Chapter 5

Cluster dynamics of hot and rotating light-mass nuclear system formed in low-energy $^{32}$S + $^{24}$Mg $\rightarrow ^{56}$Ni* reaction

5.1 Introduction

Light compound nuclei (CN) formed in low energy (E/A<15 MeV/nucleon) heavy-ion reactions are highly excited and carry large angular momenta. The compound systems so formed decay by emitting multiple light particles (n, p, α) and γ-rays, which in the statistical Hauser-Feshbach (HF) analysis is understood as an emission process from the equilibrated CN, resulting in CN fusion cross sections. Apparently, the decay process must depend on temperature and angular momentum dependent potential barriers [56]. For light compound systems with $A_{CN} \geq 40$, the above noted light-particles (LP) emission is always accompanied by intermediate mass fragments, the IMFs of Z>2 and 4<A<20, also called “complex fragments” or “clusters”, whose contribution, though small of the order of 5 to 10%, is to be included in the CN fusion cross sections. Then, the temperature- and spin-dependent potentials must
also be mass-asymmetry dependent. In other words, the structure effects of the compound system also become important.

In order to understand the IMF production, not only the HF analysis is extended to include the fragments heavier than α-particle in the BUSCO code [60] or in the Extended Hauser-Feshbach scission-point model [55], but also other statistical fission model descriptions [56] have been used that are based on either the scission-point or saddle-point configuration, in the GEMINI code [54] or the saddle-point “transition-state” model (TSM) [56,61–64], respectively. The LP emission in these fission models is still treated within the statistical HF method. It is interesting to mention that light-nucleus emission can also be qualitatively well described in the framework of a generalized rotating liquid-drop model, proposed recently by Royer and collaborators [143–145]. Since the measured angular distributions and energy spectra of emitted complex fragments are consistent with fission-like decays of the respective compound system, the fusion-fission process is now well established in light dinuclear systems [56]. The statistical fission models, for which the fission-decay of a CN is determined by the phase space (level density) available at the saddle-point [54,63] (or scission-point [55]) configuration, may however lack in terms of not including more explicitly the structure effects of the compound system. Large structure effects have been shown to be important in the $^{56}$Ni compound system through a strong resonance behaviour [146,147] of the excitation functions of large-angle $^{28}$Si+$^{28}$Si elastic and inelastic scattering yields [64,91]. Although neither similar resonant effects nor orbiting processes [151] have been evidenced in the $^{32}$S+$^{24}$Mg reaction [97,152,153] studied here, a fully dynamical theory for more complete description of the emission of both the LPs and IMFs in the framework of the statistical model of the decay of such a hot and rotating nuclear $^{56}$Ni system remains highly desirable.

A recent attempt towards the dynamical treatment of the decay of a hot and rotating nucleus is the work of Gupta and collaborators [48–50, 70, 71, 148], followed
in this thesis, where a dynamical collective clusterization process is proposed as a possible alternative of the fission process. Both the LPs and IMFs are considered as the dynamical mass motion of preformed fragments or clusters through the barrier. Note that, in terms of the barrier picture, a cluster-decay process is in fact a fission process with structure effects of the CN also included via the preformation of the fragments, but without any phase space arguments (i.e. with no level density calculations). Alternatively, the dynamical fission process has been considered by some authors [51, 52] simply as a continuous deformation of the CN [65–69].

The above noted dynamical cluster-decay model (DCM) of Gupta et al. [48–50, 70] is so-far used to calculate the decay constant and total kinetic energies (TKEs) of IMFs alone, and that too in a re-normalization procedure only. In other words, only the angular momentum $\ell = 0$ (s-wave) solution and TKE for fixed $\ell$-value are considered. Here, we used the DCM further for the calculation of actual (summed up, “total” $\ell$) cross sections and the average total kinetic energies $\overline{TKE}$'s for both the emitted light particles and complex IMFs using both the proximity pocket formula of Blocki et al. [3] and the proximity potential obtained from the semiclassical ETF approach in SEDF. Interesting enough, the light particles, though treated within the same dynamical collective clusterisation process, are found to possess different characteristic properties.

The data for IMFs chosen here, to apply the DCM, are those of the $^{32}S + ^{24}Mg \rightarrow ^{56}Ni^*$ reaction, where the mass spectra for $A = 12$ to 28 fragments, and the average total kinetic energy $\overline{TKE}$ for only the most favored (enhanced yields) $\alpha$-nucleus fragments, are measured at two incident energies $E_{lab} = 121.1$ and 141.8 MeV, or equivalently at $E_{c.m.} = 51.6$ and 60.5 MeV, respectively [62,97] (only even-even fragments are observed for $E_{c.m.} = 51.6$ MeV). In this experiment, by detecting both fragments in coincidence, it was possible to deduce the primary mass distribution for the decay process i.e. the mass distribution before the occurrence of the secondary light-particles emission from the fragments. These primary,
pre-evaporation mass distributions show that the mass-asymmetric channels are favored over the symmetric ones, with α-nucleus, A=4n, fragments having enhanced yields. The IMF emission cross section, estimated in one of these experiments [97], is $59 \pm 12$ mb. The CN fusion cross section data due to multiple LP emission at these energies are also deduced later from the same experiment [62], and as quoted in [63], are $1080 \pm 130$ and $1050 \pm 100$ mb, respectively, at $E_{c.m.} = 51.6$ and $60.5$ MeV, which fit the other available earlier measurements at similar and other energies for this system [97,149,150]. The total fusion cross section is then the sum of this cross section due to the LP emission and the fission-like IMF emission cross section.

5.2 Calculations and discussion of the results

The reaction $^{32}$S+$^{24}$Mg$\rightarrow^{56}$Ni* has been studied at two incident energies ($E_{c.m.} = 51.6$ and $60.5$ MeV) which correspond to 1.8 and 2.2 times the Coulomb barrier [62,97]. As already mentioned in the Introduction, at such energies, the fission-like IMF cross section $\sigma_{IMF}$ is about 6 % of the LP emission cross section $\sigma_{LP}$. Specifically, $\sigma_{IMF}/\sigma_{LP} = (59 \pm 12) \text{mb}/(1050 \pm 50) \text{mb}$ at $E_{c.m.} = 60.8$ MeV [97]. In the following, however, we choose to use the data of Ref. [62], and the above noted energy of Ref. [97] is close to one of the energies of the chosen data.

In the experiment of Ref. [62], for the fission-like decay of $^{56}$Ni*, a complete mass spectrum of IMFs is measured, beginning with mass $A_L=12$ fragment. Later on, the measurements of the IMF mass spectrum have been extended at $E_{lab} = 130$ MeV (equivalently, $E_{c.m.} = 55.7$ MeV) in order to include the $^8$Be fragment yields [154]. In this experiment [154], an enhanced emission yield for $^8$Be by a factor of 1.5 to 1.8, over the two α-particles, is observed, which in the Extended Hauser-Feshbach Method (EHFM) [55] is shown to be related to an increased deformation of the heavier fragment $^{48}$Cr. More recently, ternary events from a conjectured hyperdeformed $^{56}$Ni CN have been observed at $E_{lab} = 165$ MeV (equivalently, $E_{c.m.}$
Figure 5.1: The fragmentation potentials $V(A)$ for the compound system $^{56}\text{Ni}^*$ formed in the reaction $^{32}\text{S}+^{24}\text{Mg}$ at $E_{c.m.}=51.6$ MeV, calculated for different $\ell$-values at a fixed $T=3.39$ MeV (corresponding to $E_{c.m.}=51.6$ MeV) for the use of proximity pocket formula of Blocki et al. for the proximity potential and the proximity potential obtained from the semiclassical ETF approach in SEDF. The fragmentation potential $V(A) = V_{LM}(T) + U(T) + E_s(T) + V_p(T) + V_e(T)$ for fixed $R = R_a$. The $R_a$ parameter for the use of $V_p(T)$ from proximity pocket formula of Blocki et al. is $R_a = C_t + 1.28$ fm and of the semiclassical ETF approach in SEDF it is $R_a = R_t + 1.165$ fm. The $\ell_c$-value is as per Eq. (2.69).

$E_c = 70.7$ MeV [155]. The two sets of data (Ref. [154] and [155]) are consistent with the strong deformation effects found in $^{56}\text{Ni}$, as is also populated by the $^{28}\text{Si}+^{28}\text{Si}$ fusion-evaporation reaction [156, 157].

Figures 5.1 (a) and (b) give for $^{56}\text{Ni}^*$ the mass fragmentation potentials $V(A)$ at different $\ell$-values, for the fixed $T=3.39$ MeV and $R = R_a$, for the use of the proximity potential of Blocki et al. [3] and of the proximity potential obtained from the semiclassical ETF approach in SEDF. The $R_a$-value is chosen for the best fit to the cross section data. Thus, for the calculations based on DCM with use
Figure 5.2: The temperature and angular momentum dependent scattering potentials, illustrated for $^{56}Ni^* \rightarrow ^{12}C + ^{44}Ti$ at $T = 3.39$ MeV (equivalently, $E_{c.m.} = 51.6$ MeV) for the use of proximity pocket formula of Blocki et al. for the proximity potential and the proximity potential obtained from the semiclassical ETF approach in SEDF. The potential for each $\ell$ is calculated by using $V(R, T, \ell) = E_c(T) + V_p(T) + V_e(T)$, normalized to exit channel $T$-dependent binding energies $B_l(T) + B_{hl}(T)$, each defined as $B(T) = V_{LDM}(T) + \delta U(T)$. The decay path, defined by $V(R_{a}, \ell) = Q_{eff}(T, \ell)$ for each $\ell$, is shown to begin at $R_{a} = C_{t} + \Delta R$ or $R_{a} - C_{t}$ for $\ell = 0$ case, where $\Delta R$ is the average over $\eta$ of the different neck-length parameters $\Delta R(= R_{a} - C_{t})$ or $= R_{a} - R_{t}$ calculated for $V(R_{a}) = Q_{eff}(T, \ell = 0)$ for all possible fragmentations. The critical angular momentum $\ell_{c}$-value is determined from Eq. (2.69).

of proximity pocket formula of Blocki et al., the $R_{a}$-value is taken as $R = R_{a} = C_{t} + 1.28$ fm (see Fig. 5.2 (a)) while for the use of the semiclassical ETF approach in SEDF the $R_{a}$-value is $R = R_{a} = R_{t} + 1.165$ fm (see Fig. 5.2 (b)). $R_{t}$ used in the semiclassical ETF approach in SEDF, instead of $C_{t}$, are the half density radii obtained by fitting the the experimental data [37,38] to the polynomial in nuclear mass (see Eq. 2.40). Two interesting results are apparent in Fig. 5.1: (i) Because of $T$-dependent $V_{LDM}$, the non-$\alpha$, $Z=N$, even-$A$, fragments also appear at minima which are in addition to the preferred $\alpha$-nucleus structure present in the
macroscopic liquid drop energy due to the “Wigner term”. This happens because
the pairing term $\delta$ in $V_{LDM}$ goes to zero for $T > 2$ MeV (refer to the Appendix in
[50]). We notice in Fig. 5.1 that, even for $\ell=0$ case, the potential energy minima
at odd-Z ($=N$) fragments are much deeper (more so for $^{10}$B and $^{14}$N fragments),
as compared to that for even-Z ($=N$) fragments, which is apparently due to $\delta=0$
at this temperature. Also, the shell corrections $\delta U$ are already nearly zero at these
temperatures. Thus, with the addition of temperature in the potential, not only
the shell structure effects vanish but also there is no explicitly preferred $\alpha$-nucleus
structure left. It is worth noting here that the same behavior is also known for
fission calculations [56], based upon either the saddle-point picture [54,61,62] or
the scission-point picture [55]. (ii) The structure in the fragmentation potential
(the positions of minima and maxima) is independent of the $\ell$-value, though very
important effects of symmetric or asymmetric mass division are present here in Fig.
5.1. Apparently, the favored (lower in energy) asymmetric mass distribution at zero
and smaller $\ell$-values go over to the symmetric one for partial waves with angular
momenta $\ell$ near the $\ell_c$-value. In particular, at lower $\ell$-values, the light particles (plus
the corresponding heavy fragments) are strongly favored over the heavier fragments
(IMFs), but this situation gets reversed at or near the $\ell_c$-value. In this reversing
process (from favored asymmetric at $\ell=0$ to favored symmetric mass distribution at
higher $\ell$-values), the relative depths of potential energy minima at odd-Z fragments
seem to grow more, making these fragments energetically more favorable. As we
shall see in the following, these results have important consequences not only for
the relative contributions of odd-Z and even-Z fragments but also for the LP and
IMF emission at different $\ell$’s to the total decay cross section. Furthermore, it may be
relevant to mention here that we already know from the experiments of Beckerman
et al. [158] that the emission of IMFs starts only beyond a certain energy (and
hence beyond a certain angular momentum) and that, for lower energies, only LP
emission occurs which gives the complete fusion cross section. This result is already

81
given by the DCM worked out in s-wave (\(\ell=0\)) approximation [48–50].

![Diagram of Ni* decay](image)

Figure 5.3: The fragment preformation factor \(P_0(\ell, A_L)\), the penetrability \(P(\ell, A_L)\) and the decay cross section \(\sigma_\ell(A_L)\), with \(\ell\) summed over \(\ell_{\text{max}} = \ell_c\) calculated from Eq. (4.3), for the decay of \(^{56}\text{Ni}^*\) to various complex fragments (both LPs and IMFs), using the corresponding fragmentation potentials in Fig. 5.1 based on DCM.

Figures 5.3 (a) and (b) show the calculated preformation factors \(P_0(\ell, A_L)\), the penetration probability \(P(\ell, A_L)\) and the cross section \(\sigma_\ell(A_L)\), with \(\ell\) summed over \(\ell_{\text{max}} = \ell_c\), for use of the fragmentation potentials of Figs. 5.1 (a) and (b), respectively. Only the energetically most favored mass fragments are considered for both the LPs and IMFs. First of all, two important results can be drawn for \(P_0(A_L)\) (shown as solid thin line in Figs. 5.3 (a) and (b)): (i) \(P_0(A_L)\) is a strongly oscillating function with maxima only at Z=N, even-A fragments; (ii) the preformation yields are large for light-particles and the asymmetric fragments. In other words,
in agreement with experiments [62, 97], an asymmetric mass distribution of IMFs is favored by the preformation factors. On the other hand, the penetrability $P(A_L)$ (doted line) is almost a monotonically decreasing function, with $P$ being relatively small for symmetric fragments. Thus, $P(A_L)$ also support the asymmetric mass distribution. The total cross section $\sigma(A_L) = \sum_{\ell=0}^{L} \sigma_{\ell}(A_L)$ for each fragment (solid thick line) is a combined effect of these two terms, to be discussed below in comparison with experiments. Here we notice that its behavior is given more by $P_0(A_L)$ than by $P(A_L)$. (iii) from the Figs. 5.3 (a) and (b) we find that the use of the proximity potential of the semiclassical ETF approach in SEDF gives the cross-section
considerably small for LPs and much smaller for IMFs in comparison with the use of Blocki proximity pocket formula in DCM, and is near to the measured value \( \sigma_{LP} = 1050 \text{ mb} \) for the semiclassical ETF approach. However, before going over to the comparison between the calculation and data, in the following, we first study the variation of the above mentioned quantities with angular momentum \( \ell \).

Figures 5.4 (a) and (b) shows the variation of \( P_0 \) with \( \ell \) for the energetically favored LPs \((A \leq 4)\) and the even-A, \(N=Z\) IMFs (the contribution of the energetically unfavored odd-A IMFs is small at all \( \ell \)s). The maximum \( \ell \)-value, \( \ell_{\text{max}} \), is taken to be equal to the \( \ell_c \)-value. We notice that, whereas \( P_0 \) decreases for LPs with an increase of \( \ell \), it increases for IMFs as \( \ell \) increases and then starts to decrease at a large \( \ell \)-value. \( P_0(\ell) \) for \( ^4\)He behaves like the LPs and that the behaviour of all LPs \((A \leq 4)\) is different from that of the IMFs \((A > 4)\). Also, for heavier IMFs \((A \geq 16)\), \( P_0 \) is almost zero for \( \ell \leq 18 \) h. Furthermore, in Fig. 5.5, the \( P \)'s for LPs are also large, rather the largest \((P = 1\) for proton emission since there is no barrier at all \( \ell \)s), but the same for \( \ell \leq 18 \) h is nearly zero for all IMFs. Thus, for the penetrability \( P \) also, the behavior of LPs is different from those of the IMFs. This result for the cross sections means that the lower \( \ell \) values contribute mostly to the LP cross section \( \sigma_{LP} \) and that the higher \( \ell \) values \((\ell > 18 \) h) contribute to the fission-like IMFs production cross section \( \sigma_{IMF} \). This is illustrated in Fig. 5.6 for the use of proximity potential of Blocki et al., where \( \sigma_{LP}, \sigma_{IMF} \) and the total cross section \( \sigma_{\text{Total}} = \sigma_{LP} + \sigma_{IMF} \) are plotted for each \( \ell \). (The values of the three cross sections for \( \ell_{\text{max}} = \ell_c \) are also given in the brackets of the legend). We notice that \( \sigma_{LP} \) is already zero for \( \ell \geq 31 \) h (a value close to \( \ell_c \)), which means that not only the two processes of LP and IMF emissions get separated at \( \ell \approx 18 \) h, but also the decay process stops at \( \ell = \ell_c \)-value (at least for the LPs in the present calculations).

The IMF cross section starts to contribute only for \( \ell > 18 \) h and is maximum at \( \ell = \ell_c \). Furthermore, if the \( \ell \) coordinate is extended up to, say, \( \ell_{\text{jus}} \) (not shown in the figure), the contribution to LP emission remains zero but the IMF cross section...
Figure 5.5: The variation of P with $\ell$, for both the LPs (dashed lines) and even-A, N=Z IMFs (solid lines), calculated on DCM for the compound system $^{56}\text{Ni}^*$, using the scattering potentials as in Fig. 5.2 (a). For proton, there is no barrier at any $\ell$-value and, hence P=1 for proton.

This goes on increasing as $\ell$ increases. This means that if the decay process continues beyond $\ell \approx \ell_c$, only $\sigma_{IMF}$ contributes to $\sigma_{Total}$. In other words, in DCM, $\ell_{max} = \ell_c$ seems to be an automatic choice, fixed by the initial conditions of the experiment, as in Eq. (2.69). Alternatively one could choose $\ell_{max}$ at an $\ell$-value where $\sigma_{LP} \rightarrow 0$, as is discussed below.

The individual contributions of IMFs are illustrated in Fig. 5.7, for the $^8\text{Be}$ emission and the $^{28}\text{Si}$ emission. We notice that the contributions of the lighter IMFs towards $\sigma_{IMF}$ are much larger than that for heavier IMFs. This result is consistent with the observation of an asymmetric mass distribution, which is favored. It is interesting to note that the same results (as presented in Figs. 5.6 and 5.7) is obtained in the statistical fission model calculations for this reaction (see, for example, Fig. 14 in [62]). The noticeable difference is that, in the statistical fission
Figure 5.6: The variation of evaporation residue cross sections due to the LPs $\sigma_{LP}$ (dotted line), the IMFs $\sigma_{IMF}$ (dashed line), and their sum $\sigma_{Total}$ (solid line) with $\ell$, for $\ell$-values up to $\ell_c$, calculated on DCM for the compound system $^{56}\text{Ni}^*$ formed in $^{32}\text{S}+^{24}\text{Mg}$ reaction at $E_{c.m.} = 51.6$ MeV. The parameter $\Delta R$ (=1.28 fm) is the same for both the LPs and IMFs. The cross sections given in the brackets are obtained by summing for $\ell_{max} = \ell_c = 32 \hbar$. Here the LPs consist of $^{1,2}\text{H}$ and $^{3,4}\text{He}$.

Before giving the quantitative comparison of $\sigma_{LP}$ and $\sigma_{IMF}$ calculations with experimental data, we study here the role of changing $A/R$-value and the nature of the statistical fission model $[62,97]$, $\sigma_{fission}$ (≡ $\sigma_{IMF}$) also reduces to zero at $\ell = \ell_c$, which is due to the chosen phase space (the sharp cutoff approximation) in that model. Another point of interest to note in DCM is that the so-called promptly emitted LPs are really not that prompt but they do have a considerable overlap with the binary-decay process (of cluster emission) for the higher $\ell$-values. This is also consistent with the statistical fission model of $[62,97]$. It is true that LP emission starts early but continues along with the emission of IMFs till the decay process itself stops for $\ell > \ell_c$. 

86
Figure 5.7: The variation of $P_0(l, A_L)$, $P(l, A_L)$ and $\sigma_f(A_L)$ with $l$, for the emission of $^8$Be and $^{28}$Si fragments from $^{56}$Ni$^*$ formed in $^{32}$S+$^{24}$Mg reaction at $E_{c.m.} = 51.6$ MeV, calculated on DCM, as in Fig. 5.6. For $^{28}$Si-decay, the calculated $P$ is ten times and, $P_0$ one tenth, of the plotted values.

emitted light-particle(s). Fig. 5.8 shows the results of DCM with the use of different $\Delta R$-values for the LPs and IMFs, taking $\Delta R = 0.41$ fm for LPs, but keeping the same $\Delta R = 1.28$ fm (as in Fig. 5.6) for IMFs. We notice that the magnitude of (total $\ell$-summed) $\sigma_{LP}$ reduces considerably (by a factor of about 2) where as $\sigma_{IMF}$ remains nearly the same (rather an increase by about 20% can be observed). Furthermore, if we also change the proton-emission to neutron-emission, as in Fig. 5.9, the $\ell_c$-value remains the same, but the magnitude of $\sigma_{LP}$ reduces further to about 60%, keeping the $\sigma_{IMF}$ almost unchanged. These results in Figs. 5.8 and 5.9 are to be compared with the respective measured values of $\sigma_{LP}=1080$ mb and $\sigma_{IMF}=60$ mb. The agreement for $\sigma_{IMF}$ can be further improved if the $\ell$-values are summed only upto the point where $\sigma_{LP} \rightarrow 0$; then the calculated $\sigma_{IMF}=103$ or 106 mb in Fig. 5.8.
Figure 5.8: The same as for Fig. 5.6, but for use of different \( \Delta R \)-values for LPs and IMFs. or Fig. 5.9, respectively. Note that the drastic reduction of \( \sigma_{LP} \) in Fig. 5.9 occurs because the lower \( \ell \)-values (\( \ell \leq 3 \hbar \)) also do not contribute to the LP emission cross section. It is known for experiments that it is more difficult to evaporate a neutron than a proton, as is also shown to be energetically the case in Fig. 5.1. However, we do not make a search for the exact emission of LPs, since we do not attempt a one-to-one comparison with the data. Nonetheless, these results demonstrate for the first time the general success of DCM in giving the LP emission in a non-statistical formalism.

Next, Fig. 5.10 is same as Fig. 5.6, but with the use of the proximity potential of semiclassical ETF approach in SEDF at the same \( \Delta R \) parameter (=1.165 fm) for both the LPs and IMFs. We notice that in this case \( \sigma_{LP} \) goes to zero at angular momentum value \( \ell = 35 \hbar \) (a value equal to \( \ell_c \), Eq. (2.69), obtained from the initial conditions of the reaction). Also the two processes of LP and IMF emissions get separated at \( \ell \approx 24 \hbar \), while in case of the use of the proximity pocket formula.
of Blocki et al. the IMF starts contributing at a lower angular momentum ($\ell \approx 18 \hbar$). One could choose $\ell_{\max}$ as the $\ell$-value at which $\sigma_{LP} \rightarrow 0$ as is discussed below. Also, the calculated cross-section compare with experiments almost exactly (compare $\sigma_{LP}=1081$ with 1080 and $\sigma_{IMF}=59\pm12$ mb).

Figure 5.11 is same as Fig. 5.10, but at $E_{c.m.} = 60.5$ MeV with the use of different $\Delta R$ parameter 1.26 fm and 1.00 fm for the LPs and the IMFs respectively. The variation of the light particle cross section with angular momentum shows that the $\sigma_{LP} \rightarrow 0$ at $\ell = 35 \hbar$, lesser than critical angular momentum value $\ell_{c} = 39 \hbar$. So, we take this angular momentum as the maximum angular momentum, $\ell_{\max}$. The cross-sections given in the bracket are obtained by summing up to $\ell_{\max}$ and reproduces the observed light particle and IMF cross-section, nicely (compare $\sigma_{LP}=1053$ with 1050 and $\sigma_{IMF} = 99$ with $59\pm12$ mb, respectively). Fig. 5.10, as well Fig. 5.11, show that with the use of nuclear potential of the semiclassical ETF approach in SEDF in DCM, with the present choice of the parameter $\Delta R$ for LPs and IMFs.
Figure 5.10: Same as 5.6, but with the use of proximity potential of the semiclassical ETF approach in SEDF.

reproduce the decay cross-sections at both the energies. It is noted that at both the energies the LPs contribution to cross-section stops at angular momentum value $\ell = 35 \hbar$.

The total cross section $\sigma = \sum \sigma_\ell$ for decay of $^{56}\text{Ni}^*$ at $E_{\text{c.m.}} = 51.6$ MeV versus light fragments $A_L$ (for both the LPs and even-\(A\), \(N=Z\) IMFs) is plotted in Fig. 5.12 for the case of Fig. 5.8 and for another $\ell_c$-value (=36 $\hbar$) for the use of proximity pocket formula, compared with the total cross-section obtained with use of the proximity potential of semiclassical ETF approach in SEDF for the case of Fig. 5.10 and with other available calculations based either on the saddle-point transition-state model (TSM), taken from [62], or on the scission-point model, the so-called EHFM scission-point model, taken from [55], and the available experimental data.
Angular Momentum $\ell (\hbar)$

Figure 5.11: Same as 5.10, but at $E_{\text{cm}} = 60.5$ MeV, which corresponds to $T = 3.60$ MeV for the use of different $\Delta R$ for the LPs and IMFs respectively. The cross sections given in the brackets are obtained by summing up to the maximum angular momentum $\ell_{\text{max}} = 35 \hbar$.

[62,97] for $A_L \geq 12$ IMFs. Apparently, the results of our calculation using the proximity potential based on semiclassical ETF compares better with experimental data, than the one based on pocket formula.

Finally a comment on TSM and EHFM calculations: In light systems, where the saddle and scission configurations are known to be very close to each other, and only a very little damping is expected as the reaction between the two nuclei proceeds, there is hardly any reason to expect a significant difference between the TSM and EHFM calculations made, respectively, at the saddle point and scission point, as compared to the cases of heavier systems where damping can occur [56]. Indeed, the calculations based on the saddle point and scission point models in
The DCM calculations are also made for another arbitrary higher $\ell_c$-value ($=36\ h$) for the use of proximity potential of Blocki et al., and compared with two other model calculations EHFM and TSM of Refs. [55] and [62], respectively, and the experimental data [62,97] for $A_L > 12$ IMFs.

Figs. 5.12 (so also in Fig. 5.15) are found to give equivalent results. Both TSM [63] and EHFM [55] calculations start with the CN formation hypothesis and then follow the system by first chance binary fission or light charged particle emission and subsequent light-particle, neutron and/or photon emission. The calculations with TSM [63] are based upon the transition-state theory for which the fission width is assumed to depend on the available phase space of the saddle point. Here the mass asymmetry dependence of the fission barrier favors the decay into mass asymmetric exit channels. The second model EHFM [55] corresponds essentially to an extension.
Figure 5.13: The variation of the first turning point $R_a$ with light fragment mass $A_L$, for cases of $R_a = C_t$, $R_a = C_t + \Delta \bar{R}(=1.28 \text{ fm})$, and the actually calculated $R_a$ from $V(R_a) = Q_{\text{eff}}(T)$, $\ell = 0$, for the decay of $^{56}\text{Ni}$ at $T=3.39 \text{ MeV}$. Note that the actually calculated $R_a$ from $V(R_a) = Q_{\text{eff}}(T)$, $\ell = 0$ can be written as $R_a = C_t + \Delta \bar{R}(\eta)$ where $\Delta \bar{R}(\eta)$ is found to be, in general, positive. For some light fragments ($A_L=1-5$, 8), we use $\ell > 0$ since calculated $Q_{\text{eff}}(T)$ is larger than the barrier for $\ell = 0$ case.

of the Hauser-Feshbach formalism which treats $\gamma$-ray emission, light-particle ($n$, $p$, and $\alpha$) evaporation and IMF-decay as the possible decay channels in a single and equivalent way. The EHFM assumes that the fission probability is proportional to the available phase space at the scission point. The input parameters of TSM and EHFM are basically the same. In each case, the diffuse cut-off approximation was assumed for the fusion partial wave distribution using a diffuseness parameter of $\delta \ell = 1 \hbar$ and a $\ell_c$-value as calculated from the experimental total fusion cross section. A constant level density parameter value of $a = A/8$ in Eq. (2.46) has been

93
chosen for both the models in view of the, respective, experimental and theoretical results of Refs. [159] and [77] for the light heavy-ion systems. The transmission coefficients obtained in the optical model (OM) calculations are used, where the potential parameters have smooth dependence on the mass number and are kind of standard in all statistical model calculations [160]. A more complete comparison of the two models TSM and EHFM, and a detailed discussion of the input parameters in statistical model calculations, is given in the review [56].

In Fig. 5.12, the EHFM calculations for IMFs in [55] are done for \( A_l \geq 12 \) fragments and are thus joined straight from \( A_l = 4 \) to \( A_l = 12 \). Only even-A fragments are plotted since the IMF-spectra at \( E_{cm} = 51.6 \) MeV is measured for only even-A fragments [62, 97]. The TSM calculations for LPs in [62] are performed within the HF formalism, and hence are shown to be the same for EHFM model [55]. As already stated above, for the LP emission at this energy, the measured (fusion or evaporation residue) cross section is available (\( \sigma_{LP} = 1080 \pm 130 \) mb) but the separate yields for each emitted LP is not given for a possible direct comparison between the experiment and model calculations.

We notice in Fig. 5.12, that the known discontinuity at the point between \( A_l = 4 \) and 6 in both TSM and EHFM calculations, due to the use of HF formalism for LPs (\( A \leq 4 \)), is no more present in the present DCM calculations. The DCM treats both the LPs and IMFs emission in a similar manner, although the present calculations (corresponding to the case of Fig. 5.8) overestimate \( \sigma_{LP} \) by a factor of more than two, but the same for Fig. 5.10 reproduces \( \sigma_{LP} \) nicely. Note that mass one particle is same (proton) in both the calculations. However, this discrepancy get resolved if in the case of proximity of Blocki et al. the proton is replaced with neutron (Fig. 5.9), yielding the calculated \( \sigma_{LP} \) to be also in better agreement with the data. Apparently, for the calculation of the evaporation residue cross section \( \sigma_{LP} \) or \( \sigma_{LP} + \sigma_{IMF} \), it is very important to know exactly the contributing particles (i.e. their multiplicities) for comparisons with experiments. Note further that the
HF analysis gives nearly equal cross sections for each of the four emitted particles ($A_L=1-4$), whereas a decreasing function of the light-particle mass is obtained in DCM. It will be of great interest to measure the multiplicity and trends of LP cross sections in near future.

For the IMFs, in Fig. 5.12, the general comparison between the experimental data and DCM for $\ell_c = 32 \hbar$ in the case of proximity pocket formula and $\ell_c = 35 \hbar$ in case of the proximity potential of the semiclassical ETF approach in SEDF is of the same quality as for the TSM or EHFM, at least for $A_L \leq 22$. For $A_L > 22$ the TSM and EHFM predictions appear to be better, and DCM seem to require a larger $\ell_c$-value. However, the DCM calculations for arbitrary $\ell_c = 36 \hbar$ give poorer fits for the lighter fragments. Thus, a smaller $\Delta R$-value is suggested for a higher $\ell_c$-value (see Fig. 5.14 below). In any case, in general, in experiments [56] the CN component is free from deep inelastic “orbiting” (fully damped collision) yields only for lighter fragments with $A < 24$.

Note that in Fig. 5.12, for DCM calculations $\Delta R=0.41$ fm for LPs and $\Delta R=1.28$ fm for IMFs. However, a closer comparison of DCM calculations with experiments (Fig. 5.14, discussed below) favors the use of a fragment-dependent $\Delta R$ or the actual $\Delta R(\eta)$, calculated from $Q_{eff}$, presented in Fig. 5.13 for light mass fragments. We notice in Fig. 5.13 that $\Delta R(\eta)$ has an oscillatory nature, if compared to the smooth variation of $C_t$ or $C_t + \Delta R$ with $\eta$. The maxima in $\Delta R(\eta)$ correspond to $\alpha$-nuclei IMFs and the minima to $N=Z$ non-$\alpha$-nuclei IMFs, with the odd-$A_L$ fragments lying in between. For light-particles, the $\Delta R(\eta)$ values increase almost monotonically. In any case, the division between the LPs and the IMFs is clearly evident from the variation of $\Delta R$ with $\eta$.

Figure 5.14 shows the DCM calculations for use of different average $\Delta R$-values and the actual $\Delta R(\eta)$ obtained in Fig. 5.13 from calculated $Q_{eff}$. These calculations are presented here only for even-$A$ IMFs. We have also added here the DCM calculations for $\Delta R=1.28$ fm, $\ell_c = 32 \hbar$ (from Fig. 5.12). It is clear that $\Delta R=1.28$
Figure 5.14: The same as for Fig. 5.12, but for DCM alone, calculated for different average \( \Delta R \) values and the actual \( \Delta R(A_{\ell}) \) determined from \( V(R_a) = Q_{\text{eff}}(T, \ell = 0) \) (Fig. 5.13). The DCM calculations are compared with the experimental data taken from Ref. [62, 97].

fm gives the optimum fit to IMF data, though the oscillatory nature of data is almost smoothed out, particularly for the heavier IMFs. Interesting enough, this oscillatory structure of the cross section gets restored with the use of actual \( \Delta R(\eta) \) obtained from calculated \( Q_{\text{eff}} \), though the fit with data is now deteriorated. Apparently, an improvement in \( \Delta R(\eta) \) is required. This calls for an improvement in the calculations of \( Q_{\text{eff}} \) and hence in the ground-state and T-dependent binding energies.

The histograms in Fig. 5.15 show the comparisons of the absolute IMF cross sections for the best fit (\( \Delta R \)) for both the cases of proximity potentials, the experi-
Figure 5.15: The histograms of the calculated IMF cross sections $\sigma(A_L)$ on DCM with use of both the proximity potentials of Blocki et al. and of the semiclassical ETF approach in SEDF, compared with the experimental data [62, 97] and two other model calculations EHFM [55] and TSM [62], for even-$A_L$ IMFs at $E_{c.m.} = 51.6$ MeV and for both the odd- and even-$A_L$ IMFs at $E_{c.m.} = 60.5$ MeV. The predicted $A_L = 14$ fragment cross section on DCM is very large, ten times of what is plotted here.

The cross section $\sigma(A_L)$ on DCM is plotted as a function of fragment mass number $A_L$. The histograms show the calculated IMF cross sections for different model calculations compared with experimental data. The experimental data [62, 97] and the two alternate model-calculations of TSM (from [62]) and EHFM (from [55]), for $A_L \geq 12$ at both the available energies. Similarly, Fig. 5.16 shows the DCM calculated excitation functions (cross sections at different $E_{c.m.}$) for the emission of $^{12}C$ from the excited $^{56}Ni^*$ CN, using pocket formula for proximity potential. We notice in Fig. 5.16 that, independent of the choice of $\Delta R$-value (i.e. a constant or T-dependent value), the $^{12}C$ emission cross section, $\sigma(^{12}C)$, increases as the incident energy increases and reaches a maximum around $E_{c.m.} = 90$ MeV and then starts to decrease at higher incident energies. It is interesting to note that similar results are obtained in the HF calculations, using BUSCO code.
Figure 5.16: The DCM excitation functions i.e. the cross sections at different incident c.m. energies for emission of $^{12}C$ fragment from the excited $^{56}Ni^{*}$ compound nucleus, calculated for a constant and an arbitrary T-dependent $\Delta R$-value, for the use of proximity pocket formula of Blocki et al. for the proximity potential.

For the emission of $^{12}C$ from $^{114-118}Ba^{*}$ CN [161]. From Figs. 5.15 and 5.16 it is clear that the DCM contains the required features of the experimental data, as well as of other models (EHFM [55] and TSM [62]).

Furthermore, a better treatment of the binding energies and missing aspects, such as the deformations of the fragments and neck formation between them, would also ameliorate the predictions of the present model. In view of this hope, in the following, we further analyze the comparisons of the DCM calculations for average TKEs with the experimental data of Ref. [62], with the use of both the proximity potentials of Blocki et al. and the proximity potential of the semiclassical ETF approach in SEDF. Figure 5.17 and 5.18 shows the DCM calculated average total
Figure 5.17: The measured [62] and DCM calculated average total kinetic energy (TKE) for the reaction $^{32}$S+$^{24}$Mg$\rightarrow$^{56}Ni* $\rightarrow A_L + A_H$, at two incident energies $E_{c.m.} = 51.6$ and 60.5 MeV, for the use of proximity potential of Blocki et al. Also, the total kinetic energy TKE for the best fit to $\ell$-value is plotted. The average $\Delta R=1.28$ and 1.29 fm, respectively, for the two energies.

The total kinetic energy $TKE(A_i)$ for a best fit to the $\ell$-value is plotted. Apparently, TKE calculated with two different potentials, compared with the experimental data for the $^{32}$S+$^{24}$Mg reaction [62] leading to hot $^{56}$Ni* at the two chosen energies. Here, for each fragment, the TKE for each $\ell$ is averaged over its corresponding production cross section $\sigma_\ell$ w.r.t. the total cross section $\sigma(A_L) = \sum_{\ell=0}^{\ell_{max}} \sigma_\ell(A_L)$. We have also calculated the total kinetic energy $TKE(A_L)$ for a best fit to the $\ell$-value. Apparently,
Figure 5.18: Same as Fig. 5.17, but for the use of the proximity potential of the semielassieal ETF approach in SEDF. The used average $\overline{AR}$ is shown in Figs.

The comparisons with data are reasonably good for both the calculations and it is difficult to distinguish between the calculated $TKE$ and $\overline{TKE}$. The maximum $\ell$-value is nearly the same in both case. However, it is not clear why this maximum $\ell$-value is much less than the $\ell_c$-value. The simple model-dependence used for handling the deformations of the fragments and neck formation between them need further improvements.
5.3 Summary

We have applied the DCM for the decay of a hot and rotating CN, formed in light heavy-ion reactions, into multiple LP evaporation and IMF emission, with the use proximity pocket formula of Blocki et al. and the proximity potential of the semiclassical ETF approach in SEDF. The LP emission (evaporation residue) cross section $\sigma_{LP}$, constitutes the CN fusion cross section $\sigma_{fusion}$ for a negligible emission of IMFs, since $\sigma_{fusion} = \sigma_{LP} + \sigma_{IMF}$, also referred to as $\sigma_{Total}$. The statistical equilibrated CN evaporation process, successful for the emission of LPs, could not explain the IMF emission. Alternatively, the IMF emission alone could be understood as the statistical fission process in the saddle-point [62] or scission-point model [55]. On the other hand, in DCM, both the LPs and IMFs are treated identically as the dynamical collective mass motion of preformed fragments or clusters through a barrier, i.e. quantum mechanical tunneling of clusters that are considered pre-born with different probabilities before they actually penetrate the barrier. Thus, the cluster preformation probabilities contain the structure effects of the CN, that are found to be important in the description of the measured excitation functions of large-angle elastic and inelastic scattering yields in the experiments under study.

The DCM is worked out in terms of only one parameter, the neck-length parameter, that depends on the total kinetic energy of the fragments TKE(T) at the given temperature T of the CN, which itself is defined for the first time in terms of the binding energies of the emitted fragments in their ground-states and the binding energy of the hot CN. The hot CN is considered to achieve its ground-state by giving away its extra binding energy to the emitted LPs, which is shown to leave the emitted IMFs in their respective ground states with total kinetic energy TKE(T=0). The remaining (excitation) energy TXE(T) must go in to the emission of secondary light-particles from the IMFs which are otherwise already in their ground-states in the radial motion. Such an emission of secondary light-particles is not included here in the DCM so-far; rather the model predictions are compared with the primary
IMF experimental data [62], corrected for such an emission.

The DCM is applied here to the decay of $^{56}Ni^*$, formed in the $^{32}S+^{24}Mg$ reaction at two incident energies $E_{\text{c.m.}} = 51.6$ and 60.5 MeV, where both the LP cross section and IMF spectra, as well as the total average kinetic energy ($\overline{TKE}$) for only the favored $\alpha$-nucleus fragments, are measured [62]. The interesting result of DCM is that both the preformation factors and penetrabilities, as a function of angular momentum, behave differently for the LPs and the IMFs. In other words, there is an explicit division at mass-four fragment between the LPs and IMFs with $^4He$ belonging clearly to the LP regime. The preformation factor is shown to contribute more to the observed behaviour of IMF cross section $\sigma_{\text{IMF}}$, which can be compared with the experimental data reasonably well, favoring an asymmetric distribution. Furthermore, the variation of both $\sigma_{LP}$ and $\sigma_{IMF}$ with angular momentum, as well as the individual contributions of IMFs to $\sigma_{IMF}$, and the excitation functions of the emitted IMFs, match exactly the predictions of the statistical fission model, and the HF analysis. Since, unlike fission models, the DCM does not depend on the chosen phase space it has the advantage that the $\ell_{\text{max}}$-value is fixed by the initial conditions of the experiment via $\ell_c$, rather than by the available phase space. This distinguishing feature is evident in $\sigma_{IMF}$ not going to zero when $\sigma_{LP}$ goes to zero at $\ell_{\text{max}} = \ell_c$. The comparison of $\sigma_{LP}$, however, depends strongly on the type of particles involved, their multiplicities and the choice of proximity potential, as expected. The calculated $\overline{TKE}$ also reproduces the experimental data, though at an $\ell < \ell_c$-value, which has perhaps to do with the way the deformations of the fragments are included here simply through the same neck-length parameter that accounts for the temperature effects. The model is being improved both for the neglected deformation effects and neck-formation between them as well as the binding energies used to calculate this neck-length parameter.