CHAPTER - II

THEORIES OF NUCLEAR REACTION

This chapter has been divided into two parts i.e. Section 2.1 and Section 2.2. The first part explains the Coulomb excitation theory to obtain the transition probabilities of the excited states and the angular distribution of the deexcited \( \gamma \)-rays. The second part deals with the theories of Doppler shift attenuation (DSA) technique to find the lifetimes of the nuclear levels. This part also explains the theory of angular distribution on the basis of compound nucleus formation for the determination of the spins of the excited states and the multipole mixing ratios of the transitions.

2.1) Coulomb Excitation

2.1.1) Qualitative Semi-classical Discussion

When an electrically charged particle passes near a nucleus along a path described classically, the nucleus experiences a changing electric field. The distance of closest approach of a classical particle of charge \( Z_1e \) and mass \( m \) incident head-on with velocity \( v_0 \) (at infinity) on a stationary target nucleus of charge \( Z_2e \) is given by:

\[
\frac{1}{r} = \frac{Z_1Z_2e^2}{mv_0^2}, \quad a = \frac{2Z_1Z_2e^2}{4\pi\varepsilon_0mv_0^2} \tag{2.1}
\]
In quantum mechanics, the charged particle has a non-zero probability of being found in the classically forbidden region. This means that if $\lambda$ is the reduced De Brodie wavelength of incoming particle we may require that $\lambda < a/2$ or $h/\lambda < a/2$ or $\lambda \mu < 2h/a$ or

$$1/2 \mu \lambda^2 > \hbar \lambda/a$$

(2.2)

Since $\lambda/a$ has dimensions of frequency, it seems plausible to interpret $\lambda/a$ as roughly the maximum frequency $\omega_e$ of the E.M. radiation being emitted by the charged particle on account of the change in its velocity in the electric field of the nucleus. In fact, if the kinetic energy of a incident particle is well below the Coulomb barrier, it can excite a nuclear state of energy less than $\hbar \omega_e$. This process is called Coulomb excitation. The most effective charged projectiles for Coulomb excitation are the protons, alpha particles and heavy-ions. The projectiles must be moving slowly with kinetic energy well below the Coulomb barrier; otherwise they would interact with the nuclear forces and a variety of nuclear reactions will occur. To avoid this interference, one carries out Coulomb excitation at bombarding energy for which nuclear interaction effects are negligible. This maximum limit of incident energy is known as safe-energy for Coulomb excitation. It can be obtained from an empirical relation [1].

$$E_{safe} = \frac{1.44 Z_1 Z_2 (1 + A_1/A_2)}{1.25 (A_1^{1/3} + A_2^{1/3}) + s}$$

(2.3)

Where $A$ and $Z$ denote mass and atomic numbers.
where the suffix 1 and 2 refer to projectile and target nucleus, respectively and 's' denotes the separation of the projectile and target nucleus surfaces. For heavy ions 's' has been suggested to be about 5.1 fm [2] and for light ions (i.e., protons and alphas) it varies from 6 to 8 fm [3,4] depending on the target nucleus. Alternatively $E_{\text{safe}}$ has also been defined as the incident energy, where compound inelastic contribution is less than or equal to 5% of the Coulomb contribution [5]. $E_{\text{safe}}$ energy is always less than the conventionally defined coulomb barrier $E_B$. Hence the projectile does not penetrate into the target nucleus. For the excitation of a level at energy $\Delta E$ such that $\Delta E = \hbar \omega$, it is required that the nuclear time $(1/\omega) \geq$ the collision time $(a/v)$. The ratio of collision time to nuclear time is known as adiabaticity parameter $\xi$ and $v$ is the velocity of the projectile and

$$\xi = \frac{\text{Collision time}}{\text{Nuclear time}}$$

$$\xi = \frac{\omega \cdot a}{v} \times \frac{a}{v} \times \frac{\Delta E}{\hbar} \quad (2.4)$$

Where, $E$ is the bombarding energy. We expect a large excitation probability for $\xi \leq 1$. In the limiting case when $\xi \geq 1$, the collision time is so large that the rate of change of electromagnetic field with time, becomes very small and the process approaches the adiabatic limit hence, this parameter is called adiabaticity parameter.
The magnetic interaction in this process is neglected due to the small velocity \((v/c \ll 1)\) of the projectiles. Hence, coulomb excitation depends essentially on the electric multipole matrix elements. For the bombarding energy \(E < E_h\), the projectile will move along a hyperbolic orbit in the repulsive Coulomb field of the target nucleus. The most significant approximation in Coulomb excitation is to view the dynamics of the process as a classical Rutherford scattering i.e. the semi-classical approximation. The cross section of Coulomb excitation for a given final level \(f\), from the ground state can be given by a product of the probability for excitation along the orbit and the differential cross section i.e.

\[
(d\sigma/dQ)_f = P_{i \rightarrow f} (d\sigma/dQ)_R
\]

Where \((d\sigma/dQ)_f\) denotes the cross-section for Coulomb excitation for a given level \(f\) from the ground state \(i\), \((d\sigma/dQ)_R\) is the differential cross-section for Rutherford scattering and \(P_{i \rightarrow f}\) is the excitation probability of the final state \(f\). From time-dependent first order perturbation theory, the excitation probability \((P_{i \rightarrow f})\) is obtained. The expression for differential cross-section \((d\sigma/d\Omega)\) and the total excitation cross-section \(\sigma_{\text{EL}}\) of order EL are given as

\[
(d\sigma_{\text{EL}}/d\Omega) = (Z_e e/hv)^2 a^{2\lambda+2} B(E\lambda) \cdot df(\theta, \xi)/d\Omega
\]
and \( \sigma_{E\lambda} = (Z_1 e/hv)^2 \cdot a_{\lambda}^2 \cdot B(E\lambda) \cdot f_{E\lambda}(\xi) \) (2.7)

where, \( f_{E\lambda} = \int (d\sigma_{E\lambda}/d\Omega) d\Omega \) are dimensionless functions and contain all information of classical trajectory and \( B(E\lambda) \) is the reduced transition probability for the radiative transition \( E\lambda \) from nuclear state \( I_i \) to \( I_f \). Here \( M(E\lambda) \) is the reduced nuclear \( 2\lambda \) - pole matrix element [6]. To improve the expression obtained above, the projectile velocity \( v \) is replaced by some appropriate mean value of initial and final velocity \( V_i \) and \( V_f \). In preceding semi-classical treatment, the energy loss during collision process is neglected. The symmetrization of cross-section is done by making the following substitution.

\[
V^2 = V_i V_f \quad \text{and} \quad a = \frac{Z_1 Z_2 e^2}{m_0 V_i V_f} \quad \text{(2.9)}
\]

and \( \xi = \eta_f - \eta_i \) (2.10)

where \( \eta_i = \frac{Z_1 Z_2 e^2}{h V_i} \) (2.11)

and \( \eta_f = \frac{Z_1 Z_2 e^2}{h V_f} \) (2.12)
So, the cross-section given by equation (2.6) and (2.7) on symmetrization of the parameters reduced to:

\[
\left(\frac{d\sigma_{\Delta E}}{d\Omega}\right) = \eta_1^2 \cdot a^{2\lambda + 2} \cdot B(E\lambda)/(z_2e)^2 \frac{df_{\Delta E}}{d\Omega}(\theta, \xi)/d\Omega \quad (2.13)
\]

\[
\sigma_{E\lambda} = \eta_1^2 \cdot a^{2\lambda + 2} \cdot B(E\lambda)/(z_2e)^2 f_{E\lambda} (\xi) \quad (2.14)
\]

The similar expression of cross-section is also provided by the quantum mechanical approach but the ‘f’ function depends on both \(\eta_1\) and \(\xi\).

The reduced matrix elements given in equation (2.8) are symmetrical, i.e.,

\[
\langle I_i || M(E\lambda) || I_f \rangle = (-1)^{(I_f - I_i + \lambda)} \langle I_f || M(E\lambda) || I_i \rangle \quad (2.15)
\]

So the upward reduced transition probability \(B(E\lambda)^+\) is related to downward reduced transition probability \(B(E\lambda)^-\) through spin relation

\[
B(E\lambda, I_i \rightarrow I_f) = (2I_i + 1)/(2I_i + 1) B(E\lambda, I_f \rightarrow I_i) \quad (2.16)
\]

The gamma transitions between any two states are governed by the following selection rules for the angular momentum and parity

\[|I_i - I_f| \leq \lambda \leq |I_i + I_f|\]

\[
\pi_i \cdot \pi_f = \begin{cases} 
(-1)^\lambda & \text{for } E\lambda \\
(-1)^{\lambda+1} & \text{for } M\lambda
\end{cases} \quad (2.17)
\]

where \(\pi_i\) and \(\pi_f\) are the parities of the initial and final states.
In the Coulomb excitation due to lower projectile velocity \((v)\) the magnetic excitations are very weak in comparison to electric excitation, being smaller by an order of \((v/c)^2\). Therefore, in mixed transition only electric quadrupole \((E2)\) transition contributes to Coulomb excitation.

### 2.1.2) Extraction of Reduced \(E2\) Transition Probability \(B(E2)\)

The total cross section for the \(E2\) excitation of a level as function of incident projectile energy is given by the following expression

\[
\sigma_{E2} = 4.819 (1 + A_1/A_2)^2 (A_1/Z_2^2) (E_i - \Delta E') f_{E2}(\eta, \xi) \cdot B(E2) \ \text{barns}
\]

(2.18)

where \(E_i\) is the incident projectile energy in MeV and \(\Delta E'\) is the excitation energy in the lab. system and is related to the conventional excitation energy \(\Delta E\), as

\[
\Delta E' = (1 + A_1/A_2) \Delta E
\]

(2.19)

with proper modifications in equations (2.11) to (2.12), we obtain

\[
\eta_i = (Z_1 Z_2/2) (A_1/10.00 8E)^{1/2}
\]

(2.20)

and

\[
\eta_f = \eta_i (1 - \Delta E'/E)^{1/2}
\]

(2.21)

In order to extract \(B(E2)\), the thick target \(\gamma\)-ray yield \((Y_{th})\) of a given transition is required and has been given by the following relation [6].
where $N_A$ is the Avogadro's number, 'a' is the number of target nuclei/molecule, $A$ is the molecular weight of the target and $N_P$ is the number of projectiles incident on the target and $dE/dx$ is the stopping power in MeV-cm$^2$/gm.

The reduced $E2$ transition probability, $\varepsilon B(E2)$ can be obtained from the comparison of theoretical and experimental yields using the following relations and therefore from equation (2.18) and (2.22), we obtain

$$\varepsilon B(E2) = \frac{1}{K I(E2)} (\frac{Y/N_P}{\varepsilon})$$

(2.23)

where $K = \left[ \frac{4.819}{N_A \cdot a \left( \frac{A_2}{A} \right)^2 \left( \frac{A_1}{A_1 + A_2} \right) \frac{A_1}{Z_2^2}} \right]$ (2.24)

and $I(E2) = \int_0^{E_i} (E - \Delta E') f_{E2}(n, \xi) \frac{dE}{dE/dx}$

(2.25)

$Y/N_p$ = experimental yield per incident proton and the decay fraction, $\varepsilon$ for a detected $\gamma$-ray is given by

$$\varepsilon(\gamma_i) = \frac{f_i}{\Sigma(1+c_i)f_i}$$

(2.26)
where \( f_i \) are the relative intensity of all \( \gamma \)-transitions through which excited states decay and \( \alpha_i \) are the corresponding total internal conversion coefficients.

While \( f_{\text{E2}} (\eta_i, \xi) \) values have been tabulated by Alder et al. [6], the stopping power \( \frac{dE}{dx} \) as a function of projectile energy is calculated by using Bethe's formula [7]. The energy dependence for \( \frac{dE}{dx} \) has been considered as \( a \cdot E^{b \cdot c \cdot E} \) and the value of \( a, b, c \) are calculated from a plot of \( \log(\frac{dE}{dx}) \) against \( \log E \).

2.1.3) **Angular Distribution from Coulomb Excitation**

The coefficients \( a_v \) in the Legendre polynomial expansion of even order for angular distribution can be expressed in case of Coulomb excitation as;

\[
a_v = b_v^{\text{E2}} (\eta_i, \xi) A_v
\]  

(2.27)

where \( b_v^{\text{E2}} \) are the particle parameters. They have been evaluated by Alder et al [6] and Biedenharn et al [8]. The semi-classical particle parameter \( b_v^{\text{E2}} (\xi) \) differ from quantum mechanical parameters \( b_v^{\text{E2}} (\eta_i, \xi) \) by less than 5% for \( \eta_i > 20 \) and 3% for \( \eta_i > 4 \) [9]. The Coulomb excitation is predominantly \( \text{E2} \) process, so the effective angular momentum scheme contributing to the Coulomb excitation process can be expressed as

\[
L=2 \quad J_0 \rightarrow J_1
\]
\[ (L = 1) + (L' = 2) \]
\[ J_1 \xrightarrow{M1 + E2} J_2 \]

Hence, the coefficient \( A_v \) can be represented as a product of two coefficients as

\[ A_v = A_v(L_j J_0, J_1), A_v'(L'_j J_1 J_2) \]

### 2.2) Compound Nucleus

When the projectile energy is sufficient to penetrate the nucleus, the projectile will share its energy among all nucleons and compound nucleus will be formed. The compound nucleus reactions are studied to determine the spins of the excited states and mixing ratios of \( \gamma \)-ray transitions between the states through analysis of angular distribution data. Lifetime of the nuclear levels have also been measured, through Doppler Shift Attenuation method (DSAM). A brief discussion of the theories used in the analysis of the data to extract these information follows:

#### 2.2.1) Lifetime of Nuclear Levels

The bound state of an excited nucleus exists only for a certain time after which nucleus returns to a lower state or the ground state through emission of the \( \gamma \)-rays. The lifetime \( (\tau) \) and the width \( (\Gamma) \) of a bound nuclear level are related through the relation
\[ \tau \cdot \Gamma = \hbar \] (2.28)

The level width (\( \Gamma \)) is proportional to the probability of decay of the level. It is determined by the matrix elements as,

\[ \Gamma = |\langle \Psi_f | \hat{O}_{\text{decay}} | \Psi_i \rangle|^2 \] (2.29)

where \( \hat{O}_{\text{decay}} \) denotes quantum mechanical operator for the decay mode and \( |\Psi_i\rangle \) and \( |\Psi_f\rangle \) represent the nuclear wave functions for the initial and final states, respectively. The total width of the level \( \Gamma_{\text{total}} \) can be expressed in terms of individual width \( \Gamma \) for various possible decay modes of the level.

\[ \Gamma_{\text{total}} = \sum_j \Gamma_j \] (2.30)

The lifetime of the nuclear states of interest varies from \( 10^{-11} \) to \( 10^{-15} \) second. The technique for the measurement of the lifetime of nuclear levels can be broadly divided into two groups: the direct method in which the lifetime (\( \tau \)) is determined directly from the data; and indirect method where \( \tau \) is extracted indirectly from some other quantities like \( \Gamma \) and \( B(E2) \) values, obtained from the analysis of the data. These two methods can be further classified into categories depending upon the range of the lifetime. In the present study the lifetimes of the levels were measured with DSAM in the region of \( 10^{-11} \) to \( 10^{-15} \) sec.
2.2.2) **Doppler Shift Attenuation Method (DSAM)**

When a moving source emits electromagnetic radiation, the frequency of γ-ray gets changed. As the frequency of electromagnetic wave (photon) is proportional to its energy, the change in frequency of the photon due to finite velocity of source will correspond to certain amount of change in γ-ray energy. This change in energy w.r.t. the original energy is known as Doppler's shift. In the DSAM, a recoil nucleus is produced in the nuclear reaction as a source of γ-radiation. Let the recoiling nucleus moving with velocity \(v(t) \ll c\) along z-axis (beam direction) at a time \(t\) emit a γ-ray with energy \(E_y(t)\) observed by a detector (at rest) at an angle \(\theta\) w.r.t. forward direction (beam direction), given by

\[
E_y(t) = E_{y0} \left[1 + \frac{V(t)}{C} \cos \theta\right] \tag{2.31}
\]

where \(E_{y0}\) is the energy of the γ-ray when the recoiling nucleus is at rest. In fact the velocity and direction of the recoiling nucleus will change in magnitude due to the electronic and nuclear scattering.

Let \(\phi(t)\) be the angle between z-axis and the direction of the recoiling nucleus at time \(t\), so, the angle \(\chi(t)\) at time \(t\) between the direction of recoiling nucleus and the direction in which γ-ray is detected, as displayed in Fig. 2.1, through the equation

\[
\chi(t) = \theta + \phi(t) \tag{2.32}
\]
where, $\theta$ and $\phi(t)$ are denoted in the Figure. Every nucleus in an ensemble emits a $\gamma$-ray. The average $\gamma$-ray energy $<E_\gamma>$ emitted from an ensemble is given by

$$<E_\gamma> = E_{\gamma 0} \left[ 1 - <V(t) \cos \chi(t)> \right]$$  \hspace{1cm} (2.33)

The angular brackets denote the average of the enclosed quantities over time and ensemble. The factor $<V(t) \cos \chi(t)>$ can be expressed as

$$<V(t) \cos \chi(t)> = <V(t) \left[ \cos \theta \cos \phi(t) - \sin \phi(t) \sin \theta \right]>$$  \hspace{1cm} (2.34)

Since the distribution of nuclei is expected to be symmetric about z-axis, so the ensemble average of $V(t) \sin \phi(t)$ is zero.

$$<V(t) \cos \chi(t)> = \cos \theta <V(t) \cos \phi(t)>$$

$$= <V_z> \cos \theta$$  \hspace{1cm} (2.35)

where $<V_z>$ is the average recoil velocity of the ensemble (over time) of nuclei in the direction of z-axis. From equations (2.33) and (2.35), one obtains

$$<E_\gamma> = E_{\gamma 0} \left[ 1 + F(\tau) \beta_d(o) \cos \theta \right]$$  \hspace{1cm} (2.36)

where $F(\tau) = <V_z> / V(0)$  \hspace{1cm} (2.37)
and is known as attenuation coefficient and denotes the ratio of average observed Doppler's shift which corresponds to the maximum recoil velocity \( V(o) \). Here \( \tau \) denotes the life-time of nuclear level

\[
\beta_2(o) = \frac{V(o)}{C} \quad (2.38)
\]

The \( F(\tau) \) can be expressed in terms of observed energy shift

\[
\Delta E_\gamma = \langle E_\gamma(\theta_1) \rangle - \langle E_\gamma(\theta_2) \rangle \quad (2.39)
\]

at \( \theta_1 \) and \( \theta_2 \) by two different \( \gamma \)-ray detector angles as

\[
F(\tau) = \frac{\Delta E_\gamma}{E_\gamma \beta_2(o)[\cos \theta_1 - \cos \theta_2]} \quad (2.40)
\]

**Calculations of \( \beta_2(o) \):**

In a binary reaction of the type \( M_2(m_1,m_4)M_3^* \) where \( M_2, M_3, m_1 \) and \( m_4 \) denote the mass of target nucleus, excited residual nucleus, projectile and the outgoing particle, respectively, the \( z \)-component of the velocity of \( M_3^* \) at time \( t=0 \) in the laboratory system is given by

\[
\beta_2(o) = \beta_{c.m.}[1 + \gamma^{-1} \cos \theta_{c.m.}] \quad (2.41)
\]

where \( \beta_{c.m.} \), the recoil velocity in centre of mass system, is given as

\[
\beta_{c.m.} = \frac{(2m_1 E)^{1/2}}{m_1 + M_2} \quad (2.42)
\]
where various masses and projectile energy (E) are in MeV and \( \theta_{c.m} \) is the recoil angle of \( M_3^* \) in the centre of mass system and

\[
\gamma^{-1} = \frac{\text{Velocity of } M_3^* \text{ in c.m. system}}{\text{Velocity of c.m. system}}
\]  
(2.43)

and is given by

\[
\gamma^{-1} = \frac{M_3m_4}{m_1M_3^*} \left[ 1 + \frac{m_1 + M_2}{M_2} \left( \frac{Q}{E} \right) \right]^{1/2}
\]  
(2.44)

where \( Q \) is the Q-value of the excited state.

In the \((p, n\gamma)\) reaction with negative Q-values near threshold energy, the factor \( \gamma^{-1} \) becomes negligibly small so, \( \beta_z(0) \) is nearly equal to \( \beta_{c.m.} \).

\[
\beta_z(0) = \beta_{c.m.}
\]  
(2.45)

From equation (2.40) and (2.45), one can find the experimental value of \( F(\tau) \). Comparison of experimental \( F(\tau) \) with the theoretical \( F(\tau) \) value as a function of \( \tau \), provides the lifetime of nuclear state.

2.2.3) *Theoretical Calculation of F(\tau)*

The attenuation coefficient as defined by equation (2.51) lies between 0 to 1, and can be expressed \([10]\) as
\[
F(\tau) = \frac{1}{V(o)\tau} \int_0^\infty V_z(t). e^{-t/T} dt (2.46)
\]

The theoretical calculation of \( V_z(t) \) and hence \( F(\tau) \) requires the knowledge of stopping theory. The most widely used theory is by Lindhard, Scharff and Schiott (LSS) [11]. According to this theory, the rate of loss of energy of a charge particle is given by

\[
dE/dx = m_1 \frac{dV_z}{dt} = (dE/dx)_e + (dE/dx)_n (2.47)
\]

here the subscripts 'e' and 'n' refer to the electronic and nuclear stopping parts. The electronic stopping is due to interaction of recoiling nucleus with the atomic electrons. This process is dominant for recoil velocity \( \beta > 0.02 \). In nuclear stopping both coulomb and nuclear forces participate and scatter the projectile. It predominates at low recoil velocity \( \beta < 0.03 \). LSS gave the expression for both types of stopping in terms of dimensionless variables \( \varepsilon \) for energy and \( \rho \) for the path length using Thomas-Fermi type of potentials.

Two different approaches are used to calculate \( F(\tau) \) from LSS theory.

(a) **Warburton Approach:**

Here the total energy loss rate is expressed as

\[
-m_1 \left( \frac{dV_z}{dt} \right) = K_e \left( \frac{V_z}{V_o} \right) + K_n \left( \frac{V_z}{V_o} \right)^{1/2} (2.48)
\]
where the two factors on the right hand side represent the electronic and nuclear stopping, respectively. The constant $K_e$ and $K_n$ can be taken either from LSS theory or from experimental data.

Using equation (2.48) in equation (2.46) and integrating the resulting expression, one obtains the final expression for $F(\tau)$ as:

$$F(\tau) = \frac{x\gamma_i^x}{(1 + \gamma_i^2)^{\frac{3}{2}}} \int_0^1 V^2 (\gamma_i^2 + V^2)^{\frac{3}{2}} - 1 \, dv$$

(2.49)

where $x = \alpha/\tau$ ; $\alpha = \frac{m_1 V_0}{K_e \rho}$ and

$$\gamma_i^2 = \frac{K_e}{K_n} \left[ \frac{V_z(\tau)}{V_0} \right]^2$$

$\rho$ = density of the stopping material

and $V = V_z/V(\tau)$

For example, if we assume that the contribution is only from electronic stopping power, then

$$m_1 \frac{dV_z}{dt} = -K_e (V_z/V_0)$$

(2.50)

which on integration provides

$$V_z = V(\tau) e^{-\tau/\alpha}$$

(2.51)
where $\alpha$ is the slowing down time ($m_1V_0/(k_0\rho)$) and from equation (2.46) and (2.51), one obtains

$$F(\tau) = \alpha (\alpha + \tau)$$

This equation reveals that the slowing down time of the recoiling nucleus in the target and in the backing material defines the time scale for the lifetime range for DSAM. So, the lifetime can be extracted from the measurement of $F(\tau)$ if it differs measurably from two extreme values 0 and 1, otherwise a limit can be obtained for the mean lifetime.

(b) **Blaugrund Theory:**

This approach assumes that slowing down and the scattering processes can be separated. So, in order to calculate $V_z$, equation (2.35) can be written as

$$\langle V(t) \cos \chi(t) \rangle = \cos \theta \langle V(t) \rangle \langle \cos \phi(t) \rangle$$

$$= \cos \theta \langle V(t) \rangle \langle \cos \phi(t) \rangle$$

(2.53)

The quantity $\langle V(t) \rangle$ is calculated from the usual LSS theory, while the factor $\langle \cos \phi(t) \rangle$ which depends upon the ratio of masses of two nuclei is calculated with the help of expression given by Lewise [12],

$$\langle \cos \phi(t) \rangle = \exp \left[-0.5 \frac{A_2}{A_1} \cdot G(r) \cdot I\right]$$

(2.54)

where $r = A_1/A_2$ and $I = \int \frac{\varepsilon_0 (dE/d\rho)_n}{\varepsilon (dE/d\rho)} \, d\rho$ and
\[
G(r) = \begin{cases} 
1 + \frac{2}{3} r - \frac{7}{15} r^2 + 8 \sum_{n=3}^{\infty} \frac{(-r)^n}{(2n+1)(2n-1)(2n-3)}, & r < 1 \\
\frac{2}{3} + \frac{8}{15} - \frac{1}{r} - 8 \sum_{n=3}^{\infty} \frac{(-1/r)^n}{(2n+1)(2n-1)(2n-3)}, & r > 1
\end{cases}
\]

\( \varepsilon \) and \( \rho \) are universal dimensionless variables for energy and path length traversed by the ion. \( A_1 \) and \( A_2 \) are the atomic masses of the projectile and the stopping material respectively.

If there are more than one type of atoms present in the stopping material, then the above expressions can easily be modified to take into account the effect of different present atoms. In general the excited nucleus may be produced at any point in the target, so, the target is divided into various layers of equal thickness and \( F_i(t) \) is calculated for each layer by assuming that the yield is uniform across the target.

(c) **Modification of LSS Theory:**

It was found that the lifetime of the same level depends on the backing material [13]. It fluctuates when plotted against \( z \) (atomic number) of the backing. The fluctuation corresponds to electronic shell closure and depends upon the atomic number \( z_1 \) of the moving ion and the atomic number \( z_2 \) of the stopping material. To take these uncertainties into account, some workers [14] modified the LSS equation as
\frac{dE}{dx} = f_e (\frac{dE}{dx})_e + f_n (\frac{dE}{dx})_n \tag{2.55}

where the correction factor \(f_e\) and \(f_n\) are determined with the help of stopping power data due to electronic and nuclear stopping, respectively.

Warburton et al [15] modified their equation to take into account these uncertainties as follows

\begin{equation}
-m_1 \frac{dV_z}{dt} = K_e \left( \frac{V_z}{V_0} \right) + K_n \left( \frac{V_z}{V_0} \right)^{-1} - K_3 \left( \frac{V_z}{V_0} \right)^3 \tag{2.56}
\end{equation}

The last correction term becomes important for the deviation of \((\frac{dE}{dx})_e\) from \(K_e(V_z/V_0)\), for higher velocities i.e. \(V_z > z_1^{2.3} V_0\).

(d) **Correction for Cascade Feeding:**

In case where the level whose life-time has to be measured is fed from higher excited state, the accurate life-time measurement has to take the effect of this feeding into consideration. The following expression has been suggested by Hoffman et al [14] to consider this effect into account

\begin{equation}
F(\tau_1, \tau_2, \ldots \ldots \tau_n) = (P_1 + \sum_{\tau=2}^{\tau=n} P_{\tau_1}^{\tau_1} \frac{\tau_1}{(\tau_1 - \tau_1)}) F(\tau_1) - \sum_{\tau=2}^{\tau=n} P_{\tau_1}^{\tau_1} \frac{\tau_1}{(\tau_1 - \tau_1)} F(\tau_1) \tag{2.57}
\end{equation}
where $\tau_i$ and $f_i^0$ are the lifetimes of the levels and the branching ratios of the transition involved, respectively. $F(\tau_i)$ are the attenuation factors corresponding to life-time $\tau_i$. Contribution due to cascade feeding can be made negligible if the experiment is performed near threshold energy.

### 2.2.4) Angular Distribution Theory

The gamma ray angular distribution is used to extract spin and parity and multipole mixing ratios. The theory of angular distribution has been discussed from various point of views by several workers [16-23]. The differential cross-section in angular distribution of gamma rays can be expressed as a Legendre polynomial expansion of even order.

$$\frac{d\sigma}{d\Omega} = \Sigma a_v P_v(\cos \theta)$$  \hspace{1cm} (2.58)

where $v = 0, 2, 4, ..., \theta$ is the detection angle of gamma ray due to reaction. The coefficients $a_v$ are the weights of the angular dependent Legendre terms and can be decomposed into a product of energy dependent and momentum dependent terms, the latter are just appropriate transition parameters, one for each constituent step of the over-all process. The values of $a_v$ coefficients are dictated by the nature and the angular momentum of the particle effecting the transitions, and the initial and final nuclear spin involved.

In $\gamma$-ray angular distribution which having the spin sequence as shown in Fig.2.2.
the transition parameters product is

\[ \eta_v (j_1, j_1; J_0, J_1) \ U_v (j_2, j_2; J_2, J_1) \ A_v (L'L', J_3, J_2) \]  (2.59)

where the linking term for the nuclear transitions from \( J_0 \) to \( J_1 \) is given as

\[ \eta_v (j_1, j_1; J_0, J_1) = b_v (j_1, j_1, a) \ F_v (L'L', J_0, J_1) \]  (2.60)

where \( b_v (j_1, j_1, a) \) is the particle parameter and \( F_v (L'L', J_0, J_1) \) is the commensurate parameter of \( \gamma \)-transition.

The parameter of transition \( U_v (j_2, j_2; J_2, J_1) \) is for the unobserved particle \( b \) transition \( J_1 \rightarrow J_2 \). \( A_v (L'L', J_3, J_2) \) is the parameter of \( \gamma \)-ray transition for the transition \( J_2 \rightarrow J_3 \). It can be expressed in terms of generalised parameter \( F_v \) given as follows

\[ A_v (L'L', J_3, J_2) = (1 + \delta^2)^{-1} [F_v (LJ_2 J_3) + 2 \delta F_v (L'L' J_3 J_2)] \]  (2.61)

where \( \delta \) is the multipole mixing ratio given by

\[ \delta = \frac{< J_5 \parallel L'||J_2>}{< J_5 \parallel L||J_2>} \]  (2.62)
Using equation (2.60), the absolute magnitude (in mb $S^{-1}$) of gamma angular distribution differential cross-section for $(a,b\gamma)$ reactions given equation (2.58) can be written as

$$
\frac{d\sigma}{d\Omega} = \frac{1}{4} \lambda^2 \sum_{JJ'} g \eta_v (j_1 j_1 J_0 J_1) U_v (j_2 j_2 J_1 J_2) 
\times A_v (LL' J_3 J_2) T P_v (\cos \theta) \quad (2.63)
$$

where $\lambda$ is rationalized wavelength of the incident particle in the c.m. system and

$$
\lambda^2 = \frac{\hbar^2}{2 M E_1}
$$

$g$ is the statistical spin factor and is given by $g = J_1^2/(S J_o)^2$ where $S$ is the spin of the nucleon and $(S/2)^2 = 1$ for nucleons. $T$ is the Hauser-Feshback penetrability term given as;

$$
T = T_{i1} (E_1) \cdot T_{i2} (E_2) / \sum_{E} T_i (E)
$$

where $E_1$ is the energy of the incident particle, and $E$ and $E_2$ are the emergent particles energies in the c.m. system. The summation in the denominator extends to all the possible decay channel of the compound nucleus.

The transmission coefficient $T_i$ can be expressed in terms of generalised transmission coefficient [24] in order to take into account the spin-orbit interaction as following.
where $T_{1}^{(+)}$ is the transmission coefficient for a particle having spin parallel to the orbital angular momentum.

The expression for nucleon distribution or for the reactions in type of $(a,a)$, $(a,a')$, $(a,b)$, $(a,b')$, $(a,\gamma\gamma)$, $(a, b\gamma)$ and $(a, b\gamma\gamma)$ can be derived in a similar way.

It is the angular distribution characteristic in all above cases that they are exactly symmetrical above 90°. In the mathematical formalism this is expressed by the fact that, the angular dependence is given by Legendre polynomial expansions of even order. Equation (2.63) is expandable to provide the expression for $A_{0}$, $A_{2}$ and $A_{4}$ which can be normalized with $A_{0}=1$ and expressed as angular distribution function

$$W(\theta) = 1 + A_{2}P_{2}(\cos \theta) + A_{4}P_{4}(\cos \theta) \quad (2.65)$$

physically angular distribution will be symmetrical about 90° if:

i) Parity is conserved in the reaction

ii) The distribution measurement is affected in a system which makes no distinction between right and left hand co-ordinate system.

iii) The parity of wave functions describing the reaction (hence, the parity of the intermediate state) is definite.
Unless there is some preferred spin direction, as in the case of aligned nuclei or polarised particles, the angular distribution of the products of reaction proceeding by way of compound nucleus formation display this basic 90° symmetry exactly. Even for direct interaction the γ-ray distributions will exhibit this symmetry. The transmission coefficients which are required for evaluation of Hauser-Feshbach penetrability term in the compound nucleus correlation expansion are usually calculated using optical model potential of Woods-Saxon type with surface absorption (w-s derivative form factor) plus a spin-orbit term of the Thomas form factor

\[ h(r) = \left( \frac{h}{m_e c} \right)^2 \frac{1}{r} \frac{df(r)}{dr}, \]

where

\[ f(r) = \text{Wood-Saxon form factor} = \left[ 1 + \exp\left(\frac{r-R}{a}\right) \right]^{-1}, \quad V_c(r) = \text{Coulomb potential} \]

\[ V(r) = V_c(r) - \left[ V_o + \frac{Z \alpha}{A^{1/3}} V_E \pm (N-Z) \frac{V_s}{A} + i(1-t) W_{vol} \right] f(r) \]

\[ - i t W_{surf} g(r) - V_{so} (L.S) h(r), \]

\[ V_o = \text{Real optical potential}, \quad V_E = \text{Coulomb factor}, \quad V_s = \text{Asymmetry factor} \]

This is the potential of interaction for the particles ‘viewing’ either the target or residual nucleus, where \( t \) is the fraction of the surface peaked imaginary potential part \((W)\) having either surface gaussian form factor or a derivative Wood-Saxon

\[ g(r) = \frac{df(r)}{dr}. \]

The theoretical angular distribution were calculated according to the Hauser-Feschbach theory of nuclear reaction with a modified version of the computer code CINDY [25] written by Sheldon and Rogers. This code takes into account the spin-orbit interaction, presence of competing exit channel in the decay of the compound nucleus.
Experimentally, Sheldon [26] found that at lower energies partial waves having momentum \( \ell > 4 \) exert no appreciable influence upon the magnitude or structure of angular distribution. Thus we have restricted in the present analysis \( \ell \leq 4 \) only. Since compound nucleus may decay into many competing exit channels, resulting in the decrease in the cross-section, all the possible competing exit channels are taken into account. The compound nucleus decay, through additional exit channel, does not affects the angular distribution of \( \gamma \)-rays results. The transmission coefficients required for the evaluation of penetrability term were calculated automatically within the programme by subroutine SCAT using optical model parameter [27].

The CINDY output consists of theoretical \( A_2 \) and \( A_4 \) values for various \( \tan^{-1} \delta \) values within a range from \(-90^\circ\) to \(+90^\circ\) for different possible spins. A locally developed computer code 'CHISQR' [28] was used to perform \( \chi^2 \) fitting of theoretical angular distribution \( W(\theta) \) to the experimental angular distribution \( Y(\theta) \). The best fit is obtained from the minimum value of \( \chi^2 \) defined as:

\[
\chi^2 = \frac{1}{n} \sum_{i} \left[ \frac{Y(\theta_i) - W(\theta_i)}{\Delta Y(\theta_i)} \right]^2
\]  

(2.66)

where \( n \) is the degree of freedom. In angular distribution the degree of freedom means the number of experimental angles minus one. The
summation involves \((n+1)\) terms. \(\Delta Y(\theta_i)\) is the experimental error in \(Y(\theta_i)\).
Fig. 2.1: Schematic diagram to show the components of $x(t) = x_0 + mx$.
Fig. 2.2 SPIN AND ANGULAR MOMENTUM SEQUENCE

\[ J, \Omega, L, S, s \]
REFERENCES


