Chapter 6

Different nucleon-nucleon cross sections and multi-fragmentation

6.1 Introduction

The dynamics of a reaction is governed by the mean field (or mutual two- and three-body interactions) and by the nucleon-nucleon cross section. Both these (important) ingredients have different domain of dominance. At low energies, the two-body collisions are nearly absent and therefore, the mean field (mutual two- and three-body interactions) dominates the reaction whereas the nucleon-nucleon collisions take over the picture at relativistic energy. Large efforts are being made to pin down the equation of state and the magnitude of different nucleon-nucleon cross sections. The main hindrance in these attempts is that both these ingredients depend crucially on each other. Therefore, it is hard to pin down these ingredients accurately. In this chapter, we shall concentrate on the importance of nucleon-nucleon cross section in multi-fragmentation [1]. The role of different equations of state will be presented in chapter 8.

At very low energies, the free nucleon-nucleon cross section increases from 40mb to 1b. This does not mean that we have to incorporate such a large scattering cross section in the study of heavy ion collisions. At low energies, the NN collisions are blocked by Pauli principle. Hence, the free-scattering cross section should be replaced by the cross section in the nuclear medium. Due to lack of any reliable calculation, a constant cross section of 40 mb was used in the past. Now Tubingen group has developed a medium
dependent cross section (G-matrix) [2, 3, 4, 5, 6]. Several attempts are also reported in the literature, where the medium effects are incorporated by rescaling the free nucleon-nucleon cross section by certain percent [7, 8]. At low incident energies, the impact of nucleon-nucleon (NN) collisions is small. Their impact increases with increase in the bombarding energy. Though the role of different NN cross sections on observables such as transverse momentum and the disappearance of flow has been investigated extensively [9], the role of different NN cross sections on multi-fragmentation still needs a detailed investigation [1]. Here we plan to present a systematic study of the role of different NN cross sections in fragment formation by employing several NN cross sections which include the energy-dependent, a constant or even an in-medium cross sections derived from G-matrix [1]. We shall also look for the role of the angular distribution of the cross section on fragment production. The role of different nucleon-nucleon cross sections in explaining the disappearance of nuclear sideward flow will be discussed in chapter 8.

6.2 Different nucleon-nucleon cross sections

During the propagation, two nucleons can collide if they come closer than a certain distance. The scattering of these nucleons is decided by a Monte-Carlo procedure which is a stochastic scattering and hence is different from the Rutherford scattering. For nucleon-nucleon cross section $\sigma$, one can use a simple parametrization which depends on the center-of-mass energy of nucleons [10]. A more realistic cross section which takes care of the isospin of resonances and decay of resonances was also available [11]. In most of the parametrizations of NN cross section, the following processes are always there:

\[
\begin{align*}
N + N &\rightarrow N + N \quad (a), \\
N + N &\rightarrow N + \Delta \quad (b), \\
N + N &\rightarrow \Delta + \Delta \quad (c), \\
N + \Delta &\rightarrow N + N \quad (d), \\
\Delta &\rightarrow N + \pi, \\
N + \Delta &\rightarrow N + \Delta \quad (e), \\
\Delta + \Delta &\rightarrow \Delta + \Delta \quad (f).
\end{align*}
\]

(6.1)

The phase space of scattered nucleons is checked with so called classical Pauli-blocking...
method. If the phase space of scattered nucleons is already occupied, the scattering is forbidden. In the following, we first describe different nucleon-nucleon cross sections which will be used for our analysis.

6.2.1 Energy dependent cross section

For elastic channels, we use the total and differential cross section as [12]:

\[
\sigma_{nn}(\sqrt{s}) = \begin{cases} 
55 \text{(mb)}, & \text{if } \sqrt{s} < 1.8993 \\
\frac{35}{1+100(\sqrt{s}-1.8993)} + 20, & \text{if } \sqrt{s} \geq 1.8993 
\end{cases} 
\]

(6.2)

with \(\sqrt{s}\), the nucleon-nucleon center-of-mass energy given by:

\[
\sqrt{s} = \sqrt{(E_1 + E_2)^2 - (P_1 + P_2)^2}.
\]

(6.3)

The angular distribution for these channels is given by

\[
\frac{d\sigma}{dt} = a e^{bt}; \quad t = -2p^2(1 - \cos\theta).
\]

(6.4)

For inelastic channels, the total cross section is parametrized as

\[
\sigma_{nn\rightarrow n\Delta}(\sqrt{s}) = \begin{cases} 
0, & \text{if } \sqrt{s} < 2.015 \\
\frac{20(\sqrt{s}-2.015)^2}{0.015^2+(\sqrt{s}-2.015)^2}, & \text{if } \sqrt{s} \geq 2.015 
\end{cases} 
\]

(6.5)

the angular distribution for in-elastic channels is assumed to be isotropic.

The cross section for \(\Delta\) absorption, i.e. channel (d) can be obtained from equation (6.4) using detailed balance principle.

\[
\sigma_{n\Delta\rightarrow nn} = \frac{1}{8}(p_f^2/p_i^2)\sigma_{nn\rightarrow n\Delta}(\sqrt{s}),
\]

(6.6)

These parametrized forms of the cross sections are in fact a fit to the experimental measurements. The limit of \(\sqrt{s}=1.8993\) GeV (in eq. 6.2) is based on the fact that the mass of two nucleons is roughly equal to 1.876 GeV. Therefore, for two colliding nucleons with very small velocity, a constant cross section (= 55 mb) is used. The mass limit of the inelastic channel (i.e. \(\Delta\) formation in eq. 6.5) is based on the fact that the mass of \(\Delta(= N + \pi)\), is 1.076 GeV. Therefore, for \(NN \rightarrow N\Delta\) channel, the outgoing mass should
Figure 6.1: The Cugnon parametrization for the elastic (solid line) and inelastic (dashed line) cross sections of NN scattering as function of the incident energy $E_{\text{lab}}$. Figure taken from the ref. [13].
be at least 1.076+0.938 GeV.

The graphical representation of the elastic and in-elastic parts of the cross section is displayed in fig. 6.1. One sees clearly that the elastic cross section falls sharply in its initial stage and then saturate around 20 mb. Whereas, the in-elastic cross section starts increasing with increase in the bombarding energies. At very high energies, the NN cross section is fully dominated by the in-elastic channel.

6.2.2 The in-medium cross section

We shall discuss the in-medium cross section derived by Faessler and collaborator [3]. At low incident energies (E\leq 400A \text{ MeV}), the Pauli-blocking of intermediate states is quite important. This effect is reduced at higher incident energies. The Pauli-principle blocks about 4% collisions at 2A GeV. Therefore, at low incident energies it is very important to take care of the in-medium effects. The NN cross section in nuclear medium can be calculated from the G-matrix [2, 3, 4, 5] which is a solution of the Bethe-Goldstone equation [3]:

\[
\langle \hat{k}_1, \hat{k}_2 | G(W) | k_1, k_2 \rangle = \langle \hat{k} | G(W, K) | k \rangle \\
= \langle \hat{k} | V | k \rangle + \int \frac{d^3 k'}{(2\pi)^3} \langle \hat{k} | V | k' \rangle \\
\times \frac{Q_F(k', K)}{W - E(k', K) + i\eta} \langle k' | G(W, K) | k \rangle,
\]

where \( K = \frac{1}{2}(k_1 + k_2) \), \( k = \frac{1}{2}(k_1 - k_2) \), and

\[
E(k, K) = \epsilon(k_1, \rho_1) + \epsilon(k_2, \rho_2), \\
\epsilon(k, \rho) = \frac{k^2 k^2}{2m} + \text{Re}U(k, \rho).
\]

The \( Q_F \) in equation (6.7) is the Pauli operator for two (colliding) nuclear matters whose momentum distribution \( F \) is given by two overlapping Fermi spheres. The mean field potential \( U \) of the single-particle energy \( \epsilon \) is calculated from the G-matrix in a self-
consistent way:

\[ U(k,\rho) = \frac{1}{4} \sum_{\text{spin, isospin}} \int d^3k \frac{d^3\hat{k}}{(2\pi)^3} \langle k, \hat{k} | G | k, \hat{k} \rangle. \]  

(6.9)

The mean field potential derived from the G-matrix is a momentum dependent. This dependence is usually approximated by an effective nucleon mass.

By using the standard angle averaging procedure for the Pauli operator and for the single particle energy, one can obtain the decoupled partial wave Bethe-Goldstone equation [3]:

\[ \langle k, \hat{L}S\hat{J} | G | k, LSJ \rangle = \langle k, \hat{L}S\hat{J} | V | k, LSJ \rangle + \frac{2}{\pi} \sum_{L'} \int dk'' k''^2 \langle k', \hat{L}S\hat{J} | V | k'', L'SJ \rangle \times \frac{Q_F(k'', K)}{W - E(k'', K) + i\eta} \langle k', L'SJ | G | k, LSJ \rangle, \]  

(6.10)

where \( Q_F \) and \( \bar{E} \) are the angle-averaged quantities. The Bethe-Goldstone equation has the same structure as the Lippmann-Schwinger equation for the free two-body scattering. In this sense, the G-matrix (which is a solution of the Bethe-Goldstone equation) can be regarded as the two-body scattering amplitude in the presence of a nuclear medium. The differential scattering cross section can then be calculated in a straight forward way:

\[ \frac{d\sigma}{d\omega} = \frac{1}{4} \sum_{m_s, m_s', m_s''} |T_{m_s, m_s'}^{S=1}(\theta)|^2 - |T_{S=0}(\theta)|^2, \]  

(6.11)

with

\[ T_{m_s, m_s'}^{S}(\theta) = \sum_{LLJ} \sqrt{\frac{2L+1}{4\pi}} y_{L, m_s, -m_s'}(\theta, 0) \langle L0Sm_s | Jm_s \rangle \times \langle \hat{L} m_s - \hat{m}_s | S\hat{m}_s | Jm_s \rangle / \langle k, \hat{L}S\hat{J} | G | k, LSJ \rangle. \]  

(6.12)

The in-medium NN cross section obtained from the G-matrix is shown in the fig. 6.2 as a function of the nucleon energy (in the laboratory frame) for two different bombarding energies represented by the relative momentum \( K_r \). We here display the results at three different densities \( \rho / \rho_0 \) [5]. The solid line shows the free nucleon-nucleon cross section calculated using Reid soft core potential. The dashed, dash-dotted and dash-double-dotted
Figure 6.2: The in-medium $NN$ cross section based on the $G$-matrix for different incident energies (represented by the relative momentum $K_r$) and densities. The dashed, dash-dotted and dash-double-dotted lines represent, respectively, the total density $\rho/\rho_0 = 1, 1/2$ and $1/4$. Figure is taken from the ref. [5].
lines represents, respectively, the total density $\rho/\rho_0=1$, $1/2$ and $1/4$.

### 6.2.3 A constant cross section

At low energies, several calculations are reported where a constant cross section is also used [9]. Here, we shall also use an isotropic and energy dependent NN cross sections $\sigma_{NN} = 55, 40$ and $20$ mb, respectively.

With five different varieties of NN cross sections (i.e. an energy-dependent cross section, a full in-medium cross section, and three isotropic and constant cross sections), we analyze the formation of fragments, their rapidity distribution, fragment flow, etc. In this chapter, we shall present a systematic study which deals with the influence of different nucleon-nucleon cross sections on multi-fragmentation [1]. The fragments are constructed with MST [14] and SACA methods [15], respectively. To make the comparison meaningful, we report the results which comprise a wider range of impact parameters (between $b = 0$ to $10$ fm) at different bombarding energies.

### 6.3 Results and discussion

The present study uses a hard equation of state along with different types of cross sections. We carry out the simulations of the reactions of Xe + Sn nuclei at impact parameters $b = 0, 2, 4, 6, 8$ and $10$ fm and at incident energies $E_{lab} = 100A$ MeV and $400A$ MeV, respectively.

#### 6.3.1 The density distribution

We start with the time evolution of the density, defined as an average over the centroids of all nucleons given by eq. (3.4). Fig. 6.3 shows the density evolution of Xe + Sn at impact parameters $b = 0$, 4 and 8 fm, respectively. The left and right parts of fig. 6.3 are at incident energies of $100A$ MeV and $400A$ MeV, respectively. One sees several
Figure 6.3: Time evolution of nucleonic density [eq. (3.4)] for the collision of Xe+Sn. The left and right parts are at $E_{lab} = 100$ A MeV and 400 A MeV, respectively. Different nucleon-nucleon cross sections are explained in the text.
interesting results: (i) First of all, the $p_{\text{max}}$ for the collisions of Xe+Sn at 100A MeV is nearly the same at all impact parameters whereas the saturated density increases with the increase of the impact parameter. At 400A MeV, the saturation density behaves in the same manner as at 100A MeV. The maximum density ($p_{\text{max}}$) at 400A MeV collisions decreases with increase in impact parameter. (ii) One also notice that $p_{\text{max}}$ at 400A MeV is higher than at 100A MeV. This is true for all impact parameters. At 100A MeV, due to less nucleon-nucleon collisions and due to the attractive mean field, the nuclear matter remains together for a longer time. At 400A MeV, the frequent nucleon-nucleon collisions destroy the initial correlations and hence the nuclear matter disintegrates very fast.

Another interesting feature at peripheral collisions ($b = 8$ fm) is that the different NN cross sections lead to quite different saturation densities. The effect is more visible at 400A MeV. The calculation of the reaction with $\sigma_{NN} = 20$ mb leads to a higher saturation density as compared to the calculation with $\sigma_{NN} = 40$ or 55 mb. Naturally, the smaller the cross section, the smaller the destruction of the initial correlations and hence the nuclear matter remains together for a longer time. This difference in the saturation density is crucial for the fragment formation and therefore different cross sections are expected to play a role in fragment formation at peripheral collisions.

6.3.2 Effect of different cross sections on fragment multiplicity

In fig. 6.4, we show the time evolution of largest fragment $A_{\text{max}}$, emitted nucleons, medium mass fragments [MMF] ($5 \leq A \leq 20$) and heavy mass fragments [HMF] ($21 \leq A \leq 65$) for the central collision of Xe + Sn at incident energies $E_{\text{lab}} = 100$A MeV and 400A MeV, respectively. We notice that the smaller cross section ($= 20$ mb) leads to heavier largest fragments as compared to the simulations with the other cross sections. One also notices that more nucleons are emitted in the simulations with $\sigma_{NN} = 55$ mb at 400A MeV. A larger cross section (55 mb) leads to a maximal number of MMF at $E = 100$A MeV whereas at 400A MeV, all cross sections lead to nearly the same number of MMF’s. The evolution of the HMF has quite a different behaviour. We notice that though
Figure 6.4: Time evolution of heaviest fragment $A^{\text{max}}$, emitted nucleons, medium mass fragment (MMF's) $5 \leq A \leq 20$, and heavy mass fragments (HMF's) ($21 \leq A \leq 65$). Here we report the simulations of $\text{Xe} + \text{Sn}$ at impact parameter $b = 4$ fm. The left and right parts of figures are at $E_{\text{lab}} = 100$ A MeV and $E_{\text{lab}} = 400$ A MeV, respectively.
a larger cross section can produce HMF's, these HMF's are too excited to be stable and as a result they decay into MMF's after a while. The HMF's produced with smaller cross sections have small excitation energy and hence they survive in the collisions. One should keep in mind that MMF's are the fragments which survive during the collision whereas HMF's are the one which are the remnant of either the target or projectile. The overall effect of different nucleon-nucleon cross sections on the fragment formation is small at impact parameter \(b = 4\) fm. It is interesting to see that the in-medium modification of the cross section alone does not have any effect on the fragment formation.

Apart from the dependence on the incident energy, the multiplicity of fragments depends crucially on the impact parameter \([1, 15, 16, 17, 18]\). A lot of effort has been made to pin down the impact parameter experimentally. One has succeeded to couple the impact parameter with a number of charge particles emitted \([17]\).

In fig. 6.5, we demonstrate the influence of different NN cross sections on the size of the heaviest fragment \(A^{\text{max}}\), on the emitted nucleons, on the light mass fragments (LMF) \((2 < A < 4)\) and on the intermediate mass fragments (IMF) \((5 < A < 65)\) as a function of the impact parameter. Here we concentrate on fragments obtained at the end of the reaction (i.e., at 200 fm/c).

In fig. 6.5, one notices that the size of the heaviest fragment which survived the collision depends very strongly on the impact parameter. This fact has been predicted by several authors and has been confirmed experimentally \([17, 18]\). For central collisions at 100A MeV, \(A^{\text{max}}\) is \(\approx 21\), whereas for peripheral collisions \((b = 10\) fm), \(A^{\text{max}}\) is about 107. Though the \(A^{\text{max}}\) for peripheral collisions at 400A MeV is same as that at 100A MeV, the largest mass is much smaller for central collisions. The simulations with larger \(\sigma_{\text{NN}}\) \(= 55\) mb at 400A MeV leads to lighter \(A^{\text{max}}\) compared to smaller \(\sigma_{\text{NN}}\) \(= 20\) mb. This is understandable because a larger \(\sigma_{\text{NN}}\) leads to more NN collisions which will destroy the initial correlations among the nucleons. One also notices that different cross sections have the least effect at 100A MeV but a large effect for semi-central and peripheral collisions.
Figure 6.5: The heaviest fragment $A_{\text{max}}$, emitted nucleons, light mass fragments (LMF’s) ($2 \leq A \leq 4$) and intermediate mass fragments (IMF’s) ($5 \leq A \leq 65$) as a function of impact parameter. The displayed results are of Xe+Sn at $t = 200 \text{ fm/c}$.
For central collisions, the excitation energy is already large and therefore, different cross-sections do not play a decisive role. At semi-central and peripheral collisions, the overlap is small and hence the excitation energy depends directly on the number of collisions the nucleons suffer.

In central collisions which result in a complete disassembly of nuclear matter (we find a $A_{\text{max}} \approx 21$ at 100A MeV and of $\approx 6$ at 400A MeV, the NN collisions derive the reaction. On the other hand, at peripheral collisions ($b = 10$ fm) the reaction mechanism is dominated by mutual two and three body interactions. At semi-central collisions, the dynamics is governed by interplay of collisions and mean field.

If one compares fig. 6.3 (where the density evolution is shown) with fig. 6.5, one concludes that there is a close relationship between fragment formation and saturation density reached in a reaction. The second row in fig. 6.5 gives the nucleons emitted in Xe + Sn collisions. One notices a gradual decrease in the emission of nucleons with an increasing impact parameter. Similar trends have also been seen in recent experimental studies. Naturally, the central collisions lead to more collisions which enhances the emission of more nucleons. At peripheral collisions, more fragments survive and as a result less nucleons are emitted. One also notices that the emission of nucleons is more important at 400A MeV as compared to 100A MeV.

The emission of LMF’s ($2 < A < 4$) is similar to the emission of nucleons. It is clear that fewer nucleons and light fragments should be emitted at lower beam energies and at larger impact parameters. Again the influence of different $\sigma_{\text{NN}}$ on LMF’s production is quite appreciable for semi-central and peripheral collisions at 400A MeV.

The last row which displays the IMF production clearly indicates that at least for the semi-central and peripheral collisions, we cannot neglect the influence of different nucleon-nucleon cross sections. One also sees a different trend of IMF’s production as a function of the impact parameter. At 100A MeV, we observe maximum fragment production in
central collisions. The production remains constant for semi-central collisions. At 400\,\text{A}\,\text{MeV}, the picture has changed. Now we observe a maximum at semi-central collisions. This feature has been observed experimentally for collisions of Au + Au [17, 18]. One also observes that a smaller $\sigma_{NN}$ results in more IMF at central collisions. This effect is more visible at 400\,\text{A}\,\text{MeV}. In central collisions, the relative change for different cross sections is small because a small cross section already destroys all correlations and hence one sees a very small change in the multiplicity of fragments. At 400\,\text{A}\,\text{MeV}, the Pauli blocking is very small and hence even smaller cross section have a sizable influence. When one uses a smaller cross section at peripheral collisions, one sees a big fragment and a number of nucleons and light fragments which seem to be emitted from the big fragment. In other words, simulations with smaller cross section (i.e., 20 \,\text{mb}) lead to a situation where the spectator matter receives less energy and thus it cools down after emitting a few nucleons. In contrast to this, the simulations with larger cross sections at 400\,\text{A}\,\text{MeV} are able to transfer an appreciable amount of energy and momentum to the spectator matter which results in breaking of spectator into several small and medium size pieces. In a recent article, ALADIN experiment showed that the QMD model with a standard cross section (i.e., with the Cugnon cross section) fails to explain the multiplicity of the observed Au + Au collisions at 400\,\text{A}\,\text{MeV} and 600\,\text{A}\,\text{MeV} [17]. In this comparison, QMD simulations were analyzed with the normal minimum spanning tree method. Recently, we analyzed the same data with QMD coupled with a new algorithm [15] and were able to explain the experimental data. In ref. [17], one of the possible causes given was that in QMD less energy is transferred from the participant zone to the spectator zone and hence results in the emission of heavier $A_{\text{max}}$ and lesser IMF’s. From our results, it seems that the simulations with larger $\sigma_{NN}$ emit more IMF and also it peaks at larger impact parameters. These findings can help in explaining the ALADIN experimental results. Note that in ref. [17], the QMD simulation (coupled with MST) does not produces less IMF’s but at the same time it peaks at smaller impact parameters. Our analysis shows that larger $\sigma_{NN}$ produces more IMF’s at semi-central and peripheral collisions and at the same time the distribution peaks at larger impact parameters. Therefore, heavy-ion simulations with a larger cross section can explain the experimental results of the ALADIN experiments.
Note that different cross sections at 100A MeV have a very small influence on fragment production. The QMD model already explains the fragment distribution at 100A MeV nicely [18].

As discussed earlier, apart from the different dependence of cross sections on energy and the density of the medium, the associated question is to look for the effect of angular dependence of cross sections on fragment production. In fig. 6.6, we display the heaviest fragment $A_{\text{max}}$, the multiplicities of emitted nucleons, LMF ($2 \leq A \leq 4$) and IMF ($5 \leq A \leq 65$) as a function of the impact parameter using the standard Cugnon cross section (Cug) and isotropic Cugnon cross section (iso-Cug). It is worth reminding the reader that both these cross sections have the same energy dependence. Interestingly, we see that the different angular dependence of the cross section does not affect the fragment distribution. Furthermore, the rapidity distribution of fragments (not shown here) is nearly independent of the form of the angular distribution.

From fig. 6.5, we note that different cross sections have a drastic effect on the fragment distribution obtained with MST at peripheral collisions. We also analyze the Xe+Sn collision at 8 fm using SACA. We have simulated the Xe+Sn collisions at 100A and 400A MeV using two extreme cross sections $\sigma = 55$ and 20 mb. Note that the normal MST method does not transfer enough energy to the spectator matter at peripheral collisions and hence it leads to fewer IMF's with normal cross section. A larger cross section (as seen in fig. 6.5), therefore, results in much more fragments. The SACA searches for that configuration of nucleons and fragments which has the largest binding energy and hence no such problem arises in SACA. We note that the role of different cross sections in fragment production is different in SACA than in MST. Using MST, one has $2.1 (3.55)$ and $1.70 (0.24)$ IMF's at 100 (400A MeV) using $\sigma = 55$ and 20 mb, respectively. Whereas SACA (at 60 fm/c) gives $6.08 (5.0)$ and $5.4 (4.2)$ IMF's at 100A (400A MeV) using $\sigma = 55$ and 20 mb, respectively. In other words, the difference in the multiplicities of fragments at peripheral collisions using $\sigma = 55$ and 20 mb is reduced in SACA compared to MST at 400A MeV. The striking point is that a smaller cross section of 20 mb gives
Figure 6.6: Same as fig. 6.5, but using the standard Cugnon cross section and isotopic cross section.
relatively more fragments in SACA. What is its cause? Due to the width $2L$ of the Gaussian wave function, the expectation value of the two-body potential has a range of 3.6 fm which is large compared to the range of the nuclear force in free space. Therefore, MST detects the matter as a single fragment even if the velocities of the nucleons contained in it are very different, whereas with a larger cross section, the spectator matter receives enough energy and it breaks into intermediate mass fragments. On the other hand, SACA searches for the most bound configuration where each fragment is properly bound. It, therefore, groups the nucleons in a fragment if their velocities are such that the fragment is properly bound. In other words, we can have a situation (in SACA) where nucleons of different fragments have an overlap in spatial space. This type of situation will arise either at low energy or at peripheral collisions where very small part of the energy is being transferred from participant zone to the spectator zone. In this situation, the MST and SACA will give entirely different results. The MST will show a very big fragment whereas SACA will show medium size fragments. At higher energies with a larger cross section, the nucleons will suffer much collisions and hence they will be far apart from each other and therefore, SACA and MST will give similar results. In Xe+Sn collisions at 400A MeV, the SACA with $\sigma = 20$ mb yields more IMF whereas the result with $\sigma = 55$ mb is less affected compared to one in MST. Therefore, one sees that the difference in the multiplicity of fragments obtained with different cross sections is reduced if one analyze the fragments with sophisticated algorithms such as SACA which searches for the most bound structure.

### 6.3.3 Impact parameter dependence of fragment multiplicities

To understand the fragment production more carefully, in fig. 6.7, we divide the fragment distribution into smaller windows, i.e. we separate the fragments with $5 \leq A \leq 9$, $10 \leq A \leq 19$, $5 \leq A \leq 20$ and $21 \leq A \leq 65$. We see that in 100A MeV reactions, different cross sections have smaller effects. The simulations at 400A MeV yields several interesting points: (i) one sees that more fragments are emitted at peripheral collisions with larger cross sections. In addition, no heavy fragments survive in the central
Figure 6.7: Same as fig. 6.5, but for a different mass range.
reactions. One sees a shift in the peak of fragment multiplicity towards larger impact parameters. The light fragments peak at smaller impact parameter whereas heavy fragments are maximal at larger impact parameters. This is quite understandable because in central collisions, the excitation energy deposited in the system is very large and hence no heavy fragment can survived. The formation of HMF's in simulations with $\sigma = 55$ starts at semi-central collisions.

6.3.4 Mass distribution

The mass distribution for the collisions of Xe + Sn at 100A MeV and 400A MeV is shown in fig. 6.8. Here we choose the two typical impact parameters $b = 2$ and $b = 8$ fm. We find that for central collisions, the mass distribution is a continuous curve with a slope increasing with increase in incident energy. At peripheral collisions, we clearly have two parts: (i) the decreasing cross section of emitted nucleons and light particles and IMF's up to $A \approx 25$ and the increasing cross section for heavier masses (the remnants of projectile and target). A larger value of $\sigma_{NN}$ leads to more heavier fragments as compared to the reaction dynamics with low $\sigma_{NN}$.

6.3.5 The rapidity distribution

The rapidity distribution (eq. 3.5) is assumed to give information about the degree of thermalization achieved in a reaction. In fig. 6.9, we show the rapidity distribution of the emitted nucleons, MMF's and HMF's. The LMF's behave in a similar manner to emitted nucleons. We find that the simulations with larger $\sigma_{NN}$ emit more particles in the midrapidity region. One also finds that different $\sigma_{NN}$ have some effect on the rapidity distribution of nucleons and IMF's. In addition, the nucleons seems to be emitted from the participant zone. The MMF's and HMF's are emitted from the spectator matter as they peak at target and projectile rapidity. From the rapidity distribution one can also conclude that the different $\sigma_{NN}$ has some effect on the fragment rapidity distribution. It is clear from fig. 6.7, that no HMF survives in central collisions at 400A MeV simulations.
Figure 6.8: The mass distribution of Xe+Sn collisions at $E_{\text{lab}} = 100A$ MeV (left part) and $E_{\text{lab}} = 400A$ MeV (right part), respectively. The upper part is at impact parameter $b = 2$ fm, whereas the lower part is at $b = 8$ fm. The displayed results are at 200 fm/c.
Figure 6.9: The final stage rapidity distribution (obtained at $t = 200$ fm/c) as a function of $Y_{c.m.}/Y_{beam}$. Here the displayed results are for Xe+Sn at impact parameter $b = 4$ fm.
and hence the simulations with larger cross sections does not show any distribution for these fragments.

To understand the behaviour of the rapidity distribution at different impact parameters, we in fig. 6.10 display the rapidity distribution of nucleons, LMF’s, MMF’s, HMF’s using the G-matrix cross section at 400A MeV. We notice that for central collisions more nucleons and light fragments are emitted from the midrapidity region. With an increasing impact parameter the emission of nucleons starts shifting from the midrapidity region to the spectator region with peaks around projectile and target rapidity. We mention that the calculation with all other cross sections exhibits similar trends. One also sees a shift in the impact parameter at which a maximal number of fragments can be found. This is expected from fig. 6.7. The light fragments are produced when the system is highly excited which is the case in central collisions only.

6.3.6 Transverse momentum

Finally, we display the transverse momentum of fragments in fig. 6.11. Here results with different cross sections are displayed at a bombarding energy of 400A MeV. We display our results at $b = 4$ fm. We clearly see that fragments have larger flow compared to emitted nucleons. In addition, different cross sections have quite different transverse momenta. The variation of the transverse momentum with respect to the impact parameter is shown in fig. 6.12, where the directed transverse momentum is plotted as a function of the impact parameter. We notice that $\sigma_{NN} = 55$ mb results in maximum flow which is followed by $\sigma_{NN} = 40$ mb, G-matrix and Cugnon cross sections. The simulations with larger $\sigma_{NN}$ result in more NN collisions which leads to a larger transverse flow. One also notices that the difference in transverse flow obtained with the standard Cugnon cross section and isotropic Cugnon cross section is very small. These type of findings and the behaviour of directed transverse momentum has also been reported by other authors.
Figure 6.10: The final stage rapidity distribution of emitted nucleons, light mass fragments and intermediate mass fragments at different impact parameters. The displayed results are for Xe+Sn collisions at a bombarding energy of 400A MeV and using the G-matrix cross section.
Figure 6.11: The nucleonic and fragment flow \( \langle p_x/A \rangle \) as a function of \( Y_{\text{cm}}/Y_{\text{beam}} \). Here we simulate the reaction of Xe+Sn at an incident energy of 400A MeV and at impact parameter \( b = 4 \) fm.
Figure 6.12: The directed transverse momentum as a function of impact parameter. The transverse momentum is calculated by summing the directed transverse flow of nucleons and fragments. The Xe+Sn collisions are at 400A MeV.
Figure 6.13: The charge of heaviest fragment and the multiplicity of IMF's as a function of the impact parameter. The data points have been extracted from ref.[19].
6.3.7 A comparison with experimental data

We have also carried out our (preliminary) comparison of charge of heaviest fragment and the multiplicity of fragments with experimental data reported by ALADIN experiment. In fig 6.13, we display the charge of heaviest fragment ($Z_{max}$) calculated with cross sections of 40, 55 mb and Cug. The lower part of the figure is for the multiplicity of fragments as a function impact parameter. Note that no experimental filter is applied to our calculations. Very interesting, a calculation with $\sigma=55$ mb is able to reproduce the data up to some extent but one still sees large discrepancies at peripheral collision. From upper and lower parts, it is evident that a higher $Z_{max}$ is produced with $\sigma=55$ mb which results in fewer IMF's at intermediate impact parameters. This also shows that the failure of QMD+MST to reproduce the experimental data cannot purely be correlated with unrealistic cross section. Even a very large cross section does not reproduce the data fully.

6.4 Summary

Here we have investigated the influence of different cross sections on the fragments formation. The fragments are formed using the minimum spanning tree method. A variety of different nucleon-nucleon cross sections (energy-dependent, in-medium, and isotropic free cross sections) are employed. We studied the multiplicities of emitted nucleons, light mass fragments and intermediate mass fragments at two energies i.e., 100A MeV and 400A MeV and over a large impact parameter range (b= 0 to 10 fm). The results clearly indicate that the nucleon-nucleon cross section has appreciable influence on the fragment formations at higher incident energies. At low bombarding energies, the influence of different nucleon-nucleon cross section is small. This influences is maximal for semi-central and peripheral collisions at higher energies whereas this effect vanishes at central collisions. The different $\sigma_{NN}$ also affects the rapidity distribution and the fragment flow. A larger $\sigma_{NN}$ produces more IMF's at 400A MeV in semi-central and peripheral collisions with a shift towards larger impact parameters. In conclusion, the choice of a nucleon-nucleon cross section can
play a significant role in the fragment formation within the framework on the quantum molecular dynamics model coupled with the minimum spanning tree method. The choice of different nucleon-nucleon cross sections plays a small role in fragment production if one constructs the fragments using simulated annealing clusterization method which searches for the most bound configuration consisting of nucleons and fragments [1].


[12] For Cugnon parameterization, we use the original QMD program developed by J. Aichelin and Co-workers.


