# Contents

Acknowledgements iii

1 Introduction 1
   1.1 Development in mathematical programming 1
   1.2 Brief survey of relevant work 5
   1.3 Summary of Thesis 11

2 Capacitated Transportation Problems 31
   2.1 Capacitated transportation problem with bounds on rim conditions 32
      2.1.1 Theoretical development for problem (P.2.1) 34
      2.1.2 Problem (P.2.1) with restricted flow 39
      2.1.3 Unbalanced Capacitated Transportation problem 45
      2.1.4 Paradox in a capacitated transportation problem 54
      2.1.5 Numerical Illustration 56
      2.1.6 Conclusions 63
   2.2 Non-linear capacitated transportation problem 65
      2.2.1 Notations, Preliminary results 65
      2.2.2 Theoretical Development 70
      2.2.3 Sufficient condition for the existence of a paradoxical solution 71
      2.2.4 Algorithm to find a 'paradoxical solution' 73
      2.2.5 Numerical Illustration 83
      2.2.6 Concluding Remarks 90
### 3 Sensitivity Analysis

3.1 Basic Sensitivity Analysis ................................................................. 92  
   3.1.1 Dual Simplex Algorithm for bounded variables ........................... 95  
   3.1.2 Sensitivity Analysis ................................................................. 102  
   3.1.3 Numerical Illustration ............................................................. 123  
   3.1.4 Concluding Remarks .............................................................. 138  

3.2 Positive Sensitivity Analysis ............................................................... 138  
   3.2.1 Basic duality results in linear programming with bounded variables ......................................................... 139  
   3.2.2 Definition of three sensitivity analysis methods ......................... 150  
   3.2.3 The range of PSA using different optimal solutions .................. 154  
   3.2.4 The relationship between PSA and BSA under degeneracy .......... 164  
   3.2.5 Concluding Remarks .............................................................. 169  

### 4 Integer Programming Problems                                        171

4.1 Valid-cuts in integer programming ..................................................... 172  
   4.1.1 Notations, definitions and some basic results ......................... 172  
   4.1.2 Theoretical development of valid-cuts ...................................... 173  
   4.1.3 Constrained integer programming ........................................... 184  
   4.1.4 Numerical illustrations .......................................................... 188  
   4.1.5 Concluding Remarks .............................................................. 193  

4.2 Ranking of Integer feasible solutions ............................................... 194  
   4.2.1 Notations and Some basic results ........................................... 194  
   4.2.2 Theoretical development of various cuts ................................. 197  
   4.2.3 Numerical Illustration .......................................................... 209  
   4.2.4 Concluding Remarks .............................................................. 219  

### 5 Convex programming with single separable constraint                  221

5.1 Convex Programming Problem \((P.5.1)\) .............................................. 222  
5.2 Algorithm 1 for problem \((P.5.1)\) ................................................. 224  
5.3 Convergence of Algorithm 1 ......................................................... 226  
5.4 Algorithm for problem \((P.5.1^*)\) .................................................. 229
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.4.1 Characterization of the optimal solution of problem ((P.5.1^*))</td>
<td>229</td>
</tr>
<tr>
<td>5.4.2 Analysis of the solution to problem ((P.5.1^*))</td>
<td>231</td>
</tr>
<tr>
<td>5.4.3 Algorithm 2. and its convergence</td>
<td>232</td>
</tr>
<tr>
<td>5.5 Problems ((P.5.3)) and ((P.5.4))</td>
<td>233</td>
</tr>
<tr>
<td>5.6 Numerical Illustration</td>
<td>235</td>
</tr>
<tr>
<td>5.7 Conclusions</td>
<td>241</td>
</tr>
</tbody>
</table>