3.1 Introduction

In this chapter, we propose two selection procedures for selecting populations better than a control population. The bestness is defined in terms of location parameter. One of the procedures is based on two-sample linear rank statistics whereas the other one is based on a comparatively simple statistic other than linear rank statistics.

Rizvi et al. (1968) proposed non-parametric ranking procedures for comparison of treatment populations with a control in terms of $\alpha$-quantiles. Deshpande and Mehta (1983) proposed procedures while comparing populations in terms of distribution functions. Gill and Mehta (1993) developed
selection procedures for selecting populations better than a control population while restricting to only scale parameters under the non-parametric set-up.

Selection procedures based on joint ranking of sample observations from all the populations were initiated by Lehmann (1963), followed by Puri and Puri (1968,69) and Bartlett and Govindarajulu (1968). However, Rizvi and Woodworth (1970) provide counter examples that the procedures based on joint ranking do not control the probability of correct selection over both the slippage parametric configuration (used under the indifference-zone) as well as the entire parametric space (used under the subset selection approach). For a detailed discussion one may refer to the review on Rizvi-Woodworth's paper by Dudewicz (1972).

Hsu (1980,81) used pairwise ranking to propose subset selection procedures and has shown that these procedures control the PCS over the entire parametric space. Koziol and Reid (1977) have shown the asymptotic equivalence of two ranking methods while comparing the k populations in terms of either location or scale parameters. Motivated with this asymptotic equivalence of two ranking methods and the fact that the selection procedures based on the set of two-sample
rank statistics control the probability of a correct selection over the entire parametric space, while the procedures based on joint ranking fail to do so, we have used two-sample rank statistics which are non-parametric in nature. These selection procedures are seen to control the probability of a correct selection over the entire parametric space and are not subject to counter example of Rizvi and woodworth (1970).

In section 3.2 the problem has been formulated and proposed selection procedures are given in section 3.3. The proposed selection procedures are shown to be strongly monotone in section 3.4. In section 3.5 approximate implementation of the proposed procedures, with the help of existing tables, is discussed. Simulation study is made in section 3.6 in order to see the relative performance of the proposed procedures.

3.2 Statement of the Problem

Let $\pi_0, \pi_1, \ldots, \pi_k$ be $(k + 1), \ k \geq 2$ independent populations. The population $\pi_0$ is assumed to be control population and populations $\pi_1, \ldots, \pi_k$ are the treatment populations. In this chapter, we define bestness in terms of location parameters and
the problem of selecting all populations better than the control is considered in two cases. In case (i) assume that the population \( \pi_i \) has the absolutely continuous distribution function (c.d.f.) \( F_i(x) = F(x - \mu_i) \), where \( \mu_i \) is the location parameter, \( i = 0,1,...,k \) and \( F(.) \) is an (unknown) absolutely continuous distribution function. The treatment population \( \pi_i \) is said to be better than the control population \( \pi_0 \) if \( \mu_i \geq \mu_0, \ i = 1,...,k \). The goal is to select a subset of the \( k \) treatment populations which contains all the populations better than the control. Any such selection is called a correct selection (CS). In case (ii) the underlying assumption is that the \( (k+1) \) populations differ in their location parameters and have \( F(0) = p \), so that \( \mu_i \), the location parameter of the \( i \)-th population, is its \( p \)-th quantile, \( i = 0,1,...,k \). Here \( p \) is assumed to be known.

Let \( \Omega = \{ \underline{\mu}: \underline{\mu} = (\mu_0, \mu_1,...,\mu_k) \}, \ -\infty < \mu_i < \infty, \ i = 0,1,...,k \} \) be the parametric space. We have used two-sample statistics for proposing subset selection procedures. These procedures control the probability of correct selection (PCS) over the entire parametric space. These procedures are required to satisfy the \( P' \)-condition.
\[ p_i[CS] \geq p^* \quad \forall \mu \in \Omega, \quad (3.2.1) \]

where \(2^{-k} < p^* < 1\).

### 3.3 Proposed Selection Procedures

**Case (i)**

Here the populations \(\pi_0, \pi_1, \ldots, \pi_k\) are assumed to differ only in their location parameters. The selection procedures proposed in this case are based on two-sample linear rank statistics. Let \(X_{i\alpha}, \alpha = 1, \ldots, n_i\) be a random sample of \(n_i\) observations from \(\pi_i, \quad i = 0, 1, \ldots, k\) and let \(X = (X_{01}, \ldots, X_{0n_0}, X_{11}, \ldots, X_{1n_1}, \ldots, X_{k1}, \ldots, X_{kn_k})\) be the vector of all the observations.

Let \(R_{0\alpha}^{(i)}\) denote the rank of \(X_{0\alpha}\) in the combined sample \(X_{11}, \ldots, X_{in_i}, X_{01}, \ldots, X_{0n_0}\). Let

\[
S_0^{(i)} = \left(\frac{1}{n_0} + \frac{1}{n_i}\right) \sum_{\alpha = 1}^{n_\alpha} a_m(R_{0\alpha}^{(i)}) - n_0 \bar{a}_m, \quad (3.3.1)
\]

where \(m = n_0 + n_i\), \(\bar{a}_m = \frac{a_m(1) + \ldots + a_m(m)}{m}\) and \(a_m(\beta)\) are some given scores satisfying the following two assumptions:

**Assumption-1:** For any positive integer \(m\), the scores \(a_m(1), \ldots, a_m(m)\) are generated by a non-decreasing, non-constant and square integrable function \(J(u)\) (\(0 < u < 1\)) in either of the
following two ways: \( a_m(\beta) = J(\beta/(m+1)) \) or \( a_m(\beta) = E(J(U_{m}^{(\beta)})) \), where \( \beta = 1, \ldots, m \) and \( U_{m}^{(1)} \leq \ldots \leq U_{m}^{(m)} \) denote the order statistic based on a sample of size \( m \) from the uniform distribution on the interval \((0,1)\).

**Assumption-2:** \( J(u) \) (\( 0 < u < 1 \)), which is called as limiting score function, is such that

\[
\frac{1}{m-1} \sum_{\beta=1}^{m} (a_m(\beta) - \bar{a}_m)^2 \to \sigma^2 = \int_0^1 (J(u) - \bar{J})^2 \, du < \infty, \tag{3.3.2}
\]

where \( \bar{J} = \int_0^1 J(u) \, du \).

The proposed selection procedure based on the statistic \( S_0^{(i)} \) is as follows:

**R_1:** For any \( i \) (\( i = 1, \ldots, k \)), include the population \( \pi_i \) in the subset if and only if \( S_0^{(i)} \leq C_0^{(i)}(n,P^*) \), where \( n = (n_0, n_1, \ldots, n_k)^t \) and the constant \( C_0^{(i)}(n,P^*) \) are chosen such that for a pre-assigned probability \( P^* (2^{-k} \leq P^* < 1) \)

\[
P_0 \left[ S_0^{(i)} \leq C_0^{(i)}(n,P^*), \quad i = 1, \ldots, k \right] = P^*. \tag{3.3.3}
\]

Here \( P_0 \) indicates that the probability is computed under the parametric configuration \( \mu_0 = \mu_1 = \ldots = \mu_k \).
Now we shall show that the procedure $R_1$ satisfies $P^*$-condition when the scores satisfy Assumption-1.

**Theorem 3.3.1:** Under Assumption-1, procedure $R_1$ satisfies $P^*$-condition.

**Proof:** Assume without any loss of generality that population $\pi_i$ is better than the control population $\pi_0$. Since the scores satisfy Assumption-1, $\max_{l \leq k} S_{0}^{(i)}$ is non-decreasing in $X_{01}, \ldots, X_{0n_0}$ and non-increasing in other components of $X$. Hence, by Lemma 4.1 of Mahamunulu (1967), we have for any $\mu \in \Omega$,

$$P^* = P_{0}
\left[ S_{0}^{(i)} \leq C_{0}^{(i)}(n, P^*), \; i = 1, \ldots, k \right]
\leq P_{\mu}
\left[ S_{0}^{(i)} \leq C_{0}^{(i)}(n, P^*), \; i = 1, \ldots, k \right]
\leq P_{\mu}
\left[ CS|R_1 \right]$$

This proves the theorem.

**Case (ii)**

Here once again the $(k+1)$ populations differ in their location parameters and it is further assumed that $F(0)=p$, so that $\mu_i$, the location parameter of the $i$-th population, is its $p$-th quantile. For practical situations of this type, one may refer to interesting papers by Chakraborti and Desu (1988a,88b).
In this case, the selection procedure is based on the following statistics. Let

$$U_{(i)}^0 = \text{Number of observations in the } i\text{-th } (i = 1, \ldots, k) \text{ sample not exceeding } Q,$$

where

$$Q = s\text{-th order statistic in the sample from control population.}$$

Here $$s = \lfloor n_0 p \rfloor + 1$$, and $$\lfloor x \rfloor$$ is the largest integer not exceeding $$x$$.

Now $$E[U_{(i)}^0] = E[\text{Number of observation in the } i\text{-th } (i = 1, \ldots, k) \text{ sample } \leq X_{(n_0 p+1)}],$$

where $$X_{(n_0 p+1)}$$ is the sample quantile of order $$p$$ from control population. Therefore,

$$E[U_{(i)}^0] = n_i F_i(\mu_0), \quad i = 1, \ldots, k.$$  

When $$\mu_i = \mu_0 \quad \forall i$$, then

$$E[U_{(i)}^0 / n_i] = F(0) = p, \quad i = 1, \ldots, k.$$  

Let

$$W_{(i)}^0 = \frac{U_{(i)}^0}{n_i} - p, \quad i = 1, \ldots, k.$$  

In this case the proposed selection procedure is based on statistic $$W_{(i)}^0$$ and is defined as:
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\( R_2: \) For any \( i (i = 1,\ldots, k) \), include \( \pi_i \) in the subset if and only if \( W_0^{(i)} \leq d_0^{(i)}(n, \text{P}') \).

Here \( n = (n_0, n_1, \ldots, n_k) \), and the constant \( d_0^{(i)}(n, \text{P}') \) are chosen such that for a pre-assigned probability \( \text{P}' (2^{-k} < \text{P}' < 1) \), we have

\[
\text{P}' = \text{P}_0 \left[ W_0^{(i)} \leq d_0^{(i)}(n, \text{P}'), \; i = 1,\ldots, k \right],
\]

(3.3.4)

and again \( \text{P}_0 \) indicates that the probability is computed under \( \mu_0 = \mu_1 = \ldots = \mu_k \).

It can easily be verified, on the lines of arguments used in Theorem 3.3.1, that the procedure \( R_2 \) satisfies \( \text{P}' \)-condition.

In the following section we define unbiasedness, monotonicity, and the strong monotonicity of a selection procedure and then establish this property for procedures \( R_1 \) and \( R_2 \).

3.4 Strong Monotonicity of Procedures \( R_1 \) and \( R_2 \)

Gupta and Nagel (1971), and Santner (1975) have defined unbiasedness, monotonicity, and the strong monotonicity properties of a selection procedure while proposing selection procedures for parametric families of probability distributions. Now below we define unbiasedness, monotonicity and the
strong monotonicity properties of a selection procedure $R$ when the parameters of interest are location parameters. For any $\mu \in \Omega$, let

$$P_\mu(i) = P_\mu(\pi_i \text{ is included in the subset } |R|), \ i = 1, \ldots, k.$$ 

**Definition 3.4.1:** The selection procedure $R$ is said to be unbiased if and only if

$$\mu_i > \mu_j, \ j = 1, \ldots, k \text{ implies that } P_\mu(i) \geq P_\mu(j) \text{ for } j = 1, \ldots, k \ (j \neq i) \text{ and for all } \mu \in \Omega.$$ 

**Definition 3.4.2:** The selection procedure $R$ is said to be monotone if and only if

$$\mu_i > \mu_j \text{ implies that } P_\mu(i) \geq P_\mu(j) \text{ for all pairs } (i,j) \text{ and for all } \mu \in \Omega.$$ 

**Definition 3.4.3:** The selection procedure $R$ is strongly monotone in $\pi_i$ iff

$$P_\mu(i) \text{ is increasing in } \mu_i \text{ when all other components of } \mu \text{ are fixed, and }$$

$$P_\mu(i) \text{ is decreasing in } \mu_j \ (j \neq i) \text{ when all other components of } \mu \text{ are fixed.}$$
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The following theorem shows strong monotonicity of selection procedure $R_j$.

**Theorem 3.4.1:** The selection procedure $R_j$ is strongly monotone.

**Proof:** Define the indicator function $I(.)$ as

$$I(S_0^{(i)}) = \begin{cases} 1 & \text{if } S_0^{(i)} \leq C_0^{(i)}(n, P^*) \\ 0 & \text{otherwise.} \end{cases} \quad (3.4.1)$$

Then by using the arguments of Theorem 3.3.1 and Lemma 4.1 of Mahamunula (1967), we have

$$P_{ll}(i) = E_{\mu^*}[I(S_0^{(i)})] \leq E_{\mu^*}[I(S_0^{(i)})] = P_{ll}(i), \quad (3.4.3)$$

where $\mu = (\mu_0, \mu_1, \ldots, \mu_i, \ldots, \mu_k)$ and $\mu^* = (\mu_0, \mu_1, \ldots, \mu_i^*, \ldots, \mu_k)$ with $\mu_i \leq \mu_i^*$. This proves the theorem.

As the strong monotonicity implies monotonicity which in turn implies unbiasedness [see Santner (1975)]; as a consequence, we have the following Corollary:

**Corollary 3.4.1:** The selection procedure $R_j$ is monotone and unbiased.
On the lines of Theorem 3.4.1 we can easily show that the selection procedure $R_2$ is strongly monotone and hence monotone and unbiased.

In the next section, we see that with the help of existing tables selection procedures $R_1$ and $R_2$ can be approximately implemented.

### 3.5 Approximate Implementation of Procedures $R_1$ and $R_2$

Here we first establish the asymptotic normality of vectors $S = (S_0^{(i)}, \ldots, S_0^{(k)})^t$ and $W = (W_0^{(i)}, \ldots, W_0^{(k)})^t$.

The asymptotic normality of vector $S = (S_0^{(i)}, \ldots, S_0^{(k)})^t$ under the configuration $\mu_0 = \mu_1 = \ldots = \mu_k$ follows immediately from a result of Koziol and Reid (1977) and is stated below in Lemma 3.5.1:

**Lemma 3.5.1:** Under $\mu_0 = \mu_1 = \ldots = \mu_k$ and as $\min(n_0, n_1, \ldots, n_k) \to \infty$ such that $\frac{n_i}{N} \to \lambda_i$, $0 < \tilde{\lambda}_i < 1$, for $i = 0, 1, \ldots, k$, the random vector $(N/\sigma_N^2)^{1/2}S$ is asymptotically normally distributed with mean vector $0$ and dispersion matrix as
Chapter 3

\[
\left( \frac{N}{\sigma_N^2} \right) E[S_0^{(i)} S_0^{(j)}] = \begin{cases} \frac{1}{\lambda_0} + \frac{1}{\lambda_1} & \text{for } i = j \\ \frac{1}{\lambda_0} & \text{for } i \neq j, \end{cases} \quad (3.5.1)
\]

where

\[
\sigma_N^2 = \frac{1}{N-1} \sum_{\beta=1}^{N} (a_N(\beta) - \bar{a}_N)^2 \to \int_0^1 (j(u) - \bar{j})^2 du \quad (3.5.2)
\]

and

\[
N = n_0 + n_1 + \ldots + n_k.
\]

Now let \( n_1 = n_2 = \ldots = n_k = n \) (say) and as \( n(n_0) \to \infty \), suppose that

\[
\frac{n(n_0)}{N} \to \lambda_1(\lambda_0), \quad \text{where } N = n_0 + nk. \text{ Let}
\]

\[
Z_0^{(i)} = \left( \frac{1}{\lambda_0} + \frac{1}{\lambda_1} \right)^{-1/2} \left( \frac{N}{\sigma_N^2} \right)^{1/2} S_0^{(i)}, \quad i = 1, \ldots, k. \quad (3.5.3)
\]

It follows from Lemma 3.5.1, that the limiting distribution of the random vector \( Z = (Z_0^{(1)}, \ldots, Z_0^{(k)}) \) under \( \mu_0 = \mu_1 = \ldots = \mu_k \) is asymptotically multivariate normal of equally correlated normal variables. The common value of this correlation coefficient is

\[
r = \frac{1}{1/\lambda_0 + 1/\lambda_1}.
\]

The constant \( C_0^{(i)}(n, P^*) \) of the selection procedure \( R_1 \), when \( n_1 = n_2 = \ldots = n_k \) is determined such that

\[
P^* = P_0 \left[ Z_0^{(i)} \leq z, \quad i = 1, \ldots, k \right] \quad (3.5.4)
\]

\[
= P_0 \left[ \max_{1 \leq i \leq k} Z_0^{(i)} \leq z \right] \quad (3.5.5)
\]
where

\[ z = \left( \frac{1}{\lambda_0} + \frac{1}{\lambda_1} \right)^{1/2} \left( \frac{N}{\sigma^2_n} \right)^{1/2} C_0^{(i)} (n, p^*) , \quad i = 1, \ldots, k . \]

Now we can make use of Table-I of Gupta et al. (1973) [reading \( N \) as \( k \), \( \alpha \) as \( 1 - P^* \) and \( \rho \) as \( \frac{1}{\lambda_0} \frac{1}{\lambda_0 + 1} \) (with \( \lambda_0 = \frac{n_0}{N} \), \( \lambda_1 = \frac{n_1}{N} \)] in that table] to read the constant \( z \) and thereby get the value of constant \( C_0^{(i)} (n, p^*) , \quad i = 1, \ldots, k \).

The asymptotic distribution of \( W = \left( W_0^{(1)}, \ldots, W_0^{(k)} \right)' \) follows from the result in David (1981, p 255) or Mukhopadhyay (1996, p 327) or Chakraborti and Desu (1988a) and is stated below in Lemma 3.5.2.

**Lemma 3.5.2:** The asymptotic distribution of \( N^{1/2} (W - E(W)) \) as \( \min(n_0, n_1, \ldots, n_k) \to \infty \) as \( \frac{n_i}{N} \to \lambda_i, \quad 0 < \lambda_i < 1, \quad i = 0, 1, \ldots, k \) is normal with mean vector 0 and dispersion matrix \( \Sigma = \left( \sigma_{ij} \right) \), where

\[ N = n_0 + n_1 + \ldots + n_k \]

and

\[ \sigma_{ij} = \begin{cases} Q_i^2 p_0 / \lambda_0 + p_i / \lambda_i & \text{for } i = j \\ Q_i Q_j p_0 / \lambda_0 & \text{for } i \neq j \end{cases} \quad (3.5.6) \]

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where \( p_0 = p(1-p), \quad p_i = F_i(\mu_0) P(1-F_i(\mu_0)), \quad Q_i = \frac{f_i(\mu_i)}{f_i(\mu_0)}, \quad i = 1, \ldots, k, \)

and we assume that \( F_i(\mu_0) = f_i(\mu_0) \) exists and is positive for \( i = 0, 1, \ldots, k. \)

Under \( \mu_0 = \mu_1 = \ldots = \mu_k, \) it is easy to see that \( E(W) = 0 \) and

\[
\sigma_{ij} = \begin{cases} 
\frac{p_0}{\lambda_0} + \frac{p_0}{\lambda_i} & \text{for } i = j \\
\frac{p_0}{\lambda_0} & \text{for } i \neq j 
\end{cases} \quad (3.5.7)
\]

Consequently, we have the following theorem.

**Theorem 3.5.1:** Under \( \mu_0 = \mu_1 = \ldots = \mu_k \) and as

\[
\min(n_0, n_1, \ldots, n_k) \to \infty \text{ such that } \frac{n_i}{N} \to \lambda_i, \quad 0 < \lambda_i < 1, \quad \text{for } i = 0, 1, \ldots, k,
\]

the random vector \( \left( \frac{N}{p_0} \right)^{1/2} W \) is asymptotically normally distributed with mean vector 0 and dispersion matrix as

\[
\left( \frac{N}{p_0} \right) E\left[W_0(i)W_0(j)\right] = \begin{cases} 
1/\lambda_0 + 1/\lambda_i & \text{for } i = j \\
1/\lambda_0 & \text{for } i \neq j 
\end{cases} \quad (3.5.8)
\]

On the lines similar to the implementation of selection procedure \( R_1, \) it can be easily shown that the selection procedure \( R_2 \) can also be implemented by making use of Table-I of Gupta et al. (1973). In this case, Theorem 3.5.1 will be used to standardize the random vector \( \left( \frac{N}{p_0} \right)^{1/2} W. \)
In the following section, we carry out simulation study in order to see the relative performance of procedures $R_1$ and $R_2$.

3.6 Simulation Study

In this section, we present the results of our simulation study carried out to assess the relative performance of procedures $R_1$ and $R_2$. Simulation is carried out in the following steps:

(i) Three sets of parametric families namely normal, double exponential, and cauchy are considered.

(ii) Four populations say $\pi_0$, $\pi_1$, $\pi_2$ and $\pi_3$ with particular parametric configurations from each family are taken.

(iii) In procedure $R_1$ we restrict to the situation when the underlying scores are Wilcoxon scores, i.e., $J(u) = u, \ 0 < u < 1$. For procedure $R_2$, we assume that the location parameter of the $i$-th population ($i = 0,1,2,3$) is the median, i.e., $p = 1/2$.

(iv) Random samples of common sample size $n$ ($n = 6(2)40$) are generated through computer from each of the four populations in a set and the size of the selected subset along with whether correct selection or not is noted by
taking $P' = 0.90$ for both procedures $R_1$ and $R_2$. This process is repeated 10,000 times.

(v) The estimated expected subset size ($\hat{E}(S)$) and estimated probabilities of correct selection ($\hat{P}(CS)$) for the above mentioned values of $n$ are obtained for both procedures by taking the average of the subset sizes and proportion of the times selection is correct in 10,000 repetitions, respectively. These values of $\hat{P}(CS)$ and $\hat{E}(S)$ for both procedures under different parametric configurations of families of distributions considered above and various choices of $n$ ($n = 6(2)40$) are listed in the Tables 3.6.1 and 3.6.3.

(vi) As a measure of "goodness" of a subset selection procedure, we use the ratio of the estimated expected subset size $\hat{E}(S)$ to the estimated probability of correct selection $\hat{P}(CS)$, i.e., $\frac{\hat{E}(S)}{\hat{P}(CS)}$. A rule $R$ is said to be "better" than a rule $R^*$ if the ratio for $R$ is less than the ratio of $R^*$. The relative efficiency of the procedure $R_1$ relative to the
procedure $R_2$ is an inverse ratio of the measures of goodness, i.e.,

$$e(R_1, R_2) = \frac{E(S|R_2)}{E(S|R_1)} \times \frac{P(CS|R_1)}{P(CS|R_2)}.$$ 

The values of $e(R_1, R_2)$ for different parametric configurations of families of distributions considered above and various choices of $n$ ($n = 6(2)40$) are listed below in the Tables 3.6.2 and 3.6.4.

Table 3.6.1

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<th>Cauchy</th>
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Table 3.6.2
e(R₁,R₂) values for various underlying distributions

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* In Tables 3.6.1 and 3.6.2 the configurations of the various families of distributions considered are as under:

1. Normal Distribution

\[ \pi_0 \sim N(2,1), \quad \pi_1 \sim N(2.8,1), \quad \pi_2 \sim N(0.5,1), \quad \pi_3 \sim N(2.1,1). \]

2. Double Exponential Distribution

\[ \pi_0 \sim DE(2,1), \quad \pi_1 \sim DE(2.8,1), \quad \pi_2 \sim DE(0.5,1), \quad \pi_3 \sim DE(2.1,1). \]

Here the notation \( DE(\mu,\lambda) \) denotes the double exponential distribution with location parameter \( \mu \) and scale parameter \( \lambda \).

3. Cauchy Distribution

\[ \pi_0 \sim C(2,1), \quad \pi_1 \sim C(2.8,1), \quad \pi_2 \sim C(0.5,1), \quad \pi_3 \sim C(2.1,1). \]

Here the notation \( C(\mu,\lambda) \) denotes the cauchy distribution with location parameter \( \mu \) and scale parameter \( \lambda \).
### Table 3.6.3

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<th>Cauchy</th>
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</table>

In Tables 3.6.3 and 3.6.4 the configurations of the various families of distributions considered are as under:

### Table 3.6.4

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<th>Cauchy</th>
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</table>

** In Tables 3.6.3 and 3.6.4 the configurations of the various families of distributions considered are as under:
Chapter 3

1. Normal Distribution

\[ \pi_0 \sim N(1.8,1), \quad \pi_1 \sim N(0.5,1), \quad \pi_2 \sim N(1.5,1), \quad \pi_3 \sim N(2.1,1). \]

2. Double Exponential Distribution

\[ \pi_0 \sim DE(1.8,1), \quad \pi_1 \sim DE(0.5,1), \quad \pi_2 \sim DE(1.5,1), \quad \pi_3 \sim DE(2.1,1). \]

3. Cauchy Distribution

\[ \pi_0 \sim C(1.8,1), \quad \pi_1 \sim C(0.5,1), \quad \pi_2 \sim C(1.5,1), \quad \pi_3 \sim C(2.1,1). \]

From the above tables of \( e(R_1,R_2) \), we note the following:

(i) For all values of \( n \) considered here procedure \( R_1 \), restricted to Wilcoxon scores, performs relatively better than procedure \( R_2 \), restricted to the case when \( p=1/2 \), when the underlying distribution is normal under the two specified sets of configurations considered in these tables.

(ii) When the underlying distributions are double exponential and cauchy no uniform pattern of relative performance emerges for the various choices of \( n \) considered here.