectively, for \( s_0 \leq 1.2511 \) and \( s_0 \geq 1.2511 \). Here, \( s_0 \) is defined in units of \( b \), i.e. \( s_0 = s_0/b \). This function is defined for negative (the overlap region), zero (touching configuration) and positive values of \( s_0 \). \( s_0 \), distance of closest approach defined in Figure 2.1, depends on deformations and orientations of reactants/products (explained later in this section). \( b \) is the diffuseness of the nuclear surface given by

\[ b = \left[ \frac{\pi}{2\sqrt{3} \ln 9} \right] t_{10-90}, \]

where \( t_{10-90} \) is the thickness of the surface in which the density profile changes from 90% to 10%. The value of \( b \sim 1 \text{ fm} \). However, temperature dependent \( b \) is given as

\[ b(T) = 0.99(1 + 0.009T^2) \]  

(2.25)
The $\gamma$ is the specific nuclear surface tension given by

$$\gamma = 0.9517 \left[ 1 - 1.7826 \left( \frac{N - Z}{A} \right) \right] \text{MeV fm}^{-2}$$  \hspace{1cm} (2.26)

$R$ is the mean curvature radius of the reaction partners and is described in the following for deformed and oriented nuclei.

For the axially symmetric shapes, the nuclear radius parameter (to all higher multipole orders $\lambda = 2, 3, 4, \ldots$) is given by Eqs. (2.15) and (2.16). In terms of the radii of curvature $R_{11}$ and $R_{12}$ in the principal planes of curvature of each of the two nuclei ($i=1, 2$) at the points of closest approach (defining $s_0$ in Figure 2.1), the mean curvature radius $\bar{R}$ for deformed, oriented nuclei lying in different planes (non co-planar nuclei) is given by

$$\frac{1}{\bar{R}^2} = \frac{1}{R_{11}R_{12}} + \frac{1}{R_{21}R_{22}} + \left[ \frac{1}{R_{11}R_{21}} + \frac{1}{R_{12}R_{22}} \right] \sin^2 \phi$$

$$+ \left[ \frac{1}{R_{11}R_{22}} + \frac{1}{R_{21}R_{12}} \right] \cos^2 \phi.$$  \hspace{1cm} (2.27)

Here, $\phi$ is the azimuthal angle between the principal planes of curvature of two nuclei (for co-planar nuclei $\phi=0^\circ$). The four principal radii of curvature are

$$R_{11}(\alpha_i) = \frac{\left[ R_i^2(\alpha_i) + R_i^2(\alpha_i) \right]^{3/2}}{R_i(\alpha_i) + 2R_i^2(\alpha_i) - R_i(\alpha_i)R_i^2(\alpha_i)}$$

$$R_{12}(\alpha_i) = \frac{R_i(\alpha_i) \sin \alpha_i}{\cos (\pi/2 - \alpha_i - \delta_i)}.$$  \hspace{1cm} (2.28)

where, $R_i^2(\alpha_i)$ and $R_i^2(\alpha_i)$ are the first and second order derivatives of $R_i(\alpha_i)$ with respect to $\alpha_i$, respectively. For the derivation of the radius of curvature $R_{11}$, see [64], and it follows from Figure 2.2), and Ref. [65], that $R_{12} = h/\cos \omega_i$, with $h = R_i(\alpha_i) \sin \alpha_i$ and $\omega_i = \pi/2 - \alpha_i - \delta_i$. Also, for $n$ to be a normal vector

$$\tan \delta_i = \frac{R_i'(\alpha_i)}{R_i(\alpha_i)}.$$  \hspace{1cm} (2.29)

Note that $R_{11}(\alpha_i) = R_{12}(\alpha_i)$, respectively, for $\alpha_1 = 0^\circ$ or $180^\circ$ and $\alpha_2 = 180^\circ$ or
Figure 2.2: An axially symmetric (quadrupole) deformed and oriented nucleus, showing the nuclear radius parameter $R_1(\alpha_1)$ and the geometry associated with the principal radius of curvature $R_{12}(\alpha_1)$.  

For a configuration of deformed, oriented nuclei, the minimum distance $s_0$ (Figure 2.1) in Eq. (2.24) is

$$s_0 = R - X_1 - X_2$$

(2.30)

with the projections $X_i$ along the collision Z-axis given as

$$X_1 = R_1(\alpha_1)\cos(\theta_1 - \alpha_1)$$

$$X_2 = R_2(\alpha_2)\cos(180 + \theta_2 - \alpha_2)$$

(2.31)
with the minimization conditions on \( s_0 \),

\[
\frac{\partial s_0}{\partial \alpha_1} = \frac{\partial s_0}{\partial \alpha_2} = 0, \tag{2.32}
\]

resulting in

\[
\tan(\theta_1 - \alpha_1) = -\frac{R_1'(\alpha_1)}{R_1(\alpha_1)},
\]
\[
\tan(180 + \theta_2 - \alpha_2) = -\frac{R_2'(\alpha_2)}{R_2(\alpha_2)}. \tag{2.33}
\]

Comparing Eqs. (2.29) and (2.33), we get

\[
\delta_1 = \theta_1 - \alpha_1
\]
\[
\delta_2 = 180 + \theta_2 - \alpha_2. \tag{2.34}
\]

to be used in Eq. (2.28). Thus, for the given \( \theta_1 \) and \( \theta_2 \), \( X_1 \) and \( X_2 \) are obtained for the angles \( \alpha_1 \) and \( \alpha_2 \) satisfying the minimization conditions Eq. (2.33). From a detailed study [63], based upon QMFT by Gupta and collaborators, the optimum orientations are given for "cold, non-compact" and "hot, compact" fusion configurations corresponding to largest interaction radius/lowest barrier and smallest interaction radius/highest barrier respectively. The details can be seen in Table 1 of [63].

Note that the conditions Eq. (2.33) refer to perpendiculars (normal vectors) at the points \( P_1 \) and \( P_2 \). In other words, if the distance \( s_0 \) were to be shortest, the perpendicular conditions Eq. (2.33) must be used which would apparently give equations Eq. (2.31) for \( X_i \).

The Eq. (2.23) is valid for zero (touching configuration) and positive values of \( s_0 \), but is also used for negative \( s_0 \). As the two nuclei overlap \( (s_0 < 0) \), a crevice is formed and, in an adiabatic approximation, the system adjusts its shape parameters such that the two colliding nuclei form a single indented body in the form of a single hyperboloid of one sheet with a hyperboloidal crevice, as shown in Figure 2.3(a).
For such a necked system, shown in Figure 2.3(b), following Blocki et al. [61], the proximity potential is obtained by Gupta and collaborators [51],[52] as

$$V_P(s_0) = \pi \gamma b^2 \Phi_1(s_0 = 0) \frac{(c_1^2 + c_2^2 - 2\epsilon^2)}{(s_1^2 + s_2^2)}$$

(2.35)

where $\Phi_1(s_0 = 0) = -2.0306$ is the first moment of the universal function $\Phi$ at $s_0 = 0$, and $c_1$, $z_1$, and $\epsilon$ are the shape parameters in Figure 2.3(b). Apparently, for two equal nuclei, $z_1 = z_2$ and $c_1 = c_2$.

### 2.2.4 The Coulomb potential

Coulomb potential describes the force of repulsion between two interacting nuclei due to their charges. It acts along the line joining the two nuclei. The Coulomb potential [63] for two interacting hot, deformed and oriented nuclei is given as

$$V_c(Z_i, \beta_{\lambda i}, \theta_i, T) = \frac{Z_1 Z_2 e^2}{R(T)} + 3Z_1 Z_2 e^2 \sum_{\lambda = 1, 2} \frac{R_1^2(\alpha_i, T)}{(2\lambda + 1) R(T)^{\lambda + 1}} Y_\lambda^{(0)}(\theta_i) \left[ \beta_{\lambda i} + \frac{4}{l} \beta_{\lambda i}^2 Y_\lambda^{(0)}(\theta_i) \right],$$

(2.36)
with \( R_i \) from Eq. (2.15). \( Y_\lambda^{(0)}(\theta_i) \) are the spherical harmonics function.

### 2.2.5 Rotational Energy due to angular momentum

The rotational motion gives an additional energy due to the angular momentum \( \ell \), defined as

\[
V_\ell = \frac{\hbar^2 \ell(\ell + 1)}{2I(T)}
\]

with \( I_{NS}(T) = \mu R(T)^2 \), is the non-sticking limit of moment of inertia with \( \mu = \frac{A_1 A_2}{A_1 + A_2} \) as the reduced mass. \( m \) is the nucleon mass. In the complete sticking limit, the moment of inertia \( I_S \) is given as,

\[
I_S(T) = \mu R(T)^2 + \frac{2}{5} A_1 m R_1^2(T) + \frac{2}{5} A_2 m R_2^2(T).
\]

with \( R_i \) from Eq. (2.15). However, for the relative separation of interest here, we use the sticking limit.

### 2.2.6 Classical Hydrodynamical Mass Parameters

The kinetic energy part of the Hamiltonian in Eq. (2.8) enters through the mass parameters. We use here the classical mass parameters of Kroger and Scheid [66]. The model of Kröger and Scheid is based on the hydrodynamical flow, as shown in Figure 2.4. This model gives a simple analytical expression, whose predictions are shown to compare nicely with the microscopic cranking model calculations. For the \( B_{\eta\eta} \) mass we get,

\[
B_{\eta\eta} = \frac{A m R^2}{4} \left[ \frac{v_c(1 + \gamma)}{v_c(1 + \delta^2)} - 1 \right]
\]

with

\[
\gamma = \frac{R_c}{2R} \left[ \frac{1}{1 + \cos \vartheta_1} \left( 1 - \frac{R_c}{R_1} \right) + \frac{1}{1 + \cos \vartheta_2} \left( 1 - \frac{R_c}{R_2} \right) \right]
\]

\[
\delta = \frac{1}{2R} \left[ (1 - \cos \vartheta_1)(R_1 - R_c) + (1 - \cos \vartheta_2)(R_2 - R_c) \right]
\]

\[
v_c = \pi R_c^2 R
\]
and $v_t = v_1 + v_2$, is the total conserved volume. The angles $\theta_1$ and $\theta_2$ and geometry of the model are shown in Figure 2.4. For $\theta_1 = \theta_2 = 0$, $\delta = 0$ which corresponds to two touching spheres. $R_c(\neq 0)$ is the radius of a cylinder of length $R$, having a homogeneous flow in it; whose existence is assumed for the mass transfer between the two spherical fragments. We have generalized this formalism for deformed nuclei by using the radii $R_1$ and $R_2$ for deformed nuclei, given by Eq. (2.15).

2.2.7 Solution of the Schrödinger equation and the fragments preformation probability $P_0$

Once the Hamiltonian Eq. (2.7) is established, the Schrödinger equation (2.8) in mass fragmentation co-ordinate $\eta$ can be solved. On solving Eq. (2.8) numerically, $|\psi^{\eta}(\eta)|^2$ gives the probability $P_0$ of finding the mass fragmentation $\eta$ at a fixed $R$ on the decay path.

$$P_0(A_2) \propto |\psi^{\eta}(A_2)|^2$$

(2.43)

For fission studies, like the spontaneous fission and fission through the barrier, the motion in $R$ at the saddle point is adiabatically slow as compared to the $\eta$ motion. Therefore, the potential is minimized in the neck $\varepsilon$ and deformation coordinates $\beta_1$ and $\beta_2$ at each $R$ and $\eta$ values. Starting from the nuclear ground state in spontaneous
fission or cluster decay, and to have complete adiabaticity, only the lowest vibrational state \( \nu = 0 \) is occupied. Then, the mass (or charge) distribution yield, proportional to the probability \( |\psi^{(0)}(\eta)|^2 \) (or \( |\psi^{(0)}(\eta_Z)|^2 \)) of finding a certain mass (or charge) fragmentation \( \eta \) (or \( \eta_Z \)) at a position \( R \) on the decay path, when scaled to, say, mass \( A_2 \) of one of the fragments \( (d\eta = \frac{3}{A}) \) is given by:

\[
P_0(A_2) = |\psi_R^{(0)}(A_2)|^2 \frac{2}{A} \sqrt{B_{\eta}(A_2)}. \tag{2.44}
\]

However, if the system is excited or we allow interaction between various degrees of freedom, higher values of \( \nu \) would also contribute. These enter via the excitation of higher vibrational states, and through the temperature dependent potential \( V \) and masses \( B_{ij} \).

The effect of adding temperature on potential \( V \) and masses \( B_{ij} \) is to reduce the shell effects in them, resulting finally in the liquid drop potential \( V_{LDM} \) and smoothed (averaged) masses \( \bar{B}_{ij} \) for the systems to be very hot. Apparently, cold fission means taking both the potential \( V \) and masses \( B_{ij} \) with full shell effects included in them and hot fission means using the \( V_{LDM} \) and smoothed (averaged) masses \( \bar{B}_{ij} \). The possible consequence of such excitations are included here by assuming a Boltzmann like occupation of excited states

\[
|\psi^{(\nu)}(\eta)|^2 = \sum_{\nu=0}^{\infty} |\psi^{(\nu)}(\eta)|^2 \exp \left( -\frac{E_\nu}{T} \right) \tag{2.45}
\]

Note that we are dealing here with a directly measurable quantity, the mass (or charge) asymmetry, which works dynamically as mass (or charge) transfer coordinate. Thus, the calculated yields \( P_0(A_i) \) (or \( P_0(Z_i) \)) are directly comparable with experiments. It may be stressed that there is no free parameter in these calculations. The nuclear shape, once minimized in the neck \( \varepsilon \) and deformation coordinates \( \beta_1 \) and \( \beta_2 \) at a given \( R \approx R_{saddle} \), remains fixed for both the mass and charge distributions of fission or decay fragments.
2.2.8 Penetration Probability $P$

For $R$-motion, instead of solving the Schrödinger Eq.(2.9), Gupta and collaborators used the WKB approximation to calculate the penetration probability $P$. For each $\eta$-value, the potential $V(R)$ for $R \geq R_t$ is calculated by using Eq.(2.12) and for $R < R_t$ it is parameterized simply as a polynomial of degree two in $R$, as

$$V(R) = \begin{cases} a_1R + a_2R^2 & \text{for } R_0 \leq R \leq R_t \\ V_c + V_p + V_t & \text{for } R \geq R_t \end{cases} \quad (2.46)$$

A typical scattering potential, for $^{232}$U$\rightarrow^{208}$Pb$+^{24}$Ne is shown in Figure 2.5. This is calculated by using Eq.(2.46) or Eq.(2.12) for the case of $\ell=0$. The path of the penetration and the related quantities are also shown. The constants $a_i$ ($i = 1, 2$) occurring in the polynomial, are determined by using the following boundary conditions.

1. At $R = R_0$, $V(R) = Q$
2. At $R = R_t$, $V(R) = V(R_t)$

In order to avoid the use of undetermined part of the potential, i.e., $V(R_0 \leq R \leq R_t)$, the first (inner) turning point $R_a$ is chosen at the touching configuration, $R_a = R_t$, and the outer turning point is taken at $R_b$ to give the $Q$-value of the reaction, i.e., $V(R_b) = Q$. This means that the transmission probability $P$ consists of three contributions:

1. The penetrability $P_i$ from $R_t$ to $R_i$,
2. the (inner) de-excitation probability $W_i$ at $R_i$ and
3. the penetrability $P_b$ from $R_i$ to $R_b$

giving the penetration probability as

$$P = P_i W_i P_b.$$  \hfill (2.47)

The shifting of first turning point from $R_t$ to $R_0$ gives the penetrability calculations similar to Shi and Swiatecki [67] for spherical nuclei, which is known not to fit the exotic cluster decay experimental data without the adjustment of assault frequency. Following the excitation model of M. Greiner and W. Scheid [68], the de-excitation probability $W_i$ is given as

$$W_i = \exp(-bE_i).$$  \hfill (2.48)

This means that the de-excitation process is restricted to only a single transition. If the parameter $b$ were allowed to depend on $R_i$, it should then become a process of multiple de-excitation and proceed as step-like process. For a heavy cluster decay into the excited states of the daughter nucleus the $b = 0$ is assumed [68], which means

$$W_i = 1,$$  \hfill (2.49)

so that,

$$P = P_i P_b.$$  \hfill (2.50)
where $P_i$ and $P_b$ are calculated by using WKB approximation, as

$$P_i = \exp \left[ \frac{2}{\hbar} \int_{R_a}^{R_t} \left\{ 2\mu [V(R) - V(R_i)] \right\}^{1/2} dR \right]$$

(2.51)

and

$$P_b = \exp \left[ \frac{2}{\hbar} \int_{R_t}^{R_b} \left\{ 2\mu [V(R) - Q] \right\}^{1/2} dR \right].$$

(2.52)

Here $R_a$ and $R_b$ are, respectively, the first and second turning points. This means that the tunnelling begins at $R = R_a$ and terminates at $R = R_b$, with $V(R_b) = Q$-value for ground state decay.

The integrals of the Eqs. (2.51) and (2.52) are solved analytically by parameterizing the above calculated potential $V(R)$, as follows:

$$V(R) = \begin{cases} 
 a_1 R + a_2 R^2, & R_0 \leq R \leq R_t, \\
 V(R_t) + m(R - R_t), & R_t \leq R \leq R_m, \\
 V_B - \frac{1}{2} k(R - R_B)^2, & R_m \leq R \leq R_b, \\
 V(R_b) - c_1 \frac{R_b - R_h}{R_h}, & R_h \leq R \leq R_t, \\
 V(R_t) - c_2 \frac{R_b}{R_t}, & R_t \leq R \leq R_b,
\end{cases}$$

(2.53)

For a polynomial of degree higher than two, analytical solutions of WKB integrals could not be obtained. The polynomial equation above is true for any inner turning point and hence $R_t$ could be chosen empirically at any point on the polynomial part, as was shown by Kumar and Gupta [51]. Equation (2.53) means that, the first part of the potential from $R_0$ to $R_t$ (or $R_{emp}$) is a polynomial of degree two in $R$, the second part from $R_t$ to $R_m$ is a straight line of slope `$m$', the top part between $R_m$ and $R_b$ being an inverted harmonic oscillator and the rest from $R_b$ to $R_t$ and $R_t$ to $R_b$ are the Coulomb potentials of the type $1/R$. Finally, $V_B$ and $R_B$ give the height and position of the barrier. The analytical solution for the integrals
are obtained as

\[
\int_{R_t}^{R_b} V(R) dR = \int_{R_t}^{R_m} V(R) dR + \int_{R_m}^{R_b} V(R) dR + \int_{R_m}^{R_t} V(R) dR + \int_{R_t}^{R_b} V(R) dR \quad (2.54)
\]

For different components of Eq.(2.50), we have

\[
P_i = \exp \left( -\frac{2}{\hbar} \sqrt{2\mu} \left( \frac{\sqrt{\alpha_2}}{2} t_1 (t_1^2 - L^2) \right)^{\frac{1}{2}} - t_2 (t_2^2 - L^2) \right)
\]

\[
- L^2 (\cosh^{-1} \left( \frac{t_1}{L} \right) - \cosh^{-1} \left( \frac{t_2}{L} \right))
\]

\[
+ \frac{2}{3} (V(R_m) - V(R_t)) [(V(R_m) - V(R_t))^{\frac{3}{2}} - (V(R_t) - V(R_t))^{\frac{3}{2}}]
\]

\[
- \frac{1}{\sqrt{2k}} [V_B - V(R_t)][(\Theta_2 - \frac{1}{2} \sin 2\Theta_2 - \Theta_1 + \frac{1}{2} \sin 2\Theta_1]
\]

\[
+ \sqrt{C_1 R_t R_i} (\Theta_3 - \frac{1}{2} \sin 2\Theta_3)]
\]

(2.55)

with

\[
a_1 = \frac{R_0(Q - V(R_t))}{R_t(R_t - R_0)}, a_2 = -\frac{a_1}{R_0}, t_1 = R_t - \frac{1}{2} R_0, t_2 = R_{emp} - \frac{1}{2} R_0
\]

\[
L^2 = \frac{1}{4} R_0^2 + R_t(R_t - R_0) \left[ \frac{Q - V(R_t)}{Q - V(R_t)} \right]
\]

\[
\Theta_1 = \cos^{-1} \frac{R_m - R_B}{\sqrt{\alpha_2}}, \Theta_2 = \cos^{-1} \frac{R_h - R_B}{\sqrt{\alpha_2}}, \Theta_3 = \tan^{-1} \left( \frac{R_t - R_h}{R_h} \right)^{1/2}
\]

\[
\alpha_2 = \frac{2}{k} [V_B - V(R_t)]
\]

\[
k = 2 \left( [V_B - V(R_m)]^{1/2} + [V_B - V(R_h)]^{1/2} \right)^2
\]

\[
(R_m - R_h)^2, C_1 = R_t \left( \frac{V(R_h) - V(R_t)}{R_t - R_h} \right)
\]

and

\[
P_b = \exp \left[ -\frac{2}{\hbar} \sqrt{2\mu} \sqrt{C_2 R_t R_t} \left( \Theta_4 - \frac{1}{2} \sin 2\Theta_4 \right) \right]
\]

(2.56)
Figure 2.6: Scattering potentials $V(R)$ for $^{286}112^* \rightarrow ^{283}112^+3n$ for $T=1.20$ MeV (equivalently, $E_{cm}=200.6$ MeV) at two different $\ell$ values ($\ell_{max}$ and $\ell_{min}$). The decay path, defined by $V(R_a, \ell)$ for each $\ell$ is shown to begin at $R_a = R_i + R_2 + \Delta R$ for the $\ell_{min}$ value. The definition of “barrier lowering” $\Delta V_B = V(R_a) - V_B$ is also shown in this figure for the $\ell_{min}$ value.

with

$$\Theta_4 = \tan^{-1} \left[ \frac{R_b - R_i}{R_i} \right]^{1/2}$$

$$C_2 = \frac{R_b [V(R_i) - V(R_b)]}{R_b - R_i}.$$ 

Substituting $P_i$ and $P_b$ in Eq. (2.50) we get the probability of penetration or the tunnelling probability $P$.

2.3 The Dynamical Cluster-decay Model for hot and rotating compound systems

The dynamical cluster decay model (DCM) [32]-[45] for hot and rotating nuclei (i.e. angular momentum $\ell \neq 0$ and temperature $T \neq 0$) is a reformulation of the preformed cluster model (PCM) of Gupta and collaborators for ground-state decays ($\ell=0$ and $T=0$) in cluster radioactivity (CR) and related phenomena [46]-[53]. Like
PCM, DCM is also based on the dynamical (or quantum mechanical) fragmentation theory of cold phenomenon in heavy ion reactions and fission dynamics. In DCM, besides temperature and angular momentum effects in the decay of excited compound nuclei, the deformation and orientation effects of the decay products are also taken care, especially in the decay of heavy excited CN for which deformations of the decay products seem to play significant role. The DCM, worked out in terms of the collective coordinates of mass asymmetry $\eta = \frac{A_1-A_2}{A_1+A_2}$ and relative separation $R$, respectively, give

(i) the nucleon-division (or -exchange) between outgoing fragments, and
(ii) the transfer of kinetic energy of incident channel ($E_{c.m.}$) to internal excitation (total excitation or total kinetic energy, $TXE$ or $TKE$) of the outgoing channel.

It may be noted that the fixed decay point $R = R_a$ (defined later), at which the process is calculated, depends on temperature $T$ as well as on $\eta$ (i.e. $R(T, \eta)$). This energy transfer process is defined by the following equation (see Figure 2.6).

$$E^* + Q_{out}(T) = TKE(T) + TXE(T).$$ (2.57)

The CN excitation energy $E^*_{CN} = E_{c.m.} + Q_{in}$ is related to temperature $T$ (in MeV) as given by Eq.(2.22).

Using the decoupled approximation to $R$- and $\eta$-motions, the DCM defines the CN decay cross section, in terms of partial waves, as [32]-[45]

$$\sigma = \sum_{\ell=0}^{l_s} \sigma_\ell = \frac{\pi}{k^2} \sum_{\ell=0}^{l_s} (2\ell + 1)P_0P; \quad k = \frac{2\mu E_{c.m.}}{\hbar^2}$$ (2.58)

where, $P_0$, the preformation probability, refers to $\eta$-motion and $P$, the penetrability, to R-motion. Here the complex fragments (both light and heavy fragments) are treated as the dynamical collective mass motion of preformed clusters or fragments through the barrier. The structure information of the CN enters the model via the preformation probabilities $P_0$ (also known as the spectroscopic factors) of the fragments given by the solution of stationary Schrödinger equation in $\eta$, at the fixed $R=R_a$, the first turning point of the penetration path shown in Figure 2.6 for
different \( \ell \)-values,

\[
\left\{ - \frac{\hbar^2}{2\sqrt{B_{ym}}} \frac{\partial}{\partial \eta} \sqrt{B_{ym}} \frac{\partial}{\partial \eta} + V(R, \eta, T) \right\} \psi^\nu(\eta) = E^\nu \psi^\nu(\eta) ,
\]

(2.59)

with \( \nu=0,1,2,3... \) referring to ground-state (\( \nu = 0 \)) and excited-states solutions.

For the decay of a hot CN, Gupta et al. use the postulate for the first turning point

\[
R_a(T) = R_t + \Delta R(T)
\]

(2.60)

where

\[
R_t = R_1 + R_2
\]

(2.61)

\( \Delta R(T) \) is the neck-length parameter that assimilates the neck formation effects. This method of introducing a neck length parameter is similar to that used in both the scission-point [69] and saddle-point [70, 71] statistical fission models. The \( R_i \) are radius vectors, given by Eqs.(2.15) and (2.16).

The corresponding potential \( V(R_a) \) acts like an effective Q-value, \( Q_{eff} \), for the decay of the hot CN at temperature \( T \), to two exit-channel fragments observed in g.s. (\( T=0 \)), defined by

\[
Q_{eff}(T) = B(T) - \left[ B_L(T = 0) + B_H(T = 0) \right]
\]

\[
= TKE(T) = V(R_a(T))
\]

(2.62)

with \( B's \) as the respective binding energies.

The above defined decay of a hot CN into two cold (\( T=0 \)) fragments, via Eq.(2.62), could apparently be achieved only by emitting some light particle (s) (LPs), like \( n, p, \alpha, \) or \( \gamma \)-rays of energy

\[
E_x = B(T) - B(0) = Q_{eff}(T) - Q_{out}(T = 0)
\]

\[
= TKE(T) - TKE(T = 0)
\]

(2.63)

which is zero for the g.s. decay, like for exotic CR. Note that the second equality
in Eq. (2.63) is not defined for a negative $Q_{\text{out}}(T = 0)$ system since the negative 
$\text{TKE}(T=0)$ has no meaning. Apparently, Eq. (2.63) w.r.t Eq.(2.62) suggests that 
the emission of light-particles starts early in the decay process. The exit channel 
fragments in Eq. (2.62) are then obtained in the ground-state with $\text{TKE}(T=0)$, as 
can be seen by calculating $E_{CN}^* - E_x$: 
\begin{equation}
E_{CN}^* - E_x = -Q_{\text{out}}(T) + \text{TKE}(T = 0) + \text{TXE}(T). \tag{2.64}
\end{equation}

The excitation energy $\text{TXE}(T)$ (not treated here) is used in, the secondary emission 
of light particles from the fragments which are otherwise in their ground states 
with $\text{TKE}(T=0)$ in the radial motion. Thus, by defining $Q_{\text{eff}}(T)$ as in Eq. (2.62), 
in this model we treat the LP emission at par with the heavy fragments, called 
intermediate mass fragments (IMFs) emission or fusion-fission(ff)fragments. Thus, 
in this model a non-statistical dynamical treatment is attempted for not only the 
emission of IMFs and ff but also of multiple LPs, understood so-far only as the 
statistically evaporated particles in a $CN$ emission. It may be reminded here that 
the statistical model interpretation of IMFs is not as good as it is for the LP 
production [69]-[74].

In terms of $Q_{\text{eff}}(T)$, the second turning $R_b$ satisfies (see Figure 2.6)
\begin{equation}
V(R_a, \ell) = V(R_b, \ell) = Q_{\text{eff}}(T, \ell) = \text{TKE}(T). \tag{2.65}
\end{equation}

with the $\ell$-dependence of $R_a$ defined by
\begin{equation}
V(R_a, \ell) = Q_{\text{eff}}(T, \ell = 0), \tag{2.66}
\end{equation}

which means that the $R_a$, given by Eq. (2.66), is the same for all $\ell$-values, and that 
$V(R_a, \ell)$ acts like an effective Q-value, $Q_{\text{eff}}(T, \ell)$, given by the total kinetic energy 
$\text{TKE}(T)$. Then, using Eq. (2.65), $R_b(\ell)$ is given by the $\ell$-dependent scattering 
potentials, at fixed $T$ is given by Eq.(2.12), which is normalized to the exit channel 
binding energy $B_L(T) + B_H(T)$. Such a potential is illustrated in Figure 2.6, for
The second turning point $R_b$ is marked for the $\ell = 0h$ case of $R_a = R_t + \Delta R(T)$. Note that as the $\ell$-value increases, the $Q_{\text{eff}}(T)$-value ($= \text{TKE}(T)$) increases and hence $V(R_a, \ell)$ increases, since the decay path for all the $\ell$-values begins at $R = R_a$.

Finally, the $\ell_c$-value in Eq. (2.58) is the critical $\ell$-value, in terms of the bombarding energy $E_{\text{c.m.}}$, the reduced mass $\mu$ and the first turning point $R_a$ of the entrance channel $\eta_{\text{in}}$, given by

$$\ell_c = R_a \sqrt{2\mu [E_{\text{c.m.}} - V(R_a, \eta_{\text{in}}, \ell = 0)]/\hbar}, \quad (2.67)$$

or, alternatively, it could be fixed for the vanishing of fusion barrier of the incoming channel, called $\ell_{fus}$, or else the $\ell$-value ($\ell_{\text{max}}$) where the light-particle cross section $\sigma_{LP} \rightarrow 0$. This, however, could also be taken as a variable parameter [70, 75]. The parameter $\Delta R$ can also be related to the so-called barrier lowering parameter $\Delta V_B$, shown in Figure 2.6 and defined as, $\Delta V_B = V(R_a) - V_B$. Note that $\Delta V_B$ is always negative and hence is referred to as lowering of the barrier.
Bibliography


