Chapter 3

Role of static deformation and compact orientation of target nucleus in measured fusion, fusion-fission and capture cross-sections of $^{244}\text{Pu}+^{48}\text{Ca}$ reaction.

3.1 Introduction

The experimental work of the last three decades or so has shown that in the synthesis of heavy nuclei with $Z \geq 102$, both in the so-called cold and very hot fusion reaction studies, the evaporation residue cross-sections $\sigma_{ER}$ decrease rapidly with increasing $Z$ of compound nucleus. Extrapolating the dependence of measured $\sigma_{ER}$ on $Z$, we would arrive at extremely low cross-sections $\sim 1$-$10$ fb for the production of isotopes of elements $Z \geq 112$ (see Figure1.2). However, the recent, medium hot fusion reaction data on the synthesis of superheavy elements at Dubna[1–4], using
$^{48}$Ca beam on various deformed actinides $^{233,238}$U, $^{242,244}$Pu, $^{243}$Am, $^{245,248}$Cm and $^{249}$Cf, result in evaporation residue (ER) cross-section $\sigma_{ER}$ for $Z=112-118$ nuclei that are about a factor of three to five larger than for the well known cold or very hot fusion reactions for $Z\leq112$ (see Figure 1.2(d)). One of the Dubna results, the reaction $^{238}$U($^{48}$Ca,3n)$^{293}$112, was confirmed very recently at GSI [5]. Such an increased fusion threshold for an intermediate hot fusion reaction, observed in terms of a broad peak of excitation functions located at a higher excitation energy, compared to that for $^{206,208}$Pb based cold fusion reactions, is associated with the static deformation of the target nucleus, whose orientation at the point of collision should play the most important role at the very start of its path toward an almost spherical compound nucleus [3,6]. Theoretically, such a shift to higher energy (by ~20 MeV) occurs for a “compact” configuration, orientation $\theta_c \leq 90^0$ of the deformed nucleus; $^{48}$Ca being a spherical nucleus) of the entrance channel nuclei. In fact, the calculations [7,8] show that there is a distribution of the barrier in orientation degrees of freedom, characterized by (i) the minimum interaction radius (distance between the centers of the interacting nuclei) and highest interaction barrier, corresponding to the most compact configuration of the hot composite system, and (ii) the maximum interaction radius and lowest interaction barrier, depicting an elongated cold fusion configuration. All other configurations lie in between these two extremes.

In a very recent paper [9], Gupta et al. have shown that for reactions involving a deformed target nucleus with quadrupole deformation alone and/ or with small (including negative values) hexadecapole deformation, the compact configuration occurs at an orientation angle $\theta_c = 90^0$ of the deformed nucleus (the equatorial compact configurations, ec), whereas the one with large (positive) hexadecapole deformation is compact at $\theta_c < 90^0$ (not-equatorial compact, nec) configuration. This is shown to occur, respectively, for $Z \geq 114$ and $Z < 114$ nuclei, the difference between the “ec” and “nec” configurations being large ~20°. Note that for $^{48}$Ca beam the systems are of spherical-plus-deformed nuclei, and hence of co-planar nuclei (azimuthal angle $\phi = 0^0$). For a compact configuration, the compound nucleus has a higher probability of reaching its final shape and hence it would be interesting to see the role of
compactness and also of quadrupole and higher multipole deformations of the target nuclei on, say, the evaporation residue cross-sections $\sigma_{ER}$ ($=\sum_x \sigma_{xn}$, $x=3,4$ for hot fusion reactions; also $=5$ in some cases) of $^{48}$Ca-induced reactions on various actinide targets. In this chapter, we have chosen to apply our considerations to $^{244}$Pu($^{48}$Ca,xn)$^{292-114}$ reaction, where the individual light-particle decay channels $\sigma_{xn}$, $x=3,4$ and 5, are measured in Dubna experiment [1], at the (higher) excitation energies of $E^* \approx 31-53$ MeV [10]. For this reaction, $^{48}$Ca nucleus forms the compact configuration with $^{244}$Pu at $\theta_c=90^\circ$ (see Table I of Ref. [9]) since $^{244}$Pu has a small positive hexadecapole deformation. Thus, we have here a case of equatorial compact (the ec) configuration.

In addition to the evaporation residues $\sigma_{ER}$, rather the channel cross-sections $\sigma_{xn}$ here in this reaction, the excitation functions for fusion-fission (ff) and capture, i.e., $\sigma_{ff}$ and $\sigma_{cap}$, are also measured [11], though at lower excitation energies $E^* \approx 27-37$ MeV. In a later publication [3], based on the, so-called, Dubna classical model of compound nucleus (CN) formation (discussed in the next section of this chapter), these authors argued that for such hot fusion reactions $\sigma_{cap} \approx \sigma_{ff}$, the quasi-fission (qf) cross-section (see Figure 7 and the associated discussion in [3]).

The quasi-fission channel, characterized by the formation of highly asymmetric mass fragments due to a few nucleon transfer ($A_L \approx 85-89$, $A_H \approx 203-207$ for $^{244}$Pu+$^{48}$Ca$\rightarrow^{292}$114* reaction), most probably occurs at the early stage of the fusion process, and at lower excitation energies ($E^* < 30$ MeV), below the Bass barrier ($\sim 33$ MeV for the reaction under study), when the compound nucleus is still strongly deformed. At this stage, the target and projectile nuclei can be considered to have not yet lost their identity[11, 15]. For such an energy range, $T \sim 1$ MeV and hence the shell effects are not completely washed out (refer to potential energy minima in Figure 3.1, which occur only for the three processes of ER, qf and ff, and the entrance channel nuclei). There are experimental evidences[16, 17] that such shell effects are important for the quasi-fission process, which means that qf is rather a cold process compared to fusion-fission. Note, however, that the excitation energies involved in all of these studies are large, $\sim 30-40$ MeV, which equally favor the quasi-fission to
be a hot process. In the present work, we consider the quasi-fission process to mean the non-compound nucleus effects (qf, DIR, pre-equilibrium, etc.) and study the qf process for use of both the hot and cold barriers, respectively, the “equatorial” and “polar” configurations. On the other hand, the symmetric or near symmetric fission channel (fragment mass $A_i = A/2 \pm 20$) is the last stage of the process where the strongly deformed complex develops in to the neck formation.

We use here the dynamical cluster-decay model of Gupta and collaborators extended to include the deformation and orientation degrees of freedom of the colliding or outgoing nuclei. The model is quite general, describing completely the ER plus ff and the competing qf (equivalently, “capture”) process. The model uses a single parameter, the neck length parameter $\Delta R$, which, for a given temperature, takes different values for different processes. Since one of the aims of present study is to see the role of compact orientation $\theta_c$ and of multipole deformations $\beta_\lambda$, $\lambda \geq 2$, on fusion cross-sections, as already stated above, we discuss here in this chapter in detail their effects on the evaporation residues $\sigma_{ER}$ or, equivalently, the channel cross-sections $\sigma_{ch}$, as an illustrative example. We find that the channel cross-section for collisions between the spherical $^{48}$Ca beam and deformed $^{244}$Pu target nucleus at its respective compact orientation is much more for deformations included (up to hexadecapole $\beta_4$) than for the case of the two nuclei taken to be spherical, signifying the importance of static deformations and the corresponding compact orientation of the target nucleus at the point of collision in its path toward the (spherical) compound nucleus.

In the following section, (Section 3.2) the results of our calculations for the compound system $^{292}114^*$ are presented. A summary of our results is given in Section 3.3.

3.2 Calculations

Figure 3.1 shows the mass fragmentation potentials $V(A)$ at different $\ell$-values for the compound system $^{292}114^*$ at $R = R_a = R_t + \Delta R$, and $T=1.14$ MeV corresponding to
Figure 3.1: Mass fragmentation potentials $V(A)$ for the compound system $^{292}114^*$ at $R = R_{a} = R_{t} + \Delta R$ and at $T=1.14$ MeV corresponding to the $^{244}$Pu+$^{48}$Ca reaction at $E_{c.m.}=200.1$ MeV, calculated for deformations $\beta_2 + \beta_3 + \beta_4$ and (hot) compact orientations $\theta_c$ for all possible combinations of two nuclei at different $\ell$-values. The $\Delta R$-values are 1.3, 1.336, 1.734, 1.746, 0.795 and -0.661 fm, respectively, corresponding to light fragment mass regions $A_L=1-2$, 3, 4, 5, 6-112 and 113-146, for the best fit to data (see Figure 3.3 and Figure 3.4).

one of the incident energies $E_{c.m.}=200.1$ MeV of experiments[1], calculated for use of deformations up to hexadecapole, i.e., $\beta_2 + \beta_3 + \beta_4$ and compact orientations $\theta_c$. The deformation parameters $\beta_\lambda$, are from [18] and compact orientations $\theta_c$ are calculated as in [9]. The neck length parameter $\Delta R$ values, given in figure caption, are chosen for the best fit to data on light particles ($xn$, $x=3-5$) channel, quasi-fission and fusion-fission cross-sections (see Figure 3.4), with intermediate values intrapolated or kept the same as for the fitted case. At least three results are evident from this figure: (i) fragments corresponding to potential energy minima remain the same for
Figure 3.2: Same as for Figure 3.1, but for the preformation probability $P_0$, calculated by using the fragmentation potential of Figure 3.1.

...every $\ell$-value, though a detailed inspection shows that the favored (lower in energy) asymmetric mass distributions at zero and smaller $\ell$ values go over to the symmetric ones for partial waves with $\ell$ near the $\ell_{\text{max}}$ value. In other words, lower $\ell$ values contribute more to the light particles (plus the corresponding heavy fragments) emission and the higher $\ell$ values to quasi-fission and fusion-fission processes; (ii) a reaction valley (a minimum) appears at the incoming channel $^{244}\text{Pu}+^{48}\text{Ca}$, which is a "hot fusion" reaction since the potential energy surface in Figure 3.1 is calculated for conditions of "compact orientations" for hot fusion [9]. It may be reminded here that no such minimum occurs in a similar potential energy surface calculated for cold "elongated configurations" [8]. (iii) The other minima in Figure 3.1 occur around $A_L \approx 84-87, A_H \approx 208-205$ and $A_L = 125$ or 126, $A_H = 167$ or 166, i.e., $A_i = A/2 \pm 20$ or 21, corresponding to the observed quasi-fission and symmetric and near symmetric
Figure 3.3: Top panel: Excitation functions of individual 3n, 4n and 5n evaporation channels for the fusion reaction $^{244}$Pu+$^{48}$Ca. The experimental data is from Ref. [1], and the solid, dashed and dotted lines represent our calculations on DCM for the best fitted $\Delta R$ values shown in Figure 3.4 for $\ell_{\text{max}}=160\hbar$. The results for 5n are only suggestive since it is observed at one energy only.

Bottom panel: Excitation functions for the quasi-fission (qf) and fusion-fission (ff) processes observed in $^{244}$Pu+$^{48}$Ca$\rightarrow^{292}$114*. The experimental data is from [11], and the solid and dashed lines represent our DCM calculations for $\ell_{\text{max}}=160\hbar$. For qf, only the incoming channel is considered in (hot) “equatorial” or (cold) “polar” configuration, where $P_0=1$. The calculations in both the top and bottom panels are made only for the observed excitation energies $E^*$, with the polynomial extensions of the fitted $\Delta R$ values in Figure 3.4, and lines and curves are drawn for the guide of eyes.
fusion-fission processes, respectively. These minima arise due to the remaining shell effects in the potential energy surface $V(\eta)$ at temperature $T=1.14$ MeV, calculated for the hot fusion configurations. These results are better presented in Figure 3.2 for the preformation probability $P_0$. We notice in Figure 3.2 that maximum yields are obtained for the regions of symmetric and near symmetric fission peaked at $A_L=124$, $A_H=168$ (equivalently, $A/2 \pm 22$), asymmetric fragments centered around $A_L=84-87$, $A_H=208-205$, and the light particles ($x$-neutrons, $x \leq 4$), in close agreement with experiments. In the following, however, we use these results for the calculation of cross-sections due to neutron emissions, the evaporation residue cross-section $\sigma_{ER}$, and the fusion-fission cross-section $\sigma_{ff}$ only. The quasi-fission is considered as a competing process where only the incoming channel $^{244}\text{Pu}+^{48}\text{Ca}$ contributes to $\sigma_{qf}$, with $P_0=1$.

Figure 3.3 shows the excitation functions for hot evaporation channels 3n, 4n and 5n, and the fusion-fission (ff) and quasi-fission (qf) decay channels of $^{292}\text{Ni}^{*}$ formed in $^{244}\text{Pu}+^{48}\text{Ca}$ reaction. The experimental data are from [1] and [11], respectively, and the calculations are made for potentials $V(\eta)$ and $V(R)$ and hence $P_0$ and $P$, respectively, for deformations up to $\beta_4$ and compact orientations $\theta_c$ of the hot fusion process at $R = R_a = R_t + \Delta R$ with $\Delta R(E^*)$ given in Figure 3.4 for best fit to each of the three processes, using the sticking moment of inertia $I_S$ and $\ell_{max}=160h$. Apparently, the model reproduces the data nicely with in one parameter fitting. For fusion-fission, with a view to include the symmetric mass division ($A_L = A_H=146$) and noting that the maximum yields are for $A_L=124$, $A_H=168$ (equivalently, $A/2 \pm 22$), the best fit to data is obtained for fragment mass range $A_L=113-146$, $A_H=179-146$ (equivalently, $(A/2) \pm 33$). Here, it may be noted that mainly the yields around the maxima contribute. The contributions due to 1n and 2n emissions are neglected, since these are small due to their emissions lying below the fusion barrier (Bass barrier lies at an $E^* \sim 33$ MeV). Also, the excitation functions for 5n are tentative since there is only one data point. The interesting result of Figure 3.4 is that the neutrons emission occur earliest (largest $\Delta R$), then the hot quasi-fission almost competing with neutrons emission, and finally the fusion-fission
at the latest (smallest $\Delta R$). Note that the $\Delta R$'s for "cold" qf (diamonds with a dashed line) are relatively closer to the process of neutrons emission, and hence competing better than the "hot" qf, though the differences between the two (hot and cold) qf processes, for exactly similar fits in Figure 3.3, are very small of $\sim$5-7% only. Different $\Delta R$'s for the three processes of ER, ff and qf, in terms of reaction times, mean that these processes occur in different time-scales, as is indicated by measurements. It is relevant to remind here that whereas the time-scales refer to relative separation $t_{\text{ft}} (=t_{\text{ft1}} + t_{\text{ft2}} + \Delta t_{\text{ft}})$, the occurrence probabilities of the three processes are given by the mass asymmetry $\eta$ coordinate in terms of the (energetically favorable) minima in $V(\eta)$.

Figure 3.4 also illustrates the connection of $\ell_{\text{max}}$-value, due to the use of the sticking or non-sticking moment of inertia, with the length parameter $\Delta t$. All the calculations presented above refer to the sticking moment of inertia $I_S$ where $\ell_{\text{max}}$ is fixed for $\sigma_{\text{st}} \rightarrow 0$, and has a large value of $160\hbar$. In Figure 3.4, for the case of (hot) quasi-fission alone, we have also shown our results for use of the non-sticking moment of inertia $I_{NS}$ and found that for a similar fit as above for $I_S$, $\ell_{\text{max}}$ decreases considerably (from $160\hbar$ to $18\hbar$) but then $\Delta R$ increases appreciably (see solid line with crosses), approaching almost the value for neutrons emission using $I_S$. The interesting result is that for quasi-fission, an exactly the same $\ell_{\text{max}}(E^*)$ variation is also obtained for $I_S$ at $\ell_{\text{max}}=18\hbar$ (solid line with open squares) as is obtained above for $I_{NS}$ (solid line with crosses). Important enough, the $\ell_{\text{max}}=18\hbar$ is very close to $\ell_{\text{max}}=16\hbar$ value given by the finite-range liquid drop model (FRLDM) for mass $A=292$ compound nucleus (see Figure 1 in [19]). For heavy ion collisions, however, we consider the sticking moment of inertia as more appropriate, which involves a much larger limiting value for $\ell$ and hence a smaller neck length parameter $\Delta R$ required for the proximity potential ($\leq 2$ fm).

The role of the static deformation of the target nucleus is studied in Figure 3.5, which shows the calculated 4n-channel cross-section $\sigma_{4n}$ as a function of the excitation energy $E^*$ for the case of the nuclei taken to be spherical, compared with the cases of higher multipole deformations included up to $\beta_4$, i.e., $\beta_2 + \beta_3 + \beta_4$
Figure 3.4: The best fitted neck length parameter $\Delta R$ as a function of excitation energy $E^*$ for neutron channels of evaporation residue, the fusion-fission and quasi-fission processes. We refer here to calculations with sticking moment of inertia $I_S$ and $\ell_{\text{max}}=160\hbar$. For quasi-fission, calculations are also shown for best fit to $I_{NS}$ and the same for $I_S$ at the obtained lower limit of $\ell_{\text{max}}=18\hbar$, considering in all cases the (hot) “equatorial” configuration. The cold quasi-fission calculations are illustrated for the case of $I_S$ and $\ell_{\text{max}}=160\hbar$ (diamonds with a dashed line).

Apparently, at each excitation energy, the $\ell$ for collisions between deformed nuclei at their respective compact orientations for the cases $\beta_2 + \beta_3 + \beta_4$ or $\beta_2$ alone are shown to be much more (by a factor of the order of $10^3$ to $10^5$) than for the case of the nuclei taken to be spherical, thus signifying that the increase in fusion threshold for an intermediate hot fusion reaction is associated with the static deformation (and the associated compact orientation $\theta_C$) of the target nucleus at the point of collision.
Figure 3.5: The 4n-channel cross-section as a function of the excitation energy $E^*$ for $^{244}$Pu+$^{48}$Ca$\rightarrow ^{292}$114$^*$ reaction, for the cases of the nuclei taken to be spherical and with deformations included up to hexadecupole deformations, i.e., $\beta_2 + \beta_3 + \beta_4$ (same as in Figure 3.3 for 4n) and for quadrupole deformation $\beta_2$ alone, using in each case the $R_\alpha$ from $\Delta R$-value fitted (in Figure 3.4) for the case of $\beta_2 + \beta_3 + \beta_4$. The dashed and dotted lines show the same calculations, respectively, for SU(MN) and averaged masses $\bar{B}_{MP}$, both for the $\beta_2 + \beta_3 + \beta_4$ case. Calculations are made for only the observed excitation energies $E^*$ and the curves are for the guide of eyes.

in its path toward the (spherical) compound nucleus. This happens because, for the same $R_\alpha$, the nuclei with multipole deformations come much closer to each other than in the case of spherical nuclei. We have also calculated the 3n-channel cross-section $\sigma_{3n}$ for all the three cases (spherical, $\beta_2$ alone and $\beta_2 + \beta_3 + \beta_4$) and obtained the same result as for $\sigma_{4n}$ above, except that, compared to the case of $\beta_2 + \beta_3 + \beta_4$, the $\sigma_{3n}$ for the case of $\beta_2$ alone increases by a similar order of $10^2$ as it decreases for $\sigma_{4n}$, such that the evaporation residue cross-section $\sigma_{ER}$ ($=\sigma_{3n} + \sigma_{4n}$) remains the same (with in less than one order of magnitude) for the two cases of deformations.
(\(\beta_2\) alone and \(\beta_2 + \beta_3 + \beta_4\)) but is larger by an order of \(10^4\) compared to the case of spherical nuclei.

In Figure 3.5, we have also studied the sensitivity of our calculations to the use of different shell corrections and masses \(B_{\text{eff}}\). The dashed and dotted lines in Figure 3.5 show our calculations for the exit channels 4n, respectively, for use of \(\delta U(MN)\) and averaged masses \(\bar{B}_{\text{eff}}\). We notice in Figure 3.5 that the calculated cross-sections for use of different prescriptions for shell corrections do not change much, but that the shell effects in masses (to whatever extent they are present in hydrodynamical masses) play an important role. The 4n cross-sections for average masses increase by an order of magnitude \(10^2\) over the actual masses (compare dotted line with solid line for the case of \(\beta_2 + \beta_3 + \beta_4\)).

Finally, it may be noted that our approach of the dynamical cluster-decay model (DCM) is completely different from the Dubna method\cite{12-14} of formation of compound nucleus in strong competition with the quasi-fission process. In their approach, the whole process of compound nucleus formation and decay is divided into three individual reaction stages, even if connected with each other but treated and calculated separately\cite{13}: (1) colliding nuclei approach the point of contact, describing the capture process, (2) touching nuclei evolve into a spherical compound nucleus, the CN formation stage, which together with stage 1 gives the fusion cross-section, and (3) the survival probability, accounting for the emission of neutrons (and \(\gamma\)-rays) with respect to regular fission, which together with the above stated first two stages of reaction (the capture and CN formation) gives the evaporation residue or production cross-section \(\sigma_{ER}\). In DCM, however, the emission of neutrons constituting the evaporation residue and other heavier fragments like in fusion-fission are treated as the barrier penetration of preformed clusters at the point of closest approach, thereby including the dynamical and nuclear structure effects explicitly. For the competing quasi-fission, the DCM considers only the incoming channel with a preformation factor of unity.
3.3 Summary and discussion of results

The dynamical cluster-decay model (DCM), with effects of deformations of the incoming nuclei or of outgoing fragments and their “compact” orientation degrees of freedom included, is used to calculate the fusion-evaporation residue, fusion-fission and quasi-fission excitation functions of an “equatorial” compact (θ_c=90° for 244Pu) hot fusion reaction 244Pu+48Ca. The quasi-fission is also calculated for the “polar” elongated (θ_c=0° for 244Pu) configuration of cold process. Using the higher multipole deformations up to hexadecapole, i.e., β_2 + β_3 + β_4, we find that, with one parameter fitting, the DCM gives a very good description of the excitation functions for light-particle (here x_n, x=3-5) decay channels, the quasi-fission and the fusion-fission of 244Pu+48Ca reaction forming the compound nucleus 292114* of super-heavy element Z=114. For quasi-fission, both hot and cold processes give nearly the same result. The single fitting parameter used is the neck length ΔR(T), which for the sticking moment of inertia is the largest for evaporation residue due to the (prompt) emission of neutrons, smaller for the competing quasi-fission and finally the smallest (forming a necked configuration) for fusion-fission of hot compound nucleus. Different ΔR’s for the three processes (ER, ff and qf) mean to suggest different time-scales for their occurrences, in agreement with the indications by experiments. The shell effects in both the fragmentation potential and mass parameters (the kinetic energy part of Hamiltonian) are shown to be important. A sensitivity check of the calculations shows that the fusion excitation functions calculated for such hot fusion reactions at their respective compact orientations for cases of β_2 + β_3 + β_4 or of quadrupole deformation β_2 alone, are shown to be much larger than for the case of all the nuclei taken to be spherical, signifying that the increase in fusion threshold for an intermediate hot fusion reaction is associated with the static deformation of the target nucleus and its compact orientation at the point of collision in its path toward the (spherical) compound nucleus. The role of sticking versus non-sticking moment of inertia for the limiting angular momentum ℓ_{max} is also studied, which results in an increase of ΔR for a much lower ℓ_{max}-value with I_{NS} used in most ex-
Experimental analysis or (incidentally) the same small $\ell_{\text{max}}$-value for the same increased $\Delta R$ with $I_S$, which is close to the $\ell_{\text{max}}$-value predicted by the finite-range liquid drop model (FRLDM).
Bibliography


