Chapter 4

Role of Momentum correlations in multi-fragmentation

4.1 Introduction

From chapter 2, it is clear that several different models are available to study the multi-fragmentation. All dynamical models follow the dynamics of single nucleons only. No model deals with the fragments directly. After few hundreds fm/c, one constructs the fragments. The time of the construction of fragments varies with the author. One mostly chooses $t = 200 \text{ fm/c}$ [1, 2, 3]. This method of fragment's construction can vary from simple space correlations to a very complicated algorithms which depend on the minimization of energy [2, 3, 4, 5]. In recent years, fragment formation with percolation [6] or the aggregation method is also very much in use [7]. The details of these models are given in chapter 2. Though the simple space correlation method (like minimum spanning tree (MST) method), by definition, cannot address the question of the time scale of fragmentation, it is a very fast method. Note that the space correlation method will lead to a single cluster at the time of high density.

Apart from these facts, the MST method has a couple of general limitations: (i) As we know, because of frequent nucleon-nucleon collisions in central collisions, the initial nucleon-nucleon correlations are destroyed. As a result, the nucleons with large relative momentum can be close in spatial space and hence can be a part of the same fragment. These fragments will be unstable and hence will decay in the due course of time by
emitting either nucleons or light mass fragments. (ii) In contrast to this, the picture in peripheral collisions is entirely different. In peripheral collisions, we have the multifragmentation of the spectator matter. The spectator nuclei receive the energy and get excited. These excited nuclei emit light mass fragments and intermediate mass fragments. In other words, in peripheral collisions, the much of the initial correlations are preserved. In a recent experiment [8], it is found that the QMD model coupled with a simple correlation method fails to explain the experimentally observed fragment multiplicities at peripheral collisions. The multiplicity of the fragments reported in [8] with the normal space correlation method was far less than obtained in the experiments [8]. One of the failure was attributed to less transfer of energy from the participant to the spectator zone and hence the spectator nucleus does not break into fragments.

Recently, Puri et al. [2] developed a new algorithm to identify the fragments. This new algorithm (SACA) searches the most bound fragment configuration based on energy minimization and hence is able to explain the experimental data at peripheral collisions very nicely [2]. The SACA can identify the fragments at $T_{\text{min}}$ (time at which the largest fragment $A^\text{max}$ has its lowest value), whereas normal MST gives a single largest fragment. This single fragment in MST can be a bunch of fragments which are close in coordinate space and even overlapping. The SACA can identify the fragment with stable configuration. To justify our argument, we plot in fig 4.1 the coordinates of nucleons bound in fragments (detected in SACA) at $T_{\text{min}}$ for a single simulation of Au-Au at 400A MeV. For the clarity of the figure, we plot the coordinates of those nucleons which are bound in fragments with mass 5 or more. The upper part of the fig. 4.1 shows the spatial coordinates whereas the momentum coordinates are displayed in the lower part of the figure. The left part of the figure is the front view (i.e. X versus Z) whereas the side view (i.e. Y versus X) is displayed on the right hand side. Naturally, we have two heaviest fragments in MST. The most interesting point is that the $A^\text{max}$ detected by the MST method is indeed not a single fragment, but is a bunch of fragments which are close in coordinate space and even are overlapping. Due to overlapping, these fragments are identified as single fragment in MST. One also sees that each of the fragment detected in SACA comprises of nucleons
Figure 4.1: The snapshot of the coordinates of nucleons bound in a fragment with mass $\geq 5$ as detected in SACA at 80 fm/c. Here the results of a single simulation are displayed. The displayed results corresponds to a single QMD event.
which are occupying a part of the coordinate space. On the other hand, there is no clear isolated momentum space occupied by these fragments.

The similar conclusion can be drawn from the side view of the reaction. From the figure 4.1, it is clear that any method which is based on the simple coordinate space approach will not be able to detect these fragments which are overlapping and therefore, one has to wait till these fragments are well separated from each other. On contrary, the energy minimization like SACA takes care of the energy of the system and hence can detect the configuration with most favoured energy of the system. Due to mutual overlap or very closely placed in coordinate space, these fragments exchange the nucleons, and also emit the nucleons. The fragments are indeed created after the violent phase of the reaction is over, but the interactions are still active. One of the causes of these interactions could be the long range nature of the interactions in QMD. If one analyze the fragment distribution at $T_{\text{min}}$, one is able to explain the experimentally observed fragments distribution and kinetic energy spectra with SACA without any other assumptions [9]. Therefore, the methods based on the energy minimization, though are very time consuming, are of great importance for the study of the multi-fragmentation at peripheral collisions where normal clusterization method based on space correlations fails to get the proper fragments.

While considering the space correlation for fragment formation, one has argued that at a later stage of the reaction (i.e. after 200 fm/c), the correlations in momentum space do not play any role [10]. This argument is based on the assumption that at a later stage of the reaction, the nucleons with large relative momenta will disassociate themselves from each other and hence any cut in relative momentum of the nucleons should have no influence on the fragment distribution.

We shall show here that above assumption is valid only if the value of cut in relative momentum is $\geq 200\text{MeV/c}$. This value is not at all realistic as this cut in relative momentum is too large to keep nucleons bound in a fragment. If one takes the value of cut (in the relative momentum of two nucleons) about the average Fermi momentum of
nucleons (i.e., $\approx 150$ MeV/c), one finds a strong effect of cut (in the relative momentum of nucleons) on fragment multiplicity and rapidity distribution in central collisions whereas the effect is the least for peripheral collisions. The idea of imposing a cut in momentum space is to avoid the creation of fragments which are not properly bound and will either decay after a while or will emit nucleons in the course of time. To justify our argument, we also calculate the fragment multiplicities using recently advanced simulated annealing clusterization algorithm [SACA] [2]. From ref. [2], it is clear that the SACA is able to identify the fragment distribution which is most bound. We shall show here that the fragment distribution for central collisions with a cut of $150$ MeV/c in the relative momentum of nucleons matches very well with the most bound fragment distribution obtained using the SACA at asymptotic times (i.e., after about $120$ fm/c) whereas it has nearly no effect for peripheral collisions and hence one can see a large difference in the results at peripheral collisions obtained with the SACA and with the normal minimum spanning tree MST method or with cut in the relative momentum of nucleons. In this chapter, we study the role of momentum correlations in the framework of QMD model [1]. In the following first we will explain the definition of momentum correlations and then explain our results.

### 4.2 Definition of momentum correlations

The simple MST fails to find the most bound configuration of the fragment. The extended MST also takes care of the relative momentum of nucleons [11]. Along with the restriction in the coordinate space of nucleons, we also put a restriction in the momentum space of nucleons. That is, we require:

\[
|p_i - p_j| \leq p_{min},
\]

where $p_{min} = 150$ MeV/c. It shows that the nucleons with large relative momentum cannot be in the same fragment even if they are close in the spatial space. The method is dubbed...
4.3 Results and discussion

In the following, we use hard and soft equations of state with an NN cross section fitted by Cugnon (see e.g. [1]). The simulations of Au on Au are carried out at impact parameters \( b = 3 \) and \( 8 \) fm and at incident energies \( E_{\text{lab}} = 100\text{A}, 150\text{A}, 250\text{A}, 400\text{A} \) and \( 600\text{A} \) MeV, respectively, using the quantum molecular dynamics [1] model. We also simulate Nb-Nb reaction at \( E_{\text{lab}} = 100\text{A}, 150\text{A}, 250\text{A}, 400\text{A} \) and \( 600\text{A} \) MeV for the same analysis.

4.3.1 Time evolution of fragments using MST, MST(SP) and SACA

Figures 4.2 and 4.3 show the time evolution of fragment formation for Au-Au at \( 400\text{A} \) MeV and \( 150\text{A} \) MeV, respectively. The left and right parts of Fig. 4.2 are results using hard and soft equation of state, respectively. We show the evolution of largest fragment \( (A^{\text{max}}) \), emitted nucleons, light mass fragments (LMF’s) \( (2 < A \leq 4) \) and intermediate mass fragments (IMF’s) \( (5 < A \leq 65) \). Note that the results indicated by MST are the one where fragments are decided by constraints in space-coordinates only. The results indicated by MST(SP) are the ones where both space and momentum constraints are taken into account eq. (4.1). The results obtained with the SACA are labeled as SACA.

For the value of the cut in the relative momentum of two nucleons, we allow two nucleons to be in the same fragment if their relative momenta are less than the average Fermi momentum. From QMD simulations, we find that the average Fermi momentum is \( \approx 150 \) MeV/c. Therefore, we take a value of \( p_{\text{min}} = 150 \) MeV/c until stated explicitly. The effect of a variation in \( p_{\text{min}} \) is studied in figs. 4.6 and 4.7, below. Note that there can be nucleons who have large relative momentum than 150 MeV/c. But at the same time, one should keep in mind that it is only a supplementary condition. Therefore, we
can talk of an average value.

The evolution of the largest mass $A_{\text{max}}$ shows quite interesting results: (i) The MST(SP) method gives quite different time evolution of $A_{\text{max}}$ compared to MST method. Finally, both emerge to similar results. Naturally, at the start of the collision, colliding nuclei have very large relative momenta and hence the MST(SP) method gives two clusters with mass 197 each, whereas the MST method gives just a single fragment with 394 nucleons contained in it. The evolution of $A_{\text{max}}$ when analyzed with the SACA is very close to the one obtained with the MST(SP) after 50 fm/c. One also notices that SACA can identify the $A_{\text{max}}$ as early as 40-50 fm/c. This is followed closely by the MST(SP) method which identifies the $A_{\text{max}}$ at about 50-60 fm/c. A cut of 150 MeV/c in relative momentum of nucleons clearly helps in identifying the largest fragments quite early in time.

The second row which deals with the emission of nucleons shows more nucleons using the MST(SP) method compared to the MST. Because of no restriction on relative momentum in the MST method, we observe a delayed emission of nucleons and light fragments using the MST method. The delayed emission is due to fact that until about 40 fm/c, there is just a single fragment in the MST method. Because of an early emission of fragments and free nucleons in the MST(SP) method, we find that fragments saturate about 30 fm/c earlier in the MST(SP) method than in MST method. One also notices that in the SACA and MST(SP) methods, the fragment emission starts at a time when the MST method gives just a single largest fragment consisting of 394 nucleons. This is understandable because the MST approach is based on a simple space correlation and therefore, in this approach nucleons with very large relative momenta will be a part of a cluster if their centroids are less than a distance of 4 fm. One also notices that the cut in momentum space is very important for central collisions. In central collisions, because of violent nucleon-nucleon collisions, nucleons of the same fragment can have different momenta. We also see that the fragment distribution obtained with the SACA model is very close to the MST(SP) analysis in most cases. It is worth mentioning that in ref. [2],
Figure 4.2: The evolution of the central collision of Au-Au using a hard (left part) and soft equation of state (right part). The result at $E_{lab} = 400A$ MeV with the MST and MST(SP) methods are indicated by solid and dash-double-dotted lines, respectively. The results with the SACA analysis are shown by the dashed line. From top to bottom, the rows display the time evolution of the largest fragment $A_{max}$, free-nucleons, LMF's ($2 \leq A < 4$), and IMF's ($5 \leq A < 65$), respectively.
Figure 4.3: Same as Fig. 4.2, but for Au-Au at 150A MeV.
the SACA is suggested to be used at 50-60 fm/c. The SACA approach always leads to most bound structure of fragments with minimum energy. A good agreement between the MST(SP) method and SACA shows that by imposing a cut \( ( = 150 \text{ A MeV}) \) in the relative momentum of two nucleons, one gets the most bound configuration consisting of fragments of all sizes.

The time evolution of LMF’s and IMF’s show that at 400A MeV collisions, a cut in relative momentum of nucleons have strong effect on their multiplicities. The MST(SP) matches with the most bound fragment structure (obtained with SACA) after about 120 fm/c whereas the MST method fails to meet the most bound structure even after 200 fm/c.

In fig. 4.3, we display the same results as fig. 4.2, but at 150A MeV. We notice that the MST(SP) now detects the largest fragment \( A_{\text{max}} \) at same time as that of MST method. The identification of LMF’s and IMF’s using the MST(SP) method takes a longer time compared to the SACA. The MST method fails to match the results obtained with the SACA even at 200 fm/c. It is worth mentioning that the MST(SP) method seems to do a better job at higher incident energies in central collisions. The MST(SP) method at 600A MeV (not shown here) can identify the fragments earlier than obtained at 400A MeV. From figs. 4.2 and 4.3, it is also clear that different equations of state have the least effect on fragment production.

In fig. 4.4, we display the results obtained for peripheral collisions at 150A MeV and 400A MeV, respectively using a soft equation of state. We notice that the multiplicity of the fragments obtained with the MST and MST(SP) method at 200 fm/c is quite different than the one obtained with the SACA. The emission of nucleons using MST(SP) method is closer to the SACA compared to MST method. Nearly no effect of the cut on fragment production at peripheral collisions can be understood on grounds that in peripheral collisions, the fragments are emitted from a spectator matter. In other words, the fragments are not unstable and hence the MST(SP) method does not differ much from the MST.
Figure 4.4: Same as Fig. 4.3, but for 150A MeV (left part) and 400A MeV (right part). Here we use $b = 8$ fm and a soft equation of state for simulations.
method. The huge difference between the results obtained using the SACA and the MST and MST(SP) methods at peripheral collisions is discussed in detail in ref. [2].

4.3.2 The rapidity distribution

Figure 4.5 deals with the rapidity distribution of the $j^{th}$ particle defined in chapter 3 (eq. 3.5) [1, 10, 12]. The results displayed in fig. 4.5 are at 200 fm/c. We find that the fragment rapidity using the MST and MST(SP) methods shows some difference. It was also expected from multiplicity picture (figs. 4.2 and 4.3). There are some difference in the rapidity distribution of emitted nucleons, being larger for the MST(SP) method. On the other hand, IMF’s indicate their origin from spectator matter (i.e. they peak at target and projectile rapidities). The rapidity distribution of fragments using the SACA is close to the one obtained with the MST(SP) method. This clearly justifies the use of a cut in relative momenta of two nucleons.

4.3.3 Role of different cuts in relative momenta

In figs. (4.2 -4.4), we have fixed $p_{\text{min}}$ (= 150 MeV/c) and studied its influence on the fragment formation. We now vary $p_{\text{min}}$ between 1 MeV/c and 300 MeV/c at a fixed $d_{\text{min}}$ (=4 fm) and study the role of momentum constraints on the fragment dynamics. In figs. 4.6 and 4.7, we show the emitted nucleons, LMF’s (2 $\leq A \leq 4$) and IMF’s (5 $\leq A \leq 65$) obtained at 200 fm/c as a function of $p_{\text{min}}$ for Au-Au and Nb-Nb at bombarding energies $E_{\text{lab}} = 100A$, 150A, 250A, 400A, and 600A MeV, respectively. The left and right hand sides of fig. 4.6 are with impact parameter $b = 3$ fm and 8 fm, respectively, whereas the left and right parts are at impact parameter of 2 fm and 6 fm, respectively. We notice (i) for $p_{\text{min}} \leq 50$ MeV/c, there are nearly no IMF’s and LMF’s and final matter ends up in single nucleons. For $p_{\text{min}} \approx 100 – 150$ MeV/c, one sees a sharp decline in emitted nucleons and enhancement in LMF and IMF production. After $p_{\text{min}} = 200$ MeV/c, the fragment structure saturates and there is no further change in the structure. This analysis is understandable on the grounds that for very small $p_{\text{min}}$, we deny the nucleons to form
Figure 4.5: The rapidity distribution for the collision of Au-Au as a function of $Y_{\text{cm}}/Y_{\text{beam}}$. Here the left and right parts of the figure are at impact parameters $b = 3$ and 8 fm, respectively. The results obtained with the MST, MST(SP), and SACA are shown by solid, dash-double-dotted, and dashed lines, respectively.
Figure 4.6: The emitted nucleons, LMF's (2 \leq A \leq 4), and IMF's (5 \leq A \leq 65) (obtained at 200 fm/c) as a function of \(p_{\text{min}}\). The displayed results are for Au-Au at impact parameters of \(b = 3\) fm (left side) and \(b = 8\) fm (right side) using a soft equation of state.
Figure 4.7: Same as Fig. 4.6, but for Nb-Nb at $b = 2$ and 6 fm, respectively.
a fragment even if their relative momentum is very small. As a result, LMF's and IMF's production is very small. When we relax the restriction on relative momentum, IMF's production increases. Naturally, nucleons with a very large relative momentum will be far apart in space after 200 fm/c and hence further relaxation in $p_{\text{min}}$ does not change the fragment structure any more. From the above discussion, one can conclude that nucleons with relative momentum larger than 200 MeV/c are already quite far in space and hence there is no change in fragment structure. We also notice that the size of colliding nuclei has least effect on our understanding of the cut in momentum space. Naturally, a very small or very large value of $p_{\text{min}}$ has no physical relevance. The natural choice of the value of the cut in the relative momentum of two nucleons is the average Fermi momentum. We have seen that a cut of the order of Fermi momentum (in the relative momentum of two nucleons) has sizable effect on the fragment distribution and rapidity distribution of fragments. It is worth to mention here that presently $d_{\text{min}}$ is taken to be 4 fm. If one takes too small value of $d_{\text{min}}$, it will lead to large number of particle emission and thus, one will find lighter $A_{\text{max}}$ in the starting which in principle should be equal to the mass of beam nuclei.

4.4 Summary

In summary, we have carried out a systematic study of the role of the momentum correlation in fragment formation [11]. To investigate the influence of the momentum correlation on fragment production, we introduced an additional condition for fragment formation. Under this condition, the relative momentum of the nucleons is also taken into account while deciding the fragment formation. To justify our results, we also report the results of fragment formation using the SACA which gives the most bound structure of fragments. We find that with a realistic value of the cut (= 150 MeV/c) in the relative momentum of nucleons, the fragment distribution is quite different compared to no cut in momentum space for central collisions. The difference is small at peripheral collisions. The fragment distribution in central collisions obtained with a cut (= 150 MeV/c) in momentum space (at 120 fm/c) agrees with the most bound configuration obtained using
the simulated annealing clusterization algorithm, whereas normal MST method fails to match the bound structure even at 200 fm/c. Therefore, an additional condition of the cut (of the order of average Fermi momentum) in relative momentum of nucleons is important to get the most bound structure of fragments in central collisions [11]. The work of comparing the MST(SP) results with experimental data is in progress.
Bibliography


