CHAPTER 3

MECHANICAL SPRINGS

3.1 DEFINITION
A mechanical spring may be defined as an elastic body whose primary function is to deflect or distort under load (or to absorb energy) and which recovers its original shape when released after being distorted [4].

3.2 FUNCTIONS OF SPRINGS
The functions of the springs are,
- To absorb Energy and mitigate Shock.
- To apply a Definite Force or Torque
- To Support Moving Masses or Isolate Vibration
- To Indicate or Control Load or Torque

3.3 TYPES/CONFIGURATIONS OF SPRINGS
Following configuration of springs is generally available in the market.
- Helical compression or tension springs.
- Laminated or leaf springs.
- Disc or Belleville springs.
- Helical Torsion springs
- Conical and volute springs
- Special purpose springs.

3.4 HELICAL COMPRESSION SPRINGS
The most common helical compression spring has a constant coil diameter, constant pitch, round-wire spring, as shown in Figure 3.1. A helical spring may be coiled either left handed or right handed. Dimensional parameters for a standard helical compression spring are shown in Figure 3.1. The wire diameter is \( d \), the mean coil diameter is \( D \), and these two dimensions along with the free length \( L_f \) and the number of coils \( N_t \) or the coil pitch \( p \) are used to define the spring geometry for calculation and manufacturing purposes. The outside
diameter $D_o$ and the inside diameter $D_i$ are of interest mainly to define the minimum hole in which it will fit or the maximum pin over which it can be placed. They are found by adding or subtracting the wire diameter $d$ to or from the mean coil diameter $D$. The minimum recommended diametric clearances between the $D_o$ and a hole or between $D_i$ and a pin are $0.10D$ for $D < 13$ mm or $0.05D$ for $D > 13$ mm [5].

The helical springs have the following advantages:

(a) They are easy to manufacture
(b) They are available in wide range.
(c) They are reliable.
(d) They have constant spring rate.
(e) Their performance can be predicted more accurately.
(f) Their characteristics can be varied by changing dimensions.

![Diagram of helical compression spring dimensions](image)

**Fig. 3.1 Dimensional parameters for Helical Compression Springs**

### 3.5 SPRING MATERIALS

The ideal spring material would have a high ultimate strength, high yield point, high resilience, creep resistant and low modulus of elasticity in order to provide maximum energy storage. For dynamic loaded springs, the fatigue strength properties of the material are of primary importance. High strengths and yield points are attainable from medium to high-carbon and alloy steels, and these are the most common spring materials despite their high modulus of elasticity. A few stainless steel alloys are suitable for springs, as are beryllium Copper and phosphor bronze, among the copper alloys. [5]
Most light-duty springs are made of cold-drawn, round or rectangular wire or thin, cold-rolled, flat-strip stock. Heavy-duty springs, such as vehicle-suspension parts, are typically made from hot-rolled or forged forms. Spring materials are typically hardened in order to obtain the required strength. Small cross sections are work hardened in the cold-drawing process. Large sections are typically heat treated. Low temperature heat treatment (175 to 510°C) is used after forming to relieve residual stresses and stabilize dimensions, even in small-section parts. High-temperature quenching and tempering is used to harden larger springs that must be formed in the annealed condition. E.g. of spring materials are, steels for cold-wound springs, Music Wire, Oil-tempered spring Wire, Hard-drawn spring wire, Chrome-Vanadium steel wire, chrome-silicon wire, stainless-steel spring wire, carbon- and alloy-steel bars for springs. Table 3.1 gives some common spring materials and its description. Table 3.2 shows the mechanical properties of some spring wires.

3.5.1 TENSILE STRENGTH

Spring materials may be compared by an examination of their tensile strength; these vary so much with wire size that they cannot be specified until the wire size is known. The material and its processing also, have an effect on the tensile strength. The graph of tensile strength versus wire diameter is almost a straight line for some materials when plotted on log-log paper. The equation for this line is written as

\[ S_{ut} = \frac{A}{d^m} \quad d \text{ is in mm, } A \text{ is MPa. mm}^m. \]

This gives an estimation of tensile strength when the intercept A, and the slope m of the line are known. The values of these constants are given in Table 3.3.
<table>
<thead>
<tr>
<th>ASTM</th>
<th>Material</th>
<th>SAE</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A227</td>
<td>Cold-drawn wire ('hard-drawn') 0.60-0.70C</td>
<td>1066</td>
<td>Least expensive general-purpose spring wire. Suitable for static loading but not well for fatigue or impact.</td>
</tr>
<tr>
<td>A228</td>
<td>Music wire 0.80-0.95C</td>
<td>1085</td>
<td>Toughest, most widely used material for small coil springs. Highest tensile and fatigue strength of all spring wire.</td>
</tr>
<tr>
<td>A229</td>
<td>Oil-tempered wire, 0.60-0.7C</td>
<td>1065</td>
<td>General-purpose spring steel. Less expensive and available in larger sizes than music wire. Suitable for static loading but not well for fatigue or impact.</td>
</tr>
<tr>
<td>A230</td>
<td>Oil-tempered wire 0.60-0.7C</td>
<td>1070</td>
<td>Valve-spring quality-suitable for fatigue loading.</td>
</tr>
<tr>
<td>A232</td>
<td>Chrome vanadium</td>
<td>6150</td>
<td>Most popular alloy spring steel. Valve-spring quality-suitable for fatigue loading.</td>
</tr>
<tr>
<td>A313</td>
<td>Stainless steel</td>
<td>30302</td>
<td>Suitable for fatigue applications.</td>
</tr>
<tr>
<td>A401</td>
<td>Chrome silicon</td>
<td>9254</td>
<td>Valve-spring quality-suitable for fatigue loading. Second highest strength to music wire and has higher temperature resistance</td>
</tr>
<tr>
<td>8134,</td>
<td>Spring brass</td>
<td>CA-260</td>
<td>Low strength-good corrosion resistance.</td>
</tr>
<tr>
<td>260</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8159</td>
<td>Phosphor bronze</td>
<td>CA-510</td>
<td>Higher strength than brass-better fatigue resistance-good corrosion resistance.</td>
</tr>
<tr>
<td>8197</td>
<td>Beryllium copper</td>
<td>CA-172</td>
<td>Higher strength than brass-better fatigue resistance-good corrosion resistance.</td>
</tr>
<tr>
<td></td>
<td>Inconel X-750</td>
<td></td>
<td>Corrosion resistance.</td>
</tr>
</tbody>
</table>
Table 3.2 Mechanical properties of some spring wires [6]

<table>
<thead>
<tr>
<th>Material</th>
<th>Elastic Limit, percent of $S_{ut}$</th>
<th>Diameter d, mm</th>
<th>$E$ GPa</th>
<th>$G$ GPa</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tension</td>
<td>Torsion</td>
<td>&lt;0.8</td>
<td>0.8—1.6</td>
</tr>
<tr>
<td>Music wire A228</td>
<td>65—75</td>
<td>45—55</td>
<td>&lt;0.8</td>
<td>203.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.8—1.6</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.6—3.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>&gt;3.0</td>
<td></td>
</tr>
<tr>
<td>HD spring A227</td>
<td>60—70</td>
<td>45—55</td>
<td>&lt;0.8</td>
<td>18.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.8—1.6</td>
<td>197.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.6—3.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>&gt;3.0</td>
<td></td>
</tr>
<tr>
<td>Oil tempered A239</td>
<td>85—90</td>
<td>45—50</td>
<td></td>
<td>196.5</td>
</tr>
<tr>
<td>Valve spring A230</td>
<td>65—75</td>
<td>50—60</td>
<td></td>
<td>203.4</td>
</tr>
<tr>
<td>Chrome A231</td>
<td>88—93</td>
<td>65—75</td>
<td></td>
<td>203.4</td>
</tr>
<tr>
<td>vanadium A232</td>
<td>88—93</td>
<td></td>
<td></td>
<td>203.4</td>
</tr>
<tr>
<td>Chrome silicon A401</td>
<td>85—93</td>
<td>65—75</td>
<td></td>
<td>203.4</td>
</tr>
<tr>
<td>Stainless steel A31 3</td>
<td>65—75</td>
<td>45—55</td>
<td></td>
<td>193</td>
</tr>
<tr>
<td>17-7PH</td>
<td>75—80</td>
<td>55—60</td>
<td></td>
<td>208.4</td>
</tr>
<tr>
<td></td>
<td>65—70</td>
<td>42—55</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>65—75</td>
<td>45—55</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>72—76</td>
<td>50—55</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phosphor bronze 159</td>
<td>75—80</td>
<td>45—50</td>
<td></td>
<td>103.4</td>
</tr>
<tr>
<td>Beryllium copper 197</td>
<td>70</td>
<td>50</td>
<td></td>
<td>117.2</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>50—55</td>
<td></td>
<td>131</td>
</tr>
<tr>
<td>Inconel alloy X-750</td>
<td>65—70</td>
<td>40—45</td>
<td></td>
<td>213.7</td>
</tr>
</tbody>
</table>

3.5.2 SHEAR STRENGTH

Extensive testing has determined that a reasonable estimate of the ultimate strength in torsion of common spring materials is 67% of the ultimate tensile strength.

$Sus = 0.67 S_{ut}$.
Table 3.3 Constants A and m of $S_{ut} = A/d^m$ for Estimating Minimum Tensile Strength of Common Spring Wires [6]

<table>
<thead>
<tr>
<th>Material</th>
<th>ASTM No</th>
<th>Exponent m</th>
<th>Diameter mm</th>
<th>A, Mpa.mm$^m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Music wire</td>
<td>A228</td>
<td>0.145</td>
<td>0.10-6.5</td>
<td>2211</td>
</tr>
<tr>
<td>OQ&amp;T wire</td>
<td>A229</td>
<td>0.187</td>
<td>0.5-12.7</td>
<td>1855</td>
</tr>
<tr>
<td>Hard-drawn wire</td>
<td>A227</td>
<td>0.190</td>
<td>0.7-12.7</td>
<td>1783</td>
</tr>
<tr>
<td>Chrome-vanadium wire</td>
<td>A232</td>
<td>0.168</td>
<td>0.8-11.1</td>
<td>2005</td>
</tr>
<tr>
<td>Chrome-silicon wire</td>
<td>A401</td>
<td>0108</td>
<td>1.6-9.5</td>
<td>1974</td>
</tr>
<tr>
<td>302 Stainless wire</td>
<td>A313</td>
<td>0.146</td>
<td>0.3-2.5</td>
<td>1867</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.263</td>
<td>2.5-5</td>
<td>2065</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.478</td>
<td>5-10</td>
<td>2911</td>
</tr>
<tr>
<td>Phosphor-bronze wire</td>
<td>B159</td>
<td>0</td>
<td>0.1-0.6</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.028</td>
<td>0.6-2</td>
<td>913</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.064</td>
<td>2-7.5</td>
<td>932</td>
</tr>
</tbody>
</table>

3.5.3 TORSIONAL YIELD STRENGTH

The torsional yield strengths of spring wire vary, depending on the material and whether the spring has been set or not. Table 3.4 shows recommended torsional yield-strength factors for several common spring wires as a percentage of the wire’s ultimate tensile strength. These factors should be used to determine an estimated strength for a helical compression spring in static loading.

Table 3.4 Maximum Torsional Yield Strength $S_{ys}$ for Helical Compression Springs in Static Applications [5]

<table>
<thead>
<tr>
<th>Material</th>
<th>Maximum Percent of Ultimate Tensile Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before Set Removed</td>
<td>After Set Removed</td>
</tr>
<tr>
<td>Cold-drawn carbon steel (e.g., A227, A228)</td>
<td>45%</td>
</tr>
<tr>
<td>Hardened and tempered carbon and low-alloy steel (e.g., A229, A230, A232, A401)</td>
<td>50</td>
</tr>
<tr>
<td>Austenitic stainless steel (e.g., A313)</td>
<td>35</td>
</tr>
<tr>
<td>Nonferrous alloys (e.g., 8134, 8159, B 197)</td>
<td>35</td>
</tr>
</tbody>
</table>

3.5.4 TORSIONAL FATIGUE STRENGTH

Torsional fatigue strength over the $10^3$ N $10^7$ cycles range varies with the material and whether it is shot peened or not. Table 3.5 shows recommended
values for several wire materials in the peened and unpeened conditions at three points on their S-N diagrams at $10^5$, $10^6$ and $10^7$ cycles. Note that these are torsional fatigue strengths and are determined from test springs loaded with equal mean and alternating stress components (stress ratio $R = \frac{\tau_{min}}{\tau_{max}} = 0$).

So, they are not directly comparable to any of the fully reversed fatigue strengths generated from rotating-bending specimens because of both, the torsional loading and the presence of a mean stress component. We use the designation $S_{fw}$, for these wire fatigue strengths to differentiate them from the fully reversed fatigue strengths. These fatigue strengths $S_{fw}$ are nonetheless very useful in that they represent an actual (and typical) spring-fatigue-loading situation and are generated from spring samples, not test specimens, so the geometry and size are correct. Note that the fatigue strengths in Table 3.5 are declining with increasing number of cycles, even above $10^6$ cycles, where steels usually display an endurance limit.

### Table 3.5 Maximum Torsional Fatigue Strength $S_{fw}$ for Round-Wire Helical - Compression Springs in Cyclic Applications (Stress Ratio, R=0) [5].

<table>
<thead>
<tr>
<th>Fatigue Life (cycles)</th>
<th>Percent of Ultimate Tensile Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ASTM 228, Austenitic Stainless Steel and Nonferrous</td>
</tr>
<tr>
<td></td>
<td>Unpeened</td>
</tr>
<tr>
<td>$10^5$</td>
<td>36%</td>
</tr>
<tr>
<td>$10^6$</td>
<td>33%</td>
</tr>
<tr>
<td>$10^7$</td>
<td>30%</td>
</tr>
</tbody>
</table>

### 3.6 NOMENCLATURE OF SPRINGS

#### 3.6.1 SPRING LENGTHS

Compression springs have several lengths and deflections of interest, as shown in Figure 3.2. Free length $L_f$ is the overall spring length in the unloaded Condition, i.e., as manufactured. Assembled length $L_a$ is the length after installation to its initial deflection $y_{initial}$. The working load is applied to further compress the spring through its working deflection $y_{working}$. The minimum working length $L_m$ is the shortest dimension to which it is compressed in service. The shut height or solid length $L_s$ is its length when compressed such that all coils are in
contact. Once shut, spring can support much larger "indefinite" loads up to the compressive strength of wire. The clash allowance $Y_{\text{clash}}$ is the difference between the minimum working length and the shut height, expressed as a percentage of the working deflection. A minimum clash allowance of 10-15% is recommended to avoid reaching the shut height in service with out-of-tolerance springs, or with excessive deflections.

![Fig 3.2 Lengths of Helical Compression springs](image)

**Fig 3.2** Lengths of Helical Compression springs

![Fig 3.3 Four types of end details of Helical Compression Springs](image)

**Fig 3.3** Four types of end details of Helical Compression Springs

### 3.6.2 END DETAILS

Four types of end details are available on helical compression springs and they are, plain, plain-ground, squared and squared-ground as shown in fig 3.3. Plain ends result from simply cutting the coils and leaving the ends with the same pitch as the rest of the spring. This is the least expensive end detail, but provides poor alignment to the surface against which the spring is pressed. The end coils
can be ground flat and perpendicular to the spring axis to provide normal surfaces for load application. Squaring the ends involves yielding the end coils to flatten and remove their itch. This improves the alignment. A flat surface on the end coil of at least 270° is recommended for proper operation. Squaring and grinding combined provides a 270-330° flat surface for load application. It is the most expensive end treatment but is nevertheless recommended for machinery springs unless the wire diameter is very small (<0.02 in or 0.5 mm), in which case they should be squared but not ground.

3.6.3 ACTIVE COILS

The total number of coils \( N_t \) may or may not contribute actively to the spring's deflection depending on the end treatment. The number of active coils \( N_a \) is needed for calculation purposes. Squared ends effectively remove two coils from active deflection. Grinding by itself removes 1 active coil. Figure 3.3 shows the relationships between total coils \( N_t \) and active coils \( N_a \) for each of the four end-coil conditions. The calculated number of active coils is usually rounded to the nearest 1/4 coil, as the manufacturing process cannot always achieve better than that accuracy.

3.6.4 SPRING INDEX

The spring index \( C \) is the ratio of coil diameter \( D \) to wire diameter \( d \).

\[
C = \frac{D}{d} \tag{3.1}
\]

The preferred range of \( C \) is from 4 to 12. At \( C<4 \), the spring is difficult to manufacture, and at \( C>12 \) it is prone to buckling and also tangles easily when handled in bulk.
3.6.5 Spring deflection

Figure 3.4 shows a portion of a helical coil spring with compressive axial loads applied. The load on the spring is in compression, the spring wire in torsion, as the load on any coil tends to twist the wire about its axis. A helical compression spring is, in fact, a torsion bar wrapped into a helical form, which packages better. The deflection of a round-wire helical compression spring is easily obtained using Castigliano’s theorem. The total strain energy for a helical spring is composed of a torsional component and a shear component. The strain energy is

\[ U = \frac{T^2}{2GJ} + \frac{F^2}{2AG} \]  \hspace{1cm} (3.2)

Substituting \( T = FD/2 \), \( l = \pi DN \), \( J = \pi d^4/32 \), and \( A = \pi d^2/4 \) results in

\[ U = \frac{4F^2D^3N}{d^4G} + \frac{F^2DN}{d^2G} \]  \hspace{1cm} (3.3)

Then, using Castigliano’s theorem to find total deflection \( y \),

\[ y = \frac{\partial U}{\partial F} = \frac{8FD^3N}{d^4G} + \frac{4FDN}{d^2G} \]  \hspace{1cm} (3.4)

Since \( C = D/d \), Eq. 3.4 can be rearranged to yield
\[ y = \frac{8FD^3Na}{d^4G \left( 1 + \frac{1}{2C^2} \right)} = \frac{8FD^3Na}{d^4G} \]  

Where \( F \) is the applied axial load on the spring, \( D \) is mean coil diameter, \( d \) is wire diameter, \( N_a \) is number of active coils, and \( G \) is the shear modulus of the material.

3.6.6 SPRING RATE

The spring rate or stiffness or spring constant is defined as the load required per unit deflection of the spring. Spring rate, \( k = F/y \) Where \( F \) is the Load, and \( y \) = Deflection of the spring. The standard helical compression spring has a spring rate \( k \) that is essentially linear over most of its operating range, as shown in Figure 3.5. The first and last few percent of its deflection have a nonlinear rate. When it reaches its shut height \( L_s \), all the coils are in contact and the spring rate becomes the stiffness of the solid coils in compression. The spring rate should be defined between about 15\% and 85\% of its total deflection and its working deflection range \( L_a - L_m \) kept in that region.

The equation for spring rate is found by rearranging the deflection equation.

\[ k = \frac{d^4G}{8D^3Na} \]  

**Fig 3.5 Force-Deflection Curve for a standard Helical Compression Spring**
Fig 3.6 Stress distributions across wire in a helical compression spring


3.7 STRESSES IN HELICAL SPRINGS OF CIRCULAR WIRES [6]

There will be two components of stress on any cross section of a coil as shown in Figure 3.4. A torsional shear stress and a direct shear stress due to the force F. These two shear stresses have the stress distributions across the section as shown in Figure 3.6a and 3.6b. The two stresses add directly, and the maximum shear stress $\tau_{\text{max}}$ occurs at the inner fiber of the wire’s cross section, as shown in Figure 3.6c.

\[
\tau_{\text{max}} = \frac{T r}{J} + \frac{F}{A} = \frac{F(D/2)(d/2)}{p d^4/32} + \frac{F}{p d^2/4}
\]

\[
= \frac{8F D d}{p d^3} + \frac{4F}{p d^2}
\]

We can substitute the expression for spring index C from equation 3.1 in equation 3.8.

\[
\tau_{\text{max}} = \frac{8F C}{p d^2} + \frac{4F}{p d^2} = \frac{8FC + 4F}{pd^2}
\]

\[
\tau_{\text{max}} = K_S \frac{8F D}{p d^3}
\]

Where $K_S = \left(1 + \frac{0.5}{C}\right)$

To consider the effect of curvature and direct shear A M Wahl [4] determined the stress-concentration factor for this application.
Since Wahl's factor $K_w$ includes both effects, we can separate them into a curvature factor $K_c$ and the direct shear factor $K_s$ using

$$K_w = K_s K_c$$  \hspace{1cm} (3.14)

$$K_c = \frac{K_w}{K_s}$$  \hspace{1cm} (3.15)

If a spring is statically loaded then yielding is the failure criterion. If the material yields locally it will relieve the local stress concentration that is due to the curvature factor $K_c$ and equation 3.10 can be used to account just for the direct shear. But, if the spring is dynamically loaded, then failure will be by fatigue at stresses well below the yield point, and equation 3.13 should be used to incorporate both the direct shear and curvature effects. In a fatigue-loading case with both mean and alternating loads, equation 3.10 can be used to compute the mean stress component and equation 3.13 is used for the alternating stress component.

### 3.8 BUCKLING OF COMPRESSION SPRINGS

When the free length of the spring ($L_F$) is more than four times the mean or pitch diameter ($D$), then the spring behaves like a column and may fail by buckling at a comparatively low load. The critical axial load ($W_{cr}$) that causes buckling may be calculated by using the following relation, i.e.,

$$W_{cr} = k x K_B x L_F$$  \hspace{1cm} (3.16)

Where $k$ = spring rate or stiffness of the spring $= \frac{F}{y}$

$L_F$ = Free length of the spring, and

$K_B$ = Buckling factor depending upon the ratio $L_F/D$.

The buckling factor ($K_B$) for the hinged end and built-in end springs may be taken from the table 3.6.
Table 3.6 Values of buckling factor ($K_B$)

<table>
<thead>
<tr>
<th>$L_F / D$</th>
<th>Hinged end spring</th>
<th>Built-in end spring</th>
<th>$L_F / D$</th>
<th>Hinged end spring</th>
<th>Built-in end spring</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.72</td>
<td>0.72</td>
<td>5</td>
<td>0.11</td>
<td>0.53</td>
</tr>
<tr>
<td>2</td>
<td>0.63</td>
<td>0.71</td>
<td>6</td>
<td>0.07</td>
<td>0.38</td>
</tr>
<tr>
<td>3</td>
<td>0.38</td>
<td>0.68</td>
<td>7</td>
<td>0.05</td>
<td>0.26</td>
</tr>
<tr>
<td>4</td>
<td>0.20</td>
<td>0.63</td>
<td>8</td>
<td>0.04</td>
<td>0.19</td>
</tr>
</tbody>
</table>

In order to avoid the buckling of spring, it is either mounted on a central rod or located on a tube. When the spring is located on a tube, the clearance between the tube walls and the spring should be kept as small as possible, but it must be sufficient to allow the increase in spring diameter during compression.

3.9 ECCENTRIC LOADING OF SPRINGS

Sometimes, the load on the springs does not coincide with the axis of the spring, i.e., the spring is subjected to an eccentric load. In such cases, not only the safe load for the spring reduces, the stiffness of the spring is also affected. The eccentric load on the spring increases the stress on one side of the spring and decreases on the other side. When the load is offset by a distance $e$ from the spring axis, then the safe load on the spring may be obtained by multiplying the axial load by the factor $\frac{D}{2e + D'}$ where $D$ is the mean diameter of the spring.

3.10 ENERGY STORED IN HELICAL SPRINGS OF CIRCULAR WIRE

Let $F =$ Load applied on the spring, and $y =$Deflection produced in the spring due to the load $W$.

Assuming that the load is applied gradually, the energy stored in a spring is,

$$U = \frac{1}{2} F \cdot y$$

3.17

The maximum shear stress induced in the spring wire,

$$t = K_x \frac{8F \cdot D}{pd^3} \quad \text{or} \quad F = \frac{pd^3 \cdot t}{8K \cdot D}$$

3.18
We know that deflection of the spring,

\[ y = \frac{8F D^3 Na}{G d^4} = \frac{8 \pi d^3 t}{8K D} \cdot \frac{D^3 Na}{G d^4} = \frac{pt D^2 Na}{K d G} \]  

3.19

Substituting the values of \( F \) and \( y \) in equation (3.17), we have

\[ U = \frac{1}{2} x \frac{pd^3 t}{8K D} \cdot \frac{pt D^2 Na}{K d G} \]  

3.20

\[ = \frac{t^2}{4K^2 G} (pdN) \left( \frac{p}{4} xd^2 \right) = \frac{t^2}{4K^2 G} xV \]  

3.21

Where \( V \) = Volume of the spring wire

\[ = \text{Length of spring wire} \times \text{Cross-sectional area of spring wire} \]  

\[ = (pdn) \left( \frac{p}{4} xd^2 \right) \]  

3.22

3.11 COMPRESSION-SPRING SURGE

Any device with both mass and elasticity will have one or more natural frequencies; springs are no exception to this rule and can vibrate both laterally and longitudinally when dynamically excited near their natural frequencies. If allowed to go into resonance, the waves of longitudinal vibrations, called surging, cause the coils to impact one another. The large forces from both the excessive coil deflections and impacts will fail the spring. To avoid this condition, the spring should not be cycled at a frequency close to its natural frequency. Ideally, the natural frequency of the spring should be greater than about 13 times that of any applied forcing frequency.

The natural frequency \( \omega_n \) or \( f_n \) of a helical compression spring depends on its boundary conditions. Fixing both ends is the more common and desirable arrangement, as its \( f_n \) will be twice that of a spring with one end fixed and the other free. For the fixed-fixed case:

\[ \omega_n = \sqrt{\frac{kg}{W_a}} \text{ Rad/sec} \quad f_n = \frac{1}{2} \sqrt{\frac{kg}{W_a}} \text{ Hz} \]  

3.23
Where $k$ is the spring rate, $W_a$ is the weight of the spring's active coils, and $g$ is the gravitational constant. It can be expressed either as angular frequency $\omega_a$ or linear frequency $f_n$. The weight of the active coils can be found from

$$W_a = \frac{\pi^2 d^2 DN_a y}{4}$$  \hspace{1cm} 3.24

Where $\gamma$ is the material’s weight density, $G$ is shear modulus. For total spring weight substitute $N_t$ for $N_a$.

Substituting equation 3.24 in 3.23 gives

$$f_n = \frac{2}{\pi N_a D^2} \sqrt{\frac{G g}{32 \gamma}} \text{Hz}$$  \hspace{1cm} 3.25

The surge in springs may be eliminated by using the following methods.

1. By using friction dampers on the centre coils, so that the wave propagation dies out.
2. By using springs of high natural frequency.
3. By using springs having pitch of the coils near the ends different than at the centre to have different natural frequencies.

3.12 SETTING OF SPRINGS

When a wire is coiled into a helix, tensile residual stresses are developed at the inner surface and compressive residual stresses occur at the outer surface. Neither of these residual stresses is beneficial. They can be removed by stress relieving (annealing) the spring.

Beneficial residual stresses can be introduced by a process confusingly called both “set removal” and “setting the spring” by the manufacturers. Setting can increase the static load capacity by 45-65% and double the spring’s energy-storage capacity of material. Setting is done by compressing the spring to its shut height and yielding the material to introduce beneficial residual stresses. The “set” spring loses some free length but gains the benefits described above. In order to achieve the advantages of setting, the initial free length must be made longer than the desired (post set) length and should be designed to give a stress...
at shut height about 10 to 30% greater than the yield strength of the material. Less than that amount of overload will not create sufficient residual stress. More than 30% overstress adds little benefit and increases distortion.

3.13 SQUARE OR RECTANGULAR SPRINGS [4]

Helical springs of square or rectangular bar section are sometimes used for cases where a large amount of energy must be stored within a given space. Particularly if the spring is coiled flat wise, it is clear that a larger amount of material may be provided within a given outside diameter and compressed length than if a circular section were used. More energy may be stored within a given space for such a design than would be the case if a circular bar section were used. Although the rectangular bar section theoretically does not have as favorable an elastic stress distribution as does the round bar section, for static loading or loads repeated only a few times this disadvantage is of no particular importance, since local yielding of the highest stressed portions can occur without appreciably affecting the performance of the spring or the capacity for storing energy. However, where fatigue or repeated loading of the spring is present, this non uniformity of stress distribution will be a disadvantage. A further disadvantage is the fact that the quality of material used is generally not as good as would be the case where round wire is used; also, the rectangular-bar material may be difficult to procure.

In general, when bar stock of rectangular section is coiled to a helical form, a keystone or trapezoidal shape of cross section finally results, and this tends to reduce the space efficiency and energy storage capacity. Springs with rectangular cross sections having the long side of the section parallel to the axis are sometimes used in the design of precision scales in order to obtain a more nearly linear load-deflection characteristic.

In contrast to round-wire helical compression or tension springs where curvature effects can be neglected in calculating deflections, such effect are particularly important in rectangular-Wire springs coiled flat wise. In such cases neglecting curvature may result in errors 15 per cent or more.

For springs made of rectangular wire, as shown in Fig 3.7, the maximum shear stress is given by
This expression is applicable when the longer side (i.e., \( t > b \)) is parallel to the axis of the spring. But when the shorter side (i.e., \( t < b \)) is parallel to the axis of the spring, then maximum shear stress, \( t = Kw \frac{F.D(1.5t + 0.9b)}{b^2.t^2} \) 3.26

And deflection of the spring,

\[
y = \frac{2.45F.D^3.Na}{G.b^3(t-0.56b)}
\] 3.28

For springs made of square wire, the dimensions \( b \) and \( t \) are equal. Therefore, the maximum shear stress is given by

\[
t = Kx \frac{2.4F.D}{b^3}
\] 3.29

And deflection of the spring,

\[
y = \frac{5.568F.D^3.n}{G.b^4} = \frac{5.568F.C^3.n}{G.b}
\] 3.30

Where \( b = \) Side of the square

Note: In the above expressions,

\[
K = \frac{4C - 1}{4C - 4} + 0.615, \quad \text{and} \quad C = \frac{D}{b}
\]
3.14 DESIGN OF SPRINGS


The primary objective of the spring designer is to design a spring which will do the job required and at the same time be the most economical, all factors considered. This means that the spring must fit into the space available and have a satisfactory life in service not only from the standpoint of fatigue breakage but also from that of excessive relaxation or set in service.

In other cases where failure may endanger life or property, the designer may require maximum reliability even at increased cost. This is the case, for example, in aircraft-engine valve springs. In such cases, more expensive materials, such as valve-spring-quality wire, and additional processing costs, such as those due to shot peening, may be justified. For certain applications, the designer may wish to obtain a spring of minimum weight, volume, or length.

3.14.2 CHOICE OF SPRING MATERIAL

One of the most important decisions to be made by the spring designer is the choice of the proper spring material. Since the primary purpose of most springs is to store energy, and since the energy stored for a given volume of material is proportional to the square of the stress, it is of advantage to use a high-strength material which will permit operation at relatively high stresses. For example, a 10 per cent increase in working stress generally means a 20 per cent reduction in amount of spring material required. This explains the wide use of spring steel, which has relatively high tensile strength, and also explains why springs are generally stressed higher than is the case for other applications.

3.14.3 CHOICE OF WORKING STRESS

Another important decision required of the designer is the choice of working stress. Some of the more important factors governing this choice are: Material Properties, Condition of spring surface, Kind of loading (static or fatigue, number of cycles), Corrosion effects, Creep or load loss at load-elevated temperature,
3.14.4 TYPES OF LOADING

The type of loading to which the spring is subjected is the important factor to consider in designing various types of mechanical springs.

**Static loading-normal temperature.** This category refers to springs subject to a steady load or one repeated, say, less than 100 to 1,000 times. (An example is a spring used to apply gasket pressure.) In such cases the chief problem is usually to avoid excessive set or load loss. Thus, if a helical spring is compressed by a certain amount, the load may drop or relax with time; if the spring is loaded by a constant load, the spring may take a set or creep. In practical design, this relaxation must usually be limited to the initial load.

In general, at normal temperatures if the peak stress in the spring is kept below the elastic limit of the material, trouble from set or relaxation will seldom occur. In the case of springs which are preset, the nominal working stress may, in some cases, be higher than the elastic limit. This is due to favorable residual or trapped stresses which are set up during the presetting operation and which subtract from the load stress.

In the case of statically loaded springs it is common practice to neglect stress-concentration effects in calculating stresses. (Such effects occur because of sharp bends, holes, notches, etc.) This is justified since the usual spring material has sufficient ductility that stress relief may occur at localized points.

**Static loading-elevated temperatures.** At elevated temperatures, it is found that the effects of creep or relaxation become much more pronounced than at normal temperatures, i.e., the springs tend to set or lose load more rapidly. This load loss or relaxation is also a function of time. For example, for temperatures around 250 to 350°F, music or oil-tempered wire helical springs may be used, but for higher temperatures other materials such as stainless steel, Monel, Inconel and Inconel "X" are generally required. In such cases the design must be based on the creep and relaxation properties of the material at the operating temperature.
Fatigue or repeated loading. This category includes many spring applications where the load does not remain constant but varies with time. For example, an automotive-engine valve spring is initially compressed by a certain amount in assembly. During operation it is compressed periodically from a minimum shear stress $t_{\text{min}}$ to a maximum value $t_{\text{max}}$. This case is similar to the general case of fatigue loading shown in Fig. 3.8, where an alternating bending stress component $\sigma_a$ is superimposed on a static or mean component $\sigma_o$. Thus the stress cycle is from a minimum stress $s_{\text{min}}$ to a maximum stress. The difference $s_{\text{max}} - s_{\text{min}}$ is known as the stress range $\sigma_r$. Such a stress cycle may be repeated millions of times in certain spring applications (valve springs) while in others, it may occur perhaps only a few times. The allowable stresses in the two cases are considerably different. In general, the stress range $\sigma_r$ is of particular importance where fatigue is involved, since for many materials the limiting endurance range is approximately constant provided the yield stress is not exceeded.

3.14.5 SURFACE CONDITIONS, DECARBURIZATION, SHOT PEENING

Fatigue tests on helical or leaf springs show that the surface condition is particularly important where repeated loading is involved. In hot-wound helical springs especially, decarburization of a thin surface layer usually occurs, and this reduces fatigue life. Surface defects such as quench cracks, pits, coiling-tool marks, and seams are also detrimental to spring endurance. Hence, where repeated loading is present, such defects should be avoided, while the amount of decarburization should be kept to a minimum.
In general, it is also advantageous to use the shot-peening process for springs subject to fatigue loading. This process consists in propelling hardened steel shot at high velocity against the spring surface with an air blast or centrifugal-type machine. This peening action cold-works the surface and sets up beneficial compressive stresses, both of which tend to increase the endurance strength. This process, however, cannot compensate for an excessive thickness of decarburized layer or for defects present.

3.14.6 VARIATIONS IN DIMENSIONS

Another factor which the spring designer should keep in mind is that there is always an unavoidable variation in the size of wire or plate used in making springs. The effect of these variations may often be large, especially when it comes to obtaining proper load-deflection characteristics. For example, in the case of helical springs, a cumulative variation in both coil and wire diameter of only 1 per cent will result in a 7 per cent change in the load-deflection characteristic of the spring. Thus for 0.1-in, wire, a 1 per cent variation would correspond to a change in diameter of only 0.001 in. Such variations are easily possible in commercial practice. Hence, it may be necessary to allow the spring manufacturer some leeway in choosing the other spring dimensions to compensate for unavoidable variations in sizes of commercial wire stock.

3.15 DESIGNING HELICAL COMPRESSION SPRINGS FOR STATIC LOADING

The functional requirements for a spring design can be quite varied. There may be a requirement for a particular force at some deflection, or the spring rate may be defined for a range of deflection. In some cases there are limitations on the outside diameter, inside diameter or working length. The approach to design will vary depending on these requirements. In any case, spring design is inherently an iterative problem. Some assumptions must be made to establish the values of enough variables to calculate the stresses, deflections, and spring rate. Because wire size appears to the third or fourth power in the stress and deflection equations, and because material strength is dependent on wire size, the safety of the design is very sensitive to this parameter.
Many approaches may be taken to spring design, and more than one combination of spring parameters can satisfy any set of functional requirements. It is possible to optimize parameters such as spring weight for a given set of performance specifications. To minimize weight and cost, the stress levels should be made as high as possible without causing static yielding in service.

A trial wire diameter \( d \) should be assumed and a reasonable spring index \( C \) chosen, from which the coil diameter \( D \) can be calculated using equation 3.1. A trial spring material is chosen and the relevant material strengths calculated for the trial wire diameter. It is convenient to calculate the stress before computing the deflection because, both involve \( d \) and \( D \). Only the deflection depends on \( N_a \). If a required force \( F \) is defined, the stress at that force can be computed with equation 3.10 or 3.13 as appropriate. If two operating forces are defined with a specified deflection between them, they will define the spring rate.

The stress state is compared to the yield strength for static loading. The safety factor for static loading is

\[
N_s = \frac{S_{ys}}{t} \tag{3.31}
\]

If the calculated stress is too high compared to the material strength, the wire diameter, spring index, or material can be changed to improve the result. When the calculated stress at the required operating force seems reasonable compared to the material strength, a trial number of coils and a clash allowance can be assumed and further calculations for spring rate, deflection, and free length done using equations 3.5 and 3.6. Unreasonable values of any of these parameters will require further iteration with changed assumptions.

After several iterations, a reasonable combination of parameters can usually be found. Some of the things that need to be checked before the design is complete will be the stress at shut height, and the \( D_i, D_o \) and free length of the coil with respect to packaging considerations. In addition, the possibility of buckling needs to be checked.
3.16 DESIGNING COMPRESSION SPRINGS FOR FATIGUE LOADING

Helical compression springs in many applications are subjected to variable or fatigue load. For e.g., the valve spring of an IC engine may be subjected to several millions of stress reversals during its life cycles. Those springs which can carry at least 10 million stress reversal cycles before fatigue failure are called springs with infinite life. Springs subjected to fluctuating stress are designed on the basis of fatigue failure. However, there is a basic difference of the fatigue failure of the rotating shaft and that of a spring. A helical compression spring is never subjected to a completely reversed loading as observed in a rotating shaft. The load on these springs may be either a variable compressive force, or a pulsating force which varies between zero and maximum compressive force $F_{\text{max}}$. Fig 3.9 shows a variable force on a helical spring varying between $F_{\text{max}}$ and $F_{\text{min}}$, where $F_{\text{min}}$ is the force due to initial compression of the helical spring. In the worst condition $F_{\text{min}}$ may be zero as shown in fig 3.10.

![Fig 3.9 Variable load on a compression spring](image)

![Fig 3.10 Pulsating load on compression spring](image)

Let us now consider a spring subjected to a force varying between $F_{\text{max}}$ and $F_{\text{min}}$ in a load cycle the mean and amplitude forces are given as:

Mean force, $F_m = \frac{F_{\text{max}} + F_{\text{min}}}{2}$

3.32
Amplitude force, \( F_a = \frac{F_{\text{max}} - F_{\text{min}}}{2} \) \hspace{1cm} \text{(3.33)}

The mean torsional shear stress \( t_m = K_s \frac{8F_mD}{\Pi d^3} \) \hspace{1cm} \text{(3.34)}

Where \( K_s = \text{shear stress concentration factor} = 1 + 0.5/C \) It is used only for mean shear stress calculation.

The torsional shear stress amplitude

\[ t_a = K \frac{8F_aD}{\Pi d^3} \] \hspace{1cm} \text{(3.35)}

Where \( K \) is the Wahl's stress corrective factor.

The design of a spring under fatigue load conditions is based upon the modified soderberg line approach as shown in fig 3.11. Accordingly the point A can be found by drawing a line OA inclined at 45° with the X-axis, which intersects the original soderberg line at point A. The point A on this diagram indicates the limiting value of stress due to pulsating load condition. The point B on the X-axis indicates the limiting value of stress due to static load condition. Thus the line joining the points A and B is called soderberg line of failure. To take into account the effect of factor of safety (FoS), a line CD is drawn parallel to the line AB, from point D on the X-axis where OD = \( t_y/FoS \), The line CD is called the modified soderberg line of failure.

According to the soderberg hypothesis, any point lying either on line CD or within triangle O'CD is considered safe. The modified design equation is

\[ \frac{1}{FoS} = \frac{t_m - t_a}{t_y} + 2t_a/t_n \] \hspace{1cm} \text{(3.36)}

\[ 
\begin{array}{c}
\text{Fig 3.11 Modified soderberg line}
\end{array} \]