Chapter 5

Establishing the island of stability for superheavy nuclei via the dynamical cluster-decay model applied to hot fusion reaction $^{48}$Ca$+^{238}$U$\rightarrow^{286}_{112}$*: All decay processes (ER, ff and qf)

5.1 Introduction

A long standing problem of nuclear structure studies in the region of superheavy nuclei has been to establish the center of island of stability, i.e., the next doubly magic nucleus (magic Z and magic N) heavier than $^{208}$Pb. With nearly 50 years of new data at hand, since the shell model was first worked out, perhaps a new (one- or two-center) shell model is now needed, or at least it is relevant to ask the question "what differences would arise, say, for the calculated fusion cross-section in the superheavy mass region, if the magic Z, N shells were chosen as Z=120, N=184
or \( Z=126, N=184 \) or \( Z=120, N=172 \) on the basis of mean field studies, rather than \( Z=114, N=184 \) on shell model basis." In Chapter 4, using the dynamical cluster-decay model, we solved this problem for the fusion-evaporation residue cross-sections \( \sigma_{ER} \) alone, calculated for the three cases of magic \( Z=114, 120 \) or 126 and \( N=184 \), using the Dubna data [1] on hot fusion reaction \( ^{48}\text{Ca}+^{238}\text{U}\rightarrow^{286}112^{*} \) as an example [2]. This study seems to support the island of stability for superheavy nuclei to center around \( Z=126, N=184 \), rather than around \( Z=114, N=184 \), with \( Z=120, N=184 \) as the second best possibility. However, for this reaction, in addition to \( \sigma_{ER} \) (\( =\sigma_{2n}+\sigma_{4n}, \) the sum of neutron-channel cross-sections) at higher excitation energies of \( E^{*}\approx30-40 \) MeV, the excitation functions for fusion-fission (\( \sigma_{ff} \)) and capture (\( \sigma_{cap} \), equivalently, quasi-fission \( \sigma_{qf} \)) are also measured in another Dubna experiment [3], though at lower excitation energies of \( E^{*}\approx20-40 \) MeV. The quasi-fission (\( qf \)) channel is characterized by the formation of highly asymmetric mass fragments due to a few nucleon transfer (light mass fragment \( A_{2} \approx76-86 \), and the corresponding heavy mass fragment \( A_{1} \approx210-200 \), for the above said reaction), occurring most probably at the early stage of the fusion process when the compound nucleus is still strongly deformed and the target and projectile nuclei can be considered to have not yet lost their identity. On the other hand, the symmetric or near symmetric \( ff \) channel (fragment mass \( A_{i}=A/2\pm20, i=1, 2 \)) is the last stage of the process where the strongly deformed complex develops into the neck formation. Of-course, the evaporation residue occur almost promptly (largest neck-length parameter \( \Delta R \)). Therefore, in this chapter, we consider the total data [1,3] on three cross-sections (\( \sigma_{ER}, \sigma_{ff} \) and \( \sigma_{qf} \)) simultaneously for a best fit of the only parameter of the model, the neck-length \( \Delta R(E^{*}) \). We consider all the four cases of magic \( Z=114, 120 \) or 126 with \( N=184 \) and \( Z=120, N=172 \) for obtaining the shell corrections from the "empirical" formula of Myers and Swiatecki [4] with the corresponding liquid drop energies obtained from Seeger’s mass formula [5] based work of Davidson et al. [6].

The chapter is organised as follows: Our application of this model to \( ^{48}\text{Ca}+^{238}\text{U} \) reaction, with observed \( \sigma_{ER}, \sigma_{ff} \) and \( \sigma_{qf} \) cross-sections [1,3], is made is Section 5.2. \( ^{48}\text{Ca} \) nucleus is known to form a “compact” configuration with \( ^{238}\text{U} \) at an orientation
Figure 5.1: Mass fragmentation potentials $V(A)$ for the compound system $^{286}\text{U}^{112*}$ at $R = R_0 = R_t + \Delta R$ and at $T=1.20$ MeV corresponding to the $^{238}\text{U} + ^{48}\text{Ca}$ reaction at $E_{cm}=200.6$ MeV, calculated for deformations $\beta_2 + \beta_3 + \beta_4$ and (hot) compact orientations $\theta_c$ for all possible combinations of two nuclei at different $\ell$-values. The $\Delta R$-values are 1.2, 1.197, 1.039 and -0.827 fm, respectively, corresponding to light fragment mass regions $A_1=1-2$, 3-4, 5-109 and 113-143, for the best fit to the complete data $\sigma_{ER}$, $\sigma_{IF}$ and $\sigma_{Of}$ (see Figures 5.4 and 5.5).
angle $\theta_c = 72^\circ$, since $^{238}$U has a large positive hexadecapole deformation. A summary and discussion of our results is given in Section 5.3.

5.2 Calculations

We first study the role of changing the magic numbers, in superheavy mass region, on the fragmentation potential and hence on the preformation probability $P_0$, for the three processes of evaporation residue (ER), fusion-fission (ff) and quasi-fission (qf), all fitted simultaneously, for the compound system $^{286}112^*$ formed in $^{48}\text{Ca}+^{238}\text{U}$ reaction.

Figure 5.1 shows the mass fragmentation potentials $V(A_2)$ for the compound system $^{286}112^*$ at $R = R_a = R_t + \Delta R$, and $T=1.20$ MeV, corresponding to one of the incident energy $E_{c.m.} = 200.6$ MeV of the $^{48}\text{Ca}+^{238}\text{U}$ reaction [1], and for the sticking moment-of-inertia ($I = I_S$) and deformations up to hexadecapole deformations ($\beta_{2i} - \beta_n$) and “compact” orientations $\theta_{2a}$. $\Delta R$ refers to the best fitted values (for different mass regions) for the case of $Z=126, N=184$ (see Figure 5.3(a)) and, for comparisons, is kept the same for the other two cases ($Z=120, N=184$) also. Note that in chapter 4, we considered $A_R$ fixed for all $\eta$ values. Compared to our earlier calculation [7] where the neck-length parameter $\Delta R$ was fixed for the whole mass region, here it is different for each mass region of the three processes. However, just as in Chapter 4, a change in the fragmentation potentials is shown to occur only in cases where one of the nucleus in target-projectile combinations ($A_1, A_2$) is close to the new magic numbers. In other words, no change is expected to take place for pairs in the neighborhood of symmetric fragments, as is shown to be the case for fragments beyond the light fragment masses $A_2 > 70$. For $A_2 < 70$, Figure 5.1 shows that some of the minima change (due to the heavy fragment $A_1$), in particular their relative depths, as the proton closed shell for superheavy mass region is changed. One such important change is in the minima at $3\eta$ and $4\eta$ clusters (plus the corresponding heavy fragments). In Figure 5.1, the $3\eta$ and $4\eta$ fragments lie lower in energy for the case of $Z=126, N=184$ or $Z=120, N=184$, and higher in
Figure 5.2: (a) Preformation probability $P_q$ for the compound system $^{286}112^*$ at $R = R_a = R_t + \Delta R$ and at $T=1.20$ MeV corresponding to the $^{48}\text{Ca} + ^{238}\text{U}$ reaction at $E_{c.m.}=200.6$ MeV, calculated for deformations $\beta_2 \ldots \beta_4$ and (hot) “compact” orientations $\theta_{ci}$ for all possible combinations of two nuclei ($i=1, 2$) at different $\ell$-values. $\Delta R= 1.2, 1.197, 1.039$ and $-0.827$ fm, respectively, for light fragment mass regions $A_2=1-2, 3-4, 5-109$ and $113-143$ (and their corresponding heavy fragments), for a best fit to data [1, 3] on ER, qf and ff (see, Figures 5.3, 5.4 and 5.5). (b) Channel cross-sections $\sigma_\ell$, $x=3-4$, for $^{286}112^*$, plotted as a function of $\ell$, for the case of $I = I_S$. The cut-off is at $\sigma_\ell < 10^{-9}$ pb, limiting the $\ell$-value to $\ell_{min}=91 \hbar$ and $\ell_{max} 160 \hbar$. 

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the case of $Z=114, N=184$. In terms of preformation probability $P_0$, and hence the evaporation residue cross section $\sigma_{ER}$, the above result would mean that both $P_0$ and $\sigma_{ER}$ are larger for the case of $Z=126, N=184$ or $Z=120, N=184$ magic numbers and almost indistinguishable for $Z=126$ or $120, N=184$ which is different from that in Chapter 4 due to $\Delta R$ being now different for different mass regions.

A number of other new reaction valleys (target-projectile combinations) arise: For $Z=126, N=184$ case, the minima are lying at $^{24}\text{Na}+^{262}\text{Md}$, $^{29}\text{Al}+^{257}\text{Es}$, $^{35}\text{P}+^{251}\text{Bk}$ and $^{37}\text{S}+^{249}\text{Cm}$; and for $Z=120, N=184$ case, the minima are lying at $^{25}\text{Na}+^{262}\text{Md}$, $^{29}\text{Al}+^{257}\text{Es}$, $^{36}\text{S}+^{250}\text{Cm}$ and $^{51}\text{Ca}+^{235}\text{U}$; for $Z=114, N=184$ case, the minima are lying at $^{24}\text{Na}+^{262}\text{Md}$ and $^{36}\text{S}+^{250}\text{Cm}$. However the minima common to all the cases are: $^{12}\text{B}+^{274}\text{Bh}$, $^{22}\text{F}+^{264}\text{Lr}$, $^{27}\text{Mg}+^{259}\text{Fm}$, $^{32}\text{Si}+^{254}\text{Cf}$, $^{42}\text{Ar}+^{244}\text{Pu}$, $^{48}\text{Ca}+^{238}\text{U}$, $^{53}\text{Ti}+^{233}\text{Th}$, $^{54}\text{Ti}+^{233}\text{Th}$, $^{58}\text{Cr}+^{228}\text{Ra}$ and $^{67}\text{Co}+^{218}\text{At}$. In the region of $A_2 > 70$, where fragmentation potential is exactly same for all the three cases, the minima are also exactly same and are: $^{74}\text{Zn}+^{212}\text{Pb}$, $^{60}\text{Ge}+^{206}\text{Hg}$, $^{84}\text{Se}+^{202}\text{Pt}$, $^{91}\text{Kr}+^{195}\text{Os}$, $^{94}\text{Sr}+^{192}\text{W}$, $^{109}\text{Ru}+^{177}\text{Er}$, $^{114}\text{Cd}+^{172}\text{Gd}$, $^{118}\text{Pd}+^{168}\text{Dy}$, $^{128}\text{Sn}+^{158}\text{Sm}$, $^{134}\text{Te}+^{152}\text{Nd}$, $^{138}\text{Xe}+^{148}\text{Ce}$ and $^{140}\text{Ba}+^{146}\text{Ba}$. The minima common to all the three cases are shown in the Figure 5.1.

Figure 5.2(a) shoes $P_0$ plotted as a function of fragment mass number for the three cases of magic numbers ($Z=126, 120$ or $114, N=184$), calculated for the two extreme $\ell$-values, fixed for the light-particles channel cross sections $\sigma_{xn}(\ell) \to 0$, $x=3, 4$, i.e., $\sigma_{xn}(\ell) > 10^{-9}$ pb, as is illustrated in Figure 5.2(b). We fix $\ell_{\min}=91$ and $\ell_{\max}=160$ h, though the values of angular momenta at which the channel cross sections actually become negligible are slightly different for $3n$ and $4n$ emissions and for the three cases of different magic numbers. Figure 5.2(a) shows that the maximum yield ($P_0$) is obtained only for the three observed [1,3] regions of symmetric fission ($(A/2) \pm 20$), near target-projectile fragments corresponding to quasi-fission (centered around $A_2=76-86$, $A_1=210-200$), and the light particles giving rise to evaporation residues ($A_2=1-4$, but here only $A_2=3n$ and $4n$ are observed [1]; the $1n$ and $2n$ emissions lie below the fusion barrier which for $^{48}\text{Ca}+^{238}\text{U}$ reaction is at $E^* \sim 32$ MeV), in complete agreement with experiments [1,3]. Note that for fusion-fission,
with a view to include the symmetric mass division \( A_1 = A_2 = A/2 = 143 \) and noting that the maximum \( P_0 \) are for \( A_2 = 138, A_1 = 148 \) (equivalently, \( (A/2) \pm 5 \)), the best fit to data is obtained for fragment mass range \( A_2 = 113-143, A_1 = 173-143 \) (equivalently, \( (A/2) \pm 30 \)). Concerning the dependence of \( P_0 \) on magic shells, as expected, \( P_0 \) is different for the three chosen magic proton numbers 114, 120 or 126 for superheavy mass region, and the difference is more prominent for the heavy mass fragments in the superheavy mass region (and their complementary light mass fragment in the mass region \( A_2 < 58 \)) and smaller angular momenta (illustrated here for \( \ell_{\text{min}} = 91 \hbar \)). For the larger \( \ell_{\text{max}} = 160 \hbar \), however, \( P_0 \) is almost same for the three cases, more so for \( A_2 > 58 \) (and complementary fragments). Specifically, for the light particles (here 3n and 4n), \( P_0 \) is larger for \( \ell_{\text{min}} \) value compared to that for \( \ell_{\text{max}} \), whereas the same for symmetric fission and quasi-fission increase considerably in going from \( \ell_{\text{min}} \) to \( \ell_{\text{max}} \). Note, however, that the magnitude of the yields for the relevant three regions (ER, qf and ff) presented here in Figure 5.2(a) differ significantly from our earlier calculation (Figure 2 in [7]) based on ER alone, taking same \( A_R \) for the three regions of ER, qf and ff. Another maximum of interest in the \( P_0 \) graph (Figure 5.2(a)) corresponds to the incoming channel \( ^{48}\text{Ca} + ^{238}\text{U} \), which does not change due to the changed magic numbers.

The neck-length parameter \( \Delta R \) as a function of CN excitation energy \( E^* \) for the simultaneous best fit to the data on cross-sections for the three processes of ER, qf and ff, considered for the cases of four magic pairs of \( Z, N \) (\( Z = 126, 120 \) or 114, \( N = 184 \) or \( Z = 120, N = 172 \)), is shown in Figure 5.3(a). Interestingly, due to the changed magic numbers, as expected, the largest change in \( \Delta R \) occurs for ER, the 3n and 4n emissions, since the complementary heavy fragments lie in the neighborhood of superheavy region, and the \( \Delta R \) for ff is nearly the same since they refer to symmetric and nearly symmetric fragments, i.e., far away from superheavy magic shells. The qf process is independent of the magicity of shells since it refers to incoming channel alone. The important result is that the neck-length parameter \( \Delta R \) (equivalently, relative separation \( R_a \)) is different for the three processes, which means to predict that the three processes of ER, qf and ff happen in different time-
Figure 5.3: (a) The best fitted neck-length parameter $\Delta R$ as a function of CN excitation energy $E^*$ for the three processes of evaporation residue ($\sigma_{ER} = \sigma_{3n} + \sigma_{4n}$), the fusion-fission (ff) and quasi-fission (qf), considered for the four cases of magic numbers. Note that the qf process is independent of magicity of shells. (b) The barrier lowering parameter $\Delta V_B$ as a function of $E_{c.m.}$ for $\ell = \ell_{max}$ case. ER consists of $3n$ and $4n$ and for ff only the fragment $A/2 - 30$ is considered. The change in $\Delta V_B$ due to uncertainty in $3n$ and $4n$ cross-sections is also illustrated.

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scales, in agreement with the indications of experiments [1,3]. The $\Delta R$ is largest for ER and smallest for ff, which means to suggest that the neutrons emission occur earliest, then the quasi-fission process, and finally the fusion-fission process from a necked-in system.

The parameter $\Delta R$, can be translated to the so called barrier lowering parameter $\Delta V_B$, as illustrated in Figure 2.6, is plotted in Figure 5.3(b) as a function of $E_{cm}$, for the three processes at $l = l_{max}$. Interestingly, $\Delta V_B$ is nearly constant and within the error bars (calculated for 3n and 4n cross-sections at the highest energy) for both ER and ff, but shows a larger “barrier-lowering” at lower energies for qf process. The important point is that $\Delta V_B$ is negative and non-zero for all the three processes, which means to conclude that barrier lowering is essential and inbuilt in DCM, a property first noted [8] for reactions such as $^{64}$Ni+$^{100}$Mo known for the hindrance effect in coupled channel calculations.

Figures 5.4 and 5.5 show, respectively, the calculated $\sigma_{ER}$ ($=\sigma_{3n} + \sigma_{4n}$) and $\sigma_{ff}$ and $\sigma_{qf}$, compared with experimental data [1,3], for the neck length parameters $\Delta R(E^*)$ obtained in Figure 5.3(a). In each of these figures, the fitted cross-sections in panels (a), (b), (c) and (d) refer, respectively, to the cases of magic pairs $Z=126$, 120 or 114, $N=184$ and $Z=120$, $N=172$. Then, in each panel, using the same $\Delta R(E^*)$, we calculated the cross-sections ($\sigma_{ER}$ and $\sigma_{ff}$) for the cases of other three magic numbers. For example, in panel (a), we fit $\sigma_{ER}$ or $\sigma_{ff}$ (in Figure 5.4 or 5.5) to the case of $Z=126$, $N=184$, and calculate for the same $\Delta R(E^*)$, the two cross-sections for the other three cases of $Z=120$, $N=184$; $Z=114$, $N=184$ and $Z=120$, $N=172$. As already noted above, the $\sigma_{qf}$, being independent of magic shells, remains the same in each panel. Clearly, our model calculations in each panel (a), (b), (c) and (d) of both the figures (Figures 5.4 and 5.5) reproduce the experimental data for the three processes of ER, qf and ff (solid spheres with error bars) successfully in terms of a single fitting parameter $\Delta R$ at each excitation energy $E^*$ (or temperature $T$).

In Figure 5.4, the $\sigma_{ER}$ apparently remains the largest and nearly indistinguishable for $Z=126$, $N=184$ and $Z=120$, $N=184$, but then, independent of $E^*$, in Figure 5.5, the $\sigma_{ff}$ is always the highest for $Z=120$, $N=184$, with both the cross-sections...
Figure 5.4: Calculated and measured $\sigma_{ER}$ as a function of the CN excitation energy $E^*$ for $^{48}\text{Ca}+^{238}\text{U} \rightarrow ^{286}112^*$ reaction. Experimental data is from [1] and the calculations in panels (a), (b), (c) and (d) are made for use of the magic pairs $Z=126$, 120 or 114, $N=184$ or $Z=120$, $N=172$, with the neck-length parameter $\Delta R(E^*)$ taken from Figure 5.3(a), which are fitted, respectively, to the cases of $Z=126$, 120 or 114, $N=184$ or $Z=120$, $N=172$. In each panel, the same $\Delta R(E^*)$ are used for the remaining three cases.

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Figure 5.5: Same as for Figure 5.4, but for $\sigma_{ff}$ and $\sigma_{qf}$. Experimental data is from [3].
\(\sigma_{ER}\) and \(\sigma_{ff}\)) for \(Z=120, N=172\) always remaining the lowest. A closer look suggests that the two cross-sections for \(Z=120, N=184\) are the largest of all the four cases studied here and hence the strongest magic numbers (largest shell corrections), compared to the \(Z=126, N=114\), \(N=184\) or \(Z=120, N=172\) magic numbers, supporting the relativistic mean field model \([9,10]\) result.

### 5.3 Summary

In this chapter, we have studied the effect of using different magic numbers for the superheavy region on fusion-evaporation residue, fusion-fission and quasi-fission cross sections, taking \(^{48}\text{Ca}+^{238}\text{U} \rightarrow ^{286}112^*\) as an example. In other words, for seeing what differences would arise as a result of the proton magic shell being at \(Z=120, N=184\); or \(Z=126, N=184\), or \(Z=120, N=172\) instead of the commonly used \(Z=114, N=184\), with \(N=184\), we have calculated the fusion-evaporation residue cross sections \(\sigma_{ER}\), fusion-fission cross sections \(\sigma_{ff}\), and quasi-fission cross sections \(\sigma_{qf}\), for all the four cases of \(Z=114, 120\) or \(126\) and \(N=184\) and \(Z=120, N=172\) as magic numbers. For calculating the fusion excitation functions, we have used the dynamical cluster-decay model (DCM) with effects of deformations (up to hexadecapole) and “compact” orientation degrees of freedom of the incoming nuclei and/or outgoing fragments included. Note that the chosen \(^{48}\text{Ca}+^{238}\text{U}\) reaction forms a non-equatorial compact (\(\theta_c=72^\circ\) for \(^{238}\text{U}\)) hot fusion configuration. The shell corrections are calculated by using the “empirical” formula of Myers and Swiatecki for \(Z=126, 120\) or \(114\) with \(N=184\) as the closed shells, and the constants of liquid drop energy due to Davidson et al. adjusted in each case to obtain the experimental binding energies. The result of our calculation is that the evaporation residue cross-sections (which are strongly dependent on the choice of magic shells due to the complementary heavy residue) are the largest and nearly indistinguishable for \(Z=126, N=184\) or \(Z=120, N=184\), but the fusion-fission cross-sections, independent of \(E^*\), are always favoured (highest) for \(Z=120, N=184\). Also, since \(\Delta V_B\) is non-zero for all the cases, we conclude that
barrier lowering is an inherent part of DCM.

Concluding, our detailed calculations and a close look at the results suggests $Z=120, N=184$ as the strongest magic numbers (largest shell corrections) which should form a doubly-magic spherical nucleus at the center of island of stability of superheavy nuclei.
Bibliography


