Chapter 6

SH-waves at a corrugated interface between a dry sandy half-space and an anisotropic elastic half-space

6.1 Introduction

Chakraborty and Chandra (1984) investigated a problem of reflection and refraction of a plane SH-wave at a plane interface between a dry sandy layer and a sedimentary rock (anisotropic of transversely isotropy type). They obtained the reflection and the transmission coefficients and depicted them graphically against the angle of the incidence. They have also derived the energy partition equation and shown the effect of the anisotropy and the sandiness on these coefficients. In this chapter, we have extended their problem at a corrugated interface. The reflection and transmission coefficients for the first and the second order approximation of the corrugation and the energy partition equation have been derived. The effects of the sandiness and the anisotropic characteristic of the half-spaces, the corrugation of the interface, the frequency and the angle of the incidence have been studied on these coefficients.

6.2 Formulation of the problem and its solution

The geometry of the problem is same as shown in Figure 2.0. Here, the half-space $H_1$ $[-\infty < Z \leq \zeta(x)]$ is now corresponds to a dry sandy half-space and the half-space $H_2$ $[\zeta(x) \leq Z < \infty]$ corresponds to an anisotropic elastic half-space. Weiskopff (1945) has shown that in an idealized soil, the resistance to shear is much

smaller than that in a solid because of the slipping of granules on each other and the resulting shearing deflection is thus much greater. For such materials, the relation $E/\mu = 2(1+\sigma)$ valid for a purely solid, where $E$ is Young’s modulus, $\mu$ is the modulus of rigidity and $\sigma$ is Poisson’s ratio, may be modified as follows

$$\frac{E}{\mu} = 2\eta(1+\sigma).$$

Here, $\eta > 1$ corresponds to the sandy materials and $\eta = 1$ corresponds to an elastic solid. Chakraborty and Chandra (1984) applied Weiskopff’s theory to investigate the problem of reflection and transmission of a plane $SH$-wave at the plane boundary between a dry sandy layer and an anisotropic elastic medium.

Neglecting the body forces, the equation of motion for the plane $SH$-wave propagating in a sandy elastic medium $H_1$ is given by (Chakraborty et al., 1982)

$$\frac{\mu_1}{\eta} \left[ \frac{\partial^2 V_1}{\partial x^2} + \frac{\partial^2 V_1}{\partial z^2} \right] = \rho_1 \frac{\partial^2 V_1}{\partial t^2}, \quad (6.1)$$

where $V_1$ is the $y$-component of the displacement vector, $\mu_1, \rho_1$ and $\eta$ are, respectively, the rigidity, the density and the sandiness of the medium. The shear wave velocity in the half-space $H_1$ is given by $\beta_1 = \sqrt{\frac{\mu_1 \eta}{\rho_1}}$. Using the method of separation of variables, the time harmonic solution of equation (6.1) for the plane $SH$-wave propagating in the positive direction of $x$-axis is given by

$$V_x = [A_0 e^{-sz} + B_0 e^{sz}] e^{i(\omega t - k_1 x)},$$

where $A_0$ and $B_0$ are constants, $\omega$ is the angular frequency, $k_1 = \frac{\omega \sin e}{\beta_1}$, is the horizontal component of the wavenumber (Gupta, 1965), where $e$ is the angle of incidence and

$$s = \sqrt{k_1^2 - \frac{\omega^2}{\beta_1^2}}. \quad (6.2)$$

The equation of motion for the plane $SH$-wave in a transversely isotropic elastic medium $H_2$, in the absence of body forces, may be written as

$$N \frac{\partial^2 V_2}{\partial x^2} + L \frac{\partial^2 V_2}{\partial z^2} = \rho_2 \frac{\partial^2 V_2}{\partial t^2}, \quad (6.3)$$

where $V_2$ is the $y$-component of the displacement vector, $N$, $L$ and $\rho_2$ are, respectively, the elastic constants and the mass density. The shear waves velocities in the half-space
$H_2$ along the $x$ and $z$-directions are given by $\beta_2 = \sqrt{N/\rho_2}$ and $\beta_2' = \sqrt{L/\rho_2}$ respectively.

The time harmonic solution of equation (6.3) for the plane $SH$-wave propagating in the positive direction of $x$-axis is given as

$$V_2 = [C_0 e^{-qz} + D_0 e^{qz}] e^{i(\omega t-k_2z)},$$

where $C_0$ and $D_0$ are constants, $k_2$ is the wavenumber, $k_2 = \frac{\omega \sin f}{\beta_2}$, $f$ is the angle of refraction and

$$q = \sqrt{\frac{N}{L} \left( \frac{k_2^2 - \omega^2}{\beta_2^2} \right)}.$$  \hspace{1cm} (6.4)

Let us assume that a ray of plane $SH$-wave of unit amplitude propagating through the upper half space $H_1$ be incident at the corrugated interface $z = \zeta$ making an angle $e$ with the $z-$ axis. The procedure for solving the present problem is same as adopted in the problems investigated in the previous chapters.

As in earlier problems, taking into account the effect of corrugation of the interface, the total displacement in the half-space $H_1$ will be the sum of the displacements due to the displacements caused by the incident, the regularly reflected and the irregularly reflected waves, is given as

$$V_1 = [e^{-s_2} + Be^{s_2} + \sum_{n=1}^{\infty} B_n e^{s_n z} e^{-m k^* x} + \sum_{n=1}^{\infty} B'_n e^{s'_n z} e^{m k^* x}] e^{i(\omega t - \frac{\pi s_2}{s_2'})},$$

where

$$s_n = \frac{i \omega \cos e_n}{\beta_1}, \hspace{1cm} s'_n = \frac{i \omega \cos e'_n}{\beta_1}.$$  \hspace{1cm} (6.5)

$e_n$ and $e'_n$ are the angles which a spectrum of $n$th order of irregularly reflected waves makes to the left side and to the right side of the regularly reflected $SH-$ wave respectively. The quantity $k^*$ is defined in earlier chapters and is connected to the wavelength of the corrugated interface.

Similarly, the total displacement $V_2$ in the half-space $H_2$ is the sum of the displacements due to regularly refracted and the irregularly refracted waves, given by

$$V_2 = [D e^{-qz} + \sum_{n=1}^{\infty} D_n e^{-q_n z} e^{-m k^n x} + \sum_{n=1}^{\infty} D'_n e^{-q'_n z} e^{m k^n x}] e^{i(\omega t - \frac{\pi q_n}{q_n'})},$$

where

$$q_n = i \sqrt{\frac{N}{L} \frac{\omega}{\beta_2} \cos f_n}, \hspace{1cm} q'_n = i \sqrt{\frac{N}{L} \frac{\omega}{\beta_2} \cos f'_n}.$$  \hspace{1cm} (6.6)

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\( f_n \) and \( f'_n \) are the angles which a spectrum of \( n^{th} \) order of irregularly refracted waves make to the left side and to the right side of the regularly refracted wave respectively.
The constants \( B, D, B_n, D_n, B'_n \) and \( D'_n \) can be determined by satisfying the following boundary conditions at the interface.

### 6.3 Boundary conditions

The boundary conditions to be satisfied at the corrugated interface \( z = \zeta(x) \) are the same as given in earlier chapters. They are written as

\[
[V_1]_{H_1} = [V_2]_{H_2} , \quad \frac{\mu_1}{\eta} \left[ \frac{\partial V_1}{\partial z} - \frac{\partial V_1}{\partial x} \zeta' \right] = \left[ L \frac{\partial V_2}{\partial z} - N \frac{\partial V_2}{\partial x} \zeta' \right].
\]  

(6.7)

As done in the previous problems, inserting the values of the displacements given above in these boundary conditions, we obtain

\[
e^{-\alpha \xi} + B e^{\alpha \xi} + \sum_{n=1}^{\infty} B_n e^{\alpha n \xi} e^{-mk'z} + \sum_{n=1}^{\infty} B'_n e^{\alpha n' \xi} e^{mk'z} = D e^{-\alpha \xi} + \sum_{n=1}^{\infty} D_n e^{\alpha n \xi} e^{-mk'z} + \sum_{n=1}^{\infty} D'_n e^{\alpha n' \xi} e^{mk'z},
\]  

(6.8)

and

\[
\frac{\mu_1}{\eta} \left[ (-s + \frac{i \omega \sin \mu}{\beta_1} \zeta') e^{-\alpha \xi} + B(s + \frac{i \omega \sin \mu}{\beta_1} \zeta') e^{\alpha \xi} + \sum_{n=1}^{\infty} B_n e^{-mk'z} \left( s_n + i \left( \frac{\omega \sin \mu}{\beta_1} + nk' \right) \zeta' \right) e^{\alpha n \xi} \right.
\]

\[
+ \sum_{n=1}^{\infty} B'_n e^{mk'z} \left( s'_n + i \left( \frac{\omega \sin \mu}{\beta_1} - nk' \right) \zeta' \right) e^{\alpha n' \xi}\n\]

\[
= L[D(-q + \frac{i \omega \sin \mu}{L \beta_2} \zeta') e^{-\alpha \xi} + \sum_{n=1}^{\infty} D_n e^{-mk'z} \left( -q_n + \frac{i N \omega \sin \mu}{L \beta_2} \left( \frac{\omega \sin \mu}{\beta_2} + nk' \right) \zeta' \right) e^{-\alpha n \xi} \right]
\]

\[
+ \sum_{n=1}^{\infty} D'_n e^{mk'z} \left( -q'_n + \frac{i N \omega \sin \mu}{L \beta_2} \left( -nk' \right) \zeta' \right) e^{-\alpha n' \xi}. \quad (6.9)
\]

Equations (6.8) and (6.9) help us to determine the reflection and transmission coefficients to any order of approximation of the corrugated interface.

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6.4 Solution of the first order approximation

As done before, the solution for the first order approximation of the corrugation for the coefficients $B$ and $D$ can be obtained as

$$1 + B = D,$$

(6.10)

$$s \mu_1 (1 - B) = q \eta LD.$$  

(6.11)

These formulae provide the formulae for the reflection coefficient $B$ and the transmission coefficient $D$ at a plane interface between a sandy half-space and an anisotropic elastic half-space. Solving equations (6.10) and (6.11), we obtain

$$B = \frac{\mu_1 s - L \eta q}{\mu_1 s + L \eta q}, \quad D = \frac{2 \mu_1 s}{\mu_1 s + L \eta q}. $$

(6.12)

The solutions of the first order approximation of the corrugation for the coefficients $B_n$ and $D_n$ are given by

$$B_n - D_n = [(1 - B)s - qD]\zeta_n,$$

(6.13)

$$\mu_1 s_n B_n + L \eta q_n D_n = [-\mu_1 (s^2 + \frac{n k^* \omega \sin e}{\beta_1})(1 + B) + L \eta (q^2 + \frac{n k^* N \omega \sin f}{L \beta_2})]D \zeta_n.$$  

(6.14)

Similarly, the solutions for the first order approximation for the coefficients $B'_n$ and $D'_n$ are as follows

$$B'_n - D'_n = [(1 - B)s - qD]\zeta_n,$$

(6.15)

$$\mu_1 s'_n B'_n + L \eta q'_n D'_n = [-\mu_1 (s^2 - \frac{n k^* \omega \sin e}{\beta_1})(1 + B) + L \eta (q^2 - \frac{n k^* N \omega \sin f}{L \beta_2})]D \zeta_n.$$  

(6.16)

The equations (6.13)-(6.16) provide the formulae of the coefficients $B_n$, $D_n$, $B'_n$ and $D'_n$ as follows

$$B_n = \frac{\Delta B_n}{\Delta n}, \quad D_n = \frac{\Delta D_n}{\Delta n}, \quad B'_n = \frac{\Delta B'_n}{\Delta n}, \quad D'_n = \frac{\Delta D'_n}{\Delta n}, $$

(6.17)

where

$$\Delta B_n = [-\mu_1 (1 + B)(s^2 + \frac{n k^* \omega \sin e}{\beta_1}) + (1 - B)L \eta s q_n + \eta LD (q^2 - q_n + \frac{n k^* N \omega \sin f}{L \beta_2})]\zeta_n,$$

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\[ \Delta_{D_n} = [-\mu_1(1+B)(s^2 - q_s \sin \frac{nk^*\omega}{\beta_1}) - \mu_1(1-B)s\sin + \eta LD(q^2 + \frac{nk^*N\omega \sin f}{L\beta_2})] \zeta_n, \]
\[ \Delta_{B_n} = [-\mu_1(1+B)(s^2 - \frac{nk^*\omega \sin e}{\beta_1}) + (1-B)L\eta s q'_n + \eta LD(q^2 - q q'_n - \frac{nk^*N\omega \sin f}{L\beta_2})] \zeta_n, \]
\[ \Delta_{D'_n} = [-\mu_1(1+B)(s^2 - q s'_n - \frac{nk^*\omega \sin e}{\beta_1}) - \mu_1(1-B)s\sin + \eta LD(q^2 + \frac{nk^*N\omega \sin f}{L\beta_2})] \zeta_n, \]
\[ \Delta_n = \mu_1 s + \frac{\eta}{L} q n, \quad \Delta'_n = \mu_1 s'_n + \frac{\eta}{L} q'_n. \]

The values of the coefficients \( B \) and \( D \) appearing in the above expressions are given by (6.12). Here the coefficients \( B_n, B'_n \) and \( D_n, D'_n \) are the reflection and the transmission coefficients respectively, for the first order approximation of the corrugation.

### 6.5 Solution of the second order approximation

To find the solution of the second order approximation of corrugation for the coefficients \( B, D, B_n, D_n, B'_n \) and \( D'_n \), we collect the terms independent of \( x \) and \( \zeta \), the coefficients of \( e^{-mk^*x} \) and the coefficients of \( e^{mk^*x} \) to both sides of equations (6.8) and (6.9) after inserting (3.39). We obtain

\[ (1 + B)(1 + s^2 \zeta_n \zeta_{-n}) + s_n B_n \zeta_n + s'_n B'_n \zeta_{-n} \]

\[ = D(1 + q^2 \zeta_n \zeta_{-n}) - q_n D_n \zeta_n - q'_n D'_n \zeta_{-n}, \quad (6.18) \]

\[ s(1 - B)[1 + s^2 \zeta_n \zeta_{-n}] - B_n \zeta_n[s^2_n - nk^*(k + nk^*)(1 + \frac{s^2_n}{2} \zeta_n \zeta_{-n})] - B'_n \zeta_{-n}[s^2_{n'} + nk^*(k - nk^*)] \]

\[ (1 + s^2_n \zeta_n \zeta_{-n}) = \frac{L\eta}{\mu_1} [D q(1 + q^2 \zeta_n \zeta_{-n}) - D_n \zeta_n \{q^2_n - \frac{N}{L} nk^*(k + nk^*)} \]

\[ (1 + \frac{q^2_n}{2} \zeta_n \zeta_{-n}) - D'_n \zeta_{-n}\{q^2_n - \frac{N}{L} nk^*(k - nk^*)(1 + \frac{q^2_n}{2} \zeta_n \zeta_{-n})\}] \]

\[ (1 - B) \zeta_n - B_n(1 + s^2_n \zeta_n \zeta_{-n}) - B'_n \frac{s^2_n}{2} \zeta_{-n} \]

\[ = Dq \zeta_n - D_n(1 + q^2_n \zeta_n \zeta_{-n}) - D'_n q^2_n \zeta_{-n}, \quad (6.19) \]

\[ (1 + B)(s^2 + knk^*) - \frac{s^2}{2}(1 - B)knk^* \zeta_n \zeta_{-n} + B_n s_n(1 + s^2_n \zeta_n \zeta_{-n}) \]

\[ + B'_n s'_n(1 + s^2_n \zeta_n \zeta_{-n}) = \frac{L\eta}{\mu_1} [D \zeta_n \{q^2 + \frac{N}{L} knk^*(1 + \frac{q^2}{2} \zeta_n \zeta_{-n})\} \]

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Equations given by (6.18)-(6.23) enable us to give the reflection and transmission coefficients for the second order approximation of the corrugation.

6.6 Special case

In the case of the boundary surface given by \( z = c \cos \ k^*x \) and adopting the same procedure as explained in earlier chapters, we obtain the following formulae of \( B_i, D_i, B_i' \) and \( D_i' \) for the first order approximation of the corrugation from equation (6.17) are

\[
B_i = \frac{\Delta B_i}{\Delta_1}, \quad D_i = \frac{\Delta D_i}{\Delta_1}, \quad B_i' = \frac{\Delta B_i'}{\Delta_1'}, \quad D_i' = \frac{\Delta D_i'}{\Delta_1'} \quad (6.24)
\]

where the values of \( \Delta B_i, \Delta D_i, \Delta B_i', \Delta D_i' \), \( \Delta_1 \) and \( \Delta_1' \) are given as:

\[
\Delta_{B_i} = \frac{c}{2}[-\mu_1(1 + B)(s^2 + \frac{k^*\omega \sin e}{\beta_1}) + (1 - B)\eta_1q_1 + \eta_2D(q^2 - q_1 + \frac{k^*N\omega \sin f}{L\beta_2})],
\]

\[
\Delta_{D_i} = \frac{c}{2}[-\mu_1(1 + B)(s^2 - q s_1 + \frac{k^*\omega \sin e}{\beta_1}) - \mu_1(1 - B) s s_1 + \eta_2D(q^2 + \frac{k^*N\omega \sin f}{L\beta_2})],
\]

\[
\Delta_{B_i'} = \frac{c}{2}[-\mu_1(1 + B)(s^2 - q s_1' + \frac{k^*\omega \sin e}{\beta_1}) - \mu_1(1 - B) s s_1' + \eta_2D(q^2 - \frac{k^*N\omega \sin f}{L\beta_2})],
\]

\[
\Delta_{D_i'} = \frac{c}{2}[-\mu_1(1 + B)(s^2 - q s_1' - \frac{k^*\omega \sin e}{\beta_1}) - \mu_1(1 - B) s s_1' + \eta_2D(q^2 - \frac{k^*N\omega \sin f}{L\beta_2})],
\]

It is clear from the formulae given in equation (6.24) that all the coefficients for the first order approximation of the corrugation are proportional to the amplitude of the corrugated interface.
6.6.1 Particular cases

(a) If the sandiness and the anisotropy factors of the media are neglected, then both the media become isotropic elastic solid half spaces. Thus in this case, the problem reduces to the problem of reflection and refraction of SH-waves incident at a corrugated interface between two elastic half-spaces (Asano, 1960). Plugging $\eta = 1$ and $N = L = \mu_2$ into the equations (6.2), (6.5) and (6.6), the values of the quantities $s$, $q$, $s_n$, $q_n$, $s'_n$ and $q'_n$ reduce to

\[
s = i \frac{\omega}{\beta_1} \cos \epsilon, \quad q = i \frac{\omega}{\beta_1} \left( \frac{\beta_2^2}{\beta_1^2} - \sin^2 \epsilon \right)^{1/2},
\]

\[
s_n = i \frac{\omega}{\beta_1} \cos e_n, \quad q_n = i \frac{\omega}{\beta_1} \left( \frac{\beta_2^2}{\beta_1^2} - \sin^2 e_n \right)^{1/2},
\]

\[
s'_n = i \frac{\omega}{\beta_1} \cos e'_n, \quad q'_n = i \frac{\omega}{\beta_1} \left( \frac{\beta_2^2}{\beta_1^2} - \sin^2 e'_n \right)^{1/2}.
\]

The expressions of the coefficients $B_1$, $D_1$, $B'_1$ and $D'_1$, in this case, become

\[
B_1 = \frac{c}{2 \Delta_1} \left[ -\mu_1 (1 + B) \left( s^2 - \frac{k^* \omega \sin e}{\beta_1} \right) + \mu_2 s_1 (1 - B) + \mu_2 D (q^2 - q q_1 + \frac{k^* \omega \sin e}{\beta_1}) \right],
\]

\[
D_1 = \frac{c}{2 \Delta_1} \left[ -\mu_1 (1 + B) \left( s^2 - q s_1 + \frac{k^* \omega \sin e}{\beta_1} \right) - \mu_1 s s_1 (1 - B) + \mu_2 D (q^2 + \frac{k^* \omega \sin e}{\beta_1}) \right],
\]

\[
B'_1 = \frac{c}{2 \Delta_1} \left[ -\mu_1 (1 + B) \left( s^2 - q s_1 - \frac{k^* \omega \sin e}{\beta_1} \right) + \mu_2 s_1 (1 - B) + \mu_2 D (q^2 - q q_1 - \frac{k^* \omega \sin e}{\beta_1}) \right],
\]

\[
D'_1 = \frac{c}{2 \Delta_1} \left[ -\mu_1 (1 + B) \left( s^2 - q s'_1 - \frac{k^* \omega \sin e}{\beta_1} \right) - \mu_1 s s'_1 (1 - B) + \mu_2 D (q^2 - \frac{k^* \omega \sin e}{\beta_1}) \right],
\]

\[
\Delta_1 = \mu_1 s_1 + \mu_2 q_1, \quad \Delta'_1 = \mu_1 s'_1 + \mu_2 q'_1.
\]

These formulae give the reflection and refraction coefficients for the first order approximation of the corrugated interface between two uniform elastic half spaces. It can be verified that by neglecting the sandiness and the anisotropic behaviors as explained above, the boundary conditions (6.8) and (6.9) match with those of Asano (1960) for the corresponding problem.

Further, if we replace the corrugated interface by a plane interface, the problem reduces to the problem of reflection and refraction SH-waves incident at a plane interface between two homogeneous isotropic elastic half-spaces. In this case, we put $c = 0$ into...
equation (6.24), the coefficients $B_1$, $D_1$, $B'_1$ and $D'_1$ will vanish, since they are proportional to $c$, the amplitude of corrugated interface. Setting $\frac{\mu_1}{\mu_2} = m_1$ and $\frac{\beta_1}{\beta_2} = m_2$, the reflection and transmission coefficients at the plane interface, as given by equation (6.12), become

$$B = \frac{m_1 \cos \theta - \sqrt{m_2^2 - \sin^2 \theta}}{m_1 \cos \theta + \sqrt{m_2^2 - \sin^2 \theta}}, \quad D = \frac{2m_1 \cos \theta}{m_1 \cos \theta + \sqrt{m_2^2 - \sin^2 \theta}}.$$ 

These are identical to those as given in Savarensky (1975, pp-284) for the corresponding problem.

(b) When we set the sandy parameter $\eta = 1$, the half-space $H_1$ becomes homogeneous elastic. Thus, in this case, plugging the values of $s$ and $q$ and using the notations $N^* = N/\mu_1$ and $L^* = L/\mu_1$ into equations (6.12) and (6.24), the reflection and the refraction coefficients at the plane interface $B$, $D$ and at the corrugated interface $B_1$, $D_1$, $B'_1$ and $D'_1$ for the first order approximation of the corrugated interface between a uniform elastic half-space and a transversely isotropic elastic half-space can be obtained easily.

### 6.7 Energy partition equation

The expressions for the energy flux for each of the incident, the reflected and the refracted waves are obtained by multiplying the total energy per unit volume (which is twice the mean of the kinetic energy density) by the velocity of propagation and the area of the wave front involved. The area of the wave front is proportional to the cosine of the angle between the normal and the vertical. Thus, using Snell’s law given by equations in (2.17), Spectrum theorem given by equation (2.22) and dividing by the energy flux of the incident wave, the energy equation for the incident, the regularly reflected, the irregularly reflected $SH$-wave for the $n^{th}$ order approximation of the corrugation can be written as (Abubakar, 1962b)

$$1 = |B^2| + \sum_{n=1}^{\infty} \frac{\cos \theta_n}{\cos \theta} |B_n^2| + \sum_{n=1}^{\infty} \frac{\cos \theta'_n}{\cos \theta} |B'_n^2| + \frac{\rho_2 \beta_2 \cos f}{\rho_1 \beta_1 \cos \theta} |D^2|$$

$$+ \sum_{n=1}^{\infty} \frac{\rho_2 \beta_2 \cos f_n}{\rho_1 \beta_1 \cos \theta} |D_n^2| + \sum_{n=1}^{\infty} \frac{\rho_2 \beta_2 \cos f'_n}{\rho_1 \beta_1 \cos \theta} |D'_n^2| \quad (6.25)$$

The partition of energy at a plane interface between the sandy and anisotropic half-spaces can be readily reduced from equation (6.25) by putting the values of the coefficients $B_n$, $D_n$, $B'_n$ and $D'_n$ equal to zero (as they are proportional to the amplitude of
the corrugated interface). Thus, we obtain

\[ 1 = | B^2 | + \frac{\rho_2 \beta_2^2 \tan e}{\rho_1 \beta_1^2 \tan f} \cdot | D^2 |. \]

This relation is identical to the relation of energy partition due to incident \( SH \)-wave at a plane interface between the sandy layer and the anisotropic half-spaces, given in (Chakraborty and Chandra, 1984) for the corresponding problem.

In the present formulation, when \( n=1 \), from equation (6.25), it follows that

\[ \sum_{i=1}^{6} E_i \simeq 1, \]

where \( E_1 \) and \( E_2 \) are the ratios of the energy transmission along the regularly reflected and regularly refracted waves to the energy transmission along the incident wave. Similarly, \( E_3, E_5 \) and \( E_4, E_6 \) are the energy ratios of the energy transmission along the irregularly reflected and irregularly refracted waves to that of the incident wave for the first order approximation of the corrugation at the corrugated interface respectively.

The expressions for these energy ratios are given as

\[ E_1 = | B^2 |, \quad E_2 = \frac{\rho_2 \beta_2 \cos f}{\rho_1 \beta_1 \cos e} \cdot | D^2 |, \quad E_3 = \frac{\cos \epsilon_1}{\cos e} \cdot | B_1^2 |, \quad E_4 = \frac{\rho_2 \beta_2 \cos f_1}{\rho_1 \beta_1 \cos e} \cdot | D_1^2 |, \quad E_5 = \frac{\cos \epsilon_1}{\cos e} \cdot | B_1^2 |, \quad E_6 = \frac{\rho_2 \beta_2 \cos f_1}{\rho_1 \beta_1 \cos e} \cdot | D_1^2 |. \]

### 6.8 Numerical results and discussion

In order to study the effects of the sandy parameter, the anisotropy, the corrugation of the interface, the frequency and the angle of the incidence on the reflection and transmission coefficients, when a plane \( SH \)-wave is obliquely incident at the corrugated interface between the two half spaces \( H_1 \) and \( H_2 \), we have computed these coefficients for the model considered in Section 6.6. For this purpose, we have selected the following numerical values of the relevant elastic parameters (Chakraborty and Chandra, 1984)

\[
\frac{N}{\mu_1} = 2.95, \quad \frac{L}{\mu_1} = 2.75, \quad \frac{\rho_2}{\rho_1} = 2.5, \text{ and } \omega/c/\beta_1 = 0.10.
\]

We have taken the sandy parameter \( \eta = 1.15 \), the angle of incidence \( e = 45^\circ \) and \( k^*c = 0.00125 \), wherever not mentioned.
(i) The effect of sandiness of the half-space $H_1$: To study this effect on the reflection and transmission coefficients at both plane and corrugated interface, we have selected $\eta = 1.00, 1.10, 1.15, 1.20$ and $1.30$. Figures 6.1 and 6.2 show the variations of the modulus of reflection and transmission coefficients as functions of the angle of incidence at a plane interface between the half-spaces $H_1$ and $H_2$. We observe that there is a significant effect of the sandy parameter $\eta$ on the reflection and transmission coefficients with respect to the angle of incidence. It is noticed that the values of these coefficients decrease slowly with the increase of the sandiness. Also, with the increase of the angle of incidence and that of sandiness $\eta$ from 1.00 to 1.15, the values of these

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coefficients increase. Furthermore, when the sandiness $\eta$ varies from 1.20 to 1.30, the

values of the reflection and the transmission coefficients decrease considerably with the increase of the angle of incidence.

Figures 6.3-6.6 depict the variations of the reflection and the transmission coefficients as functions of the angle of incidence for the first-order approximation of the corrugated interface. The effect of the sandiness on these coefficients is clearly visible. The values of the reflection and the transmission coefficients $B_1, B'_1$ and $D_1, D'_1$ decrease monotonically with the increase of the angle of incidence ($\theta$) and increase monotoni-
cally with the increase of the sandiness. The behavior of all these coefficients at the corrugated interface is similar. We have also noticed from these figures that the values of the coefficients $D_1$ and $D'_1$ are less than those of the coefficients $B_1$ and $B'_1$. However, the critical angles are different for different values of the parameter $\eta$.

(ii) Effect of corrugation: As the coefficients $B$ and $D$ are the reflection and the
transmission coefficients, respectively, at the plane interface, it is obvious that they are independent on the corrugation parameter $k^*c$. This is also clear from the analytical results shown in equation (6.12). Here, we have found that the coefficients $B_1$, $B'_1$, $D_1$ and $D'_1$ are strongly affected by the corrugation $k^*c$. Figures 6.7 and 6.8 show the variation of $B_1$, $D_1$, $B'_1$ and $D'_1$ as functions of the corrugation parameter $k^*c$. We have noticed that the values of the coefficients $B_1$ and $D_1$ increase with increase of $k^*c$, while the values of the coefficients $B'_1$ and $D'_1$ decrease with the increase of parameter $k^*c$. Also, the values of the coefficients $B_1$ and $D_1$ are greater than those of the coefficients $D_1$ and $B'_1$.

(iii) Effect of frequency:

Figures 6.9-6.13 show the effect of the frequency parameter $\omega c/\beta_{11}$ on the reflection and the transmission coefficients at both the plane and the corrugated interfaces.

Fig. 6.9. Variation of the Reflection Coefficient $B$ and the Transmission Coefficient $D$ with the angle $\theta$ for different values of frequency $F = \omega c/\beta_1$. 

Amplitude Ratios ($B$, $D$)
We have noticed that the amplitude of the transmitted wave at the plane interface is greater than that of the reflected wave at the plane interface, whereas reverse behavior is found in the case of the reflection and the transmission coefficients for the first order approximation of corrugation. Also, the amplitude of each of the coefficients $B_i$, $D_i$, $B'_i$ and $D'_i$ decreases with the increase of the angle of incidence and increases with the increase of the frequency parameter $\omega c/\beta_{h_1}$.
(iv) Effect of densities of the half-spaces:

Figures 6.14-6.16 show the variations of the coefficients $B$, $D$ and $B_1$, $D_1$ and $B'_1$, $D'_1$ with the angle of incidence ($e$) respectively, when the density of the upper half-space increases slightly. It is found that these coefficients are influenced by the density of the half-space. The values of the coefficients $B$ and $D$ increase while that of the coefficients $B_1$, $D_1$, $B'_1$ and $D'_1$ decrease with the increase of the density of the upper half-space.

(v) Effect of anisotropy:

Figures 6.17-6.19 show the variations of the coefficients $B$, $D$, $B_1$, $D_1$, $B'_1$ and $D'_1$ with the angle of incidence ($e$) for different values of the anisotropy factor ($N/L$) in the lower half-space. These values are taken as $N/L = 1.0$, $1.2$ and $1.3$. It is noticed from these figures that there is a significant effect of the anisotropy of the half-space on each coefficient. The reflection and the transmission coefficients $B_1$, $D_1$, $B'_1$ and $D'_1$…
decrease with angle of incidence for each value of the anisotropic factor chosen. However, when the anisotropic factor increases, the values of $B_1$ and $B'_1$ decrease, while that of the coefficients $D_1$ and $D'_1$ increase. The behavior of the coefficients $B$ and $D$ for plane interface behave alike for different values of the anisotropic factor.

(vi) **Partition of energy:**

Figures 6.20-6.22 depict the variations of the reflected and the transmitted energy ratios of $SH$-wave at the plane and the corrugated interface for the first order approximation of corrugation with the angle of incidence ($\theta$). It is observed from these figures that the values of the energy ratio ($E_2$) corresponding to the refracted wave at the plane interface is maximum as compared to all other energy ratios in the entire range of the angle of incidence, except in the vicinity of grazing incidence. Also, the values of the energy ratios of the reflected waves with the angle of incidence, decrease, while that of the refracted waves increase. However, these energy ratios increase with increase of the sandiness almost at each angle of incidence. The sum of the energy ratios is found to be less than unity (but close to unity) at each angle of incidence. It is obvious, since we are considering only the coefficients of the first order approximation of the corrugation.
Amplitude Ratios ($B_{ll}$, $D_{ll}$)

Fig. 6.17. Variation of the Reflection Coefficient $B$ and the Transmission Coefficient $D$ with the angle $\theta$ when the anisotropy factor ($N/L$) = 1.0, 1.2, 1.3.

Fig. 6.18. Variation of the Reflection Coefficient $B_1$ and Transmission Coefficient $D_1$ with the angle $\theta$ when the anisotropy factor ($N/L$) = 1.0, 1.2, 1.3.

Fig. 6.19. Variation of the Reflection Coefficient $B_1$ and the Transmission Coefficient $D_1$ with the angle $\theta$ when the anisotropy factor ($N/L$) = 1.0, 1.2, 1.3.

Fig. 6.20. Variation of the energy ratios $E_1$ and $E_2$ with different sandy factor $E (=\eta) = 1.0, 1.2, 1.3$.

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Fig. 6.21. Variation of energy ratios $E_1$ and $E_4$ with different sandy factor $E = \eta = 1.0, 1.2, 1.3$.

Fig. 6.22. Variation of energy ratios $E_5$ and $E_6$ with different sandy factor $E = \eta = 1.0, 1.2, 1.3$. 