Chapter 1

INTRODUCTION

Historical Background of Mathematics & Mathematics Education

Mathematics and mathematics education are quite different disciplines although they have many things in common, for example their origin and development.

Mathematics as we know of it today is the science of numbers and their operations, interrelations and combination of space configuration and their structure, measurement etc.

Mathematics started with the paradoxes of Zeno from Greece who was, the first mathematician of any note. His is famous for proving three theorems: namely (i) Motion is impossible, (ii) Achilles can never catch the tortoise and (iii) Half the time may be equal to double of the time. With many similar paradoxes came, Russell's paradoxes like, "The class of all classes which are not member of themselves is either a member of itself or not".

The growth of mathematics can be seen through the work of mathematicians by solving such paradoxes. Sometimes it happens they are forced to refute the prevailing beliefs and axioms. George Cantor said the essence of mathematics lies in its freedom. It is viewed that new horizon of thinking started when Copernicus challenged the statement that the earth is the centre of the universe. Similarly, Galileo Galilea challenged the prevailing belief, "Heavier body falls the faster". Eves (1983, p. 472) writes,

"New Geometries first came about through the denial of Euclid's parallel postulates and that new algebra first came about through the denial of the commutative law of multiplication. In similar fashion, the new so-called many-valued logics first came about by denying Aristotle's law of the excluded middle".

New geometry, new algebra and new many-valued logics came into existence with the work of Bolyai, Hamilton and Lukasiewicz. Bolyai challenged Euclid's fifth postulates, Hamilton challenged the axiom of multiplication, and Lukasiewicz challenged the axiom of Aristotle in logic respectively.
In the origin and development of mathematics and mathematics education, Hindus played a vital role. Hindus have a long history of teaching and learning mathematics dating back to the Vedic age (1500 to 200 B.C.). During the period of AD 400 to 1200, a new branch knowledge known as Ganit came into existence with three separate components namely, Ank Ganit (Arithmetic), Beej Ganit (Algebra) and Rekha Ganit (Geometry). But mathematics received prominence as a separate subjects only in the 12th century with the publication of Leelavati of Bhaskaracharya.

Similar to mathematics as a science, an interesting analogy could be observed in mathematics education. Mathematics education did have paradox about gaining mathematical knowledge, "that mathematics cannot be separated from empirical experience and yet cannot be explained by empiricist epistemology".

The turning point in the development of mathematics education can be seen with the work of Jean Piaget. He gave a new impetus to mathematics education by challenging the previous epistemologies, empiricism and rationalism, by introducing the word "Action" as a prime source of knowledge. Delton Kappen challenged the Descartian philosophy "that knowledge is viewed as a set of universal "truths" with a set of "working hypotheses". The detail description and argument behind above refutation is given in chapter 2.

But the situation with regard to methods of teaching mathematics remained unchanged through the ages in India and in Nepal as well. Under the impact of Gurukul style of teaching, knowledge was imparted by oral methods. Shishyas (Students) had to remember everything by heart and it was the duty of Brahmin, usually a priest, to transmit the knowledge. The teacher demonstrated a procedure for a specific problem to solve and thereafter students followed the same procedure which is said to be the characteristics of conventional (traditional) way of teaching.

During those days, speed and accuracy were the goals of mathematics teaching. Students learnt by heart what ever they had to learn. They didn't question on the validity of the procedure but did follow the rule "Do this and that you will get the result".

As the time changed, the style of teaching also changed due to the research in psychology. People started giving importance to findings of basic research to improve the teaching learning
situations. In different time different movements came with different outlooks. Changes in psychological thought over the century have been reflected in major reforms known as movement in mathematics education. These different movements have been characterized by changes in different methods that are used to impart knowledge.

The Reform Movements in Mathematics Education

The problems of developing teaching-learning practices in schools in compatible with societal need of formal curricula, became a crux from the very beginning of school education. Skott (2000) writes:

"To talk about the form of mathematics education is obviously problematic considering the diversities of the educational initiatives in different countries and at different educational levels and also considering the different foci in different parts of the mathematics education community. Nonetheless a set of developments has evolved over the two last decades that justifies the use at least of the concept of reform movement. These developments encompass a set of changes in both the conception of mathematics, the understandings of the teaching and learning mathematics".

With regard to learning theory, the idea of Thorndike came first. Thorndike’s era, for example, was noted for drill and practice activities reflecting his primary laws of learning in the first three decades of 20th century. These laws had impact on teaching of mathematics through oral drill and practice.

The period of meaningful learning that came as progressive education movement of the 1930’s and 1940’s emphasized learning for living (Kroll, 1989). There was a shift from meaningless drill methods where speed and accuracy were the criteria for measuring learning, to a focus on developing mathematical concepts in a meaningful way. William Brownell (1935, 1945) was one of the major proponents of this view.

New-Maths movement brought major changes in the content of mathematics in the elementary schools, emphasizing structure and including new topics in mathematics in 1960s. The restructuring of the mathematics curriculum followed the 1959 Woods Hole Conference in
which recommendations were in favour of new maths. One strong supporter of this approach was Jerome Bruner (1960), whose text, *The Process of Education*, was widely acknowledged.

**Back to Basic movement** came in 1970s as an anti-new-mathematics. Teachers, administrators, and parents were astonished by declining test scores, low performance of high school graduates, disruptive classrooms, and indisciplined students. They blamed the "New-maths Movement" with a slogan "old are gold". Classrooms that are characterized by strong teacher control, structure, convergence on learning activities, less pupil freedom, less exploration of ideas, and less experimental teaching activities tend to be associated with the greatest pupil gains in basic skills achievement (Evertson and Brophy, 1973; Medley, 1977; Soar, 1972; Soar and Soar, 1976). By the end of 70s experimental evidence did not support the above assertions.

So, **Back-to-Basic movement** came to answer and reflect a return to traditional practices that pervades not only what was offered in school but how it was offered. This resurrected views of proper school practices had at least the following attributes:

- the role of the student was to pay attention and follow directions;
- the role of the teacher was to effect a controlled, structured environment within which direct instruction (drill and practice) was the dominant activity; and
- the principal outcomes of interest were the skills and items of knowledge that comprise the subject matter.

Subsequently in the 1980s **problem solving**, launched by George Polya, came as the central goal of mathematics instruction.

After 1990 the recent movement **Ethno-mathematics** came which studies the mathematics of ethnic-group or minority group such as mathematics of carpenter, fisheries and grocery etc. One of the major proponents of this movement is U. D'Ambrosio. *Ethno-Maths* is based on epistemological framework of constructivism.
Despite the conflicting views of different movements there does seem to be a convergence of views when considering the practices of mathematics teaching in schools. These is a hope for a new method in which social and individual factors can shape and balance each other. The new method includes an increased awareness towards learning opportunities and intersubjective collaboration among students. The term constructivism is also recently used as a new "method of teaching" (NCTM 1995).

The Curriculum and Evaluation Standards for school mathematics, a National Commission of USA, prepared by working group and published by the National Council of Teachers of Mathematics (NCTM, 1989) does highlight the importance of children being actively involved in their learning, that is, they should construct, modify, and integrate ideas by interacting with the physical world, materials, and other children.

The constructive perspective is derived from the work of Piaget (1954, 1970, 1980). It asserts that conceptual knowledge cannot be transferred, carefully packaged, from one person to another. Rather, it must be constructed by each child solely on the basis of his own experience.

**Present Scenario of Mathematics Education in Nepal**

Now, the scenario of the mathematics instruction has changed with the change in the goal of mathematics teaching all over the globe. The national policy of education both of Nepal and of India emphasized the importance of mathematics in general education and suggest that mathematics should be visualized as the vehicle to train a child to think, reason, analyse and to articulate logically.

Recently Nepal has adopted a policy of free and universalization of primary education. Primary education includes teaching of Nepali, Mathematics, Social Studies, English and Environmental Science. Mathematics has always held a key position in the school curriculum because it has been considered indispensable knowledge to the educated man. If mathematics is an integral part of primary school curriculum, and it enjoys a second place next to the language at the primary level, then strengthening of mathematics teaching becomes imperative in universalization of primary education.
Mathematics is sequential (hierarchical) in nature, and learning is a growth process that involves children to learn. If mathematics is not learned with understanding in primary grade, then it would be very difficult for later learning. It is strongly believed that without providing quality primary education to all people, social and economical development of the country is not possible. Despite many efforts made by Faculty of Education, T.U. and Basic and Primary Education Project (BPEP) of Nepal, the achievement level in mathematics has not been remarkable.

Despite many training programme conducted by different organizations such as Faculty of Education, T.U., BPEP and others, the performance of students and teaching behaviour of the teacher have not changed. It is ironic that much of mathematics (at present) is taught through chalk and talk methods in primary grades by giving definitions, symbols and prescribed rules etc. This type of method of teaching has been handed down from the very beginning of education in Nepal. The students learn by their own devices. The effect of this traditional style of teaching can be seen from the evaluation of different researches conducted in Nepal.

A research conducted at Educational Development Service Centre (EDSC1999) in Nepal on "National Assessment of grade V students" with a view to collect a baseline information about the performance of grade V students in Nepali language, Mathematics and Social Studies. The overall mean performance of the students at national level was found to be 27.25 with a s.d. 17.08. This research raised a major concern about the low performance in mathematics and recommended for a special programme which could ensure the effectiveness in teaching mathematics.


One of the studies "Performance level of grade V students" conducted by Research Centre for Educational Innovations and Development (CERID), Tribhuvan University in 1998, revealed the fact that it is imperative to place more emphasis on mathematics teaching skills. Keeping in view the low scores of the students in mathematics in all the tests, the study suggests that the teacher-training programme should provide adequate emphasis on the methods and skills of teaching mathematics.
Another study "The effect of new curriculum on the achievement of grade V students" conducted by Basic Primary Education Project (BPEP), HMG Nepal in 1998, reports that overall mean achievement level of students has remained below 50% (except in Nepali language). In some cases, especially in Mathematics and Social Studies it is much lower than the 50%.

One of the study, "1996 Student Achievement Test: Grade 6 Mathematics" conducted by Basic Monitoring and Evaluation Unit of Secondary Education Development Project (BMEU, 1997) identifies most difficult curricular area of grade 6. It further describe. "Many responses indicate that the pupils are able to manipulate numbers and know skills and technique but have little understanding of what they are to do". It recommends, "There must be more emphasis put on teaching for understanding of concepts".

In survey research on achievement of 1845 grade 3 students of 120 schools, representing eight districts under BPEP (1995) it was found that a grand mean of mathematics was 15.24 which was considered as a poor score.

Curriculum Development Centre (CDC, 1994) did a research on, "Assessment of basic competencies of 11-12 years children of Nepal" with an objective to investigate the percentage of children (School going as well as non-going) satisfying the basic education criteria in numeracy. Working on the sample size of 1680 it was found that 68 percents of the students satisfied the basic education criteria.

Besides these reports, it is believed that the poor achievement in mathematics may be attributed to methods of teaching and classroom environment. The prevailing methods of teaching is a bi-product of large classes.

For many years traditional theories have shaped the way the classrooms are built: the way the courses are taught, and the way the students' knowledge is assessed. In many classrooms, the chairs are bolted to the floor so that the students all face the instructor, who is presumed to be the sole source of the knowledge.

DePree (1998) conducted an experiment on "Small-Group Instruction: Impact on Basic Algebra Students" and found that students who participated in the experimental (small-group)
method of instruction experienced significantly greater increase in confidence level, and higher completion rate, than students who participated in the control (lecture) method of instruction. However, no significant differences in achievement were found.

Similarly one of the study, "Teaching mathematics to large classes in Nepal: A study with Practical Suggestion", conducted by Shrestha et al (1985) identifies different problems perceived by students and teachers caused by large class size when traditional teaching methods are used.

In this context, it is essential to take the different opinions of the concerning personnel's about the status of teaching mathematics in Nepal.

Observations Concerning Teaching Mathematics in Nepal

It is imperative to consider some of the observations made by concerned individuals regarding the art of teaching and the outcomes of educational investment. For this investigator's observation as a teacher educator, teacher educator's observation as a trainer were collected and classroom teacher's views were also gathered.

Investigators' Views About the Status of Teaching Mathematics

Regarding the present practices followed in schools, the investigator's personal experience is that the teaching of mathematics is not very exciting at all. While giving training to twenty primary teachers, the investigator observed his trainees helplessly looking towards him in order to accomplish a task to measure an angle drawn upside down on the blackboard. What was more frustrating was that they did not even discuss among themselves how to find the solution of the problem. First the investigator thought that they did not understand the problem, but it was not the case. When the investigator asked to place the protector on the base line of another angle which was usual in drawing, having one arm parallel to the horizon, almost all trainees knew how to measure the previous angle. This was one of the vignette that showed that learning could take place if the learner is given proper guidance.
This is one instance to cite a few which has been a matter of concern for the investigator for a long time. In this context, it was pertinent to ask the teacher-trainer as well as the trainees the problems they usually face in teaching mathematics. Their views are given below:

**Inservice Trainees’ Views Regarding the Problems of Actual Teaching Mathematics**

- A large number of students;
- The scarcity of materials; and
- A number of classes to teach

The points given in three bullets above were the problems faced by the teachers which refrained them to sustain their interest and in teaching the way they were trained to teach. So, based on these views the problems of teaching mathematics as perceived by the teacher in Nepalese classroom are listed below.

**Problems of Teaching Mathematics in Nepalese Schools**

**Large Classes**

It is believed that a single factor, large size class, generates lot of problems for both the students and the teachers.

**Problems faced by the students**

Overcrowded Classroom: Sitting close to one another, not being able to move, write on their copies, see the blackboard, and listen to the teacher were the difficulties in overcrowded classrooms.

**Problems Faced by the Teacher**

- To get feedback from each and every student in the class;
- To maintain discipline.
Methods of Teaching Mathematics

In broad sense there are two methods of instructions i) traditional methods of teaching ii) new methods of teaching.

Traditional Methods of Teaching

Text book instruction through conventional way of teaching includes various methodologies, which focus primarily on rote learning and practice of skills.

New Methods of Teaching

But at the moment, instruction is envisioned through which students construct their own meaning for mathematical concepts and procedures. They are engaged in meaningful problem-solving activities through new method of teaching known as constructivism.

Constructivism as a new Method of Teaching

This research work started with the help of trained teachers in four schools. Trained teachers did know the students' centred teaching but their teaching behaviours were almost teachers-centred. So, there always arises such questions : What is constructivism? How is it different from conventional way of teaching? What is a sharp line that distinguishes the conventional and constructivism ways of teaching, one on the left and another on the right? What are the similarities and differences between these two methods?

To answer the first question, the standard source a dictionary was consulted. Page & Thomas (1977 p. 84) defines constructivism as a:

"viewpoint in learning theory and child development which holds that a child actively constructs his/her own ways of thinking as a result of innate capabilities interacting with his/her experience".
The aforementioned activities are not new prescription for teaching mathematics. It is simply a set of activities which are necessary in constructivism. Constructivist theory posits that students make sense of the world by synthesizing new experiences into what they have previously understood. They form rules through reflection on their interaction with objects and ideas. When they encounter an object, idea, or relationship that does not make sense to them, they either interpret what they see to conform to their rules or they adjust their rules to better account for the new information.

So, a brief characteristics of the constructivist method are outlined below:

A constructivist approach includes:

- taking student's former knowledge into account;
- maximising social interactions;
- providing a variety of sensory experiences;
- interactive discussion, and problem centred work.

To answer the second and third questions, related literature were reviewed. Review of literature shows that the above two methods are similar in the view that our knowledge of the world is based on the experience of our senses. They differ, however, on their beliefs about the extent to which the world is knowable. Hidden assumption of conventional method is "Knowledge can be transferred intact from the mind of the teacher to the mind of the learner" whereas the assumption of constructivism is "Knowledge is seldom transferred intact from the mind of the teacher to the mind of the learners."

Jaeger (1992) presented a paper, "The Constructivism of Meaning From Experience" at the Annual Meeting of the National Council of Teachers. He further stated that tenets of a constructivist perspective include: (1) learners come to school with a wealth of prior knowledge, (2) learners make meaning of their world by logically linking pieces of knowledge, communication, and experiences, (3) these belief systems are resistant to change; (5) direct instruction is unlikely to change belief systems, (6) learning takes place when confrontation with new experience yields dissonance; (7) a social context facilities these processes; and (8) learning takes place best in a meaningful context. All these can be addressed by constructivist way of teaching.
In this context it is imperative to discuss in what way these two methods differ. In conventional method the teacher cannot check on individual students, it becomes comparatively easy for a student to idle away his time. A large class situation demands more self-motivation and self-discipline from the student than a small class where the teacher is able to be more aware of what his students are doing. One of many solutions of the above problems is constructivism, a new way of teaching.

Constructivism, to a greater or lesser degree, provide solutions to the above problems. The class members seldom come together on mass but meet in small groups or work individually, thus the mass disciplinary problems can be avoided. Because many of the tests are self marked, marked in the group, the overall marking burden on the teacher is reduced.

It is more appropriate, if we read what Piaget, the progenitor of constructivism, says. One of Piaget's followers, Aebli (1951), cited in Hilgard & Bower (1975, p. 340), made some suggestions about how to use Piaget's theory in relation to teaching. He translated the epigram made by Piaget into English language as "To think to operate". Hence, all teaching should stress pupil initiative, an active experimentation with environment in which overt actions gradually become translated into mental operations.

A second important point is that interaction with peers is important, if for no other reason than to liberate the child from his egocentrism. In group activities he must eventually learn to take the perspective of the other, leading what Piaget calls decentration.

Constructivist teachers encourage student inquiries by asking thoughtful, open-ended questions and encouraging students to ask questions of each other. The questions are designed to challenge students to look beyond the apparent, delve into issues deeply and broadly, and form their own understanding. Often, there is no one "right" interpretation, even though some analyses are more sophisticated and useful than others.

Several guidelines can be derived from effective teaching according to constructivism:

• Pupils must talk in order to learn;
• Pupils need to use manipulative materials as they learn maths;
• Pupils need to work with real objects and real problems rather than with contrived ones;
• Pupils must be given the challenge to use their brains in creative and pattern detecting ways.

These days, National Council of Teachers of Mathematics (NCTM Standards) is working hard to support the mathematics teachers for teaching mathematics meaningfully. NCTM advocates that bridges from the concrete to the symbolic could be established meaningfully through mathematical relationship. These connections may tie the teaching of mathematics with applications and actual problems-solving activities. These connections may also link real world models with abstract mathematical ideas. The teacher's role is not to build a single bridge from the concrete to the abstract, but rather to provide a learning climate that facilitates a continual process of bridge building. Many different researches have consistently documented the teachers essential role, not only in establishing these bridges but also in helping children make the transition from concrete to abstract.

In preparing for a class, a teacher selects task which have a high probability of calling the students to action. The "call to action" serves as an invitation both to the problem solver and to the teacher. While the problem solver (individually or in group) is invited to engage in action and reflection, the teacher is invited to see understanding of the problem being solved, the action chosen, and the nature of the reflections. Reflection serves to provide feedback on the results of the action. Actions and reflections occur in a subgroup and whole class discussion when they share the meaning. The role of the teacher in this process is that of a facilitator and every effort is made to be non-judgemental, always encouraging to accomplish the task. Hence the essence of constructivism is highlighted below:

"In constructivism the role of the teachers changes from talking & describing to listening & asking questions to the students".