Chapter 8

The flow of nucleons and fragments

8.1 Introduction

The nuclear reaction dynamics at incident energies between 10A MeV and 2A GeV yields several interesting phenomena: The reaction dynamics at low energies is governed by the attractive mean field whereas the repulsive interactions decide the fate of the reaction at higher incident energies. It has been observed that the interactions at higher incident energies are dominated by the nucleon-nucleon scattering which causes the particle emission in forward center-of-mass angles and hence flow is positive. On contrary, the interaction at low incident energies is dominated by the attractive part of the nuclear mean field which results in the particle emission at backward angles and hence collective flow is negative. While going from low to high bombarding energies, the attractive and the repulsive part of the interaction balances each other at some bombarding energy and hence flow disappears. This energy at which flow disappears is termed as the balance energy [1].

The origin of collective flow is based on the assumption that the relative momentum between the interacting nucleons is rather small at the start of reaction. As soon as two nuclei overlap, the particles with very large relative momenta are closely packed in the configuration space as shown in fig 8.1 (a & b). When two nuclei approach each other, they feel an attractive potential due to the geometry causing the collective transverse momentum in the direction of the impact parameter. As soon as the nuclei overlap, the potential becomes repulsive and accelerates the particles in transverse direction as shown
Figure 8.1: Transverse momentum caused by momentum dependent forces. (a) The reaction in the beam-impact parameter plane. (b) The potential along the z-axis. We see in the overlap region a strong repulsive and outside it an attractive potential. Hence the particles want to leave the overlap plane by gaining transverse momentum. Figure is taken from the ref. [2].

Figure 8.2: Pictorial display in the in-plane bounce off caused by compression. One can see that the forward moving particles have opposite transverse momentum to the backward moving particles. Also displayed is the squeeze out, the enhanced emission of light particles perpendicular to the reaction plane close to mid-rapidity. Figure is taken from the ref. [2].
in fig 8.2. Due to very strong potential gradient, the deflected particles attain an appreciable amount of the transverse momenta. This transverse momentum causes the system to expand radially, thus decreasing density.

In this chapter, we would like to understand the collective flow of nucleons and fragments and its disappearance. We shall use different colliding nuclei apart from large number of impact parameters and incident energies. We shall also make an attempt to discuss the role of different nucleon-nucleon cross sections and equations of state.

8.2 Results and discussion

8.2.1 Effect of different NN cross sections on nuclear flow

In fig. 8.3, we plot the \( \langle p_x/A \rangle \) as a function of \( Y_{cm}/Y_{beam} \). Here simulations are carried out for Au-Au reaction at impact parameter of 4 fm and at incident energy 600A MeV. Once the reaction is over, the clusterization is performed within MST method. The upper, middle and lower parts of the figure represent the free-nucleons, LMF's (2 ≤ A ≤ 4) and IMF's (5 ≤ A ≤ 65), respectively. As reported by other authors [3], the nucleons exhibit less flow compared to fragments. These findings are in agreement with experimental results. In addition, the nuclear flow increases drastically with an increase in the cross section. We see that a larger cross section of 55 mb allows more nucleon-nucleon collisions which results in much higher transverse momentum.

8.2.2 Effect of different equations of state on nuclear flow

The role of different equations of state (eos) and their momentum dependence on flow is depicted in figure 8.4. Here Xe+Sn reaction is simulated with Hard, Soft, Hard with momentum dependence (HMD) and Soft with momentum dependence (SMD), respectively. The Soft eos produces lesser particle flow compared to Hard equation of state. It is also interesting to note that the momentum dependence leads to quite different nuclear flow. The main cause is that the momentum dependence produces repulsion in the
Figure 8.3: Averaged $\langle p_x/A \rangle$ as a function of $Y_{cm}/Y_{beam}$. The various curves indicate the results with different cross sections. The solid line is for Cugnon, long dashed line for $\sigma = 40$ mb and dotted line for $\sigma = 55$ mb, respectively.
Figure 8.4: Averaged \( \langle p_x / A \rangle \) as a function of \( Y_{\text{cm}} / Y_{\text{beam}} \). The various curves indicate the results with different equations of state. The solid line is for Hard, long dashed line for Soft, dotted line for HMD and dash-double-dotted line for SMD, respectively.
medium which results in diverting the nucleons towards transverse direction during the early phase of the reaction. This deflection leads to larger transverse flow at all incident energies and impact parameters.

In the following, we shall first demonstrate that the \( \langle p_x/A \rangle \) and directed transverse momentum \( \langle p_x^{\text{dir}} \rangle \) are equivalent quantities. Then, we shall discuss the role of different nucleon-nucleon cross sections in disappearance of flow.

### 8.2.3 Time evolution of collective flow and \( \langle p_x/A \rangle \)

There are several methods of defining the nuclear (sideways) flow. Usually, the balance energy is extracted from \( \langle p_x/A \rangle \) plots i.e., the in-plane transverse momentum as a function of the normalized rapidity \( (Y_{cm}/Y_{beam}) \). By a linear fit to the slope of these curves, one can define the so called reduced flow (F). By plotting F as a function of the beam energy, one obtains the balance energy \( E_{bal} \) as a point where a linear fit of the energy dependence of F passes through zero. This \( \langle p_x/A \rangle \) plot is similar to function of the rapidity bins. One can have a rather more integrated quantity called the "Directed transverse momentum" \( \langle p_x^{\text{dir}} \rangle \) which is given by eq. 5.2 [2, 4]. In transverse momentum, all rapidity bins are taken into account. Therefore, this provides one value as a measure of the in-plane flow instead of a complicated function such as the \( \langle p_x/A \rangle \) plot.

In fig. 8.5, we show the time evolution of \( \langle p_x^{\text{dir}} \rangle \) at different bombarding energies using nucleon-nucleon cross sections taken from Cugnon, a constant cross section \( \sigma = 55 \) mb, 40 mb and 20 mb, respectively. Note that the different curves in each part of the figure 8.5 are at different incident energies. The a, b, c and d parts of the figure 8.5 are, respectively, with cross section of 55 mb, 40 mb, Cugnon, and 20 mb. We see that the flow first becomes negative and finally it saturates either at -ve or +ve values depending on the bombarding energy. The negative flow at the start of reaction signifies the attractive nuclear interactions at the start. At low incident energies, due to the lack of available phase space, we have few nucleon-nucleon collisions and as a result, the nuclear interactions are
Figure 8.5: The time evolution of averaged $\langle p_{x}^{\text{dir}} \rangle$ as a function of time. Here we simulate $^{64}\text{Zn} + ^{27}\text{Al}$ for 500-2000 events at each bombarding energy. (a), (b), (c), and (d) are the simulations using $\sigma=55$, 40 mb, Cugnon and 20 mb, respectively. Note that different lines in different parts of the figure represent different energies.
still attractive. At higher incident energies, however, one has frequent nucleon-nucleon collisions and therefore, nuclear flow is repulsive at the end of the simulations. One also notices that a larger cross section can have more (allowed) collisions and thus, the flow becomes positive at lower incident energies compared to that of smaller cross section. The flow \( \langle p_x^{dir} \rangle \) becomes zero at 60A, 70A, 100A and 120A MeV for simulations with \( \sigma = 55 \) mb, 40 mb, Cug/GMC, and 20 mb, respectively. One also notices that it take more time at low incident energies before the flow saturates. For example, the simulations with 55 mb at 40A MeV takes about 120 fm/c before they saturate whereas the same reaction at 200A MeV yields the saturated flow as early as 40 fm/c.

In fig. 8.6, we plot \( \langle p_x/A \rangle \) as a function of \( Y_{cm}/Y_{beam} \). We show each plot at a fixed energy using five different nucleon-nucleon cross sections. The a,b,c, and d parts of the figure 8.6 are at incident energy of 60A, 70A, 100A and 120A MeV, respectively. Note that \( \langle p_x^{dir} \rangle \) vanishes at 60A, 70A, 100A and 120A MeV, respectively, for the reactions with \( \sigma = 55 \) mb, 40 mb, Cug, and 20 mb. (see fig. 8.5). The slope of the \( \langle p_x/A \rangle \) with \( \sigma = 55 \) mb is almost zero at 60A MeV whereas all other cross sections yield a negative flow. In the same way, the slope of the \( \langle p_x/A \rangle \) vanishes at 70A, 100A and 120A MeV, respectively, for simulations with \( \sigma = 40 \) mb, Cug/GMC and 20 mb. One also notices that the maximum flow is seen with \( \sigma = 55 \) mb. This is followed by \( \sigma = 40 \) mb and Cug/GMC. Comparing figs. 8.5 and 8.6, it is clear that \( \langle p_x^{dir} \rangle \) and the slope of \( \langle p_x/A \rangle \) vanishes at same incident energies. The \( \langle p_x^{dir} \rangle \) is negative at incident energies where the \( \langle p_x/A \rangle \) plot yields a negative slope. From this analysis, it is evident that the \( \langle p_x/A \rangle \) and \( \langle p_x^{dir} \rangle \) are equivalent quantities. This analysis is also in agreement with previous analysis of ref. [4].

### 8.2.4 The balance energy

In recent years, several efforts have been made to pin down balance energy accurately. Several different kind of experimental attempts are made. First category of experiments deal with the central collisions and one determines the balance energy for collisions in-
Figure 8.6: Averaged $\langle p_x/A \rangle$ as a function of $Y_{cm}/Y_{beam}$. Here each part of the figure is a result at fixed incident energies. The various curves in each part of the figure indicate the results with different nucleon-nucleon cross sections. The solid line is for Cugnon, long dashed line for GMC, dotted line for 55 mb, short dashed line for 40 mb, and dash-dotted for 20 mb. (a), (b), (c) and (d) are at incident energies of 60A, 70A, 100A, and 120A MeV, respectively.
volving a variety of colliding partners [5, 6, 7]. The colliding partners can be as light as carbon or as heavy as La. The balance energy in central collisions is found to vary as \( A_{\text{tot}}^{\text{-1/3}} \) (where \( A_{\text{tot}} \) is the total mass of target + projectile)[7]. The (measured) balance energy for the central collision of C + C is 127 ± 5 A MeV whereas it is about 50 A MeV for La + La [5, 6, 7]. Apart from the mass dependence of the disappearance of flow, another factor which affects the balance energy is the impact parameter [3, 8, 9, 10, 11, 12]. Due to less compression in peripheral collisions, a large value of incident energy is needed to compensate the attractive mean field and hence the flow disappears at larger incident energy compared to central collisions. Recently, several systematic experiments have been carried out where impact parameter dependence of the disappearance of flow is investigated [8, 10, 11] He et al. did the experiment of \(^{64}\text{Zn} + ^{27}\text{Al}\) [8] whereas Pak et al. performed \(^{40}\text{Ar} + ^{45}\text{Sc}\) [10, 11]. The unique outcome of these experiments is that the balance energy increases linearly with change in the impact parameter.

A study using the BUU model was also carried out by He et al. [8]. They simulated the \(^{64}\text{Zn} + ^{27}\text{Al}\) reaction using the BUU model by varying the cross section between 25 and 55 mb. They found that a smaller cross section explains the data in central collisions whereas a larger cross section is needed to explain the data at semi-central/peripheral collisions.

In fig. 8.7, we show the nucleonic flow (i.e. the transverse nucleonic flow \( p_x^{\text{dir}} \)) as a function of incident energy \( E \). Here we perform the simulations with a stiff equation of state. The results shown with open circles, triangles, filled triangles, filled squares and open diamonds show the simulations with nucleonic cross sections taken from Cugnon, G-matrix, a constant \( \sigma = 55 \text{ mb}, 40 \text{ mb} \) and 20 mb, respectively. We also show the experimental data of He et al [8]. We see several points: (i) At \( b = 1 \text{ fm} \), the standard energy dependent cross section or in-medium G-matrix cross section explains the experimental \( E_{\text{bal}} \) nicely. The energy dependent/G-matrix cross section starts deviating from the experimental data as we shift towards larger impact parameters. Note that the experimental \( E_{\text{bal}} \) is 75 ± 5, 79 ± 5, and 90 ± 5 MeV/c, respectively at \( b = 1, 2.5 \) and
Figure 8.7: The average $\langle p_x^{\text{dir}} \rangle$ as a function of incident energy. The results obtained with $\sigma = \text{Cug, G-matrix, 55, 40 and 20 mb}$ are shown, respectively, by circle, triangle, filled triangles, filled squares, and diamonds. The experimental data is indicated by an asterisk. The upper, middle and lower parts of the figure are at impact parameter $b = 1, 2.5$ and 4 fm, respectively.
4 fm. Whereas the simulations with the Cugnon (G-matrix) cross section predict the $E_{bal}$ at 72 MeV/c (74 MeV/c), 102 MeV/c (98 MeV/c) and 230 MeV/c (180 MeV/c), respectively, at $b = 1, 2.5$ and 4 fm. As the observed balance energy at $b = 4-5$ fm is an estimation, we simulate the collisions at $b = 4$ fm (rather than at 4.5 fm). We see that the balance energy obtained with in-medium cross section does not differ much compared to the energy dependent cross section. Therefore, the effect of the in-medium cross section is rather small as far as the vanishing of flow is concerned. The $E_{bal}$ obtained with $\sigma = 55$ mb overestimates the experimental $E_{bal}$ at $b = 1$ fm, whereas it explains $E_{bal}$ at $b = 4$ fm nicely. $E_{bal}$ at 2.5 fm can be explained nicely with a constant cross section of 40 mb.

The failure of the standard cross section to reproduce the balance energy at larger impact parameters has been reported in several other references [8, 10, 11, 13]. In ref. [10], the QMD simulations (for the Ca-Ca system) explain the $E_{bal}$ at smaller impact parameters. Similar results are also reported for the collision of $^{64}Zn + ^{27}Al$ [8]. This failure of standard cross section to reproduce the experimental balance energy at large impact parameter could be resolved if one takes the momentum dependence of nuclear force into account [11]. In our case, we have an additional repulsion due to large nucleon-nucleon cross section. Both the momentum dependent interaction and large cross section lead to the same effect: i.e. both produce more repulsion. In the case of multi-fragmentation, both are found to give more fragments [14, 15].

8.3 Summary

The nuclear flow and its sensitivity to different ingredients of the model is discussed in present chapter. From our study, it is very clear that the magnitude of cross section and different forms of equation of state and their momentum dependence influences the collective flow drastically. All these ingredients are used in all dynamical models as their inputs. Our results also indicate that $\langle p_z^{dir} \rangle$ is equally good quantity to study the vanishing of flow. The nuclear flow studied within the QMD [2, 16] model gives a well known behaviour i.e. the nuclear flow is negative at low incident energies and decreases with
the increase in the bombarding energy and vanishes at some bombarding energy. With further increase in the bombarding energy, the nucleonic flow increases. The balance energies is quite sensitive towards different forms of nucleon-nucleon cross sections. We also notice that the standard energy dependent (and in-medium) cross sections results in similar balance energy and explain the experimental data at semi-central collisions, whereas a larger value of the nucleon-nucleon cross section is needed at peripheral collisions to explain the data.
Bibliography


