Chapter 6

Role of various model ingredients used in the proximity potential for heavy-ion fusion studies

6.1 Introduction

It is now well accepted that the Coulomb interactions alone cannot define a fusion barrier. Nuclear interactions play an equally important role in deciding the fate of a reaction [1-3, 6, 7, 156, 159, 165, 168-197, 200-213]. At the same time, Coulomb part of the interaction potential is well known, whereas nuclear part is not clearly understood. This is perhaps the compelling cause of so many new potentials one sees in the literature [1-3, 6, 7, 156, 159, 165, 168-197, 200-213]. Among various nuclear potentials, one has the potentials within the proximity concept [1, 149, 168-172, 179, 180, 182, 189, 190, 195, 210-213, 293, 294], as well as within the energy density formalism [2, 3, 6, 159, 177, 196, 200] as discussed in Chapter 2-5. As many as two dozen potentials and their different versions are being used in the literature. It is also evident from the literature that every author has tried to justify the validity of the potential by showing that it reproduces the proximity values [179, 180]. In Chapters 3 and 4, as many as sixteen proximity based versions and their parametrizations were studied and compared with huge amount of experimental data on fusion barriers and cross sections for symmetric [210] as well as for asymmetric [211] colliding nuclei.

All proximity potentials are based on the proximity force theorem stated in Chapter 2. According to that, one can write the force between two gently curved surfaces as a
function of the separation degree of freedom $s$ as

$$F(s) = - \left( \frac{\partial V_N}{\partial s} \right) = 2\pi \bar{R} e(s) \text{ MeV}, \quad (6.1)$$

where $\bar{R} = \left( \frac{C^2 + C_2}{C_1 + C_2} \right)$ is the reduced radius and $C_i$ being the central radii [given by Eq. (2.4)]. Here $2\pi$ being the proportionality factor. In the original version of the proximity potential (labeled as Prox 1977 in Chapter 2) [168], $V_N(s)$ can be written as;

$$V_N(s) = 2\pi \bar{R} \int e(s) ds \text{ MeV}, \quad (6.2)$$

where $e(s)$ is the interaction potential per unit area between two flat surfaces made of semi-infinite nuclear matter. Blocki et al. [168] introduce a dimensionless function known as universal function $\Phi(\xi)$ as

$$\Phi(\xi) = \frac{e(\xi b)}{2\gamma b}, \quad (6.3)$$

where $\xi = \left( \frac{s - r - C_1 - C_2}{b} \right)$, $b$ is the surface width, and $\gamma$ is the surface energy coefficient. The surface width $b = 1$ fm. Finally, the nuclear part of the interaction potential $V_N(s = r - C_1 - C_2)$ is given by Eq. (2.1). In this potential, the strength of the nuclear potential depends on the relative neutron excess of the target/projectile through surface energy coefficient $\gamma$ and on the mass and surface diffuseness through the reduced radius $\bar{R}$. This concept did introduce a great amount of simplification in nuclear potential studies [2, 7, 178, 180, 182, 189, 191, 195, 200]. This method is applicable to leptodermous systems, in which the surface region is relatively thin, compared to the size of the object under consideration.

As pointed out by various authors [1], original form of the proximity potential Prox 1977 overestimates the experimental data by 4% for fusion barrier heights. The observed deviation of 4% is quite significant for heavier colliding nuclei. Furthermore, it was reported in Chapter 3 that the original proximity potential overestimated the barriers by 6.73% for symmetric colliding nuclei [210]. Similarly, large deviation was noted for the asymmetric colliding nuclei [211] in Chapter 4.

With the passage of time, several improvements/modifications were made over the original proximity potential Prox 1977 to remove the gray part of the potential. It includes either the better form of the surface energy coefficient or universal function and/or nuclear radius [1, 169, 210, 211]. A careful look reveals that these modifications/improvements are not able to explain the experimental data [175, 210]. A deep survey also pointed out that
these technical parameters (i.e. surface energy coefficient, nuclear radius, and universal function) were chosen quite arbitrarily in the literature. In the last 30 years, considerable progress has been made in the accurate determination of nuclear properties, including the surface energy coefficient and nuclear radii. Among them, the surface energy coefficient is available in a large variety of forms from time to time [213]. Also, different authors used different forms of nuclear radius [210, 211, 213]. These forms vary either in terms of their coefficients or either different mass or isospin dependence. The third technical parameter, the universal function, was also parametrized in different improved forms [1, 168, 202, 210, 211]. Unfortunately, no systematic study is available in the literature, where one can explore the role of these technical parameters in fusion barrier heights, positions, and cross sections. Alternatively, a best set of the above mentioned parameters is still missing.

In the present chapter, our aim is to pin down the role of above stated technical parameters in fusion barriers and cross sections and therefore, constituting a new proximity potential by using up-to-date value of potential parameters. In addition, we also aim to compare the final outcome with huge experimental data and its earlier available versions. This will definitely give a direct check to test the accuracy of our new proximity potential. The present systematic study includes the reactions with combined mass between $A = 19$ and $A = 294$ units. In total, 400 experimentally studied reactions including both symmetric as well as asymmetric colliding partners are taken into account. This work is based on our publications [213, 214]. Section 6.2 describes the various model ingredients used in proximity potential. Our results and calculations, including new proximity potential are presented in Section 6.3 and summary is presented in Section 6.4.

6.2 The various model ingredients used in proximity potential

6.2.1 The surface energy coefficient ($\gamma$)

In the original proximity potential [Eq. (2.1)], the surface energy coefficient $\gamma$ was taken from the work of Myers and Świątecki [225] which reads as:

$$\gamma = \gamma_0 \left[ 1 - k_\gamma \left( \frac{N - Z}{A} \right)^2 \right].$$

(6.4)
where $N$, $Z$, and $A$ refer to the neutron, proton, and total mass of two colliding nuclei.

In the above formula, $\gamma_0 = \frac{\alpha_2}{4\pi r_0^2}$; where $\alpha_2$ is usual liquid drop model surface energy coefficient and $r_0$ is the nuclear radius constant is the surface energy constant and $k_s$ is the surface-asymmetry constant. Firstly, these constants were parametrized by Myers and Świątek [225] by fitting the experimental binding energies resulting in values $\gamma_0 = 1.01734$ MeV/fm$^2$ and $k_s = 1.79$. These authors further modify the mass formula and yield new values $\gamma_0 = 0.9517$ MeV/fm$^2$ and $k_s = 1.7826$ [226]. This particular set of $\gamma$ values was used in the original proximity potential Prox 1977. This value of $\gamma$ is referred as $\gamma$-MS.

In another attempt, Möller and Nix [227] fitted the surface energy coefficient $\gamma$ with the value $\gamma_0 = 1.460734$ MeV/fm$^2$ and $k_s = 4.0$ in nuclear macroscopic energy calculations. Naturally, this will lead to more attraction compared to $\gamma$-MS. This version of $\gamma$ is labeled as $\gamma$-MN1976.

Later on, due to the availability of a better mass formula due to Möller et al. [229], $\gamma_0$ and $k_s$ were refitted to a strength of $1.25284$ MeV/fm$^2$ and $2.345$, respectively. This particular set of values were obtained directly from a least-squares adjustment to the ground-state masses of 1654 nuclei ranging from $^{16}$O to $^{233}$U and fission barrier heights [229]. This modified $\gamma$ is labeled as $\gamma$-MN1995.

In the modified proximity potential version [1], Myers and Świątek, chose $\gamma$ that also depends on the neutron skin of the interacting nuclei. The expression of $\gamma$ obtained from the droplet model [254] reads as:

$$\gamma = \frac{1}{4\pi r_0^2} \left[ 18.63(\text{MeV}) - \frac{Q}{2r_0^2} \left( \frac{t_1^2 + t_2^2}{r_0^2} \right) \right],$$

where $t_i$ is the neutron skin calculated using Eq. (2.11). This version of the surface energy coefficient is labeled as $\gamma$-MSNew.

As we see, all the previous four versions of $\gamma$ have different strengths. In fact, in the work of Möller and Nix [228], as many as five different sets of $\gamma$ parameters were listed. This will, of course, lead to different values of surface energy coefficients as well as potentials. This study [228] was based on the calculations of fission barrier heights of 28 nuclei and ground-state masses of 1323 nuclei. Royer and Renaud [295] used the $\gamma_0$ value the same as $\gamma$-MS with a different $k_s$ value (= 2.6). Recently, Pomorski and Dudek [230] obtained different surface energy coefficients by including different surface
curvature effects in the liquid drop model. All the above stated surface energy coefficients are listed in Table 6.1. It is clearly visible from the table that a large number of surface energy coefficients are available in the literature depending upon the advancement in theories and/or in experiments. Definitely, the original proximity potential [168] used the value of $\gamma$ that was proposed four decades ago. Therefore, an up-to-date knowledge of this parameter is essential so that a best set of this parameter can be used in further fusion studies.

Table 6.1: The different surface energy coefficients available in the literature are displayed. The different sources from where the corresponding values are obtained are also listed.

<table>
<thead>
<tr>
<th>S. No.</th>
<th>$a_0$(MeV)</th>
<th>$r_0$(fm)</th>
<th>$\gamma_0$(MeVfm$^{-2}$)</th>
<th>$\kappa_x$</th>
<th>source of information</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>18.56</td>
<td>1.2049</td>
<td>1.01734</td>
<td>1.79</td>
<td>mass formula/fission barriers</td>
<td>[225]</td>
</tr>
<tr>
<td>2</td>
<td>17.9439</td>
<td>1.2249</td>
<td>0.9517</td>
<td>1.7826</td>
<td>-do-</td>
<td>[226]</td>
</tr>
<tr>
<td>3</td>
<td>24.7</td>
<td>1.16</td>
<td>1.460734</td>
<td>4.0</td>
<td>fission barrier heights</td>
<td>[227]</td>
</tr>
<tr>
<td>4</td>
<td>21.7</td>
<td>1.18</td>
<td>1.2402</td>
<td>3.0</td>
<td>-do-</td>
<td>[203]</td>
</tr>
<tr>
<td>5</td>
<td>20.57</td>
<td>1.18</td>
<td>1.1756</td>
<td>2.2</td>
<td>-do-</td>
<td>[228]</td>
</tr>
<tr>
<td>6</td>
<td>21.53</td>
<td>1.16</td>
<td>1.27326</td>
<td>2.5</td>
<td>-do-</td>
<td>[228]</td>
</tr>
<tr>
<td>7</td>
<td>21.14</td>
<td>1.16</td>
<td>1.2502</td>
<td>2.4</td>
<td>-do-</td>
<td>[228]</td>
</tr>
<tr>
<td>8</td>
<td>21.13</td>
<td>1.16</td>
<td>1.2496</td>
<td>2.3</td>
<td>-do-</td>
<td>[282]</td>
</tr>
<tr>
<td>9</td>
<td>17.9439</td>
<td>1.2249</td>
<td>0.9517</td>
<td>2.6</td>
<td>-do-</td>
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</tr>
<tr>
<td>10</td>
<td>21.1866</td>
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<td>1.25284</td>
<td>2.345</td>
<td>-do-</td>
<td>[229]</td>
</tr>
<tr>
<td>11</td>
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<td>1.18995</td>
<td>1.08948</td>
<td>1.9830</td>
<td>surface curvature effects</td>
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<td>12</td>
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<td>0.9180</td>
<td>0.7546</td>
<td>-do-</td>
<td>[230]</td>
</tr>
<tr>
<td>13</td>
<td>16.9707</td>
<td>1.21725</td>
<td>0.911445</td>
<td>2.2938</td>
<td>-do-</td>
<td>[230]</td>
</tr>
</tbody>
</table>

6.2.2 The nuclear radius (R)

As earlier stated, the nuclear radius was also taken in the literature arbitrarily. For example, proximity potentials 1977 [168,210] and 1988 [169,210] use the equivalent sharp radius as:

$$R_i - 1977 = 1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3} \text{ fm} \quad (i = 1, 2). \quad (6.6)$$

This formula is a semi-empirical expression supported by assuming the finite compressibility of nuclei. As a result, lighter nuclei squeeze more by the surface-tension forces, whereas heavy nuclei dilate more strongly due to the Coulomb repulsion. In both potentials, the nuclear surface diffuseness enters via reduced radius $\overline{R}$ that is used in Eq. (2.1).
The central radius \( C \) is calculated from the relation:

\[
C_i - 1977 = R_i \left[ 1 - \frac{b^2}{R_i^2} + \cdots \right] \text{ fm} \quad (i = 1, 2). \tag{6.7}
\]

For the present study, we also used the radius due to Aage Winther in Eq. (6.7) which reads [172]

\[
R_i - AW1995 = 1.20A_i^{1/3} - 0.09 \text{ fm} \quad (i = 1, 2), \tag{6.8}
\]

and the corresponding central radius [Eq. (6.7)] is denoted as \( C_{\text{AW1995}} \).

The modified version of the proximity potential uses a different form of the radius [1]:

\[
R_i - 2000 = 1.240A_i^{1/3} \left\{ 1 + \frac{1.646}{A_i} - 0.191 \left( \frac{A_i - 2Z_i}{A_i} \right) \right\} \text{ fm} \quad (i = 1, 2). \tag{6.9}
\]

This formula indicates that radius depends not only on the mass number, it also has a dependence on the relative neutron excess. Actually this formula is valid for even-even nuclei with \( Z \geq 8 \) [256]. To calculate the matter central radius \( C_r \), the neutron skin is also added in Ref. [1] using the relation

\[
C_r - 2000 = c_i + \frac{N_i}{A_i} \text{ fm} \quad (i = 1, 2). \tag{6.10}
\]

where \( c_i \) denotes the half-density radii of the charge distribution and is given by Eq. (2.10). This central radius is marked as \( C_{\text{2000}} \). Recently, a new form of Eq. (6.9) with slightly different constants is also reported [252]

\[
R_i - 2000.N = 1.2332A_i^{1/3} + \frac{2.8961}{A_i^{2/3}} - 0.18688 \left( \frac{N_i - Z_i}{A_i} \right) \text{ fm} \quad (i = 1, 2). \tag{6.11}
\]

The accuracy of the above formula is mainly improved by adding the Coulomb diffuseness correction or the charge exchange correction to mass formula [252]. By using this form of the radius in Eqs. (2.10) and (6.10), we can again calculate the central radius \( C \) denoted by \( C_{\text{2000N}} \). A large number of other radius formulas are also available in the literature but the above mentioned are the extreme one.

Bass 1973 and Ngô 1975 potentials used simple expression for radius of the form

\[
R_i = r_0A_i^{1/3} \text{ fm} \quad (i = 1, 2), \tag{6.12}
\]

where \( r_0 \) having values 1.07 and 1 fm, respectively. On the other hand, Bass 1977 [170] and CW 1976 [171] used the radius of the form

\[
R_i = a_iA_i^{1/3} - b_i \frac{1}{A_i^{1/3}} \text{ fm} \quad (i = 1, 2). \tag{6.13}
\]
with different value of $a'$ and $b'$ i.e. 1.16 and 1.39 for Bass 1977 potential, whereas 1.233 and 0.978 for CW 1976 potential. In Ngô Ngô 1980 potential, nuclear radius $R_i$ read as

$$R_i = \frac{NR_{m} + ZR_{p}}{A_i} \text{ fm } \quad (i = 1, 2), \quad (6.14)$$

where $R_{m}$ and $R_{p}$ are the equivalent sharp radii for neutron and proton and are given by Eqs. (2.38) and (2.39). The effective nuclear radius used in Denisov 2002 potential is read as

$$R_i = R_{ip} \left(1 - \frac{3.413817}{R_{ip}^2}\right) + 1.284589 \left(I_i - \frac{0.4A_i}{A_i + 200}\right) \text{ fm } \quad (i = 1, 2), \quad (6.15)$$

where, proton radius $R_{ip}$ is given by Eq. (6.9).

Recently [296], Angeli presented a set of 799 ground state nuclear charge radii. It includes the experimental data from different sources like; electron scattering, muonic atom X-rays, $k_{\alpha}$ isotope shift etc. Similarly, Nadjakov, Marinova, and Gangrusky [296] presented experimental values of root mean square (rms) nuclear charge from two different sources for 42 elements from $^{23}\text{Na}$ to $^{213}\text{Am}$. The measured rms values of the charge distribution, $\langle r^2 \rangle^{1/2}$ can be expressed in terms of equivalent rms radii using relation $R_i^{rms} = \sqrt{\frac{2}{3}} \langle r^2 \rangle^{1/2}$. Recently, Royer presented different expressions for $R_i$ that reproduce the experimental nuclear charge radius [297]. Now, to convert $R_i$ to half-density radius one can use Eq. (2.10).

De Vries et al. [255] also give experimental information about the rms and half-density radius over wider ranges of masses.

The above discussion clearly indicates that the different forms of nuclear radius available in the literature should be properly analyzed so that one can investigate the role of different terms used in these formulas in the fusion process. At the same time, due to the availability of new experimental data on nuclear charge radius a new radius formula is also in demand.

### 6.2.3 The universal function ($\Phi$)

The universal function $\Phi(\frac{C_{Cl:C_{O}}-C_{Ca}}{b})$ used in Eq. (2.1) has been derived by several authors in different forms [1, 168, 202]. In the original proximity potential Prox 1977, the universal function have a simple analytical representation in the form of a cubic-exponential pocket formula given by Eq. (2.6). This universal function was calculated using the nuclear Thomas-Fermi model with Seyler-Blanchard phenomenological nucleon-nucleon interaction [254, 298, 299]. We marked this universal function as $\Phi$-1977. Later
Blocki and Świątecki [202] generalize the proximity force theorem [168] to include surfaces that may have large curvatures and presented an improved form of Eq. (2.6) for gap and crevice configurations. In the gap configuration, two overlapping surfaces are either separated or overlapping resulting in density doubling (see figure 6.1). On the other hand, a configuration resulting from first overlapping two surfaces, erasing the overlapping portion and then filling the resulting single shape with matter, without any density doubling is known as crevice configuration (see figure 6.2). For high energy collisions the gap configuration is relevant, whereas for low energy collisions crevice configuration is much useful (Here, ‘high’ or ‘low’ means collisions energies per nucleon high or low compared to typical Fermi-motion energies in the colliding nuclei, some 20 MeV in order of magnitude). Both configurations give same barrier heights and positions because they differ only when $s < 0$, whereas barrier formation takes place for $s > 0$. The modified form of the universal function is read as

$$\Phi(\xi) = \begin{cases} 
-1.7817 + \xi, & \text{for } \xi \leq 0.0 \text{ (c)} \\
-1.7817 + 0.9270\xi + 0.143\xi^2 - 0.09\xi^3, & \text{for } \xi \leq 0.0 \text{ (g)} \\
-1.7817 + 0.9270\xi + 0.01696\xi^2 - 0.05148\xi^3, & \text{for } 0.0 \leq \xi \leq 1.9475 \\
-4.41\exp\left(-\frac{4}{0.7176}\right), & \text{for } \xi \geq 1.9475 
\end{cases} \quad (6.16)$$

where (c) correspond to crevice configuration and (g) correspond to gap configuration. The proximity potential predicts an exponential shape at the surface region [see Eqs. (2.6) and (6.16)], but with a diffuseness parameter (0.7 fm) greater than the value (0.63

Figure 6.1: The schematic representation of gap configuration (taken from Ref. [202]).
Figure 6.2: The schematic representation of crevice configuration (taken from Ref. [202]).

This universal function is marked as $\Phi$-1981.

The above discussion shows that due to the large variety of various technical parameters, a careful choice of these parameters is therefore necessary. In the following, we calculate the fusion barrier heights, positions, and cross sections using the above stated model ingredients in original proximity potential Prox 1977. Our calculations are made for 400 reactions involving both symmetric and asymmetric reactions.

6.3 Calculations and discussion of the results

6.3.1 Role of different surface energy coefficients, nuclear radii, and universal functions in fusion barriers, and cross sections

As noted in the previous subsection 6.2.1, the surface energy coefficient $\gamma$ depends strongly on the asymmetry of the reaction. In figure 6.3, we display the nuclear potential as a function of internuclear distance “r” for the reactions of $^{12}$C + $^{12}$C (in the upper panel) and $^6$He + $^{238}$U (in the lower panel) using the Prox 1977 with different versions of surface energy coefficient $\gamma$. We see that $\gamma$-MS leads to a shallow potential compared to other sets of $\gamma$, whereas $\gamma$-MN1976 leads deepest potential. This leads to deeper pocket in case of $\gamma$-MN1976 and hence has larger probability of fusion. Since fusion is a
Figure 6.3: The nuclear part $V_N$ (MeV), of the interaction potential as a function of internuclear distance “r” (in fm) using Prox 1977 with different values of surface energy coefficients $\gamma$. 

Surface phenomenon, these differences at surface have drastic effects. We also tested the previously highlighted surface energy coefficients but their value lies between the extreme limits.

In figure 6.4, we display the fusion barrier heights $V_B$ and fusion barrier positions as a function of the product of the charges of colliding nuclei $Z_1Z_2$. For the clarity of the figure, only 155 reactions are displayed. We show the results of implementing $\gamma^{-1}$.
Figure 6.4: The fusion barrier heights $V_B$ (MeV) and positions $R_B$ (fm) as the product of the charges $Z_1Z_2$ using different values of surface energy coefficient as well as different radii in the Prox 1977.

We see some mild effects in the outcome. These effects are monotonous in

$\gamma$-MN1976, $\gamma$-MN1995, and $\gamma$-MSNew as well as different radii in the Prox 19
to the wide acceptability of the radius used in Prox 1977 [Eq. (6.6)], we shall stick to the same formula in this section. The results of different γ values are quantified in figure 6.5, where we display the percentage difference ΔVb (%) and ΔRb (%) defined by Eqs. (3.3) and (3.4) for different values of surface energy coefficients γ. The experimental data are taken from the Refs. [13–68, 70–72, 78–84, 91–100, 117–136]. Actually, it is clear from the literature that no experiment can extract information about the fusion barriers directly. All experiments measure the fusion differential cross sections and then with the help of a theoretical model, one can extract the fusion barriers. We see that the use of γ-MS, which is used in the original proximity potential 1977, yield considerable deviations (±10%). Further, the use of γ-MN1976 and γ-MN1995 yield much improved results. The average deviations over 400 reactions for the fusion barrier heights are 3.99%, 0.77%, 1.77%, and 2.37%, for γ-MS, γ-MN1976, γ-MN1995, and γ-MSNew, respectively. Whereas, for fusion barrier positions, its values are -1.74%, 1.95%, 0.73%, and 0.0%, respectively over 272 reactions (barrier positions are not available for all reactions).

It is clear from the previous study that surface energy coefficients γ-MN1976 and γ-MN1995 may be better choices. To further strengthen the choice we calculate the fusion cross sections using the Wong formula [260] discussed in Chapter 2.

In figure 6.6, we display the fusion cross section σ_{fus} (in mb) as a function of the center-of-mass energy E_{c.m.} for the reactions of 26Mg + 30Si [121], 28Si + 28Si [25, 59, 61], 16O + 46Ti [70], 12C + 92Zr [95], 40Ca + 58Ni [57], and 16O + 144Sm [98], respectively. We see that γ-MN1976/γ-MN1995 give better results over the original proximity potential Prox 1977. Note that both fusion barrier height and curvature affect the sub-barrier fusion probabilities. From the previous analysis, it is clear that the effect of technical parameters, that is the surface energy coefficient γ as well as nuclear radius of the target/projectile is of the order of 10%-15%. The use of surface energy coefficient γ-MN1976/γ-MN1995 improves the results of the Prox 1977 potential considerably.

The role of third technical parameter i.e. the universal function in fusion barriers is analyzed in figure 6.7. Here, we display ΔVb (%) and ΔRb (%) as a function of the product of the charges of colliding nuclei Z_1Z_2 using two sets of universal functions [Eqs. (2.6) and (6.16)]. It is clear from the figure that deviations are significantly reduced by the use of Φ-1981 compared to original form Φ-1977. The universal function Φ-1981 reduces the average deviation over 390 reactions by 1% for fusion barriers. The experimental values are taken directly from the literature [13–68, 70–72, 78–84, 91–100, 117–136].
6.3.2 New proximity potential (Prox 2010)

The accurate knowledge of the ion-ion interaction potential is essential ingredient in the understanding of heavy-ion fusion reactions between nuclei and in the analysis of elastic and inelastic scattering. In this section, we shall modify the original proximity potential Prox 1977 by using a suitable set of the surface energy coefficient, nuclear radius, and
Figure 6.6: The fusion cross sections $\sigma_{\text{fus}}$ (mb) for the reactions of $^{26}\text{Mg} + ^{30}\text{Si}$, $^{28}\text{Si} + ^{28}\text{Si}$, $^{16}\text{O} + ^{46}\text{Ti}$, $^{12}\text{C} + ^{92}\text{Zr}$, $^{40}\text{Ca} + ^{58}\text{Ni}$, and $^{16}\text{O} + ^{144}\text{Sm}$ as a function of center-of-mass energy $E_{\text{c.m.}}$ (MeV) using different versions of $\gamma$ in Prox 1977. The original Prox 1 is also shown for comparison. The experimental data are taken from Morsad 1990 [61], Gary 1982 [59], DiCenzo 1981 [59], Aguilera 1986 [25], Neto 1990 [70], Newton 2001 [60], Sikora 1979 [57], and Leigh 1995 [98].
universal function by keeping its basic structure same. The nuclear part of the interaction potential $V_N(r)$ is calculated in the same way as Eqs. (6.1) or (2.1). As is clear from the subsection 6.2.2, huge amount of experimental data are available in the literature on rms values of the charge distribution in the recent time. By using this experimental
information we obtain a new nuclear radius $R_i$ as

$$R_i^{0\text{a}} = 1.171 A_i^{1/3} + 1.427 A_i^{-1/3} \text{ fm} \quad (i = 1, 2). \quad (6.17)$$

This form of the nuclear radius is similar to one given in Refs. [170, 171]. This is obtained by fitting 792 measured rms values of the charge distribution $(r^2)^{1/2}$ given in Ref. [296] with $3 \leq Z \leq 96$ and $6 \leq A \leq 248$. We marked this new radius as $R_i^{0\text{a}}$. Here we first convert the measured rms values to equivalent rms radii by using relation $R_i^{\text{exp}} = \sqrt[3]{\frac{5}{3} \langle r^2 \rangle^{1/2}}$. The corresponding radius is labeled as $R_i^{\text{exp}}$. After this, we apply the least-squares fitting procedure to obtain Eq. (6.17) [Shown in figure 6.8(a)]. Using Eq. (6.17), in Eqs. (6.9) and (2.10), we calculate reduced radius $\bar{R} \left( = \frac{C_1 C_2}{C_1 + C_2} \right)$ used in Eq. (2.1).

In the previous section, in total, fourteen surface energy coefficients were highlighted and the role of extreme four was analyzed in depth. Out of all these, two best forms of surface energy coefficients are further advocated. Here, we shall restrict to the latest $\gamma$ values i.e., $\gamma_0 = 1.25284$ MeV/fm$^2$ and $k_s = 2.345$ only.

For the universal function, we shall choose the modified form due to Blocki and Świątecki [202], i.e., $\Phi$-1981 for gap configuration only.

By using the above mentioned parameters i.e., nuclear radius, $R_i^{0\text{a}}$, surface energy coefficient, $\gamma$-MN1995, and universal function, $\Phi$-1981, we constructed a new proximity potential and labeled it as “Prox 2010”. Along with the above new form, we shall also use the original proximity potential Prox 1977 and its recently modified form Prox 2000 for comparison.

### 6.3.3 Comparison of new proximity potential with experimental data and older proximity potentials

As one sees from the preceding section, three different factors governing the success of proximity potential are: (i) surface energy coefficient, (ii) nuclear radius, and (iii) the universal function. We analyzed the literature very carefully and also found that the latest information on these three factors can reshape the old proximity potential. In the previous section, the role of surface energy coefficient, nuclear radius, and universal function was analyzed. Whereas, for new radius we fitted the measured rms values of the charge distribution [296] using least-squares procedure and our results are displayed in figure 6.8(a). This type of form is well known in the literature [170, 171]. We check the accuracy of the fitted formula in figure 6.8(b), where, we display the percentage difference.
Figure 6.8: (a) The variation of equivalent rms nuclear radius $R_i^{\text{expt}}$ (fm) (defined in text) as a function of the total mass of nuclei $A$. The solid line represents the straight-line least-squares fit made over the data points. The measured rms values of the charge distribution are taken from Ref. [296]. (b) The variation of $\Delta R$ (%) (defined in text) as a function of the total mass of nuclei $A$.

between fitted and experimental values defined as

$$\Delta R \, (\%) = \frac{R_i^{\text{fit}} - R_i^{\text{expt}}}{R_i^{\text{expt}}} \times 100.$$  

(6.18)

as a function of the total mass $A$ of nuclei. Interestingly, our fitted radius formula reproduces the experimental values within ±5% on average. Slight scattering for lighter mass
region is visible, whereas, for heavy mass range the agreement is perfect. We shall use this new radius formula [Eq. (6.17)] in the further analysis.

A new form of the proximity potential is constructed using the above set of parameters. By using the new version of the proximity potential along with its earlier versions i.e., Prox 1977 and Prox 2000, fusion barriers are calculated for 400 reactions.

In figure 6.9, we displayed the theoretical fusion barrier heights $V_B^{\text{theor}}$ (MeV) and positions $R_B^{\text{theor}}$ (fm) versus its corresponding experimental values. Very encouragingly, we note that Prox 2010 reproduces experimental fusion barrier heights within 1.4%. This results in close agreement with other recently parametrized potentials presented in Chapter 3 for symmetric systems. Whereas, the fusion barrier positions show some scattering around the central line (marked by shaded area). This is in agreement with our previous results presented in Chapters 3 and 4 for symmetric and asymmetric nuclei (see figures 3.5 and 4.4). This scattering may be due to the variation in the experimental set-ups and theoretical methods used to extract these values [13, 20, 51, 55, 62, 65, 80].

In figure 6.10, we displayed the theoretical fusion barrier heights $V_B^{\text{theor}}$ (MeV) calculated using different proximity versions versus corresponding experimental values. Very encouragingly, we note that Prox 2010 potential reproduces the experimental fusion barrier heights within 1.4%, whereas, its old and modified versions reproduces the same within 4.1% and 3.0%, respectively. However, the original form of the proximity potential presented in Chapter 3 overestimates the data by 6.73% for symmetric colliding nuclei [210]. Its recently modified version Prox 2000 was also not good enough to explain the data (within 5.34% for symmetric colliding nuclei [210]). This may be due to the improper use of the radius, surface energy coefficient, and universal function values. Similarly, large deviations were obtained for asymmetric colliding nuclei [211] in Chapter 4.

We quantify our outcome in figures 6.11 and 6.12. In figure 6.11, the percentage deviation $\Delta V_B$ (%) and $\Delta R_B$ (%) between the theoretical and experimental values using different proximity potentials is presented. The original proximity potential Prox 1977 along with its recently modified form Prox 2000 are also displayed. We note from the upper panel of figure 6.11 that Prox 2010 potential on the average gives better results compared to its older versions. The fusion barrier heights are reproduced within ±5% on average. Especially for the heavier colliding nuclei, we see that Prox 2010 reproduces the experimental data much better as compared to other versions. However, slight deviations are visible for fusion barrier positions. This may be due to the fact that in proximity
potential Prox 2010, we use the value of surface energy coefficient that gives stronger
attraction compared to the one used in Prox 1977 and Prox 2000 potentials. Therefore,
in Prox 2010 potential, the counterbalance between the repulsive Coulomb and attractive
nuclear part of the interaction potential occurs at larger distances and hence pushes the
barrier outwards. The average deviation for the fusion barrier heights over 400 reactions

Figure 6.9: The theoretical fusion barrier heights $V_B^{\text{theor}}$ (MeV) and positions $R_B^{\text{theor}}$ (fm) as
a function of corresponding experimental values using our modified proximity potentials
Prox 2010. For the clarity of figure only 155 systems are displayed. The experimental
values are taken from Refs. [13–68, 70–72, 78–84, 91–100, 117–136].

\[
V_B^{\text{theor}} = 1.01 V_B^{\text{expt}}
\]

\[
R_B^{\text{theor}} = (0.25 \pm 0.8) + 0.98 R_B^{\text{expt}}
\]
is reduced to 1.2% using our modified potential Prox 2010, whereas, Prox 1977 and Prox 2000 potential give 3.99% and 4.45%, respectively. This shows that our modified proximity potential explains the experimental data nicely.

In figure 6.12, we display the difference between the theoretical and experimentally
Figure 6.11: The percentage difference between the fusion barrier heights $\Delta V_B$ (%) and positions $\Delta R_B$ (%) as a function of the product of the charges of colliding nuclei $Z_1Z_2$ using different versions of the proximity potential. The experimental values are taken from Refs. [13–68, 70–72, 78–84, 91–100, 117–136].

extracted fusion barrier heights and positions. We further note that Prox 2010 potential gives better results. The deviations however reduce for heavier nuclei. This was the problem with original as well as its recently modified form as pointed out by several authors [1,175]. It is clear from figures 6.11 and 6.12, that Prox 2010 potential is able to reproduce the experimental data much better than its older versions. In figures 6.
Figure 6.12: The variation of $\Delta V_B = V_B^{\text{theor}} - V_B^{\text{expt}}$ (MeV) and $\Delta R_B = R_B^{\text{theor}} - R_B^{\text{expt}}$ (fm) as a function of the product of the charges of colliding nuclei $Z_1Z_2$. The experimental values are taken from Refs. [13-68, 70-72, 78-84, 91-100, 117-136].

6.5, 6.7, and 6.9-6.12, only 155 reactions are displayed to maintain the clarity. The experimental data are taken from Refs. [13-68, 70-72, 78-84, 91-100, 117-136]. The small difference is not significant because of the uncertainties in the analysis of the experimental data.

Finally, we test our newly modified proximity potential Prox 2010 on fusion probabilities. In figure 6.13, we display the fusion cross sections $\sigma_{\text{fus}}$ (in mb) as a function of
Figure 6.13: The fusion cross sections $\sigma_{\text{fus}}$ (mb) as a function of center-of-mass energy $E_{\text{c.m.}}$ (MeV) using earlier versions of proximity potential (Prox 1977 and Prox 2000) along with new version (Prox 2010). The experimental data are taken from Newton 2001 [95] and Stefanini 2009 [82].

The fusion cross sections are calculated using the well known Wong model [260]. The earlier versions that is, Prox 1977 and Prox 2000 potentials show...
similar results. It means no improvement is seen in Prox 2000 potential as was claimed in Ref. [1].

6.4 Summary

In this Chapter as a first part, we attempt to understand the role of surface energy coefficient, nuclear radii, and universal function in fusion dynamics. Our analysis reveals that these technical parameters can affect the nuclear potential as well as fusion barriers by the same amount as different potentials and one should be careful while choosing these technical parameters. We also propose two best set of surface energy coefficients $\gamma$-MN1976/$\gamma$-MN1995 which yields closer agreement with experimental data.

Further, by using the up-to-date knowledge of surface energy coefficient and universal function, a new proximity potential was constructed. At the same time, a new nuclear radius formula was also obtained by fitting the recent data on the measured root mean square values and was used in the construction of Prox 2010 potential. Our newly constructed proximity potential Prox 2010 reproduces the fusion barrier heights within $\pm 5\%$ on average. The average deviation of 4% reported in the literature was significantly reduced to 1.2% for fusion barrier heights. A comparison with experimental data reveals that our new proximity potential reproduces the experimental data very accurately as compared to earlier versions and hence can be used in further fusion studies.