CHAPTER 6

INDOOR AIR POLLUTION MODELING USING A SPECIFIC DECAY RATE
6.1 Introduction

Most of us usually think of air pollution as being outdoors, but the air inside any building, house or office could also be polluted. The air inside an enclosed environment becomes stagnant and saturated with germs, chemicals, air pollutions, and other harmful particles which are inhaled when we breath. Indoor air pollution cause discomfort and most of the people feel better as soon as they eliminate the indoor source of the pollution. Some of those pollutants act as slow poison since the symptoms of diseases caused by them appeared after a long period, such as respiratory diseases, allergies or cancer (Ref. Mu et al. 2011).

The main sources that affect the indoor air quality include combustion sources such as oil, gas, coal, kerosene, wood, and tobacco products, building materials and furnishings as diverse as deteriorated, asbestos, wet or damp carpet, furniture made of certain pressed wood products, central heating, cooling systems, a humidification devices, radon, pesticides, and outdoor air pollution entering a closed building.

The relevance of any source relies on the intensity of the particular pollutant that it emits and risk associated with that particular pollutant. The characteristics associated with a source are quite important, e.g., how old the source is and whether it is properly maintained or not. For example, an improperly adjusted gas stove can emit significantly more carbon monoxide than the one that is properly adjusted. Keeping in view all the health hazards associated with indoor air pollution, it is important to see the level or concentration of the hazardous pollutants inside any building to work upon the strategies to improve the indoor air quality.

In literature, many models have been proposed which are derived from basic mass balance equation to model the indoor air quality (Ref. Traynor et al. 1982, Ott 1999, Klepeis et al. 1999). The simplest among all these is to use the Box model approach. This model treats the building as a single, well mixed box with sources and sinks for the pollutants in question. The simple Box model can be expanded according to the requirement, to include several boxes in
which every single box is characterized by uniform pollutant concentration. A simple, single Box model of a building is shown in Fig. 6.1.

The single Box model is defined with various parameters and assumptions in the literature. In this type of model, the most simplified assumptions can be uniform air exchange rate throughout the duration of the study, zero initial concentration or constant decay rate etc.

**Figure 6.1**

*Box model for indoor air pollution*

In this chapter, the single Box model is modified with a particular monotone decay rate function. The modified Box model is then checked for its prediction efficiency by considering a real life data related to indoor air pollution from cookstove emissions in developing countries (Johnson et al. 2011).

This chapter is organized as follows: Section 6.2 includes the description of the indoor air pollution models under study. In Section 6.3, details of the real life data are provided and Section 6.4 deals with the illustration of the working of modified indoor air pollution model derived by using monotone decay rate and comparison of its prediction efficiency with the existing model based on constant decay rate assumption.
6.2 Modified Box model

First we consider the existing single Box model and its steady state behaviors under certain assumptions.

6.2.1 A single Box model

There are many sources of pollution which are characterized with various emission rates inside any building. In addition, ambient air entering the building may bring in new sources of air pollution which adds to whatever is generated inside. These pollutants may be removed from the building by infiltration or ventilation or they may be non-conservative and decay with time. If there is mechanical air cleaning system, some pollutants may be removed as indoor air is passed through the cleaning system and returned. To keep the model simple, such mechanical filtrations are ignored.

Assuming well mixed conditions, basic mass balance equation for pollution inside building which helps to derive the single Box model can be written as

\[
V \frac{dC(t)}{dt} = S + C_a I V - C(t) I V - K C(t) V, \quad (6.1)
\]

where

\( V \) = Volume of the conditioned space in the building \((m^3/\text{air change})\),

\( I \) = air exchange rate \((\text{ach})\),

\( S \) = Pollutant source strength \((\text{mg/hr})\),

\( C(t) \) = indoor concentration of a pollutant at time \( t \) \((\text{mg/m}^3)\),

\( C_a \) = ambient concentration \((\text{mg/m}^3)\),

\( K \) = pollutant decay rate or reactivity \((1/\text{hr})\).

Here it is assumed that \( I, C_a \) and \( K \) are constants. If \( \frac{dC(t)}{dt} = 0 \), the steady state solution for Eqn. (6.1) can be written as

\[
C(\infty) = \frac{S + C_a I}{I + K}. \quad (6.2)
\]

A general solution for \( C(t) \) can be derived from differential Eqn. (6.1) as explained in the following steps.
Indoor Air Pollution Modeling using a specific decay rate function

\[
\frac{dC(t)}{dt} = \frac{S}{V} + C_a.I - C(t).I - K.C(t) \\
\Rightarrow \frac{dC(t)}{dt} + (I + K).C(t) = (\frac{S}{V} + C_a.I).
\]

(6.3)

Let \( \frac{S}{V} + C_a.I = A \) for the sake of simplicity. The integrating factor (I.F.) of this differential equation is

\[
(I.F.) = e^{(I+K)t}.
\]

Now the solution can be found by solving

\[
C(t).e^{(I+K)t} = \int A. e^{(I+K)t} + C_1,
\]

where \( C_1 \) is the constant of integration.

At boundary condition \( t = 0, \)

\[
C_1 = C(0) - \frac{A}{(I+K)}.
\]

Following these steps the general solution for the Eqn. (6.3) is

\[
C(t) = \frac{I + C_a.I}{(I+K)} \left( 1 - e^{-(I+K)t} \right) + C(0)e^{-(I+K)t},
\]

(6.5)

where \( C(0) \) is the initial concentration inside the building.

**Remark:** For simplification, the initial condition \( C_a = 0 \) and \( K = constant(zero) \) incorporated in the above indoor air quality model gives indoor concentration level at time \( t \) as

\[
C(t) = \frac{S}{V(I+K)} \left( 1 - e^{-(I+K)t} \right) + C(0)e^{-(I+K)t}.
\]

(6.6)

### 6.2.2 Proposed Box model

In many practical situations the simplifying conditions \( C_a = 0 \) and \( K = constant(zero) \) do not hold. For example, pollution concentration may increase with time due to the poor sources of cross ventilation or one may also consider a case when pollution concentration decrease due to the introduction of fresh air or any other mechanical air cleaning system.
Keeping in view these situations, a monotone decay rate function \( K = K(t) \), is chosen as
\[
K(t) = 1 \pm \theta (1 - e^{-t}); \quad 0 < \theta < 1. \quad (6.7)
\]
We take \(+\) if the decay rate is increasing (decreasing) with time. With this form of \( K(t) \) and taking \( l \) and \( C_a \) as fixed constants, the model (6.1) is modified.

The general solution for the specific form of \( K(t) \) given in Eqn (6.7) can be evaluated using the following steps. Eqn. (6.1) with the \( K(t) \) can be written as
\[
V \frac{dC(t)}{dt} = S + C_a IV - C(t) IV - [1 \pm \theta (1 - e^{-t})] VC(t).
\]
This differential equation can be written as
\[
\frac{dC(t)}{dt} + \left[ l + \left( 1 \pm \theta (1 - e^{-t}) \right) \right] C(t) = \frac{S}{v} + C_a l.
\]
Let \( \frac{S}{v} + C_a l = A \) for the sake of simplicity. The integrating factor (I.F.) of this differential equation is
\[
(I. F.) = e^{(l+1)t \pm \theta (t + e^{-t})}.
\]
Now the solution can be found by solving
\[
C(t) e^{(l+1)t \pm \theta (t + e^{-t})} = \int A e^{(l+1)t \pm \theta (t + e^{-t})} dt + C_1.
\]
If the initial condition \( t = 0 \) is posed on the Eqn. (6.10), then \( C_1 \) will be
\[
C_1 = C(0). e^{\pm \theta} - \frac{A e^{\pm \theta}}{(l+1)}.
\]
Moving on these steps, the general solution with specific decay rate function \( K(t) \) is given by
\[
C(t) = \left( \frac{S}{v} + C_a l \right) \left[ \frac{1}{(l+1) \pm \theta (1 - e^{-t})} - e^{-[(l+1) \pm \theta (1 - (t + e^{-t})]} \right] + C(0). e^{-[(l+1) \pm \theta (1 - (t + e^{-t})]}.
\]
(6.11)

The accuracy of the proposed modified model in the prediction of \( C(t) \) has been compared with the Monte Carlo single Box model proposed by Johnson et al. (2011) by using the real life data reported by these authors. A brief description of Monte Carlo Box model proposed by Johnson et al. (2011) and the real life data represented by them are given in the following section.
6.2.3 A Monte Carlo single Box model

The Monte Carlo single box model predicts the concentration of pollution based on the stove emission and kitchen characteristics in the room. It is assumed that room is well mixed and emission from the given source is constant. Here it is assumed that removal of the pollutant is dominated by ventilation due to the instantaneous mixing with zero backflow to the room.

The mathematical model is

$$C(t) = \frac{G}{\alpha V} (1 - e^{-\alpha t}) + C_0 e^{-\alpha t},$$  \hspace{1cm} (6.12)

where

- $C(t)$ = concentration at time $t$ (mg m$^{-3}$);
- $G$ = emission rate (mg min$^{-1}$);
- $\alpha$ = nominal air exchange rate (min$^{-1}$) (assumed constant);
- $V$ = the kitchen volume (m$^3$) (assumed constant);
- $t$ = time (min);
- $C_0$ = concentration from preceding time unit (mg m$^{-3}$) and
- $F$ = fraction of emission in the kitchen environment.

Emission rate in Eqn (6.12) and amount of energy delivered for cooking are the function of several other factors. The emission rate $G$ will be calculated as

$$G = \frac{E_p}{E_0} p,$$  \hspace{1cm} (6.13)

where

- $E_p$: fuel based emission factor (mg pollutant per kg fuel),
- $E_0$: energy density of the fuel (micro joule(MJ) kg$^{-1}$) and
- $P$: stove power (MJ per minute).

Cooking energy required for the purpose was split up into three equal parts with the time duration $T_c$ given by

$$T_c = \frac{E_{DC}}{P_\eta},$$

where $E_{DC}$ is the total daily cooking energy required (MJ) and $\eta$ is stove thermal efficiency(\%).
Johnson et al. (2011) have used the model (6.12) to predict the indoor air pollution from a cookstove in the developing countries. While modeling the pollutant concentration it was assumed that $f$ is set equal to one and initial kitchen concentration $C_0$ is set equal to zero.

**Remark:** It is interesting to note that model (6.12) will become the special case of model (6.6) with $K=0$, if we put $\alpha = I$ and $G = S$ (notation wise).

To maintain the uniformity in the notations of parameters with Indoor Monte Carlo single Box model, the Box model represented by Eqn. (6.6) can be written as

$$C(t) = \frac{G}{V(a+K)} \left(1 - e^{-(a+K)t}\right) + C(0)e^{-(a+K)t}. \quad (6.14)$$

Similarly, the proposed Box model Eqn. (6.11), in terms of notations of Monte Carlo Box model with $C_a = 0$, can be written as

$$C(t) = \left(\frac{G}{V}\right)\left[\frac{1}{(a+1)\pm(1-e^{-t})} - e^{-[(a+1)\pm(1-(t+e^{-t}))]}\right] + C(0)e^{-[(a+1)\pm(1-(t+e^{-t}))]} \quad (6.15)$$

After maintaining the uniformity in notations, we now compare the Monte Carlo Box model 6.12 due to Johnson et al. (2011) with the proposed model (6.15). Comparison has been made in terms of prediction of $C(t)$ by both the models using the real life data of eleven studies reported by Johnson et al. (2011). These real life data are reproduced in the following section.

### 6.2.4 Real life Data

Johnson et al. (2011) have developed the Monte Carlo Box model to predict two pollutants namely Particulate Matter (PM$_{2.5}$) and Carbon Monoxide (CO) emitted by four different categories of Chulhas. They have used real life data of 11 studies to develop the model. These data are presented in Table 6.1. In this table, the predicted emission concentration given by Monte Carlo Box model and the actual observed emission concentration based on the 11 studies are provided. Hence to compare the efficiency of the proposed indoor model (6.15), one can consider the emission concentration of any pollutant from any type of Chulha. It is worth noting here that Johnson et al. (2011) predicted $C(t)$ from model (6.12) in which $K = 0$. 

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## Table 6.1
Data for the different parameters used in Monte Carlo single Box model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Units</th>
<th>Mean</th>
<th>Min</th>
<th>Max</th>
<th>COV</th>
<th>Basis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air exchange rate (a)</td>
<td>Hr⁻¹</td>
<td>25</td>
<td>3</td>
<td>60</td>
<td>0.6</td>
<td>(ARC, 2006; Bhangar, 2006; Brant et al., 2010)</td>
</tr>
<tr>
<td>Kitchen volume (V)</td>
<td>m³</td>
<td>30</td>
<td>3</td>
<td>100</td>
<td>0.5</td>
<td>(Bhangar, 2006; Brant et al., 2010; Brant et al., 2009; Saksena et al., 2003)</td>
</tr>
<tr>
<td>Fraction of emissions entering room (f)</td>
<td>Unit less</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Cooking energy required</td>
<td>MJ-delivered</td>
<td>11</td>
<td>3</td>
<td>30</td>
<td>0.5</td>
<td>(Habib et al., 2004)</td>
</tr>
<tr>
<td><strong>Stove Power</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Traditional Chulha</td>
<td>KJ s⁻¹</td>
<td>4.9</td>
<td>2</td>
<td>15</td>
<td>0.7</td>
<td>(Brant et al., 2010)</td>
</tr>
<tr>
<td>G3300 In-home CCT</td>
<td>KJ s⁻¹</td>
<td>3.8</td>
<td>2</td>
<td>10</td>
<td>0.3</td>
<td>(Brant et al., 2010)</td>
</tr>
<tr>
<td>G3300 Lab WBT</td>
<td>KJ s⁻¹</td>
<td>3.1</td>
<td>2</td>
<td>10</td>
<td>0.1</td>
<td>(EECL, 2009)</td>
</tr>
<tr>
<td>LPG</td>
<td>KJ s⁻¹</td>
<td>1.6</td>
<td>0.5</td>
<td>5</td>
<td>0.1</td>
<td>(Smith et al., 2000)</td>
</tr>
<tr>
<td><strong>Thermal Efficiency</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Traditional Chulha</td>
<td>%</td>
<td>14</td>
<td>5</td>
<td>35</td>
<td>0.1</td>
<td>(Brant et al., 2010)</td>
</tr>
<tr>
<td>G3300 In-home CCT</td>
<td>%</td>
<td>22</td>
<td>10</td>
<td>45</td>
<td>0.3</td>
<td>(Brant et al., 2010)</td>
</tr>
<tr>
<td>G3300 Lab WBT</td>
<td>%</td>
<td>29</td>
<td>20</td>
<td>45</td>
<td>0.1</td>
<td>(EECL, 2009)</td>
</tr>
<tr>
<td>LPG</td>
<td>%</td>
<td>54</td>
<td>40</td>
<td>60</td>
<td>0.1</td>
<td>(Smith et al., 2000)</td>
</tr>
<tr>
<td><strong>Emission factors</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Traditional Chulha</td>
<td>PM₂ (g kg⁻¹)</td>
<td>5.2</td>
<td>1</td>
<td>10</td>
<td>0.2</td>
<td>(Brant et al., 2010)</td>
</tr>
<tr>
<td></td>
<td>CO (g kg⁻¹)</td>
<td>64</td>
<td>10</td>
<td>100</td>
<td>0.2</td>
<td>(Brant et al., 2010)</td>
</tr>
<tr>
<td>G3300 In-home CCT</td>
<td>PM₂ (g kg⁻¹)</td>
<td>5.0</td>
<td>0.2</td>
<td>10</td>
<td>0.2</td>
<td>(Brant et al., 2010)</td>
</tr>
<tr>
<td></td>
<td>CO (g kg⁻¹)</td>
<td>47</td>
<td>10</td>
<td>90</td>
<td>0.2</td>
<td>(Brant et al., 2010)</td>
</tr>
<tr>
<td>G3300 Lab WBT</td>
<td>PM₂ (g kg⁻¹)</td>
<td>1.6</td>
<td>0.5</td>
<td>5</td>
<td>0.5</td>
<td>(EECL, 2009)</td>
</tr>
<tr>
<td></td>
<td>CO (g kg⁻¹)</td>
<td>34</td>
<td>5</td>
<td>80</td>
<td>0.3</td>
<td>(EECL, 2009)</td>
</tr>
<tr>
<td>LPG</td>
<td>PM₂ (g kg⁻¹)</td>
<td>0.36</td>
<td>0.05</td>
<td>1</td>
<td>0.4</td>
<td>(Habib et al., 2008; Smith et al., 2000)</td>
</tr>
<tr>
<td></td>
<td>CO (g kg⁻¹)</td>
<td>15</td>
<td>2</td>
<td>40</td>
<td>0.2</td>
<td>(Smith et al., 2000)</td>
</tr>
</tbody>
</table>

**Notes:**

- The coefficient of variation on a home-by-home basis was not available for this distribution. 0.5 is used as a reasonable estimate for this parameter.
- PM emission factors from these studies were measured as total suspended particulates. Size distributions for combustion particles are generally small with almost all mass from particles less than 2.5 mm in diameter, so we report these as PM₂ for simple comparison with AQGs.

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Hence, comparison between Monte Carlo Box model which assumes $K = 0$ and the proposed model with $K = K(t)$ forms motivation of this study.

6.3 Illustration

In the illustration, the concentration of PM$_{2.5}$ for the emission of Traditional Chulha (wood), is considered.

The comparison between the Monte Carlo Box model and proposed model involves the three cases.

6.3.1 Comparison based on averages

Johnson et al. (2011) while proposing Monte Carlo Box model have reported estimated average values of the parameters involved in their model by using the data of 11 studies. These estimated values, reported in Table 6.1, are

\[ E_F (\text{average value}) = 5.2, \quad P(\text{average value}) = 4.9, \quad E_D (\text{average value}) = 18, \]
\[ a(\text{average value}) = 25 \quad \text{and} \quad V(\text{average value}) = 30. \]

With these parameters values, the estimated value of $C(t)$ at time $t = T_c = \frac{E_D C(\text{average value})}{P(\eta)} = .262$, where $E_D C(\text{average value}) = 11$ and $P(\eta)(\text{average value}) = 14$, obtained from Monte Carlo Box model (Johnson et al. (2011)) is $1975 \mu g/m^3$ whereas the actual value was $1313 \mu g/m^3$.

For proposed model, herein the values of $C(t)$ at time $t = T_c = .262$ and same values of the other parameters as used in Monte Carlo Box model are reported in the following Table 6.2 with respect to both forms of $K(t)$ with $\theta = 0.1(0.1)0.9$, and $p = 0$. Here $p$ is taken as zero since Monte Carlo Box model is based on this assumption that $E_F$ and $P$ are independent.
Indoor Air Pollution Modeling using a specific decay rate function

Table 6.2

Values of \( C(t) \) obtained from the proposed model for \( \theta = 0.1(0.1)0.9, \rho = 0 \)

\( (E_F \text{ and } P \text{ are independent}) \)

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( K(t) = 1 + \theta(1 - e^{-\rho}) )</th>
<th>( K(t) = 1 - \theta(1 - e^{-\rho}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1443.165</td>
<td>1445.726</td>
</tr>
<tr>
<td>0.2</td>
<td>1441.889</td>
<td>1447.099</td>
</tr>
<tr>
<td>0.3</td>
<td>1440.614</td>
<td>1448.295</td>
</tr>
<tr>
<td>0.4</td>
<td>1439.342</td>
<td>1449.583</td>
</tr>
<tr>
<td>0.5</td>
<td>1438.072</td>
<td>1450.873</td>
</tr>
<tr>
<td>0.6</td>
<td>1436.805</td>
<td>1452.166</td>
</tr>
<tr>
<td>0.7</td>
<td>1435.539</td>
<td>1453.461</td>
</tr>
<tr>
<td>0.8</td>
<td>1434.276</td>
<td>1454.758</td>
</tr>
<tr>
<td>0.9</td>
<td>1433.015</td>
<td>1456.058</td>
</tr>
</tbody>
</table>

One can see from the Table 6.2 that for both forms of \( K(t) \) with \( \theta = 0.1(0.1)0.9 \), the estimated values of \( C(t) \) computed from the proposed model are very close to the actual value (1313 mg/m\(^3\)) than its estimated value (1975 mg/m\(^3\)) given by Monte Carlo Box model.

In some practical situations the data on random quantities involved in the prediction model may not be available instantly but their averages and standard deviations may be available from past studies. With these estimated values of parameters of the underlying random variables, i.e., \( E_F \) and \( P \), one can estimate \( C(t) \) of PM\(_{2.5}\) and CO using simulation technique. This simulation process is explained below for estimating \( C(t) \) of PM\(_{2.5}\) using proposed model in following two situations.

6.3.2 First Situation: only \( E_F \) is Random

In this case, we assume that \( E_F \) is random variable and \( P \) takes values in some range. Lognormal distribution of the pollutants is a common assumption for environmental data sets (Ref. Johnson et al. (2011)). Thus, it can be conveniently assumed that random variable \( E_F \) (say \( X \)), involved in the computation of \( G = \frac{E_F}{E_D} P \), follows Lognormal distribution and that \( P \) takes values in this interval \([2, 15]\) (see Table 6.1) in which minimum value of \( P \) is 2 and average value is 4.9. To compute \( C(t) \), one can generate data on \( E_F \) which follow Lognormal
distribution with approximate mean and variance as 5.2 and 1.0816 respectively (i.e. \( \mu_s \approx 5.2 \) and \( \sigma_s^2 \approx 1.0816 \)).

The simulated values of \( C(t) \) corresponding to randomly generated values of \( X \) for various fixed values of \( P \) with \( \theta = 0.1(0.1)0.9 \) and \( t = T_c = .262 \) are provided in Table 6.3.

**Table 6.3**

<table>
<thead>
<tr>
<th>( P )</th>
<th>( K(t) = 1 + \theta(1 - e^{-t}) )</th>
<th>( K(t) = 1 - \theta(1 - e^{-t}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0=0.1</td>
<td>1086.36 1272.4 1373.27 1408.08</td>
<td>1105.75 1225.8 1366.7 1448.49</td>
</tr>
<tr>
<td>0=0.2</td>
<td>1085.4 1271.28 1372.06 1406.83</td>
<td>1106.73 1226.89 1367.91 1449.78</td>
</tr>
<tr>
<td>0=0.3</td>
<td>1084.44 1270.15 1370.85 1405.99</td>
<td>1107.71 1227.98 1369.13 1451.07</td>
</tr>
<tr>
<td>0=0.4</td>
<td>1083.48 1269.03 1369.64 1404.34</td>
<td>1108.7 1229.07 1370.35 1452.36</td>
</tr>
<tr>
<td>0=0.5</td>
<td>1082.53 1267.91 1368.43 1403.11</td>
<td>1109.69 1230.17 1371.57 1453.65</td>
</tr>
<tr>
<td>0=0.6</td>
<td>1081.57 1266.8 1367.22 1401.87</td>
<td>1110.67 1231.26 1372.79 1454.94</td>
</tr>
<tr>
<td>0=0.7</td>
<td>1080.62 1265.68 1366.02 1400.63</td>
<td>1111.66 1232.36 1374.01 1456.24</td>
</tr>
<tr>
<td>0=0.8</td>
<td>1079.67 1264.57 1364.81 1399.4</td>
<td>1112.66 1233.46 1375.24 1457.54</td>
</tr>
<tr>
<td>0=0.9</td>
<td>1078.72 1263.45 1363.61 1398.17</td>
<td>1113.65 1234.56 1376.47 1458.84</td>
</tr>
</tbody>
</table>

It is clear from Table 6.3 that values of \( C(t) \) are quite close to the actual value \( C(t) = 1313 \text{ mgm}^{-3} \) if the values of \( P \) are in range specified by Johnson et al. (2011), i.e., minimum value of \( P \) is 2 and average value is 4.9.

### 6.3.3 Second Situation: Both \( EF \) and \( P \) are Random

For notational convenience we take \( EF = X \) and \( P = Y \) and assume that data on \( X \) and \( Y \), required to compute \( G = \frac{EF}{Ep} \rho \), are not available and that both are random. Theses random variables can be independent or dependent. To account for the dependency, we assume that they have bivariate Lognormal distribution with joint probability density function

\[
g(x,y) = \frac{1}{2\pi \sigma_x \sigma_y \sigma \sqrt{1-\rho^2}} \exp \left[ -\frac{1}{2(1-\rho^2)} \left( \frac{\log x - \mu_x}{\sigma_x} \right)^2 - 2\rho \left( \frac{\log x - \mu_x}{\sigma_x} \right) \left( \frac{\log y - \mu_y}{\sigma_y} \right) + \left( \frac{\log y - \mu_y}{\sigma_y} \right)^2 \right].
\]

(6.16)
Indoor Air Pollution Modeling using a specific decay rate function

where \( \rho \) is the correlation coefficient.

Now

\[
g(x,y) = \frac{1}{2\pi \sigma_x \sigma_y \sqrt{1 - \rho^2}} \exp \left[ -\frac{1}{2(1-\rho^2)} \left( \frac{\log x - \mu_x}{\sigma_x} \right)^2 + \frac{2\rho}{1-\rho^2} \left( \frac{\log x - \mu_x}{\sigma_x} \right) \left( \frac{\log y - \mu_y}{\sigma_y} \right) + \frac{1}{1-\rho^2} \left( \frac{\log y - \mu_y}{\sigma_y} \right)^2 \right]
\]

\[
= \frac{1}{2\pi \sigma_x \sigma_y \sqrt{1 - \rho^2}} \exp \left[ -\frac{1}{2\sigma_x^2} (\log x - \mu_x)^2 \right] \cdot \frac{1}{\sqrt{2\pi} \sigma_y \sqrt{1 - \rho^2}} \exp \left[ -\frac{1}{2(1-\rho^2)} \left( \frac{\log y - \mu_y}{\sigma_y} \right)^2 \right]
\]

\[
= r(x) h(y|x),
\]

where

\[
r(x) = \frac{1}{\sqrt{2\pi} \sigma_x} \exp \left[ -\frac{1}{2\sigma_x^2} (\log x - \mu_x)^2 \right]
\]

\[
h(y|x) = \frac{1}{\sqrt{2\pi} \sigma_y \sqrt{1 - \rho^2}} \exp \left[ -\frac{1}{2(1-\rho^2)} \left( \frac{\log y - \mu_y - \frac{\sigma_x}{\sigma_y} \rho (\log x - \mu_x)}{\sigma_y} \right)^2 \right].
\]

By definition of Lognormal distribution, we observe that \( r(x) \) is Lognormal with mean \( \mu_x \) and variance \( \sigma_x^2 \). Similarly the conditional density \( h(y|x) \) is Lognormal with mean

\[
(\mu_y - \frac{\sigma_y}{\sigma_x} \rho (\log(x) - \mu_x)) \]

and variance \((1 - \rho^2)\sigma_y^2\). Here \( \rho \) is correlation coefficient between \( X \) and \( Y \) and \(|\rho| \leq 1\).
In order to generate the data on \( G \), we generated the value of \( X \) with the help of Lognormal distribution with p. d. f. (6.18) using the estimated values 5.2 and 1.0816 of \( \mu_x \) and \( \sigma_x^2 \) respectively as reported by Johnson et al. (2011) (see Table 6.1).

Now, corresponding to each value of \( X \) so generated, the value of \( Y \) is generated from Lognormal distribution using p. d. f. (6.19) with \( \mu_y = 7.242, \sigma_y^2 = 11.294, \mu_x = 5.2 \) and \( \sigma_x^2 = 1.0816 \) by taking any value of \( \rho \in [0,1) \).

The other parameters involved in \( C(t) \), like \( \sigma \) and \( V \) are assumed to be constants as reported by Johnson et al. (2011). The simulated values of \( C(t) \) can be generated separately for \( K(t) = 1 + \theta(1 - e^{-t}); \ 0 < \theta < 1 \) and \( K(t) = 1 - \theta(1 - e^{-t}); \ 0 < \theta < 1 \) for any value of \( \theta \) in the interval \([0,1)\). Some of the simulated values of \( C(t) \) with \( K(t) = 1 + \theta(1 - e^{-t}) \) are presented in Table 6.4 with \( \theta = 0.2 \). In this Table, the simulated values of \( P = Y \) are also reported to keep track whether they are above 2 (minimum value reported by Johnson et al. (2011)) or not.

### Table 6.4

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( t=1 )</th>
<th>( t=1.5 )</th>
<th>( t=2 )</th>
<th>( t=2.5 )</th>
<th>( t=3 )</th>
<th>( t=3.5 )</th>
<th>( t=4 )</th>
<th>( t=4.5 )</th>
<th>( t=5 )</th>
</tr>
</thead>
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<td>0</td>
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<td>699.84</td>
<td>699.55</td>
<td>699.38</td>
<td>699.28</td>
<td>699.21</td>
<td>699.17</td>
<td>699.15</td>
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<tr>
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<td>1193.20</td>
<td>1192.72</td>
<td>1192.42</td>
<td>1192.24</td>
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<td>1371.68</td>
<td>1370.76</td>
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<td>1369.54</td>
<td>1369.46</td>
<td>1369.42</td>
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<td></td>
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<tr>
<td>1.77</td>
<td>659.11</td>
<td>658.38</td>
<td>657.94</td>
<td>657.67</td>
<td>657.51</td>
<td>657.41</td>
<td>657.35</td>
<td>657.31</td>
<td>657.29</td>
</tr>
<tr>
<td>3.17</td>
<td>1171.19</td>
<td>1169.89</td>
<td>1169.11</td>
<td>1168.63</td>
<td>1168.34</td>
<td>1168.17</td>
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</tr>
<tr>
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<td>1285.29</td>
<td>1285.10</td>
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<tr>
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<td>643.10</td>
<td>644.38</td>
<td>643.95</td>
<td>643.69</td>
<td>643.53</td>
<td>643.43</td>
<td>643.37</td>
<td>643.34</td>
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<td>3.29</td>
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<td>1158.13</td>
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<td>1156.88</td>
<td>1156.59</td>
<td>1156.42</td>
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<td>1346.87</td>
<td>1346.83</td>
</tr>
</tbody>
</table>

It can be observed from Table 6.4 that \( C(t) \) is decreasing with the time for all the values of correlation coefficient \( \rho \). One can see that values of \( C(t) \) predicted through proposed model
Indoor Air Pollution Modeling using a specific decay rate function

are quite close to its actual value \( C(t) = 1313 \text{mgm}^{-3} \) when \( P \geq 2 \). The departure from actual value of \( C(t) \) is observed when \( P < 2 \) and this departure is justified since the studies of Johnson et al. (2011) are based on the fact that \( 2 \text{KJs}^{-1} \leq P \leq 15 \text{KJs}^{-1} \) with mean value of \( P = 4.9 \text{KJs}^{-1} \).

Some of the simulated values of \( C(t) \) with \( \kappa(t) = 1 - \theta(1 - e^{-\theta}) \) are presented in Table 6.5 with \( \theta = 0.2 \).

### Table 6.5

<table>
<thead>
<tr>
<th>( P )</th>
<th>( t=1 )</th>
<th>( t=1.5 )</th>
<th>( t=2 )</th>
<th>( t=2.5 )</th>
<th>( t=3 )</th>
<th>( t=3.5 )</th>
<th>( t=4 )</th>
<th>( t=4.5 )</th>
<th>( t=5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p=0 )</td>
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<td>704.1</td>
<td>704.5</td>
<td>704.8</td>
<td>705.0</td>
<td>705.1</td>
<td>705.2</td>
<td>705.2</td>
<td>705.2</td>
</tr>
<tr>
<td>( p=2.91 )</td>
<td>1115.9</td>
<td>1117.1</td>
<td>1117.9</td>
<td>1118.4</td>
<td>1118.6</td>
<td>1118.8</td>
<td>1118.9</td>
<td>1119.0</td>
<td>1119.0</td>
</tr>
<tr>
<td>( p=3.82 )</td>
<td>1442.7</td>
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<td>1445.3</td>
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<td>1446.3</td>
<td>1446.5</td>
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<td>696.8</td>
<td>697.1</td>
<td>697.3</td>
<td>697.4</td>
<td>697.5</td>
<td>697.5</td>
<td>697.5</td>
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<tr>
<td>( p=2.82 )</td>
<td>1008.2</td>
<td>1009.3</td>
<td>1010.0</td>
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<td>1010.6</td>
<td>1010.8</td>
<td>1010.9</td>
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<tr>
<td>( p=3.86 )</td>
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<td>1365.0</td>
<td>1365.9</td>
<td>1366.5</td>
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<td>1367.3</td>
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<tr>
<td>( p=0.3 )</td>
<td>653.3</td>
<td>654.0</td>
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<td>654.9</td>
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<td>655.0</td>
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<tr>
<td>( p=2.81 )</td>
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<td>( p=3.39 )</td>
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<td>1306.8</td>
<td>1306.9</td>
<td>1306.9</td>
</tr>
</tbody>
</table>

It can be observed from Table 6.5 that \( C(t) \) is increasing with the time for all the values of correlation coefficient \( \rho \). The accuracy in prediction of \( C(t) \) increases when simulated values of \( P = \kappa \) are greater than its minimum value \( 2 \text{ KJs}^{-1} \). Departures in the predicted values of \( C(t) \) from the actual value through the proposed model are observed only in cases when simulated values of \( P = \kappa \) are less than its minimum value 2.

### 6.4 Conclusion

There are many air pollution models which are used to model the outdoor air pollution. In this chapter an indoor air quality model is considered. The simplest is to choose the Box model approach. Using various assumptions, model is set to predict the indoor air quality. Along
with the various parameters involved in this model, a monotone decay rate function \( K(t) = 1 \pm \theta(1 - e^{-t}) \) is used to modify it. We note that \( K(t) = 1 + \theta(1 - e^{-t}) \) corresponds to increasing failure rate of Makeham family of life distributions. The proposed single Box model is compared with the existing Monte Carlo single box model due to Johnson et al. (2011) and is found better to predict indoor air quality.