Chapter IV

Texture 6 Zero Lepton Mass Matrices

In chapter III, we have investigated all possibilities of hermitian texture 6 zero and texture 5 zero quark mass matrices for their compatibility with the quark mixing data. It was found that texture 6 zero quark mass matrices are unambiguously ruled out, however, it remains to be seen whether a similar conclusion can be drawn for leptons also. Thus, the broad purpose of the present chapter is to investigate all combinations of texture 6 zero hermitian lepton mass matrices for their viability against neutrino mixing data [97]. Various cases have been considered in the analysis, for example, normal hierarchy, inverted hierarchy and degenerate scenarios for both Majorana as well as Dirac neutrinos. Before discussing the specific problems to be probed in the present chapter, it is desirable to discuss briefly the attempts made by other authors regarding the texture 6 zero lepton mass matrices.

In the literature, most of the analyses have been carried out in the flavor basis [89], whereas in non flavor basis, when both charged lepton mass matrix and neutrino mass matrix have 3 zero pattern, the number of possibilities for texture 6 zero mass matrices becomes very large. Some of these possibilities have been explored in the literature [92], however, adequate attention has not been given to the cases of inverted hierarchy and degenerate neutrinos. Also, it is perhaps desirable to note that Dirac neutrinos have not yet been ruled out by the experiments [111], therefore it becomes important to carry out a detailed comparison for Dirac like as well as Majorana like neutrinos for the cases of normal, inverted and degenerate scenario.
Specifically, the purpose of the present chapter is to update the analysis of Zhou et al. [92] for all possibilities of hermitian texture six zero lepton mass matrices as well as to extend this analysis to the case of inverted and degenerate neutrino masses in addition to normal hierarchy cases. To preserve the parallelism between quarks and leptons, only those neutrino mass matrices have been considered which are consistent with the requirement of non zero and distinct neutrino masses. Also, following our analysis in the quark sector [96], the mass matrices for leptons and neutrinos are taken to be hermitian. For the sake of completion, we have also investigated the cases corresponding to charged leptons being in the flavor basis. Further, several phenomenological quantities, such as effective neutrino mass ($m_{\text{ee}}$) related to neutrino less double beta decay, Jarlskog’s rephasing invariant parameter in the leptonic sector $J_{\text{i}}$ and the corresponding Dirac like CP violating phase $\delta_{\text{CP}}$ have been calculated for the viable cases.

The detailed plan of the chapter is as follows. In Section (4.1), we have presented the essentials of the formalism connecting the mass matrix to the neutrino mixing matrix. The inputs used in the present analysis have been presented in Section (4.2). In Section (4.3) all possibilities of texture 6 zero lepton mass matrices have been given. In Sections (4.4) and (4.5), for Majorana and Dirac neutrinos respectively, the detailed calculations pertaining to normal, degenerate and inverted hierarchy have been discussed. Finally, Section (4.6) summarizes our conclusions with regard to this chapter.

4.1 Construction of $V_{\text{PMNS}}$ matrix from mass matrices

To begin with, we present the Fritzsch like hermitian texture 6 zero lepton mass matrices, e.g.,

\[
M_l = \begin{pmatrix}
0 & A_l & 0 \\
A_l^* & 0 & B_l \\
0 & B_l^* & C_l
\end{pmatrix}, \quad M_{\nu D} = \begin{pmatrix}
0 & A_\nu & 0 \\
A_\nu^* & 0 & B_\nu \\
0 & B_\nu^* & C_\nu
\end{pmatrix}.
\]  

(4.1)

$M_l$ and $M_{\nu D}$ respectively corresponding to Dirac-like charged lepton and neutrino mass matrices. It may be noted that each of the above matrix is texture 3 zero type.
with \( A_{lR} = |A_{lR}| e^{i\alpha_{lR}} \) and \( B_{lR} = |B_{lR}| e^{i\beta_{lR}} \).

For Majorana neutrinos, the neutrino mass matrix \( M_\nu \) is given by seesaw mechanism [46], for example,

\[
M_\nu = -M_{\nu D}^T (M_R)^{-1} M_{\nu D},
\]

(4.2)

where \( M_{\nu D} \) and \( M_R \) are respectively the Dirac neutrino mass matrix and the right-handed Majorana neutrino mass matrix. It may be mentioned that for both Majorana as well as Dirac neutrinos the texture is imposed only on \( M_{\nu D} \), with no such restriction on \( M_\nu \) for Majorana case. In the absence of any guidelines for the right handed Majorana mass matrix \( M_R \) as well as to keep the number of parameters under control, it would be desirable to keep its structure as simple as possible, therefore we take \( M_R = m_R I \) where \( I \) is the unity matrix and \( m_R \) denotes a very large mass scale.

To fix the notations and conventions as well as to facilitate the understanding of inverted hierarchy case and its relationship to normal hierarchy case, we detail the formalism connecting the mass matrices to the neutrino mixing matrix. The mass matrices \( M_l \) and \( M_{\nu D} \) given in equation (4.1), for hermitian as well as symmetric case, can be exactly diagonalized. To facilitate the diagonalization of \( M_k \), where \( k = l, \nu D \), the complex mass matrix \( M_k \) can be expressed as

\[
M_k = Q_k M_k^* P_k
\]

(4.3)

or

\[
M_k^* = Q_k^T M_k P_k^T
\]

(4.4)

where \( M_k^* \) is a real symmetric matrix with real eigenvalues and \( Q_k \) and \( P_k \) are diagonal phase matrices. For the hermitian mass matrix \( Q_k = P_k^T \). In general, the real matrix \( M_k^* \) is diagonalized by the orthogonal transformation \( O_k \), e.g.,

\[
M_k^{\text{diag}} = O_k^T M_k^* O_k,
\]

(4.5)

which on using equation (4.4) can be rewritten as

\[
M_k^{\text{diag}} = O_k^T Q_k^T M_k P_k^T O_k.
\]

(4.6)
(4.21) by replacing $m_1, m_2, m_3$ with $m_{\nu_1}, m_{\nu_2}, m_{\nu_3}$ for normal hierarchy and with $m_{\nu_1}, m_{\nu_2}, -m_{\nu_3}$ for inverted hierarchy for the case of Dirac neutrinos. The phase matrix $Q_lP_{\nu D}$ and the scanned ranges of lightest neutrino mass and phases $\phi_1$ and $\phi_2$ have already been mentioned in Section (4.2).

To check the compatibility of the particular combination given in equation (4.19) by assuming normal hierarchy of Dirac neutrinos, in Figure (4.3) we have plotted the allowed parameter space for $\theta_{12}$ and $\theta_{23}$ in $m_{\nu_1}-\theta_{13}$ plane represented respectively by dots and crosses. One can immediately find from the figure that there is no common parameter space available to $\theta_{12}$ and $\theta_{23}$, concluding that this particular combination given in equation (4.19) is not viable for Dirac neutrinos. This result remains unaffected even if the input parameter ranges are extended further. Also, one can easily check that mixing angle $\theta_{13}$ is coming out to be very small. Thus, normally hierarchical Dirac neutrinos are ruled out for the texture 6 zero combination given in equation (4.19).

![Figure 4.3: Allowed parameter space of $\theta_{12}$ and $\theta_{23}$ in 'lightest neutrino mass - $\theta_{13}$ plane' for normal hierarchy of Dirac neutrinos for texture 6 zero combination $\Pi_{446}$ given in equation (4.19). The dots and crosses represent the allowed parameter space for $\theta_{12}$ and $\theta_{23}$ respectively.](image)

The combinations which are viable for Majorana neutrinos, are not viable for Dirac neutrinos primarily because of mixing angle $\theta_{23}$. For example, for the combi-
nations given in the first row as well as the combinations given in the third row of the Table (4.1), \( \theta_{23} \) for Dirac neutrinos comes out < 33° i.e. below the experimental limits. Similarly, for the combinations given in the second row and in the fourth row of the Table (4.1), \( \theta_{23} > 55° \) lies above its experimental limits. Thus, we conclude that Dirac neutrinos are ruled out for these 16 combinations. A similar analysis carried out for the rest of the 128 combinations for normally hierarchical as well as for degenerate Dirac neutrinos shows that there is no viable texture 6 zero lepton mass matrices for normally hierarchical Dirac neutrinos.

Similarly, in the case of inverted hierarchy of Dirac neutrinos the explored ranges of the mixing angle \( \theta_{12} \) and \( \theta_{23} \) are completely out of the experimental limits for the combination discussed above. Analysis has been carried out for the rest of the combinations as well. We find that inverted hierarchy is completely ruled out for Dirac neutrinos. Thus, there are no viable texture 6 zero lepton mass matrices for normally hierarchical, inverted hierarchical as well as degenerate Dirac neutrinos, ruling out Dirac neutrinos completely for texture 6 zero lepton mass matrices.

For the sake of completion, we have also analyzed the cases corresponding to charged leptons being in the flavor basis for Dirac as well as Majorana neutrinos and one finds that none of matrices give results within the experimental ranges.

### 4.6 Summary and conclusions

To summarize, for Majorana as well as Dirac neutrinos, we have carried out detailed calculations pertaining to all combinations (144) of hermitian texture 6 zero lepton mass matrices. For each combination there are 6 cases, for example, normal hierarchy, inverted hierarchy and degenerate neutrinos for both Majorana as well as Dirac neutrinos, which have to be analyzed, leading to a total of 864 cases for texture 6 zero lepton mass matrices. In addition to these cases for each type of neutrinos, we have also considered those cases when the charged leptons are in the flavor basis. Detailed dependence of mixing angles on the lightest neutrino mass as well as the parameter space available to the mixing angles have been investigated for texture 6 zero cases. Also, several phenomenological quantities such as the viable ranges of neutrino masses, mixing angles \( \theta_{12} \), \( \theta_{13} \) and \( \theta_{13} \), Jarlskog’s rephasing invariant parameter \( J \), the Dirac-like CP violating phase \( \delta \) and the effective neutrino mass \( \langle m_{ee} \rangle \) have also been calculated for different cases.
The analysis leads to several interesting conclusions. We have found that for Majorana neutrinos with normal hierarchy out of 144, only 16 combinations are compatible with current neutrino oscillation data at $3\sigma$ C.L.. Out of these 16 combinations, 6 parallel combinations $I_aI_a$, $I_bI_b$, $I_cI_c$, $I_dI_d$, $I_eI_e$, $I_fI_f$ are isomeric i.e. these combinations are although structurally different from each other, their predictions for lepton masses, mixing matrix elements and for other phenomenological quantities mentioned above are exactly same. Similarly, the non parallel combinations such as $I_aI_b$, $I_bI_a$, $I_cI_d$, $I_dI_c$, $I_eI_d$ and $I_dI_e$ are again isomeric to each other. The above two sets (parallel and non parallel combinations of category 1) can be distinguished from each other on the basis of mixing angle $\theta_{23}$ as the parallel combinations yield $\theta_{23}$ below its maximal value while non parallel combinations yield $\theta_{23}$ above its maximal value. Similarly, the viable combinations $II_{I_a}$ isomeric to $II_{I_a}$ and $II_{I_b}$ isomeric to $II_{I_b}$ can be distinguished from each other again on the basis of $\theta_{23}$. Further, these four combinations yield a very constrained range of $\theta_{13}$ e.g., $7^\circ - 9^\circ$ in comparison to the ranges of $\theta_{13}$ obtained in the case of above mentioned 12 (parallel and non parallel) combinations. The ranges of neutrino masses and mixing angle $\theta_{12}$ are more or less similar in all 16 combinations. Further, $J_\ell$, $\delta_1$ and $\langle m_{ee} \rangle$ are in good agreement with the ranges obtained in other such analyses [27,91,93,94]. The above mentioned texture combinations are found to be compatible with current data even at $2\sigma$ C.L.

Assuming inverted hierarchy of Majorana neutrinos for texture 6 zero lepton mass matrices, we have found that none of the combination is compatible with the latest neutrino oscillation data. Similarly, all the considered cases pertaining to degenerate neutrinos are also found to be ruled out.

Similar investigations have been carried out for Dirac neutrinos and we have found that there is no viable texture 6 zero lepton mass matrices for normal hierarchy, inverted hierarchy as well as degenerate neutrinos, thus ruling out Dirac neutrinos completely for texture 6 zero lepton mass matrices.

In conclusion, it may be stated that the analysis pertaining to texture 6 zero lepton mass matrices for both Majorana and Dirac neutrinos reveals interesting implications for the different hierarchies of neutrino masses, particularly ruling out inverted hierarchy and degenerate scenarios for Majorana neutrinos and also ruling out Dirac neutrinos completely. This, therefore, provides motivation to extend the analysis further to other lower order texture specific mass matrices.