CHAPTER 3

NON-LINEAR SOIL-REINFORCEMENT INTERACTION
MODEL / PROGRAM

3.1 GENERAL

In case of reinforced soil beds mutual shear interaction between the soil and the reinforcement is responsible for the tensile force development in the reinforcement. This tensile force induces confining stress in the surrounding soil mass enhancing the load carrying capacity of the soil system. It is also well-established fact that the degree of improvement in load carrying capacity due to the inclusion of the reinforcement is a function of type of soil, type of reinforcement and imposed footing load. To completely understand the behaviour of reinforced soil beds, one requires a model, which takes into account the non-linear behaviour of soil, reinforcement and soil-reinforcement interaction. More importantly the model should be able to predict the state of stress and deformation in the entire soil system both at working and ultimate loads. Although empirical and closed form solutions provide useful simple solutions in many practical situations, they cannot yield realistic solutions for problems involving complexities such as non-homogeneous media, non-linear behaviour, arbitrary geometries and discontinuities etc. It is almost impracticable to carryout model studies to understand each of these factors independently or jointly as these are quite time consuming and expensive. Under these circumstances non-linear finite element model comes in handy for the solution of reinforced earth problems considering reinforcement as inclusions incorporated in the soil matrix, in which both soil and reinforcement interact with each other. A non-linear elastic confining stress dependent model following a hyperbolic relationship proposed by Duncan and Chang (1970) is used in the present study for analysis of reinforced soil beds. The analysis employs a general finite element formulation referred as Non-Linear Soil-Reinforcement Interaction Program (NLSRIP) developed from an existing non-linear soil-structure interaction program (NLSSIP) coded by Byrne and Duncan (1979).
In this chapter the salient features of the finite element code NLSRIP is presented along with its various components describing the soil, interface and reinforcement elements. Validation literature of the NLSRIP is also included.

3.2 DESCRIPTION OF THE SOIL ELEMENT

A two dimensional 4 noded quadrilateral elements is used in the analysis to represent the soil medium. It has 4 nodes with two degrees of freedom per node (u, v). Figure 3.1 shows the element used in the analysis schematically. The geometry and displacement in the element are expressed as

\[
\begin{bmatrix} x \\ y \end{bmatrix} = [N] \begin{bmatrix} \sum x_n \\ \sum y_n \end{bmatrix}
\]

(3.1)

and

\[
\begin{bmatrix} u \\ v \end{bmatrix} = [N] \{q\}
\]

(3.2)

where

\[
\{q\} = \begin{bmatrix} v_1 u_1 \\ v_2 u_2 \\ v_3 u_3 \\ v_4 u_4 \end{bmatrix}, \quad \{x\} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix},
\]

\[
\{y\} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}
\]

\[
[N] = \begin{bmatrix} N_1 & 0 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & N_1 & 0 & N_3 & 0 & N_4 & 0 \end{bmatrix}
\]

(3.3)

in which x, y, are the global coordinates, u_i and v_i are the nodal displacements. The matrix [N] contains interpolation functions N_1, N_2, N_3 and N_4.

\[
N_1 = \frac{1}{4} (1 - \xi) (1 - \eta)
\]
\[
N_2 = \frac{1}{4} (1 + \zeta)(1 - \eta) \\
N_3 = \frac{1}{4} (1 + \zeta)(1 + \eta) \\
N_4 = \frac{1}{4} (1 - \zeta)(1 + \eta) \\
\]

where, \(\zeta\) and \(\eta\) are the local coordinates as shown in Figure 3.1.

The strain-displacement relation is given by

\[
\{\varepsilon\} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{yx} \end{bmatrix} = [B] \{q\} \\
\]

Where \(B\) is the strain-displacement matrix

\[
[B] = \begin{bmatrix}
\frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial x} & 0 & \frac{\partial N_3}{\partial x} & 0 & \frac{\partial N_4}{\partial x} & 0 \\
0 & \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial y} & 0 & \frac{\partial N_3}{\partial y} & 0 & \frac{\partial N_4}{\partial y}
\end{bmatrix}
\]

and

\[
\{\varepsilon\}^T = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{yx} \end{bmatrix}
\]

The stress-strain relation is given by

\[
\{\sigma\} = [C] \{\varepsilon\} \\
\]

Where \(\{\sigma\}^T = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}\)

\([C]\) is the constitutive matrix given by,

\[
[C] = \frac{E}{\sqrt{1 + \nu^2}} \begin{bmatrix}
1 - \nu^2 & \nu \tau & 0 \\
\nu & 1 - \nu^2 & 0 \\
\nu \tau & 0 & (1 - 2\nu) \frac{\sqrt{1 - 2\nu}}{2}
\end{bmatrix}
\]

35
Figure 3.1 Representation of Four Noded Soil Element
Equation (3.6) is updated for each load increment with corresponding values of $E_t$ and $\nu_t$.

Element stiffness matrix $[K]$ is given by

$$[K] = \int \int [B]^T [C] [B] \, d\xi d\eta \quad (3.8)$$

$$[K] = \int \int [B]^T [C] [B] [J] \, d\xi d\eta \quad (3.9)$$

Where $[J]$ is the determinant of Jacobian matrix.

Use of principle of minimum potential energy yields element equation as:

$$[K] \{d\} = \{F\} \quad (3.10)$$

The stiffness of all the elements are calculated and assembled to obtain global stiffness equation. Theoretically, Poisson's ratio ($\nu$) may range between $-1$ to $0.5$. The practical range for soil is $0$ to $0.5$ and to keep equation (3.7) finite, a maximum value of $0.495$ is used in the analysis. Performances of these plain quadrilateral elements have been improved by adding bending modes as internal freedoms and carrying out modified integration technique (Cook, 1981), using a reasonable fine mesh. Gaussian elimination technique is used for the solution of equilibrium equations.

### 3.3 DESCRIPTION OF TWO DIMENSIONAL REINFORCEMENT ELEMENT

The reinforcement is considered to comprise of a collection of interconnected straight beam members which under the load may be subjected both axial force and bending moment. The beam element has two nodes with three degrees of freedom per node as shown in Figure 3.2 (a). The beams may behave in a non-linear fashion due to: (a) the non-linear nature of the stress-strain relations of the material at higher stress levels, and (b) significant changes in geometry. The stress-strain relations are assumed to be bilinear. Bilinear stress-strain behaviour of the beam element is modelled using the
parameters viz., Young’s modulus, $E$ and yield load, $F_y$ in elastic and plastic range respectively, as shown in Figure 3.2(b) (Byrne and Duccan, 1979).

The force-deflection relationship or stiffness matrix of the reinforcement element is given by:

$$
\begin{bmatrix}
\frac{AE}{L} & 0 & 0 & -\frac{AE}{L} & 0 & 0 \\
0 & \frac{12R_t}{L^3} & \frac{6R_t}{L^2} & 0 & -\frac{12R_t}{L^3} & \frac{6R_t}{L^2} \\
0 & \frac{6R_t}{L^3} & \frac{4R_t}{L^2} & 0 & -\frac{6R_t}{L^3} & \frac{2R_t}{L} \\
0 & -\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\
0 & -\frac{12R_t}{L^3} & -\frac{6R_t}{L^2} & 0 & \frac{12R_t}{L^3} & -\frac{6R_t}{L^2} \\
0 & \frac{6R_t}{L^3} & \frac{2R_t}{L^2} & 0 & -\frac{6R_t}{L^3} & \frac{4R_t}{L} \\
\end{bmatrix}$$

(3.11)

$\langle dX_1, dY_1, dM_1, ..., dM_2 \rangle$ is force vector

$\langle du_1, ..., d\theta_1, ..., d\theta_2 \rangle$ is deformation vector

The physical interpretation of stiffness coefficient given by equation (3.11) is illustrated in Figure 3.2(a) in which,

- $A$ is the cross sectional area of the beam member,
- $L$ is length of the beam member, and
- $R_t$ is tangent flexural stiffness of the beam members.
This tangent stiffness, $R_t$ will vary along the length of the beam, but for the problem considered in the present analysis, $R_t$ is taken as the average value along the length of the beam and the equation (3.11) is used for the force-deflection relationship for the beam.
3.4 VALIDATION OF FINITE ELEMENT FORMULATION

Validation of finite element formulation has been carried out by Raghavendra (1996) for reinforced soil beds using the results of model experiments conducted by Singh (1988) in the laboratory. Model tests were conducted on a sand bed of dimension 390 X 390 X 210 cm using three horizontal layers of aluminium strips as reinforcement. The density of the sand was kept at 17 KN/m³ throughout the experiment. The uniformity coefficient for the sand is 3.54 and coefficient of curvature is 0.97.

Model footing dimension are 15.24 X 91.4 cm. The load-displacement curves obtained in this experiment is compared with the load-displacement curves obtained from NLSRIP for validation using the same soil and reinforcement properties as used in the model experiments conducted by Singh (1988). Hyperbolic constants of soil are obtained from the laboratory results furnished by Raghavendra (1996) through laboratory tests. Properties of soil are listed in Table 1. Reinforced properties are presented in Table 2. The load settlement curves of unreinforced and reinforced soil beds both from finite element analysis and the tests are presented in Figure 3.3

<table>
<thead>
<tr>
<th>Table 3.1 Soil Properties for Validation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit weight, $\gamma$</td>
</tr>
<tr>
<td>Cohesion, $c$</td>
</tr>
<tr>
<td>Angle of internal friction, $\phi$</td>
</tr>
<tr>
<td>Modulus number, $K$</td>
</tr>
<tr>
<td>Exponent, $n$</td>
</tr>
<tr>
<td>Bulk modulus number $K_b$</td>
</tr>
<tr>
<td>Exponent, $m$</td>
</tr>
<tr>
<td>Failure ratio, $R_f$</td>
</tr>
</tbody>
</table>
Table 3.2 Reinforcement Properties for validation

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness, $t$</td>
<td>0.054 cm</td>
</tr>
<tr>
<td>Length, $L$</td>
<td>45.6 cm</td>
</tr>
<tr>
<td>Area of cross section, $A$</td>
<td>0.054 cm$^2$/cm</td>
</tr>
<tr>
<td>Young's Modulus, $E$</td>
<td>$1.5 \times 10^6$ kg/cm$^2$</td>
</tr>
<tr>
<td>Yield stress, $f_y$</td>
<td>1056 kg/cm$^2$</td>
</tr>
</tbody>
</table>

After Raghavendra (1996)

Figure 3.3 Load-Displacement Responses: Finite Element And Experimental Analysis.
It was observed during the analysis that settlement of nodes representing the footings is almost equal. This testifies the modelling of rigid footing by assigning very high modulus for the elements representing the rigid footing. It is clear from the Figure 3.3 at the load-displacement curves obtained from finite element analysis and model tests compared very well up to the footing settlement of about 10% of footing width. Beyond this settlement experimental curve is steeper in case of unreinforced soil bed. Theoretical predictions for reinforced soil bed also agree quite reasonably as can be seen in figure.

3.5 CONCLUDING REMARKS

The method employs a general-purpose program. The program is referred as Non Linear Soil Reinforcement Interaction Program (NLSRIP, Raghavendra (1996)). The program is development of an existing Non Linear Soil Structure Interaction Program (NLSSIP), coded by Byrne and Duncan (1980). Program uses non-linear confining stress dependent hyperbolic relationship for the soils. The soil matrix is represented by four nodded quadrilateral elements and reinforced by two dimensional beam / bar elements.

Simple confining stress dependent non-linear hyperbolic model can be used for analysing the behaviour of reinforced and unreinforced soil beds. This has been study the behaviour of soil bed reinforced with vertical/inclined strips of reinforcement at different embedment depth. The method is successful in giving complete information regarding deformations at any stage of loading in the entire system.