CHAPTER I

INTRODUCTION

1.1 OBJECTIVE OF THE THESIS

The flow of fluids past and through porous media or mixing of the fluids from a narrow slit with the surrounding saturated fluid in porous media have several important applications in the recovery of oil or gas, extraction of energy in geothermal region, the filtration of solids from liquids, the flow of liquids through ion exchange beds, drugs permeation through human skin, chemical reactors, cryogenic industry, space applications, porous bearings, cooling problems and thermal insulation in high temperature reactor technology. Of which, the petroleum industry has shown most extensive interest in the flow of fluids through porous media, in connection with the production of crude oil from the underground reservoirs where it occurs naturally. These reservoirs consists of porous sedimentary formations such as sandstone, limestone and dolomite in which the oil
is entrapped [Longwell, 1966; Richardson, 1961; Yih, 1965].

The scope of this thesis is:

(i) To investigate the structure of free jets in porous media with and without rotation with a motive to understand the dissipation mechanisms, the dynamics of the jets, the momentum transport and the effect of permeability on the spreading of jets.

(ii) To examine the influence of surface tension on velocity, temperature and stability of the flow in a system consisting of a fluid layer overlaying a layer of porous medium. This study leads to the relative importance of buoyancy and surface tension forces and their effect on inducing convection under microgravity conditions.

(iii) To study the importance of variation of permeability in explaining hydrodynamic channeling and its effect on heat transfer rate or Nusselt number by considering different physical configurations and situations.

A survey of background literature related to the above problems is advantageous to build up the proper framework for the thesis.
1.2 REVIEW OF WORK ON JETS

A jet can be defined as a flow in which the width or the cross stream scale is much smaller than the downstream scale. Such flows occur along solid boundaries as in the case of wall jets or in the absence of solid boundaries as in the case of free jets. Schlichting (1933) first obtained the solutions to the propagation of a jet issuing from a small hole into stationary fluid in two cases, namely, plane and circular jets. In the case of plane jets (i.e., when the fluid issues from a long narrow orifice and the motion is taken to be two dimensional) the equations were integrated by approximate numerical method whereas, in the case of circular jets, the equations were integrated in closed form. Later, Dickley (1937) showed that exact closed form solutions can be obtained for the plane case also. Yih (1950) studied the temperature distribution in a steady laminar preheated jet neglecting the heat due to dissipation. The solutions obtained by him are also applicable to other similar problems of diffusion. For instance, if oxygen is being discharged through a small slit or a hole into an open space filled with nitrogen, the concentration of oxygen at different points in the jet can be
obtained directly from the corresponding solution by making the necessary changes of physical constants. Bansal (1974) extended the above problem by including the viscous dissipation term and he obtained exact solution for Prandtl number of unity. For other values of Prandtl numbers, solutions are obtained by the method of successive approximations.

The jets of conducting fluids in the presence of magnetic field was first studied by Peskin (1963), who obtained the numerical solutions of the problem. Smith and Cambel (1965) obtained an analytical perturbation solution to Peskin problems. Pozzi and Bianchini (1972) investigated the velocity and temperature distribution in a laminar plane jet in the presence of magnetic field and obtained first order perturbation solutions by reducing the governing equations to a non-homogeneous Legendre equation. The same problem was extended by Bansal (1975) by including Joule heating and viscous dissipation terms to obtain perturbation solutions up to second order for arbitrary values of initial momentum in jet. Bansal (1977) studied hydromagnetic laminar and turbulent plane free jets of conducting fluids using Prandtl and Von-Mises transformation. Sulochana Gadgil (1971) examined the structure of jets in rotating systems and found
that when the rate of rotation is small, the dynamics is shown to be identical to that of the jet in non-rotating systems.

As the rate of rotation increases, Ekman layers become important and in strongly rotating case, friction in Eckman layers dominates lateral dissipation and the jet ejects fluid at its edges and the downstream momentum flux decreases with the downstream distance due to dissipation in Ekman layers.

The laminar two dimensional wall jet, issued from a narrow slit was first investigated by Andrade (1937) using water. He found that when Reynolds number was small, the observed mean velocity distribution is in agreement with the theoretical solutions. The theory of radial and plane wall jets was studied by Glauert (1956) for flows in two and in three dimensions with axial symmetry. He pointed out that it has features common to both free jet and ordinary boundary layer, thus the spreading of fluid is retarded by frictional resistance of the wall and the inner part of the flow may be expected to show certain structural similarity to a boundary layer. Moreover the entrainment of quiet fluid occurs near the outer edge of the flow which accordingly is likely to resemble a free jet in character. Bakke (1957) made an experimental
investigation of a turbulent low speed jet of air spreading out radially over a flat smooth plate and found that the velocity profiles are similar and the rate of change of velocity and the width of the jet can be expressed in simple power laws. These results support the Glauert's theoretical prediction. Kruka and Eskinazi (1964) extended the above problem to a wall jet in a moving stream.

The brief survey of the jets given above reveals that not much attention has been given to the structure of jets involving a porous medium. In this thesis we have investigated the structure of plane free jets, with and without rotation in porous media, with the motive of understanding the effect of permeability on the spreading and momentum transport of jets.

1.3 MARANGONI CONVECTION OR SURFACE TENSION INDUCED CONVECTION

The study of influence of surface tension on the velocity, temperature and on stability of the flow is of importance in space applications particularly in material processing in space. It is well established in the crystal growth studies that the perturbations of the solidification process due to
gravity-driven convection are detrimental to the perfection of most of the earthgrown crystalline materials. It is generally anticipated that a significant reduction in gravity should result in improved perfection of crystals grown from either the liquid or vapour phase [Proc. ESRO 1974]. This was satisfactorily confirmed by Skylab Missions, which have also shown that a near zero gravity environment could permit new growth techniques and the achievement of new materials [NASA Report, 1973]. However, material processing in space raises the specific problems bound to the particular behaviour of fluids in a reduced gravity convective motions induced by otherwise secondary effects on earth, e.g., surface tension driven convection, may interfere with diffusional heat and mass transfer and become predominant, thus overcoming the advantage of suppressing buoyancy effects [Carruthers & Grasso, 1972; Carruthers, 1974; Chang, 1976].

Convection driven by surface tension gradient is inevitable in material science experimental configurations in space missions, because such configuration often involve fluid interfaces. Hence, for material science processing in space, it is of importance to evaluate the critical Marangoni number
below which the convection cannot occur and to suggest the mechanisms to suppress convection.

In what follows, a brief survey and mechanism of surface tension induced convection is given. Pearson (1958) has developed a linearised theory and has concluded that surface tension forces are sufficient to induce convection in a liquid layer with a free surface provided there is a temperature or concentration gradient of proper sense and sufficient magnitude. Pearson's theory agrees in many essentials with experimental findings of Block (1956). The work of Pearson and Block has illuminated the neglected type of convection induced by surface tension. Later Scriven and Sterning (1964) have extended Pearson's small disturbance analysis to a still idealised, yet more realistic model of the fluid interface, establishing the effects of finite mean surface tension and surface viscosity. Their analysis is based on a Newtonian fluid interface in which the local departure from equilibrium interfacial stress is directly proportional to the local rate of interfacial strain. By accounting for the possibilities of shape deformations of the free surface, Scriven and Sterning (1964) found that there is critical Marangoni number for the onset of stationary instability and
that the limiting case of zero-wave number is always unstable. They have shown that the effect of surface viscosity of the Newtonian interface is to inhibit stationary instability.

The physical mechanism of generating surface tension convection is very simple. When a layer of liquid, bounded below by a rigid wall and above by a free surface, is heated uniformly below, the hot liquid rises to the free surface and cools as it moves along the surface. Since, the surface tension depends on the temperature (for a liquid it decreases with increasing temperature), there is also a surface tension gradient along the free surface. This induces a surface tension force which either tends to pull the fluid along leading to instability or restrain the fluid motion leading to stability. In shallow layers of fluid, the surface tension instability can be produced at temperature gradients which are much smaller than those required for buoyancy driven convection. In fact, Koschmieder (1974) found experimentally, dealing with a shallow layer of silicon-oil on a plane circular copper plate uniformly heated from below, the surface tension forces can produce an array of hexagonal cells of much greater regularity than that observed in buoyancy driven
convection. This fluid moves towards the surface at the centre of each of the hexagons and away from it around their peripheries. Benard's (1900, 1901) original observations of the ordered hexagonal cells are inconsistent with the buoyancy mechanism because of very low temperature gradients, but are consistent with the surface tension mechanism. In the surface tension driven mechanism, the setting up of convection is expressed by a dimensionless number,

\[ M = \frac{\sigma \frac{dT}{d}}{\gamma k^*} \]

called the Marangoni number where \( \sigma \) is the derivative of surface tension (force/length) with respect to temperature and \( k^* \) is thermal diffusivity. The convection sets in at the critical Marangoni number.

In the present thesis, we have investigated the onset of convection in a system consisting of a fluid layer overlying a layer of porous medium with the motive of understanding the nature of coupling between buoyancy and surface tension forces and their relative importance. Further, we have studied the effect of surface tension force on velocity and temperature distribution on a flow field.
1.4 INFLUENCE OF VARIABLE PERMEABILITY ON FLUID MOTION

In most of the investigations on flows through or past a porous medium, the permeability and porosity are taken as constants. It has been observed that when porosity and permeability are considered to be constant the velocity profiles are flat with a thin boundary near the walls. Several investigators [Smith and Morales, 1951; Smith and Schwartz, 1953; Schertz and Bischoff, 1969] have measured velocity distribution in a packed bed. Their experimental measurements indicate that the velocity profiles exhibit channeling of velocity near the walls in contradiction to the theoretical results. Attempts to correlate the data of this type have been hampered by a lack of knowledge of the detailed internal composition of packed beds which in turn influence the velocity perturbation within the bed. Gorton and Fraser (1935) showed that in the case of stacked spheres that unit cell voidages vary from 26 to 48% depending upon the particular stacking arrangement. Large randomly packed uniform beds of uniform spheres tend to pack with an average of void fraction of 39%. However, locally the voidage varies from point to point. The void fraction near the wall of the container will be larger than at the centre of the container, since at the wall the
spheres must confirm to the wall's curvature. Immediately adjacent to the wall the void fraction should approach unity, because each particle can make only one point contact with the wall. In the centre of the bed, that is far removed from the wall, the stacking arrangement should not be influenced by the presence of the wall, and a minimum voidage should be observed.

Tierney et al. (1958) made experimental measurements on void fraction distribution in packed beds and later Benenati and Brosilow (1962) also reported more accurate method of measuring of void fraction distribution in packed beds which can be easily adoptable to virtually any type of packing material. The results of the above measurements show that the voidage of porosity is maximum at the wall and approaches a constant value around 0.4 at a distance of five particle diameter from the wall.

Attempts have been made to relate the wall effect mathematically by Furnas (1929), Leva et al. (1947), Neale and Nader (1974) with generally unsatisfactory results, since the assumption involved in developing the mathematical expression depart appreciably from actual condition of the system.
Schertz and Bischoff (1969) attributed the hydrodynamic channeling of velocity distribution to the variation of porosity of the medium confined between the boundaries without correlating experimental results with a theoretical model. In the study of flow mal-distribution in packed beds, Stanek and Szekely (1972, 1973, 1974), Szekely and Poveromo (1975), Manoj Choudhary et al. (1976), have made allowance for high value of porosity at the wall by assuming the porosity around 0.49. But they have not allowed for continuous variation of porosity from the wall to the interior. Chandrasekhara and Vortmeyer (1979) investigated velocity distribution in fixed porous beds under isothermal conditions incorporating the variation of porosity up to a distance of 1/4 sphere diameter of the particle from the wall. For their analysis, they selected the maximum value of porosity to be 0.588 at 1/4 sphere diameter based on experimental results of Benenati and Brosilow (1962). They obtained physically realistic velocity distribution which is in fair agreement with the experimental measurements of Schertz and Bischoff (1969).

The above review on porosity variation points out, that the porosity variation has to be incorporated in flow problems.
involving porous media. Further, the variation of porosity in turn influences the thermal resistance since they are related by the expression \( k = k_f \Theta + (1 - \Theta)k_s \). It is therefore necessary, in all heat transfer problems, to include the variation of permeability and thermal resistance to get realistic values for heat transfer rates.

In this thesis we have investigated the flow and heat transfer problems making allowance for variation of permeability and thermal resistance.

1.5 PRESENT WORK

The outline of work of the present thesis is as follows: The basic equations governing the motion of the fluid in porous media and the boundary conditions are explained in Chapter II. The dynamics of the jets in porous media with and without rotation is discussed in Chapter III. In Section 3.1 we have first discussed the general characteristics of jets in porous media. The solutions for jet equations are obtained by three approaches. In the first approach we have discussed Schlichting type solution and its limitations. In second and third approaches we have used Von-Mises transformations of the type \( \Psi_y = H(x,\Psi) \) and \( \eta = \Psi \) respectively.
The second approach leads to the general solutions which yields solutions to the limiting cases. In the third case, we observe that closed form similarity solutions are possible only when both Darcy and viscous resistance terms are present. The solutions so obtained corresponds to the Darcy jet solution obtained by second approach. When Darcy resistance alone is present, the basic equation leads to a singular solution which does not satisfy the boundary conditions.

In Section 3.2, we have investigated the structure of jets in rotating porous systems. The problem is formulated using the generalised Ergun equation and introducing the non-linear bed friction term. The characteristic downstream scales which govern three dissipation mechanisms are discussed. The similarity solution for the problem is obtained using the Von-Mises transformation $\Psi = H(x,\Psi)$. It is found that the permeability of the medium accelerates the process of spreading of jets.

In Chapter IV we have discussed the influence of surface tension and buoyancy forces on convection. In Section 4.1 we have examined the surface tension induced convection or Marangoni convection in a fluid saturated porous media due to axial temperature gradient in the presence of buoyancy force.
The expressions for velocity are obtained by direct and singular perturbation methods. The energy equation is solved with and without dissipation term. The results are discussed in terms of Bond number and we observe that for low Bond numbers surface tension or Marangoni effect is dominant. In Section 4.2 we have investigated the convective instability of a fluid layer overlying a layer of porous medium including the Marangoni effect at a deformable upper surface using Brinkman-Oberbeck-Boussinesq equation for the porous layer. The solution is obtained for adiabatic boundaries by perturbing the wave number. The results show that surface tension and buoyancy forces are tightly coupled. Under microgravity conditions, it is found that the surface tension is the dominant force which induces convection.

In all the above problems we have treated the permeability as constant in the first approximation. In the following chapters we have allowed for the variation of permeability and thermal resistance to explain the experimental observations.

In Chapter V we have investigated the effect of variable permeability on velocity, temperature distribution and heat transfer. In Section 5.1, we have examined the influence of
variable permeability (K) on basic flows. We have assumed that K varies as $K(y) = K_0 (1 + y/h)^2$. The velocity profiles for the basic flows exhibit hydrodynamic channeling which is observed experimentally.

Natural convection in a saturated porous medium adjacent to impermeable horizontal surfaces with variable permeability is discussed in Section 5.2. The system of non-linear equations are solved numerically using automatic initial value technique. The application of the results to a geothermal reservoir are discussed. In Section 5.3, the problem of mixed convection in a saturated porous medium adjacent to horizontal impermeable surfaces with variable permeability is examined. The criterion for free and forced convection are established. It is found that the variation of permeability increases heat transfer rate by about 70%. The problem admits the similarity solution only for aiding flows.

The Chapter VI provides discussion and plan for future work.