CHAPTER - 3

CONTROL CHARTS FOR VARIABLES

3.1 Introduction

A control chart is a graphical device that detects variations in any variable quality characteristic of a product. Given a specified target value of the quality characteristic say $\mu_0$, production of the concerned product has to be designed so that the associated quality characteristic for the products should be ideally equal to $\mu_0$, if not, very close to $\mu_0$ on its either side i.e, if the products are showing variations in the desirable quality, the variations must be within control in some admissible sense. There should be two limits within which the allowable variations are supposed to fall. Whenever this happens, the production process is defined to be in control. Otherwise, it is out of control. Based on this principle it is necessary to think of the control limits on either side of the target value in such a way that under normal conditions the limits should include most of the observations. With this backdrop, the well known Shewhart control charts are developed under the assumption that the quality characteristic follows a normal distribution.

If $x_1, x_2, x_3, ..., x_n$ is a collection of observations of size $n$ on a variable quality characteristic of a product, $t_n$ is a statistic based on this sample, the control limits of Shewhart variable control chart are $E(t_n) \pm 3 \text{S.E}(t_n)$. Under repeated sampling of size $n$ at each time (say $k$ times) the graph of the points $(i, t_n(i))$, $i=1$ to $k$ along with three lines parallel to horizontal axis at $E(t_n) - 3 \text{S.E}(t_n)$, $E(t_n)$, $E(t_n) + 3 \text{S.E}(t_n)$
is called control chart for the statistic $t_n$. For instance, if $t_n$ is $\bar{x}$, the graph is control chart for mean. If $t_n$ is range the graph is control chart for range and so on. Assuming normality of the quality data, we can get the control limits for $\bar{x}$ chart. But the limits for other charts like range, median and mid range if derived on the above principle may not be acceptable because of the fact that the distribution of $t_n$ may not be normal. Even if asymptotic normality of $t_n$ is made use of, it is valid only in large samples. However, in quality control studies data is always in small samples only. Therefore if the population is not normal there is a need to develop a separate procedure for the construction of control limits. In this chapter we assume that the quality variate follows inverse Rayleigh model and develop control limits for such a data on par with the presently available control limits.

Skewed distributions to develop control charts are attempted by many authors. Some of them are Edgeman (1989) – Inverse Gaussian distribution, Gonzalez and Viles (2000) – Gamma distribution, Kantam and Sriram (2001) - Gamma distribution, Kantam et al. (2006a) – Log logistic distribution, Betul and Yaziki (2006) – Burr distribution and references therein. Chan and Cui (2003) have developed control chart constants for $\bar{x}$ and R charts in a unified way for a skewed distributions where the constants are dependent on the coefficient of skewness of the distribution. Inverse Rayleigh distribution is another situation of a skewed distribution that was not paid much attention with respect to development of control charts. At the same time it is one of the probability models applicable for life testing and reliability studies. Accordingly, if a lifetime data is considered as a quality data development of control charts for the same is desirable for the use by practitioners. The basic theory and the
development of control charts for the statistics – mean, median, midrange and range are presented in Section 3.2. The comparative study of the developed control limits in relation to the shewart limits is given in Section 3.3. The notion of control charts for individual observations is made use of to develop a graphical technique called analysis of means (ANOM) is presented in Section 3.4. A comparative study with ANOM of normal population is also made for some examples in Section 3.5.

3.2. Variable control charts for inverse Rayleigh distribution

(i) Mean chart

Let \( x_1, x_2, x_3, ..., x_n \) be a random sample of size n supposed to have been drawn from an inverse Rayleigh distribution with scale parameter \( \sigma \) and location parameter zero. If this is considered as a subgroup of an industrial process data with a targeted population average, under repeated sampling the statistic \( \bar{x} \) gives whether the process average is around the targeted mean or not. Statistically speaking, we have to find the ‘MOST PROBABLE’ limits within which \( \bar{x} \) falls. Here the phrase ‘MOST PROBABLE’ is a relative concept which is to be considered in the population sense. The existing procedures for normal distribution take 3-\( \sigma \) limits as the ‘MOST PROBABLE’ limits. It is well known that 3-\( \sigma \) limits of normal distribution include 99.73% of probability. Hence, we have to search two limits of the sampling distribution of sample mean in inverse Rayleigh distribution such that the probability content of these limits is 0.9973. Symbolically, we have to find \( L, U \) such that

\[
P(L \leq \bar{x} \leq U) = 0.9973
\]  \hfill (3.2.1)
Taking the equi-tailed concept L, U are respectively 0.00135 and 0.99865 percentiles of the sampling distribution of $\bar{x}$. But sampling distribution of $\bar{x}$ is not mathematically tractable in the case of inverse Rayleigh distribution. We therefore resorted to the empirical sampling distribution of $\bar{x}$ through simulation thereby computing its percentiles. These are given in the following Table 3.2.1.
Table 3.2.1: Percentiles of distribution of sample mean in inverse Rayleigh distribution

<table>
<thead>
<tr>
<th>p</th>
<th>0.99865</th>
<th>0.995</th>
<th>0.99</th>
<th>0.975</th>
<th>0.95</th>
<th>0.05</th>
<th>0.025</th>
<th>0.01</th>
<th>0.005</th>
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<tbody>
<tr>
<td>2</td>
<td>16.859790</td>
<td>8.181874</td>
<td>5.990661</td>
<td>3.9438906</td>
<td>2.883175</td>
<td>0.700722</td>
<td>0.688199</td>
<td>0.676906</td>
<td>0.6713296</td>
<td>0.665217</td>
</tr>
<tr>
<td>3</td>
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<td>3.513999</td>
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</tr>
<tr>
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<td>2.533028</td>
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<td>0.687445</td>
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<tr>
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<td>5.660109</td>
<td>4.460068</td>
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<td>2.420531</td>
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<td>0.725997</td>
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<td>0.693882</td>
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<tr>
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<tr>
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<td>8.461920</td>
<td>5.066530</td>
<td>3.899644</td>
<td>2.805676</td>
<td>2.221908</td>
<td>0.787661</td>
<td>0.759306</td>
<td>0.735635</td>
<td>0.724673</td>
<td>0.714127</td>
</tr>
<tr>
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<td>8.340324</td>
<td>5.021459</td>
<td>3.764907</td>
<td>2.697892</td>
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<td>0.801136</td>
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<td>8.248611</td>
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<td>3.716181</td>
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<td>2.155152</td>
<td>0.811946</td>
<td>0.783068</td>
<td>0.748746</td>
<td>0.735735</td>
<td>0.718930</td>
</tr>
</tbody>
</table>
The required percentiles from Table 3.2.1 are used in the following manner to get the control limits for sample mean. The mean of the inverse Rayleigh distribution is 1.7728. For a given sample size \( n \), Table 3.2.1 indicates that,

\[
P(z_{0.00135} \leq \bar{z} \leq z_{0.99865}) = 0.9973
\] (3.2.2)

where \( \bar{z} \) is the mean of a sample of size \( n \) from a standard inverse Rayleigh distribution and \( z_p \) are given in Table 3.2.1 for selected values of \( n \) and \( p \).

If \( \bar{x} \) is the mean of a data following an inverse Rayleigh distribution with scale parameter \( \sigma \), \( \bar{x} = \sigma \bar{z} \). Using this in equation (3.2.2), we get

\[
P(z_{0.00135} \leq \bar{x}/\sigma \leq z_{0.99865}) = 0.9973
\] (3.2.3)

\[
P(\sigma z_{0.00135} \leq \bar{x} \leq \sigma z_{0.99865}) = 0.9973
\] (3.2.4)

Unbiased estimate of \( \sigma \) is \( \bar{x}/1.7728 \). From equation (3.2.4) over repeated sampling, for the \( i^{th} \) subgroup mean we have

\[
P(z_{0.00135} \bar{x}/1.7728 \leq \bar{x}_i \leq z_{0.99865} \bar{x}/1.7728) = 0.9973
\] (3.2.5)

This can be written as

\[
P(A^*_2 \bar{x} \leq \bar{x}_i \leq A^{**2}_2 \bar{x}) = 0.9973
\] (3.2.6)

where \( \bar{x} \) is the grand mean, \( \bar{x}_i \) is the \( i^{th} \) subgroup mean where

\[
A^*_2 = Z_{0.00135}/1.7728, \quad A^{**2}_2 = Z_{0.99865}/1.7728.
\]

These constants are given in Table 3.2.2 and are named as percentile constants of \( \bar{x} \) chart.
Table 3.2.2  Percentile constants of $\bar{x}$ - chart

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<tr>
<th>$n$</th>
<th>$A_{2p}^*$</th>
<th>$A_{2p}^{**}$</th>
</tr>
</thead>
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<td>9.510205</td>
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<td>0.382878</td>
<td>9.466822</td>
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<td>0.396815</td>
<td>5.671639</td>
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<td>10</td>
<td>0.405515</td>
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</tbody>
</table>

(ii) Median chart

For this chart also we have to search for two limits of the sampling distribution of sample median in inverse Rayleigh distribution such that the probability content of these limits is 0.9973. Symbolically, we have to find $L$, $U$ such that

$$P(L \leq m \leq U) = 0.9973,$$

where $m$ is the median of sample of size ‘$n$’.

Taking the equi-tailed concept, $L$, $U$ are respectively 0.00135 and 0.99865 percentiles of the sampling distribution of median. But sampling distribution of median is not mathematically tractable in the case of inverse Rayleigh distribution. We therefore resorted to the empirical sampling distribution of median through simulation thereby computing its percentiles. These are given in the following Table 3.2.3.
Table 3.2.3: Percentiles of distribution of sample median in inverse Rayleigh distribution

<table>
<thead>
<tr>
<th>p</th>
<th>n</th>
<th>0.99865</th>
<th>0.995</th>
<th>0.99</th>
<th>0.975</th>
<th>0.95</th>
<th>0.05</th>
<th>0.025</th>
<th>0.01</th>
<th>0.005</th>
<th>0.00135</th>
</tr>
</thead>
<tbody>
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<td>2</td>
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<td>8.181874</td>
<td>5.990661</td>
<td>3.943890</td>
<td>2.883175</td>
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<td>0.704860</td>
<td>0.694226</td>
<td></td>
</tr>
</tbody>
</table>
The required percentiles from Table 3.2.3 are used in the following manner to get the control limits for median. We know that for an inverse Rayleigh distribution with scale parameter \( \sigma \) (for an even sample size) 

\[
\frac{2m}{(\alpha_n + \alpha_{n+1})/2}
\]

is an unbiased estimator for \( \sigma \) where \( m \) is the median of sample of size ‘n’ and \( \alpha_i \) is expected value of \( i^{th} \) standard order statistic in a sample of size n in inverse Rayleigh distribution. When n is odd 

\[
\frac{m}{(\alpha_{(n+1)/2})}
\]

is an unbiased estimator for \( \sigma \). From the distribution of median - \( m \), consider

\[
P(z_{0.00135} \leq m \leq z_{0.99865}) = 0.9973 \quad (3.2.7)
\]

where \( z_p \), \( 0 < p < 1 \) are given in Table 3.2.3. From equation (3.2.7), for the \( i^{th} \) subgroup median, we have

\[
P(F_{2p}^* \frac{\bar{m}}{\alpha_{n/2} + \alpha_{n/2+1}} \leq m_i \leq F_{2p}^* \frac{2\bar{m}}{\alpha_{n/2} + \alpha_{n/2+1}}) = 0.9973, \quad \text{if } n \text{ is even} \quad (3.2.8)
\]

\[
P(F_{2p}^* \frac{\bar{m}}{\alpha_{(n+1)/2}} \leq m_i \leq F_{2p}^* \frac{2\bar{m}}{\alpha_{(n+1)/2}}) = 0.9973, \quad \text{if } n \text{ is odd} \quad (3.2.9)
\]

where \( \bar{m} \) is the mean of all subgroup medians and the constants take the appropriate forms according as n is even or odd.

These can be written as

\[
P(F_{2p}^* \bar{m} \leq m_i \leq F_{2p}^* \bar{m}) = 0.9973 \quad (3.2.10)
\]

where \( F_{2p}^* = \frac{\alpha_{(n+1)/2}}{Z_{0.00135}} \) and \( F_{2p}^* = \frac{Z_{0.99865}}{\alpha_{(n+1)/2}} \).
The constants $F_{2p}^*$, $F_{2p}^{**}$ are the constants of median chart for a inverse Rayleigh distribution process data. These are given in Table 3.2.4 and are named as percentile constants of Median chart.

Table 3.2.4: Percentile constants of median chart

<table>
<thead>
<tr>
<th>n</th>
<th>$F_{2p}^*$</th>
<th>$F_{2p}^{**}$</th>
</tr>
</thead>
<tbody>
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</tbody>
</table>

(iii) Mid range chart

On lines of construction of range chart we have to search for two limits of the sampling distribution of sample mid range in inverse Rayleigh distribution such that the probability content of these limits is 0.9973. Symbolically, we have to find $L$, $U$ such that $P(L \leq M \leq U) = 0.9973$, where $M$ is the mid range of sample of size ‘$n$’ defined as $M = \frac{x_{(1)} + x_{(n)}}{2}$. Taking the equi-tailed concept $L$, $U$ are respectively 0.00135 and 0.99865 percentiles of the sampling distribution of mid range. But sampling distribution of mid range is not mathematically tractable in the case of inverse Rayleigh distribution.
We therefore resorted to the empirical sampling distribution of mid range through simulation, thereby computing its percentiles. These are given in the following Table 3.2.5.
Table 3.2.5: Percentiles of distribution of sample mid range in inverse Rayleigh distribution

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<th>0.975</th>
<th>0.95</th>
<th>0.05</th>
<th>0.025</th>
<th>0.01</th>
<th>0.005</th>
<th>0.00135</th>
</tr>
</thead>
<tbody>
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<td>5.990661</td>
<td>3.943891</td>
<td>2.883175</td>
<td>0.700722</td>
<td>0.688199</td>
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<td>11.222485</td>
<td>6.851959</td>
<td>4.793768</td>
<td>0.868243</td>
<td>0.792572</td>
<td>0.735222</td>
<td>0.727522</td>
<td>0.718415</td>
</tr>
<tr>
<td>9</td>
<td>34.043602</td>
<td>18.848043</td>
<td>11.954387</td>
<td>7.371525</td>
<td>5.146961</td>
<td>0.907561</td>
<td>0.834208</td>
<td>0.758697</td>
<td>0.733386</td>
<td>0.723957</td>
</tr>
<tr>
<td>10</td>
<td>35.69058</td>
<td>19.233511</td>
<td>12.347066</td>
<td>7.886197</td>
<td>5.468972</td>
<td>0.938505</td>
<td>0.865377</td>
<td>0.782502</td>
<td>0.739317</td>
<td>0.728013</td>
</tr>
</tbody>
</table>
The required percentiles from Table 3.2.5 are used in the following manner to get the control limits for mid range. We know that for an inverse Rayleigh distribution with scale parameter \( \sigma \), \( 2M/(\alpha_n + \alpha_i) \) is an unbiased estimator for \( \sigma \) where \( M \) is the mid range of sample of size ‘n’ and \( \alpha_i \) is expected value of \( i^{th} \) standard order statistic in a sample of size \( n \) in inverse Rayleigh distribution. Under repeated sampling of size \( n \), \( \bar{M} \) - the mean of mid ranges of all samples of size ‘n’, \( \frac{2\bar{M}}{\alpha_n + \alpha_1} \) is also an unbiased estimate of \( \sigma \). From the distribution of \( M \), consider

\[
P(z_{0.00135} \leq M \leq z_{0.99865}) = 0.9973 \tag{3.2.11}
\]

where \( z_p \) (0 < \( p \) < 1) are given in Table 3.2.5. From equation (3.2.11), for the \( i^{th} \) subgroup mid range, we have

\[
P(Z_{0.00135} \leq 2M/(\alpha_n + \alpha_i) \leq Z_{0.99865} \leq 2\bar{M}/(\alpha_n + \alpha_1)) = 0.9973 \tag{3.2.12}
\]

This can be written as

\[
P(G_{2p}^* \leq M_i \leq G_{2p}^{**}) = 0.9973 \tag{3.2.13}
\]

where \( G_{2p}^* = \frac{2Z_{0.00135}}{\alpha_n + \alpha_i} \), \( G_{2p}^{**} = \frac{2Z_{0.00135}}{\alpha_n + \alpha_i} \)

and \( \bar{M} \) is the mean of mid ranges, \( M_i \) is the \( i^{th} \) subgroup mid range. These are given in Table 3.2.6 and are named as percentile constants of mid range chart.
Table 3.2.6: Percentile constants of mid range chart

<table>
<thead>
<tr>
<th>n</th>
<th>(G_{2p}^*)</th>
<th>(G_{2p}^{**})</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.375233</td>
<td>9.510204</td>
</tr>
<tr>
<td>3</td>
<td>0.382756</td>
<td>13.358067</td>
</tr>
<tr>
<td>4</td>
<td>0.387655</td>
<td>14.286680</td>
</tr>
<tr>
<td>5</td>
<td>0.391483</td>
<td>14.293865</td>
</tr>
<tr>
<td>6</td>
<td>0.397491</td>
<td>14.470478</td>
</tr>
<tr>
<td>7</td>
<td>0.402451</td>
<td>16.518253</td>
</tr>
<tr>
<td>8</td>
<td>0.405241</td>
<td>16.744556</td>
</tr>
<tr>
<td>9</td>
<td>0.408367</td>
<td>19.203181</td>
</tr>
<tr>
<td>10</td>
<td>0.410654</td>
<td>20.132201</td>
</tr>
</tbody>
</table>

(iv) Range chart

As mentioned in the previous charts, we have to search for two limits of the sampling distribution of sample range in inverse Rayleigh distribution such that the probability content of these limits is 0.9973. Symbolically, we have to find \(L\), \(U\) such that

\[P(L \leq R \leq U) = 0.9973\]

where \(R\) is the range of sample of size ‘\(n\)’.

Taking the equi-tailed concept, \(L\), \(U\) are respectively 0.00135 and 0.99865 percentiles of the sampling distribution of range. But sampling distribution of range is not mathematically tractable in the case of inverse Rayleigh distribution. We therefore resorted to the empirical sampling distribution of range through simulation thereby computing its percentiles. These are given in the Table 3.2.7.
Table 3.2.7: Percentiles of distribution of sample range in inverse Rayleigh distribution

<table>
<thead>
<tr>
<th>p</th>
<th>0.99865</th>
<th>0.995</th>
<th>0.99</th>
<th>0.975</th>
<th>0.95</th>
<th>0.05</th>
<th>0.025</th>
<th>0.01</th>
<th>0.005</th>
<th>0.00135</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>31.063782</td>
<td>13.584499</td>
<td>9.804784</td>
<td>5.459278</td>
<td>3.615187</td>
<td>0.010579</td>
<td>0.005164</td>
<td>0.001839</td>
<td>0.000878</td>
<td>0.000141</td>
</tr>
<tr>
<td>3</td>
<td>45.871502</td>
<td>17.766344</td>
<td>11.936316</td>
<td>7.344730</td>
<td>4.806969</td>
<td>0.044301</td>
<td>0.030934</td>
<td>0.019251</td>
<td>0.013613</td>
<td>0.006255</td>
</tr>
<tr>
<td>4</td>
<td>48.800000</td>
<td>21.019660</td>
<td>14.238816</td>
<td>8.633285</td>
<td>5.679380</td>
<td>0.080783</td>
<td>0.060868</td>
<td>0.040165</td>
<td>0.031287</td>
<td>0.019245</td>
</tr>
<tr>
<td>5</td>
<td>49.325714</td>
<td>22.739237</td>
<td>16.118628</td>
<td>9.829440</td>
<td>6.465505</td>
<td>0.107999</td>
<td>0.082969</td>
<td>0.061026</td>
<td>0.049935</td>
<td>0.036933</td>
</tr>
<tr>
<td>6</td>
<td>49.954660</td>
<td>24.235191</td>
<td>17.819733</td>
<td>10.862133</td>
<td>7.032875</td>
<td>0.148962</td>
<td>0.108822</td>
<td>0.083995</td>
<td>0.072523</td>
<td>0.051706</td>
</tr>
<tr>
<td>7</td>
<td>56.878740</td>
<td>31.233740</td>
<td>19.967863</td>
<td>11.728809</td>
<td>7.732683</td>
<td>0.292359</td>
<td>0.140916</td>
<td>0.106117</td>
<td>0.091474</td>
<td>0.073916</td>
</tr>
<tr>
<td>8</td>
<td>58.051254</td>
<td>31.911390</td>
<td>21.068237</td>
<td>12.355273</td>
<td>8.209303</td>
<td>0.367563</td>
<td>0.230313</td>
<td>0.121917</td>
<td>0.105204</td>
<td>0.083508</td>
</tr>
<tr>
<td>9</td>
<td>66.741700</td>
<td>36.304234</td>
<td>22.518719</td>
<td>13.334203</td>
<td>8.898969</td>
<td>0.453051</td>
<td>0.303860</td>
<td>0.156926</td>
<td>0.121616</td>
<td>0.094590</td>
</tr>
<tr>
<td>10</td>
<td>70.024410</td>
<td>37.114456</td>
<td>23.276375</td>
<td>14.396691</td>
<td>9.605666</td>
<td>0.518204</td>
<td>0.372996</td>
<td>0.227190</td>
<td>0.133392</td>
<td>0.101308</td>
</tr>
</tbody>
</table>
We know that for inverse Rayleigh distribution with scale parameter \( \sigma \), \( \frac{\overline{R}}{(\alpha_n - \alpha_1)} \) is an unbiased estimator for \( \sigma \), where \( R \) is the range of sample of size ‘\( n \)’ and '\( \alpha_n \)' is the maximum ordered statistics and '\( \alpha_1 \)' is the minimum ordered statistics of the sample considered. Under repeated sampling of size \( n \), \( \frac{\overline{R}}{(\alpha_n - \alpha_1)} \) is also an unbiased estimate of \( \sigma \). From distribution of \( R \), consider

\[
P(z_{0.00135} \leq R \leq z_{0.99865}) = 0.9973 \quad (3.2.14)
\]

where \( z_p (0<p<1) \) are given in Table 3.2.7 for selected values of \( p \). From Equation (3.2.14), for the \( i^{th} \) subgroup range, we have

\[
P(Z_{0.00135} \frac{\overline{R}}{(\alpha_n - \alpha_1)} \leq R \leq Z_{0.99865} \frac{\overline{R}}{(\alpha_n - \alpha_1)}) = 0.9973 \quad (3.2.15)
\]

This can be written as

\[
P(D_{3p}^* \overline{R} \leq R \leq D_{3p}^{**} \overline{R}) = 0.9973 \quad (3.2.16)
\]

where

\[
D_{3p}^* = \frac{Z_{0.00135}}{(\alpha_n - \alpha_1)} \quad \text{and} \quad D_{3p}^{**} = \frac{Z_{0.99865}}{(\alpha_n - \alpha_1)}
\]

and \( \overline{R} \) is the mean of ranges, \( R_i \) is the \( i^{th} \) subgroup range. Thus \( D_{3p}^* , D_{3p}^{**} \) are the constants of \( R \)-chart for inverse Rayleigh distribution process data. These are given in Table 3.2.8 and are named as percentile constants of range chart.
Table 3.2.8: Percentile constants of Range chart

<table>
<thead>
<tr>
<th>n</th>
<th>$D_{3p}$</th>
<th>$D_{3p}^{**}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7.975033</td>
<td>17.522336</td>
</tr>
<tr>
<td>3</td>
<td>0.003528</td>
<td>25.875016</td>
</tr>
<tr>
<td>4</td>
<td>0.010855</td>
<td>27.526916</td>
</tr>
<tr>
<td>5</td>
<td>0.020834</td>
<td>27.823455</td>
</tr>
<tr>
<td>6</td>
<td>0.029166</td>
<td>28.178227</td>
</tr>
<tr>
<td>7</td>
<td>0.041694</td>
<td>32.083935</td>
</tr>
<tr>
<td>8</td>
<td>0.047105</td>
<td>32.745323</td>
</tr>
<tr>
<td>9</td>
<td>0.053356</td>
<td>37.647395</td>
</tr>
<tr>
<td>10</td>
<td>0.057145</td>
<td>39.499092</td>
</tr>
</tbody>
</table>

3.3 Comparative study – Control charts

The control chart constants for the statistics – mean, median, midrange and range developed in Section 3.2 are based on the population modeled by inverse Rayleigh distribution. In order to use this for a data, the data is required to follow IRD. Therefore, the power of the control limits can be assessed through their application for a true IRD data in relation to the application of Shewart limits for a true inverse Rayleigh data. With this backdrop we have made this comparative study by simulating random samples of size $n=2(1)10$ from IRD and calculated the control limits using the constants of Section 3.2 for mean, median, midrange and range in succession. The count of the number of statistic values that have fallen within the respective control limits is evaluated. Similar counts for control limits using Shewart constants available in quality control manuals are also calculated. These are named as shewart coverage probability. The coverage probabilities under the two schemes
namely true inverse Rayleigh, Shewart limits are presented in the following Tables 3.3.1, 3.3.2, 3.3.3 and 3.3.4.

**Table 3.3.1: Coverage probabilities of \( \bar{X} \) - chart**

<table>
<thead>
<tr>
<th>n</th>
<th>IRD Coverage Probability</th>
<th>SHEWART Coverage Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.9966</td>
<td>0.9570</td>
</tr>
<tr>
<td>3</td>
<td>0.9976</td>
<td>0.9559</td>
</tr>
<tr>
<td>4</td>
<td>0.9968</td>
<td>0.9544</td>
</tr>
<tr>
<td>5</td>
<td>0.9958</td>
<td>0.9563</td>
</tr>
<tr>
<td>6</td>
<td>0.9954</td>
<td>0.9574</td>
</tr>
<tr>
<td>7</td>
<td>0.9940</td>
<td>0.9590</td>
</tr>
<tr>
<td>8</td>
<td>0.9938</td>
<td>0.9595</td>
</tr>
<tr>
<td>9</td>
<td>0.9944</td>
<td>0.9628</td>
</tr>
<tr>
<td>10</td>
<td>0.9941</td>
<td>0.9636</td>
</tr>
</tbody>
</table>

**Table 3.3.2: Coverage probabilities of median chart**

<table>
<thead>
<tr>
<th>n</th>
<th>IRD Coverage Probability</th>
<th>SHEWART Coverage Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.9966</td>
<td>0.9570</td>
</tr>
<tr>
<td>3</td>
<td>0.9804</td>
<td>0.9893</td>
</tr>
<tr>
<td>4</td>
<td>0.9767</td>
<td>0.9944</td>
</tr>
<tr>
<td>5</td>
<td>0.9740</td>
<td>0.9973</td>
</tr>
<tr>
<td>6</td>
<td>0.9700</td>
<td>0.9986</td>
</tr>
<tr>
<td>7</td>
<td>0.9683</td>
<td>0.9994</td>
</tr>
<tr>
<td>8</td>
<td>0.9375</td>
<td>0.9997</td>
</tr>
<tr>
<td>9</td>
<td>0.9216</td>
<td>0.9999</td>
</tr>
<tr>
<td>10</td>
<td>0.9000</td>
<td>0.9999</td>
</tr>
</tbody>
</table>
Table 3.3.3: Coverage probabilities of mid range chart

<table>
<thead>
<tr>
<th>n</th>
<th>IRD Coverage Probability</th>
<th>SHEWART Coverage Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.9966</td>
<td>0.9656</td>
</tr>
<tr>
<td>3</td>
<td>0.9974</td>
<td>0.9531</td>
</tr>
<tr>
<td>4</td>
<td>0.9969</td>
<td>0.9322</td>
</tr>
<tr>
<td>5</td>
<td>0.9958</td>
<td>0.9279</td>
</tr>
<tr>
<td>6</td>
<td>0.9955</td>
<td>0.9129</td>
</tr>
<tr>
<td>7</td>
<td>0.9951</td>
<td>0.9116</td>
</tr>
<tr>
<td>8</td>
<td>0.9951</td>
<td>0.8547</td>
</tr>
<tr>
<td>9</td>
<td>0.9953</td>
<td>0.8326</td>
</tr>
<tr>
<td>10</td>
<td>0.9952</td>
<td>0.7398</td>
</tr>
</tbody>
</table>

Table 3.3.4: Coverage probabilities of range chart

<table>
<thead>
<tr>
<th>n</th>
<th>IRD Coverage Probability</th>
<th>SHEWART Coverage Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.9961</td>
<td>0.9438</td>
</tr>
<tr>
<td>3</td>
<td>0.9974</td>
<td>0.9216</td>
</tr>
<tr>
<td>4</td>
<td>0.9969</td>
<td>0.9142</td>
</tr>
<tr>
<td>5</td>
<td>0.9959</td>
<td>0.9103</td>
</tr>
<tr>
<td>6</td>
<td>0.9957</td>
<td>0.9033</td>
</tr>
<tr>
<td>7</td>
<td>0.9953</td>
<td>0.8676</td>
</tr>
<tr>
<td>8</td>
<td>0.9955</td>
<td>0.8412</td>
</tr>
<tr>
<td>9</td>
<td>0.9956</td>
<td>0.8128</td>
</tr>
<tr>
<td>10</td>
<td>0.9953</td>
<td>0.7844</td>
</tr>
</tbody>
</table>
Summary and conclusions

The coverage probabilities tables of mean, mid range and range proved that for a true inverse Rayleigh data if the Shewart limits are used in a mechanical way it would result in less confidence coefficient about the decision of process variation. Hence if a data is confirmed to follow inverse Rayleigh data or IRD is a best fit for a given data, then usage of Shewart constants is not advisable and exclusive evaluation and application of IRD constants is preferable, whereas the coverage probabilities table referred to median indicates that the usage of Shewart constants is advisable.

3.4 Analysis of means (ANOM)

The Shewart control chart is a common tool of statistical quality control for many practitioners. When these charts indicate the presence of an assignable cause (of non-random variability), an adjustment of the process is made if the remedy is known. Otherwise the suspected presence of assignable cause is regarded to be an indication of heterogeneity of the subgroup statistic for which the control chart is developed. For instance the statistic is sample mean, this leads to heterogeneity of process mean indicating departures from target mean. Such an analysis is generally carried out with the help of means (ANOM) to divide a collection of a given number of subgroup means into categories such that means within a category are homogeneous and those between categories are heterogeneous, under the assumption that the probability model of the variate is normal. We have already noticed that any statistical method if needs to be applied for an non-normal data separate evaluation is essential. In this section we made an attempt to
develop the ANOM procedure of Ott (1967) when the data variate is supposed to follow inverse Rayleigh distribution.

Suppose $\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_k$ are arithmetic means of $k$ subgroups of size ‘$n$’ each drawn from an IRD model. If these subgroup means are used to develop control charts to assess whether the population from which these subgroups are drawn is operating with admissible quality variations depending on the basic population model, we may use the control chart constants developed by us or the popular Shewart constants given in any SQC text book. Generally, we say that the process is in control if all the subgroup means fall within the control limits, otherwise we say the process lacks control. If $\alpha$ is the level of significance of the above decisions, we have the following probability statements.

$$P\{LCL < \bar{x}_i, \forall i = 1 \text{ to } k < UCL\} = 1 - \alpha$$ \hspace{1cm} (3.4.1)

Using the notion of independent subgroups (Equation 3.4.1) becomes

$$P\{LCL < \bar{x}_i < UCL\} = (1 - \alpha)^{1/k}$$ \hspace{1cm} (3.4.2)

With equi-tailed probability for each subgroup mean we can find two constants say $L^*$ and $U^*$ such that

$$P\{\bar{x}_1 < L^* \} = P\{\bar{x}_1 < U^* \} = \frac{1 - (1 - \alpha)^k}{2}$$

In the case of normal population $L^*$ and $U^*$ satisfy $U^* = -L^*$. For the skewed populations like IRD, we have to calculate $L^*$, $U^*$ separately from the sampling distribution of $\bar{X}$. Accordingly, these depend on the subgroup size ‘$n$’ and the number of subgroups ‘$k$’. Since sampling
distribution of \( \bar{X} \) on the basis of sample size of ‘n’ can be obtained from IRD population and \( L^*, U^* \) can be tabulated. These are given in Tables 3.4.1 and 3.4.2.

A control chart for averages giving ‘In Control’ conclusion indicates that all the subgroup means though vary among themselves are homogenous in some sense. This is exactly the null hypothesis in an analysis of variance technique. Hence the constants of Tables 3.4.1 and 3.4.2 can be used as an alternative to analysis of variance technique. For a normal population one can use the tables of Ott (1967). For an IRD our tables can be used. We therefore present below some examples for which the goodness of fit of IRD model assessed with Q–Q plot technique (strength of linearity between observed and theoretical quantiles of a model) and tested the homogeneity of means involved in each case.

Example 1: Consider the following data of 25 observations on a manufactures of metal products that suspected variations in iron content of raw material supplied by five suppliers. Five ingots were randomly selected from each of the five suppliers. The following table contains the data for the iron determinations on each ingots in percent by weight. Test whether the iron content of the material from the five suppliers is same.

<table>
<thead>
<tr>
<th>Suppliers</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.46</td>
<td>3.59</td>
<td>3.51</td>
<td>3.38</td>
<td>3.29</td>
</tr>
<tr>
<td>2</td>
<td>3.48</td>
<td>3.46</td>
<td>3.64</td>
<td>3.40</td>
<td>3.46</td>
</tr>
<tr>
<td>3</td>
<td>3.56</td>
<td>3.42</td>
<td>3.46</td>
<td>3.37</td>
<td>3.37</td>
</tr>
<tr>
<td>4</td>
<td>3.39</td>
<td>3.49</td>
<td>3.52</td>
<td>3.46</td>
<td>3.32</td>
</tr>
<tr>
<td>5</td>
<td>3.40</td>
<td>3.50</td>
<td>3.49</td>
<td>3.39</td>
<td>3.38</td>
</tr>
</tbody>
</table>
Example 2: Three brands of batteries are under study. It is suspected that the life (in weeks) of the three brands is different. Five batteries of each brand are tested with the following results. Test whether the lives of these brands of batteries are different at 5 % level of significance.

<table>
<thead>
<tr>
<th>Weeks of life</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brand 1</td>
</tr>
<tr>
<td>100</td>
</tr>
<tr>
<td>96</td>
</tr>
<tr>
<td>92</td>
</tr>
<tr>
<td>96</td>
</tr>
<tr>
<td>92</td>
</tr>
</tbody>
</table>

Example 3: Four catalysts that may affect the concentration of one component in a three component liquid mixture are being investigated. The following concentrations are obtained. Test whether the four catalysts have the same effect on the concentration at 5 % level of significance.

<table>
<thead>
<tr>
<th>Catalyst</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>58.2</td>
</tr>
<tr>
<td>57.2</td>
</tr>
<tr>
<td>58.4</td>
</tr>
<tr>
<td>55.8</td>
</tr>
</tbody>
</table>
The goodness of fit for the data in these three examples as revealed by Q–Q plot (correlation coefficient) are summarized in the following table, which shows that inverse Rayleigh distribution is a better model, exhibiting significant linear relation between sample and population quantiles.

<table>
<thead>
<tr>
<th>Example</th>
<th>inverse Rayleigh distribution</th>
<th>Normal distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (n = 25)</td>
<td>0.917132</td>
<td>0.206750</td>
</tr>
<tr>
<td>2 (n = 15)</td>
<td>0.958024</td>
<td>0.414920</td>
</tr>
<tr>
<td>3 (n = 16)</td>
<td>0.965096</td>
<td>0.444710</td>
</tr>
</tbody>
</table>

Treating these observations in each table as a single sample, we have calculated the decision limits for the IRD population and verified homogeneity of means.

3.5 Comparative study – ANOM

Our conclusions are summarized in the Tables 3.5.3 and 3.5.4.

Table: 3.5.3

<table>
<thead>
<tr>
<th>Example</th>
<th>(LDL, UDL)</th>
<th>No. of subgroups fall</th>
<th>Normal distribution</th>
<th>Coverage probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(3.379, 3.517)</td>
<td>3</td>
<td>0.6</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>(87.82, 95.52)</td>
<td>2</td>
<td>0.7</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>(26.14, 82.84)</td>
<td>2</td>
<td>0.5</td>
<td>2</td>
</tr>
</tbody>
</table>
Table: 3.5.4

<table>
<thead>
<tr>
<th>inverse Rayleigh distribution</th>
<th>No. of subgroups fall</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Within the decision lines</td>
</tr>
<tr>
<td>( LDL, UDL )</td>
<td></td>
</tr>
</tbody>
</table>

| Example 1 n = 5, k = 5, α = 0.05 | (5.63, 39.53) | 5 | 1.0 | 0 | 0 |
| Example 2 n = 5, k = 3, α = 0.05 | (106.443,913.263) | 3 | 1.0 | 0 | 0 |
| Example 3 n = 4, k = 4, α = 0.05 | (59.083,632.713) | 4 | 1.0 | 0 | 0 |

Since IRD is found to be better test for the data of the three examples, ANOM gave a larger (complete) homogeneity of data than those of Ott (1967).
<table>
<thead>
<tr>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.00597</td>
<td>0.94274</td>
<td>0.89226</td>
<td>0.87278</td>
<td>0.85451</td>
<td>0.84893</td>
<td>0.84036</td>
<td>0.82878</td>
<td>0.82594</td>
<td>0.82368</td>
<td>0.80118</td>
<td>0.79594</td>
<td>0.78552</td>
<td>0.76043</td>
<td>0.75749</td>
</tr>
<tr>
<td>4</td>
<td>1.14055</td>
<td>1.07491</td>
<td>1.03026</td>
<td>0.99689</td>
<td>0.97805</td>
<td>0.96691</td>
<td>0.95948</td>
<td>0.94550</td>
<td>0.93312</td>
<td>0.92445</td>
<td>0.89154</td>
<td>0.88189</td>
<td>0.85594</td>
<td>0.84258</td>
<td>0.82135</td>
</tr>
<tr>
<td>6</td>
<td>1.22500</td>
<td>1.14909</td>
<td>1.11631</td>
<td>1.08430</td>
<td>1.06491</td>
<td>1.05514</td>
<td>1.04624</td>
<td>1.03367</td>
<td>1.01805</td>
<td>1.01672</td>
<td>0.99274</td>
<td>0.98573</td>
<td>0.96830</td>
<td>0.95618</td>
<td>0.95589</td>
</tr>
<tr>
<td>8</td>
<td>1.30963</td>
<td>1.23930</td>
<td>1.20703</td>
<td>1.18611</td>
<td>1.16366</td>
<td>1.15021</td>
<td>1.14144</td>
<td>1.11824</td>
<td>1.10598</td>
<td>1.10329</td>
<td>1.08328</td>
<td>1.07093</td>
<td>1.04636</td>
<td>1.01763</td>
<td>1.00201</td>
</tr>
<tr>
<td>10</td>
<td>1.35342</td>
<td>1.27741</td>
<td>1.23807</td>
<td>1.20922</td>
<td>1.19087</td>
<td>1.17466</td>
<td>1.16683</td>
<td>1.16054</td>
<td>1.15180</td>
<td>1.14989</td>
<td>1.10814</td>
<td>1.10073</td>
<td>1.08807</td>
<td>1.03263</td>
<td>1.01549</td>
</tr>
<tr>
<td>11</td>
<td>1.43083</td>
<td>1.35618</td>
<td>1.31410</td>
<td>1.28171</td>
<td>1.26203</td>
<td>1.23536</td>
<td>1.21879</td>
<td>1.21086</td>
<td>1.19877</td>
<td>1.19573</td>
<td>1.16356</td>
<td>1.14950</td>
<td>1.13843</td>
<td>1.10672</td>
<td>1.08669</td>
</tr>
<tr>
<td>13</td>
<td>1.44695</td>
<td>1.38760</td>
<td>1.34869</td>
<td>1.32480</td>
<td>1.30813</td>
<td>1.29380</td>
<td>1.27270</td>
<td>1.25842</td>
<td>1.25063</td>
<td>1.24432</td>
<td>1.21376</td>
<td>1.20128</td>
<td>1.17876</td>
<td>1.15409</td>
<td>1.14853</td>
</tr>
<tr>
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<td>1.49412</td>
<td>1.42014</td>
<td>1.39235</td>
<td>1.37377</td>
<td>1.36327</td>
<td>1.34805</td>
<td>1.34141</td>
<td>1.33147</td>
<td>1.31711</td>
<td>1.31249</td>
<td>1.27015</td>
<td>1.25249</td>
<td>1.24421</td>
<td>1.22679</td>
<td>1.22313</td>
</tr>
<tr>
<td>17</td>
<td>1.53157</td>
<td>1.46659</td>
<td>1.42323</td>
<td>1.39821</td>
<td>1.38339</td>
<td>1.36621</td>
<td>1.35741</td>
<td>1.35547</td>
<td>1.34542</td>
<td>1.34188</td>
<td>1.31966</td>
<td>1.30464</td>
<td>1.27891</td>
<td>1.27155</td>
<td>1.26801</td>
</tr>
</tbody>
</table>

**TABLE 3.4.1**: Inverse Rayleigh distribution constants for analysis of means (σ = 1 with 1-α = 0.95)
<table>
<thead>
<tr>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<th>9</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.85448</td>
<td>0.82339</td>
<td>0.79941</td>
<td>0.79589</td>
<td>0.78588</td>
<td>0.78062</td>
<td>0.77741</td>
<td>0.76043</td>
<td>0.75896</td>
<td>0.75749</td>
<td>0.72465</td>
<td>0.71382</td>
<td>0.70425</td>
<td>0.64107</td>
<td>0.57789</td>
</tr>
<tr>
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<td>0.92399</td>
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<td>0.87709</td>
<td>0.85812</td>
<td>0.85585</td>
<td>0.84462</td>
<td>0.84258</td>
<td>0.83196</td>
<td>0.82135</td>
<td>0.81851</td>
<td>0.81717</td>
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<td>0.8171</td>
<td>0.8166</td>
</tr>
<tr>
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<td>0.97370</td>
<td>0.96215</td>
<td>0.95936</td>
<td>0.95604</td>
<td>0.95589</td>
<td>0.94176</td>
<td>0.93938</td>
<td>0.92586</td>
<td>0.91257</td>
<td>0.89929</td>
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</tr>
<tr>
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<td>1.01763</td>
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<td>1.08879</td>
<td>1.07830</td>
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<td>1.03263</td>
<td>1.02406</td>
<td>1.01549</td>
<td>1.01513</td>
<td>1.01512</td>
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<td>1.18365</td>
<td>1.16649</td>
<td>1.16082</td>
<td>1.15409</td>
<td>1.15131</td>
<td>1.14853</td>
<td>1.13918</td>
<td>1.13666</td>
<td>1.13136</td>
<td>1.11799</td>
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<td>1.243179</td>
<td>1.215127</td>
<td>1.18591</td>
<td>1.183857</td>
<td>1.201832</td>
<td>20.01832</td>
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<td>71.02315</td>
</tr>
<tr>
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<td>1.25119</td>
<td>1.32493</td>
<td>1.22933</td>
<td>1.22679</td>
<td>1.22496</td>
<td>1.22313</td>
<td>1.20260</td>
<td>1.19902</td>
<td>1.17335</td>
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<td>1.15494</td>
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<tr>
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<td>1.27588</td>
<td>1.27326</td>
<td>1.27155</td>
<td>1.26978</td>
<td>1.26801</td>
<td>1.19279</td>
<td>1.17624</td>
<td>1.15606</td>
<td>1.13160</td>
<td>1.10714</td>
</tr>
</tbody>
</table>
import java.io.*;
import java.util.*;
class IrdMean
{
    public static void main(String args[])throws IOException
    {
        BufferedReader br=new BufferedReader(new InputStreamReader(System.in));
        Random rd=new Random(14L);
        int t, z, m, c, count=0,l,mn=0,p,i,n,j;
        int a[]=new int[1010];
        float tp,k,res=0;
        float x[]=new float[1100];
        float mns[]=new float[1010];
        float asc[]=new float[10002];
        float temp[]=new float[10];
        float q[]=new float[100];
        float low[]=new float[50];
        float upp[]=new float[50];
        System.out.print("n Enter how many 1000 numbers : ");
        m=Integer.parseInt(br.readLine());
        c=m*10;
        t=1;
        l=1000;
        while(t<=c)
        {
            for(i=1;i<=1000;i++)  a[i]=rd.nextInt(214748);
            if(count>0)
            {
                j=(l-(l%m))+1;
                for(i=1;i<=count;i++)
                {
                    q[i]=x[j];j++;
                }
            }
        }
    }
}/
l=1000+count;
j=1;
for(i=1,p=1;i<=l&p<=1000;i++)
{
    if(count>0)
    {
        x[i]=q[j];
        j++;
        count--;
    }
    else
    {
        if(a[p]<=99)
            k=(float)a[p]*0.01f;
        else if(a[p]<=999)
            k=(float)a[p]*0.001f;
        else if(a[p]<=9999)
            k=(float)a[p]*0.0001f;
        else if(a[p]<=99999)
            k=(float)a[p]*0.00001f;
        else
            k=(float)a[p]*0.000001f;
        x[i]=(float)Math.sqrt(1.0/Math.log(1/k));p++;
    }
}
count=0;
for(i=1,j=1;i<=(l-l%m);j++)
{
    for(z=0;z<m;z++)
    {
        temp[z]=x[i];
        i++;
    }
    asc[mn]=mean(temp,m);
    mns[j]=asc[mn];
    res+=mns[j];
mn++; 
} 
count=l%m; 
t++; 
} 
System.out.println("n xbar= "+res/10000); 
for(i=0;i<mn-1;i++) 
for(j=i+1;j<mn;j++) 
if (asc[i]>asc[j]) 
{ 
    tp=asc[i]; 
    asc[i]=asc[j]; 
    asc[j]=tp; 
} 
double r=Math.sqrt(22.0/7.0); 
System.out.println("n 14 element="+asc[13]+"n50 element="+asc[49]+"n \
100 element="+asc[99]+"n 250 element="+asc[249]+"n 500 \
 element="+asc[499]); 
System.out.println("n 9987 element="+asc[9986]+"n 9950 \
element="+asc[9949]+"n9900 element="+asc[9899]+ \
"n 9750 element="+asc[9749]+"n 9500 element="+asc[9499]); 
System.out.println(asc[13]/r+" n+asc[9986]/r); 
double v1=asc[13]/r;  double v2=asc[9986]/r; 
int cnt=0; 
for(i=0;i<10000;i++) 
if(asc[i]>=v1 && asc[i]<=v2) cnt++; 
System.out.println("Count="+cnt); 
} 
static float mean(float x[],int m) 
{ 
    int i; 
    float sum=0; 
    for(i=0;i<m;i++) 
        sum=sum+x[i]; 
    return(sum/m); 
} 
}
import java.io.*;
import java.util.*;
class Ird Median
{
    public static void main(String args[])
    {
        BufferedReader br=new BufferedReader(new InputStreamReader(System.in));
        Random rd=new Random(14L);
        int t,z,m,c,count=0,l,mn=0,p,i,n,j;
        int a[]=new int[1010];
        float tp,k,res=0;
        float x[]=new float[1100];
        float mns[]=new float[1010];
        float asc[]=new float[10002];
        float temp[]=new float[10];
        float q[]=new float[100];
        float low[]=new float[50];
        float upp[]=new float[50];
        System.out.println("n Enter how many 1000 numbers : ");
        m=Integer.parseInt(br.readLine());
        c=m*10;
        t=1;
        l=1000;
        while(t<=c)
        {
            for(i=1;i<=1000;i++) a[i]=rd.nextInt(214748);
            if(count>0)
            {
                j=(l-(l%m))+1;
                for(i=1;i<=count;i++)
                {
                    q[i]=x[j];j++;
                }
            }
        }
l=1000+count;
j=1;
for(i=1,p=1;i<=l&p<=1000;i++)
{
    if(count>0)
    {
        x[i]=q[j];
        j++;
        count--;
    }
    else
    {
        if(a[p]<=99)
            k=(float)a[p]*0.01f;
        else if(a[p]<=999)
            k=(float)a[p]*0.001f;
        else if(a[p]<=9999)
            k=(float)a[p]*0.0001f;
        else if(a[p]<=99999)
            k=(float)a[p]*0.00001f;
        else
            k=(float)a[p]*0.000001f;
        x[i]=(float)Math.sqrt(1.0/Math.log(1/k));p++;
    }
}
count=0;
for(i=1,j=1;i<=(l-l%m);j++)
{
    for(z=0;z<m;z++)
    {
        temp[z]=x[i];
        i++;
    }
    asc[mn]=median(temp,m);
    mns[j]=asc[mn];
    res+=mns[j];
mn++;  
}  
count=\%m;  
t++;  
}  
System.out.println("n\n xbar= "+res/10000);  
for(i=0;i<mn-1;i++)  
for(j=i+1;j<mn;j++)  
if (asc[i]>asc[j])  
{  
  tp=asc[i];  
  asc[i]=asc[j];  
  asc[j]=tp;  
}  
double r=Math.sqrt(22.0/7.0);  
system.out.println("n9987element="+asc[9986]+"n 9950 element="+asc[9949]+"n 9900 element="+asc[9899]+"n 9750 element="+asc[9749]+"n 9500 element="+asc[9499]"};  
System.out.println (asc[13]/r+ " +asc[9986]/r);  
double v1=asc[13]/r;  
double v2=asc[9986]/r;  
int cnt=0;  
for(i=0;i<10000;i++)  
if(asc[i]>=v1 && asc[i]<=v2) cnt++;  
System.out.println("Count="+cnt);  
}  
static void sort(float x[],int m)  
{  
  for(int i=0;i<m-1;i++)  
  for(int j=0;j<m-1-i;j++)  
  if(x[j]>x[j+1])  
  {  
    float t=x[j];  
    x[j]=x[j+1];  
    x[j+1]=t;  
  }  
}
x[j+1]=t;
}
}
static float median(float x[],int m)
{
    sort(x,m);
    if(m%2==0)
        return(x[(m-1)/2]+x[m/2])/2;
    else
        return x[m/2];
}
/* PROGRAM FOR CALCULATING PERCENTILES OF DISTRIBUTION OF
SAMPLE MIDRANGE IN INVERSESE RAYLEIGH DISTRIBUTION*/

import java.io.*;
import java.util.*;
class Ird MidRange
{
    public static void main(String args[])
    throws IOException
    {
        BufferedReader br=new BufferedReader(new InputStreamReader(System.in));
        Random rd=new Random(14L);
        int t,z,m,c,count=0,l,mn=0,p,i,n,j;
        int a[]=new int[1010];
        float tp,k,res=0;
        float x[]=new float[1100];
        float mns[]=new float[1010];
        float asc[]=new float[10002];
        float temp[]=new float[10];
        float q[]=new float[100];
        float low[]=new float[50];
        float upp[]=new float[50];
        System.out.print("n Enter how many 1000 numbers : ");
        m=Integer.parseInt(br.readLine());
        c=m*10;
        t=1;
        l=1000;
        while(t<=c)
        {
            for(i=1;i<=1000;i++) a[i]=rd.nextInt(2147483647);
            if(count>0)
            {
                j=(l-(l%m))+1;
                for(i=1;i<=count;i++)
                {
                    q[i]=x[j];j++;
                }
            }
            System.out.print("n Enter how many 1000 numbers : ");
            m=Integer.parseInt(br.readLine());
            c=m*10;
            t=1;
            l=1000;
            while(t<=c)
            {
                for(i=1;i<=1000;i++) a[i]=rd.nextInt(2147483647);
                if(count>0)
                {
                    j=(l-(l%m))+1;
                    for(i=1;i<=count;i++)
                    {
                        q[i]=x[j];j++;
                    }
                }
            }
        }
    }
}
l=1000+count;
j=1;
for(i=1,p=1;i<=l&i<=1000;i++)
{
    if(count>0)
    {
        x[i]=q[j];
        j++;
        count--;
    }
    else
    {
        if(a[p]<=99)
            k=(float)a[p]*0.01f;
        else if(a[p]<=999)
            k=(float)a[p]*0.001f;
        else if(a[p]<=9999)
            k=(float)a[p]*0.0001f;
        else if(a[p]<=99999)
            k=(float)a[p]*0.00001f;
        else
            k=(float)a[p]*0.000001f;
        x[i]=(float)Math.sqrt(1.0/Math.log(1/k));p++;
    }
}
count=0;
for(i=1,j=1;i<=(l-l%m);j++)
{
    for(z=0;z<m;z++)
    {
        temp[z]=x[i];
        i++;
    }
    asc[mn]=midRange(temp,m);
    mns[j]=asc[mn];
    res+=mns[j];
mn++; 
    }
    count=l%m;
    t++;
}
System.out.println("\n\nxbar= "+res/10000);
for(i=0;i<mn-1;i++)
    for(j=i+1;j<mn;j++)
        if (asc[i]>asc[j])
            {
                tp=asc[i];
                asc[i]=asc[j];
                asc[j]=tp;
            }
    double r=Math.sqrt(22.0/7.0);
System.out.println(asc[13]/r+" "+asc[9986]/r);
    double v1=asc[13]/r;
    double v2=asc[9986]/r;
    int cnt=0;
    for(i=0;i<10000;i++)
        if(asc[i]>=v1 && asc[i]<=v2) cnt++;
    System.out.println("Count=":cnt);
}
static void sort(float x[],int m)
{
    for(int i=0;i<m-1;i++)
        for(int j=0;j<m-1-i;j++)
            if(x[j]>x[j+1])
                {
                    float t=x[j];
                    x[j]=x[j+1];
                }
x[j+1]=t;
}
}
static float midRange(float x[],int m)
{
    sort(x,m);
    return(x[m-1]+x[0])/2;
}
}
```java
import java.io.*;
import java.util.*;
class Ird Range
{
    public static void main(String args[])throws IOException
    {
        BufferedReader br=new BufferedReader(new InputStreamReader(System.in));
        Random rd=new Random(14L);
        int t,z,m,c,count=0,l,mn=0,p,i,n,j;
        int a[]=new int[1010];
        float tp,k,res=0;
        float x[]=new float[1100];
        float mns[]=new float[1010];
        float asc[]=new float[10002];
        float temp[]=new float[10];
        float q[]=new float[100];
        float low[]=new float[50];
        float upp[]=new float[50];
        System.out.print("n Enter how many 1000 numbers : ");
        m=Integer.parseInt(br.readLine());
        c=m*10;
        t=1;
        l=1000;
        while(t<=c)
        {
            for(i=1;i<=1000;i++)  a[i]=rd.nextInt(214748);
            if(count>0)
            {
                j=(l-(l%m))+1;
                for(i=1;i<=count;i++)
                {
                    q[i]=x[j];j++;
                }
            }
        }
    }
}*/
*/
*/
l=1000+count;
j=1;
for(i=1,p=1;i<=l&i<=1000;i++)
{
    if(count>0)
    {
        x[i]=q[j];
        j++;
        count--;
    }
    else
    {
        if(a[p]<=99)
            k=(float)a[p]*0.01f;
        else if(a[p]<=999)
            k=(float)a[p]*0.001f;
        else if(a[p]<=9999)
            k=(float)a[p]*0.0001f;
        else if(a[p]<=99999)
            k=(float)a[p]*0.00001f;
        else
            k=(float)a[p]*0.000001f;
        x[i]=(float)Math.sqrt(1.0/Math.log(1/k));p++;
    }
}
count=0;
for(i=1,j=1;i<=(l-l%m);j++)
{
    for(z=0;z<m;z++)
    {
        temp[z]=x[i];
        i++;
    }
    asc[mn]=range(temp,m);
    mns[j]=asc[mn];
    res+=mns[j];
mn++; 
}
count=1%m;
t++; 
} 
System.out.println("\n\n xbar= "+res/10000); 
for(i=0;i<mn-1;i++) 
for(j=i+1;j<mn;j++) 
if (asc[i]>asc[j]) 
{ 
    tp=asc[i]; 
    asc[i]=asc[j]; 
    asc[j]=tp; 
} 
double r=Math.sqrt(22.0/7.0); 
System.out.println(asc[13]/r+" \nasc[9986]/r); 
double v1=asc[13]/r; 
    double v2=asc[9986]/r; 
    int cnt=0; 
    for(i=0;i<10000;i++) 
        if(asc[i]>=v1 && asc[i]<=v2) cnt++; 
    System.out.println("Count="+cnt); 
} 
static void sort(float x[],int m) 
{ 
for(int i=0;i<m-1;i++) 
for(int j=0;j<m-1-i;j++) 
if(x[j]>x[j+1]) 
{ 
    float t=x[j]; 
    x[j]=x[j+1]; 
}
x[j+1]=t;
}
}
static float range(float x[],int m)
{
    sort(x,m);
    return(x[m-1]-x[0]);
}