Chapter 5

Plane $SH$– waves at a corrugated interface between two dissimilar perfectly conducting self-reinforced elastic half-spaces$^4$

5.1 Introduction

Chattopadhyay and Choudhury (1990) investigated a problem of reflection and transmission of magnetoelastic shear waves from a plane interface between two dissimilar self-reinforced elastic half-spaces. They derived the reflection and transmission coefficients corresponding to the reflected and transmitted waves. In this chapter, we aim to extend their problem to a corrugated interface. We shall discuss the reflection and transmission of plane shear wave incident upon a corrugated interface between two different perfectly conducting self-reinforced elastic half-spaces. Rayleigh’s method of approximation will be applied to derive the reflection and transmission coefficients for the first and second order approximation of the corrugation. These coefficients will be obtained in closed form for a periodic interface and the effect of various parameters will be studied for a specific model.

5.2 Problem formulation and its solution

We consider two perfectly conducting self-reinforced elastic half-spaces with different elastic properties, say $M_1 : [-\infty < z \leq \zeta(x)]$ and $M_2 : \zeta(x) \leq z < \infty$, separated by a corrugated interface $z = \zeta(x)$, where $\zeta(x)$ is a function of $x$, independent of $y$ whose mean value is zero. The geometry of the model considered is similar as chosen in the previous chapters and is shown through Figure 5.1.

![Figure 5.1: Geometry of the Problem](image)

Fourier series representation of function $\zeta(x)$ is given in equation (3.1). In the medium $M_m (m = 1, 2)$, the two-dimensional equation of motion for $SH$ wave propagation in $x - z$ plane and having displacement along $y$ direction, is given by

$$Q_m \frac{\partial^2 V_m}{\partial x^2} + S_m \frac{\partial^2 V_m}{\partial x \partial z} + P_m \frac{\partial^2 V_m}{\partial z^2} = \rho_m \frac{\partial^2 V_m}{\partial t^2},$$  \hspace{1cm} (5.1)
where $V_m$ is the $y-$ component of displacement vector in the medium $M_m$ and

$$Q_1 = \mu_T + a_1^2(\mu_L - \mu_T) + \mu_e H_0^2 \cos^2 \Phi, \quad Q_2 = \mu'_T + a_2^2(\mu'_L - \mu'_T) + \mu'_e H_0^2 \cos^2 \Phi',$$

$$S_1 = 2a_1a_3(\mu_L - \mu_T) + \mu_e H_0^2 \sin 2\Phi, \quad S_2 = 2a_1'a_3'(\mu'_L - \mu'_T) + \mu'_e H_0^2 \sin 2\Phi',$$

$$P_1 = \mu_T + a_3^2(\mu_L - \mu_T) + \mu_e H_0^2 \sin^2 \Phi, \quad P_2 = \mu'_T + a_3'(\mu'_L - \mu'_T) + \mu'_e H_0^2 \sin^2 \Phi',$$

where various entities used here are defined earlier in Chapter - 1. Note that the quantities having prime and used in the above expressions correspond to medium $M_2$, while those without prime correspond to medium $M_1$.

For a plane harmonic $SH-$ wave propagating in the positive direction of $x-$ axis, we assume that

$$V_m(x, z, t) = U_m(z) \exp\{-i(kx - \omega t)\}, \quad (5.2)$$

where $k$ and $\omega$ are defined earlier. Inserting equation (5.2) into (5.1) and solving the resulting equation for $U_m(z)$, we obtain the solution of equation (5.1) as

$$V_m(x, z, t) = |A \exp(-i k \gamma_m z) + B \exp(i k \gamma_m z)| \exp\{-i(kx - \omega t)\}, \quad (5.3)$$

where $A$ and $B$ are constants and

$$\gamma_m = \sqrt{\alpha_m^2 + \mu_T \left( \frac{1}{\sin^2 \Phi} - \frac{Q_m}{\mu_T} \right)}, \quad \alpha_m = \frac{S_m}{2P_m}.$$ 

Let a unit amplitude $SH-$ wave propagating through the upper medium $M_1$ be incident at the corrugated interface. This incident $SH-$ wave at the interface will give rise to ‘irregularly-reflected’ and ‘irregularly-refracted’ waves due to the corrugation of the interface, in addition to the regularly-reflected and regularly-refracted waves. The various reflected and transmitted waves are shown in Figure 5.1.

The concerned component of the displacement in the upper medium $M_1$ due to the incident, regularly and irregularly reflected $SH-$ waves, is given by

$$V_1 = |\exp(-iq z) + B_0 \exp(i q_0 z)| \exp\{-i \omega(x \sin \theta / \beta_1 - t)\} + \sum_{n=1}^{\infty} |B_n \exp(i q_n z)|$$

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\begin{align}
\times \exp\{-i\omega\left(\frac{x\sin \theta_n}{\beta_1} - t\right)\} + B'_n \exp\{i\theta'_n x\} \exp\{-i\omega\left(\frac{x\sin \theta'_n}{\beta_1} - t\right)\}, & \quad (5.4) \\
\text{and in the medium } M_2 \text{ due to the regularly and irregularly transmitted } SH- \text{ waves, is given by} & \\
V_2 &= D_0 \exp(-irx) \exp\{-i\omega\left(\frac{x\sin \delta}{\beta_2} - t\right)\} + \sum_{n=1}^{\infty} \left[ D_n \exp(-irnx) \exp\{-i\omega\left(\frac{x\sin \delta_n}{\beta_2} - t\right)\} + D'_n \exp(-ir'_nx) \exp\{-i\omega\left(\frac{x\sin \delta'_n}{\beta_2} - t\right)\} \right], & \quad (5.5) \\
\text{where } B_0 \text{ is the amplitude of the regularly reflected } SH- \text{ wave at an angle } \theta, D_0 \text{ is the amplitude of the regularly refracted } SH- \text{ wave at an angle } \delta, B_n \text{ and } B'_n \text{ are amplitudes of irregularly reflected waves at angles } \theta_n \text{ and } \theta'_n \text{ respectively, } D_n \text{ and } D'_n \text{ are amplitudes of the irregularly refracted waves at angles } \delta_n \text{ and } \delta'_n \text{ respectively, and} & \\
q_0 = k(\gamma_1 + \alpha_1), \ & q = k(\gamma_1 - \alpha_1), \ & q_n = k(\gamma_n + \alpha_1), \ & q'_n = k(\gamma'_n + \alpha_1), \ & r = k(\gamma_2 - \alpha_2), & \\
r_n = k(\gamma_2 - \alpha_2), \ & r'_n = k(\gamma'_2 - \alpha_2), & \ & \alpha_1 = \frac{S_1}{2P_1}, \ & \beta_1 = \sqrt{\frac{\mu_T}{\rho_1}}, \ & \alpha_2 = \frac{S_2}{2P_2}, \ & \beta_2 = \sqrt{\frac{\mu'_T}{\rho_2}}; & \\
\gamma_1 = \sqrt{\alpha_1^2 + \frac{\mu_T}{P_1} \left(\frac{1}{\sin^2 \theta} - \frac{Q_1}{\mu_T}\right)}, \ & \gamma_2 = \sqrt{\alpha_2^2 + \frac{\mu'_T}{P_2} \left(\frac{1}{\sin^2 \delta} - \frac{Q_2}{\mu'_T}\right)}, & \\
\gamma_1^n = \sqrt{\alpha_1^2 + \frac{\mu_T}{P_1} \left(\frac{1}{\sin^2 \theta_n} - \frac{Q_1}{\mu_T}\right)}, \ & \gamma_2^n = \sqrt{\alpha_2^2 + \frac{\mu'_T}{P_2} \left(\frac{1}{\sin^2 \delta_n} - \frac{Q_2}{\mu'_T}\right)}, & \\
\text{and} & \\
\gamma_1'^n = \sqrt{\alpha_1'^2 + \frac{\mu_T}{P_1} \left(\frac{1}{\sin^2 \theta'_n} - \frac{Q_1}{\mu_T}\right)}, \ & \gamma_2'^n = \sqrt{\alpha_2'^2 + \frac{\mu'_T}{P_2} \left(\frac{1}{\sin^2 \delta'_n} - \frac{Q_2}{\mu'_T}\right)}. & \\
\text{The Snell’s law and Spectrum theorem relating to angles of various reflected and transmitted waves with that of the incident wave are given by} & \\
\frac{\sin \theta}{\beta_1} = \frac{\sin \delta}{\beta_2} = \frac{k}{\omega} & \quad (5.6) \\
\text{and} & \\
\sin \theta_n - \sin \theta = \frac{np \beta_1}{\omega}, \ & \sin \theta'_n - \sin \theta = -\frac{np \beta_1}{\omega} , &
\end{align}
5.3 Boundary conditions

The boundary conditions at the corrugated interface are the continuity of displacement and traction. Mathematically, these boundary conditions can be written as:

At \( z = \zeta(x) \)

\[
V_1 = V_2, \quad (5.8)
\]

\[
(\sigma_{21}^{01})^{1Mx} + \tau_{23}^{1} - \zeta'((\sigma_{21}^{01})^{1Mx} + \tau_{21}^{1}) = (\sigma_{21}^{02})^{2Mx} + \tau_{23}^{2} - \zeta'((\sigma_{21}^{02})^{2Mx} + \tau_{21}^{2}), \quad (5.9)
\]

where the quantities having '1' in the superscript correspond to medium \( M_1 \) and that of having '2' in the superscript correspond to medium \( M_2 \). \( (\sigma_{21}^{01})^{1Mx} \) and \( \tau_{ij} \) are the components of Maxwell stress tensor and force stress tensor respectively. Their expressions are given earlier in Chapter-1. Exploiting the expressions given in equations (1.3), (1.44), (1.50) and (1.55), the requisite components of stresses in medium \( M_1 \) are

\[
\tau_{23}^{1} = \mu_T \frac{\partial V_1}{\partial z} + a_3(\mu_L - \mu_T)(a_1 \frac{\partial V_1}{\partial x} + a_2 \frac{\partial V_1}{\partial z}),
\]

\[
\tau_{21}^{1} = \mu_T \frac{\partial V_1}{\partial x} + a_1(\mu_L - \mu_T)(a_1 \frac{\partial V_1}{\partial x} + a_2 \frac{\partial V_1}{\partial z}),
\]

\[
(\sigma_{21}^{01})^{1Mx} = \mu_e H_0^2 (\cos^2 \Phi \frac{\partial V_1}{\partial x} + \sin \Phi \cos \Phi \frac{\partial V_1}{\partial z}),
\]

\[
(\sigma_{21}^{01})^{2Mx} = \mu_e H_0^2 (\sin^2 \Phi \frac{\partial V_1}{\partial z} + \sin \Phi \cos \Phi \frac{\partial V_1}{\partial x}), \quad (5.10)
\]

and in medium \( M_2 \) are

\[
\tau_{23}^{2} = \mu_T' \frac{\partial V_2}{\partial z} + a_3'(\mu_L' - \mu_T')(a_1' \frac{\partial V_2}{\partial x} + a_2' \frac{\partial V_2}{\partial z}),
\]

\[
\tau_{21}^{2} = \mu_T' \frac{\partial V_2}{\partial x} + a_1'(\mu_L' - \mu_T')(a_1' \frac{\partial V_2}{\partial x} + a_2' \frac{\partial V_2}{\partial z}),
\]

\[
(\sigma_{21}^{02})^{1Mx} = \mu_e' H_0'^2 (\cos^2 \Phi' \frac{\partial V_2}{\partial x} + \sin \Phi' \cos \Phi' \frac{\partial V_2}{\partial z}),
\]

\[
(\sigma_{21}^{02})^{2Mx} = \mu_e' H_0'^2 (\sin^2 \Phi' \frac{\partial V_2}{\partial z} + \sin \Phi' \cos \Phi' \frac{\partial V_2}{\partial x}),
\]
Using the equations (5.10) and (5.11) into the second boundary condition given by (5.9), we can write

\[(P_1 - \frac{S_1Q'}{2}) \frac{\partial V_1}{\partial z} + (S_1 - Q') \frac{\partial V_1}{\partial x} = (P_2 - \frac{S_2Q'}{2}) \frac{\partial V_2}{\partial z} + (S_2 - Q') \frac{\partial V_2}{\partial x},\]

(5.12)

where \(P_m\), \(Q_m\) and \(S_m\), \((m = 1,2)\) are defined earlier in equation (5.1). Putting equations (5.4) and (5.5) into the boundary conditions given by (5.8) and (5.12), we obtain

\[e^{-\nu q} + B_0e^{iq \nu} + \sum_n \{B_n e^{-\nu q \nu} \exp(-mpx) + B'_n e^{-\nu q \nu} \exp(mp)\} = D_0e^{-\nu q} + \sum_n \{D_n e^{-\nu q \nu} \exp(-mpx) + D'_n e^{-\nu q \nu} \exp(mp)\},\]

(5.13)

In writing these equations, the use of (5.6) and (5.7) have been made. Equations (5.13) and (5.14) provide us the formulae corresponding to the reflection and transmission coefficients for any order of approximation of the corrugation.

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5.4 Solutions of the first order approximation

As in the previous chapters, the amplitude and slope of the corrugated interface $z = \zeta(x)$ are assumed to be very small such that $\exp(\pm i\omega_0 \zeta) \approx 1 \pm i\omega_0 \zeta$. Inserting equation (3.1) and using equations (5.6), (5.7) into (5.13) and (5.14) and the above approximation of the corrugation, the expressions for $B_0$ and $D_0$ can be obtained by collecting the terms independent of $x$ and $\zeta$ as

\[1 + B_0 = D_0.\]  

\[
\frac{S_1 \omega \sin \theta}{2\beta_1} + P_1 q + \left(\frac{S_1 \omega \sin \theta}{2\beta_1} - P_1 q_0\right)B_0 = \left(P_2 r + \frac{S_2 \omega \sin \theta}{2\beta_1}\right)D_0. \tag{5.16}
\]

Next, we equate the coefficients of $\exp(-i\pi x)$, as in the previous chapters, to both sides of the equations and obtain

\[B_n - D_n = \kappa_{-n}[q - q_0 B_0 - r D_0], \tag{5.17}\]

\[
\left[P_1 q_n - \left(\frac{\omega \sin \theta}{\beta_1} + np\right)\right]B_n + \left[r_n P_2 + \frac{S_2 \omega \sin \theta}{\beta_1} + np\right]D_n
= \kappa_{-n}\left[-P_1 q^2 + \frac{S_1 n pq}{2} - \left(S_1 q - Q_1 np\right)\frac{\omega \sin \theta}{\beta_1} + \left(\frac{S_2 \omega \sin \theta}{\beta_1}\right)\left(Q_1 np + q_0 S_1\right)\right]
\]

\[
-\frac{S_1 n pq_0}{2} - P_1 q_0^2 \right]B_0 + \left(P_2 r^2 - \frac{S_2 np\theta}{2} + \frac{\omega \sin \theta\beta_1}{\beta_1} - np\right)D_0. \tag{5.18}\]

Similarly, to find the solution of the first order approximation of the corrugation for the coefficients $B'_n$ and $D'_n$, we equate the coefficients of $\exp(i\pi x)$ and obtain

\[B'_n - D'_n = \kappa_{n}[q - q_0 B_0 - r D_0], \tag{5.19}\]

\[
\left[P_1 q'_n - \left(\frac{\omega \sin \theta}{\beta_1} - np\right)\right]B'_n + \left[r'_n P_2 + \frac{S_2 \omega \sin \theta}{\beta_1} - np\right]D'_n
= -\kappa_{n}\left[P_1 q^2 + \frac{S_1 n pq}{2} + \left(\frac{S_1 q}{2} + Q_1 np\right)\frac{\omega \sin \theta}{\beta_1} + \left((Q_1 np - q_0 S_1)\frac{\omega \sin \theta}{\beta_1}\right)\right]
\]

\[
-\frac{S_1 n pq_0}{2} + P_1 q_0^2 \right]B_0 + \left(P_2 r^2 + \frac{S_2 np\theta}{2} + \frac{\omega \sin \theta\beta_1}{\beta_1}\right)D_0. \tag{5.20}\]

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On solving equations (5.15) and (5.16), we obtain the reflection and transmission coefficients corresponding to plane interface, as

\[ B_0 = \frac{1 - M}{1 + M}, \quad D_0 = \frac{2}{1 + M}, \]  

(5.21)

where \( M = \frac{P_2\gamma_2}{P_1\gamma_1} \).

These are the same expressions of reflection and transmission coefficients as obtained earlier by Chattopadhyay and Choudhury (1990) for the relevant problem.

On solving equations (5.17) - (5.20), we obtain the solution of the first order approximation of corrugation for the coefficients \( B_n, D_n, B'_n \) and \( D'_n \) as

\[ B_n = \frac{\Delta B_n}{\Delta_n}, \quad D_n = \frac{\Delta D_n}{\Delta_n}, \quad B'_n = \frac{\Delta'B_n}{\Delta'_n}, \quad D'_n = \frac{\Delta'D_n}{\Delta'_n}, \]  

(5.22)

where

\[ \Delta_n = \gamma^n_n + \frac{P_2\gamma_n^2}{P_1} + \frac{np}{2kP_1} (S_2 - S_1), \quad \Delta'_n = \gamma^n'_n + \frac{P_2\gamma_n'^2}{P_1} - \frac{np}{2kP_1} (S_2 - S_1), \]

\[ \Delta_{B_n} = \kappa_n k [(1 + B_0) [-\gamma^2_n + \frac{Q_1 np}{P_1 k} S_1 \left( \frac{np}{2kP_1} (S_1 + S_2) + \frac{P_2\gamma_n^2}{P_1} \right)] \left( 1 - B_0 \right) \gamma_n [\frac{S_1}{2P_1} + \frac{P_2\gamma_n^2}{P_1} - \frac{np}{2kP_1} (S_2 - S_1)], \]

\[ \Delta_{D_n} = \kappa_n k [(1 + B_0) [-\gamma^2_n + \frac{Q_1 np}{P_1 k} S_1 \left( \frac{np}{2kP_1} (S_1 + S_2) + \frac{P_2\gamma_n^2}{P_1} \right)] \left( 1 - B_0 \right) \gamma_n [\frac{S_1}{2P_1} + \frac{P_2\gamma_n^2}{P_1}], \]

\[ \Delta_{B'_n} = \kappa_n k [(1 + B_0) [-\gamma^2_n + \frac{Q_1 np}{P_1 k} S_1 \left( \frac{np}{2kP_1} (S_1 + S_2) + \frac{P_2\gamma_n^2}{P_1} \right)] \left( 1 - B_0 \right) \gamma_n [\frac{S_1}{2P_1} + \frac{P_2\gamma_n^2}{P_1}], \]

\[ \Delta_{D'_n} = \kappa_n k [(1 + B_0) [-\gamma^2_n + \frac{Q_1 np}{P_1 k} S_1 \left( \frac{np}{2kP_1} (S_1 + S_2) + \frac{P_2\gamma_n^2}{P_1} \right)] \left( 1 - B_0 \right) \gamma_n [\frac{S_1}{2P_1} + \frac{P_2\gamma_n^2}{P_1}], \]

The coefficients \( B_0 \) and \( D_0 \) appearing in the expressions of \( \Delta_{B_n}, \Delta_{D_n}, \Delta'_{B_n} \) and \( \Delta'_{D_n} \) are given by (5.21). Formulae given by (5.22) provide us the reflection and transmission
coefficients for the first order approximation of the corrugation. It is noted that the reflection and transmission coefficients are functions of the angle of incidence, elastic constants, amplitude of the corrugation and frequency of the incident waves.

5.5 Solution of the second order approximation

If the terms of the higher order than $\zeta^2$ are neglected, so that

$$\exp(-iq\zeta) = 1 - iq\zeta - \frac{q^2\zeta^2}{2}, \quad \exp(iq_0\zeta) = 1 + iq_0\zeta - \frac{q_0^2\zeta^2}{2}. \tag{5.23}$$

Inserting (5.23) into the boundary conditions given by equations (5.13) and (5.14), making use of the relations given by (5.6) and (5.7) and equating the terms independent of $x$, coefficients of $\exp(-mpx)$ and coefficients of $\exp(mpx)$ in the resulting equations, we obtain the following equations

$$1 - q^2\zeta_n\zeta_{-n} - q_0^2\zeta_n\zeta_{-n}B_0 + i\zeta_n\zeta_{-n}B_n + i\zeta_n\zeta_{-n}B'_n,$$

$$= (1 - r^2\zeta_n\zeta_{-n})D_0 - i\zeta_nD_n + i\zeta_nD'_n, \tag{5.24}$$

$$(1 - q^2\zeta_n\zeta_{-n})(p_1q + \frac{\omega S_1\sin\theta}{2\beta_1}) + (\frac{S_1\omega\sin\theta}{2\beta_1} - q_0P_1)(1 - q_0^2\zeta_n\zeta_{-n})B_0$$

$$+ i\zeta_n\left[\frac{S_1\eta_0}{2} - q_0^2P_1 + \left(\frac{S_1\eta_0}{2} - Q_1np\right)(\frac{\omega\sin\theta}{\beta_1} + np)\right]B_n + i\zeta_n\left[\frac{\omega\sin\theta}{\beta_1}\right]B'_n = (1 - r^2\zeta_n\zeta_{-n})(P_2r + \frac{S_2\omega\sin\theta}{2\beta_1})D_0$$

$$- np(Q_1np + S_1\eta_0) - q_0^2P_1 + \frac{S_1\eta_0}{2})B'_n = (1 - r^2\zeta_n\zeta_{-n})(P_2r + \frac{S_2\omega\sin\theta}{2\beta_1})D_0$$

$$- i\zeta_n[r_n(P_2 + \frac{S_2np}{2}) + (\frac{\omega\sin\theta}{\beta_1} + np)(Q_2np + \frac{S_2np}{2})]D_n + i\zeta_n[r'_n(P_2 + \frac{S_2np}{2})]D'_n$$

$$+ \left[\frac{\omega\sin\theta}{\beta_1} - np\right](Q_2np + \frac{S_2np}{2})]D'_n, \tag{5.25}$$

$$- i\zeta_n + i\zeta_nB_0 + (1 - q_0^2\zeta_n\zeta_{-n})B_n - \frac{q_0^2\zeta_n}{2}B'_n$$

$$= - i\zeta_nD_0 + (1 - r^2\zeta_n\zeta_{-n})D_n - \frac{r^2\zeta_n}{2}D'_n, \tag{5.26}$$

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\[-\zeta_n q(qP_1 - npS_1) - \frac{\omega \sin \theta}{\beta_1}(npQ_1 - \frac{S_1 q}{2})\] \\
\[-\frac{\omega \sin \theta}{\beta_1}(npQ_1 + \frac{qS_1}{2})]B_0 + (1 - q^2_2 \zeta_n \zeta_n)\left[\frac{S_1}{2}(\frac{\omega \sin \theta}{\beta_1} + np) - P_1 q_0\right]B_n\] \\
\[+ \left[\frac{q^2_2}{2}(S_1 np + P_1 q'_n) - q'_n\left(\frac{\omega \sin \theta}{\beta_1} - np\right)(\frac{S_1 q'_n}{4} - Q_1 np)\right]c_n B'_n\] \\
\[= \zeta_n D_0\left[\frac{-r^2}{2}(P_2 + \frac{S_2 np}{2}) - \frac{\omega \sin \theta}{\beta_1}\left(\frac{S_2 r}{2} - Q_2 np\right)\right] + \left[P_2 r_n + \frac{S_2}{2}\left(\frac{\omega \sin \theta}{\beta_1} + np\right)\right](1)
\]

\[-r^2_2 \zeta_n \zeta_n)D_n - \frac{r^2}{2}(P_2 r'_n - npS_2) + \frac{\omega \sin \theta}{\beta_1}\left(\frac{S_2 r'_n}{4} - Q_2 np\right)\right]c_n D'_n,\] (5.27)

\[-q_0 \zeta_n + q_0 q_0 B_n - \frac{q^2_2}{2} B_n + (1 - q^2_2 \zeta_n \zeta_n)B'_n\]

\[= -\zeta_n r D_0 - \frac{r^2}{2} c_n D_n + (1 - r^2_2 \zeta_n \zeta_n)D'_n,\] (5.28)

\[-q_0 q_1 q_0 + \frac{npS_1}{2} \frac{\omega \sin \theta}{\beta_1}\left(\frac{npQ_1 + S_1 q}{2}\right)\]

\[+ \frac{\omega \sin \theta}{\beta_1}\left(\frac{q_0 q_0}{2} - npQ_1\right)B_0 + \frac{q^2_2}{2}(q_0 P_1 - S_1 np) - q_0\left(\frac{\omega \sin \theta}{\beta_1} + np\right)(\frac{S_1 q_0}{4})\]

\[-\frac{Q_1 np}{2}B_n + \left[\left(\frac{\omega \sin \theta}{\beta_1} - np\right) - P_1 q'_n\right](1 - q^2_2 \zeta_n \zeta_n)B'_n\]

\[= \zeta_n D_0\left[\frac{-r^2}{2}(P_2 + \frac{S_2 np}{2}) - \frac{\omega \sin \theta}{\beta_1}\left(\frac{S_2 r}{2} + Q_2 np\right)\right] + \left[P_2 r_n + \frac{S_2}{2}\left(\frac{\omega \sin \theta}{\beta_1} - np\right)\right](1 - r^2_2 \zeta_n \zeta_n)D'_n.\] (5.29)

Equations (5.24) - (5.29) provide us the reflection and transmission coefficients for the second order approximation of the corrugation.

### 5.6 Special case: A periodic interface

As did in the previous chapters, the formulae of reflection and refraction coefficients $B_1$, $D_1$, $B'_1$ and $D'_1$ for the first order approximation of the corrugated interface $z = d \cos px$,
are given by

\[ B_1 = \frac{\Delta B_1}{\Delta_1}, \quad D_1 = \frac{\Delta D_1}{\Delta_1}, \quad B'_1 = \frac{\Delta B'_1}{\Delta'_1}, \quad D'_1 = \frac{\Delta D'_1}{\Delta'_1}, \]  

(5.30)

where

\[ \Delta_1 = \gamma_1' + \frac{P_2 \gamma_1'}{P_1} + \frac{p}{2kP_1} (S_2 - S_1), \quad \Delta'_1 = \gamma'_1 + \frac{P_2 \gamma'_1}{P_1} - \frac{p}{2kP_1} (S_2 - S_1), \]

\[ \Delta B_1 = \frac{kd}{2} [(1 + B_0)\gamma_1 - \gamma_1^2 + \frac{Q_1 P}{P_1} - \frac{S_1}{2P_1} (\frac{P_2 \gamma_1}{P_1}) + \gamma_1 (1 - B_0) \gamma_1 (S_1 + S_2)] + \frac{P_2 \gamma_1}{P_1}, \]

\[ \Delta D_1 = \frac{kd}{2} [(1 + B_0)\gamma_1 - \gamma_1^2 + \frac{Q_1 P}{P_1} - \frac{S_1}{2P_1} (\frac{P_2 \gamma_1}{P_1} - \gamma_1)] + (1 - B_0) \gamma_1 (S_1 + S_2) \]

\[ + D_0 (\frac{P_2}{P_1} (\gamma_2 - \frac{S_2}{2P_2})^2 - \frac{Q_2}{P_1} (\gamma_2 - \frac{S_2}{2P_2}) \frac{S_2}{2P_2} - \frac{p}{2kP_1} (S_2 + S_1))], \]

\[ \Delta B'_1 = \frac{kd}{2} [(1 + B_0)\gamma_1 - \gamma_1^2 + \frac{Q_1 P}{P_1} - \frac{S_1}{2P_1} (\frac{P_2 \gamma_1}{P_1}) + \gamma_1 (1 - B_0) \gamma_1 (S_1 + S_2)] + \frac{P_2 \gamma_1}{P_1}, \]

\[ \Delta D'_1 = \frac{kd}{2} [(1 + B_0)\gamma_1 - \gamma_1^2 + \frac{Q_1 P}{P_1} - \frac{S_1}{2P_1} (\frac{P_2 \gamma_1}{P_1}) - \gamma_1)] + (1 - B_0) \gamma_1 (S_1 + S_2) \]

\[ + D_0 (\frac{P_2}{P_1} (\gamma_2 - \frac{S_2}{2P_2})^2 + \frac{Q_2}{P_1} (\gamma_2 - \frac{S_2}{2P_2}) \frac{S_2}{2P_2} + \frac{p}{2kP_1} (S_2 + S_1)]. \]

It is clearly seen from (5.30) that the reflection and refraction coefficients \( B_1, D_1, B'_1 \) and \( D'_1 \) are proportional to the amplitude of the corrugated interface \( d \) and wavenumber \( k \) of the incident wave.

### 5.7 Particular cases

(a) If the direction of the magnetic field is parallel to the direction of wave propagation, then \( \phi = \phi' = 0 \). Also, if the direction of self-reinforcement is along the \( z \)-axis, then \( (a_1, a_2, a_3) = (a'_1, a'_2, a'_3) = (0, 0, 1) \). In this case, the quantities \( P_m, S_m \) and \( Q_m \) defined in equation (5.1) reduce to

\[ P_1 = \mu_L, \quad Q_1 = \mu_e H_0 + \mu_T, \quad P_2 = \mu'_L, \quad Q_2 = \mu'_e H_0 + \mu'_T, \quad S_m = 0 \]
and the quantities $\gamma_m$ and $\gamma'_m$ reduce to

$$
\gamma_1 = \sqrt{\frac{\mu_T}{\mu_L} (\cot^2 \theta - \epsilon_H)}, \quad \gamma'_1 = \sqrt{\frac{\mu_T}{\mu_L} (\cot^2 \theta'_1 - \epsilon_H)},
$$

$$
\gamma_2 = \sqrt{\frac{\mu_T}{\mu_L} (\cot^2 \delta - \epsilon_H)}, \quad \gamma'_2 = \sqrt{\frac{\mu_T}{\mu_L} (\cot^2 \delta'_1 - \epsilon_H)},
$$

where $\epsilon_H = \frac{\mu_H H_0^2}{\mu_T}$ and $\epsilon'_H = \frac{\mu'_T H_0^2}{\mu'_T}$. Thus in this particular case, the expressions of the coefficients $B_0$ and $D_0$ are given by equation (5.21) with $M = \frac{\mu_L \gamma_2}{\mu_L \gamma'_1}$, and the expressions of the coefficients $B'_1$, $D'_1$, $B'_1$, and $D'_1$ are given by equation (5.30) with the following modified values

$$
\Delta_1 = \gamma_1 + \frac{\mu'_L}{\mu_L} \gamma'_2, \quad \Delta'_1 = \gamma'_1 + \frac{\mu'_L}{\mu_L} \gamma'_2,
$$

$$
\Delta_{B_1} = \frac{ikd}{2} \left[ \left( 1 + B_0 \right) [ - \gamma_1^2 + \frac{\mu_T}{k \mu_L} (\epsilon_H + 1) - (1 - B_0) \frac{\mu'_L}{\mu_L} \gamma_2 \gamma'_2 - D_0 \frac{\mu'_L}{\mu_L} \gamma_1 \gamma'_2 - \frac{\mu'_T}{k \mu'_L} (\epsilon'_H + 1) ] \right],
$$

$$
\Delta_{D_1} = \frac{ikd}{2} \left[ \left( 1 + B_0 \right) [ - \gamma_1^2 + \frac{\mu_T}{k \mu_L} (\epsilon_H + 1) - (1 - B_0) \frac{\mu'_L}{\mu_L} \gamma_1 \gamma'_2 + D_0 \frac{\mu'_L}{\mu_L} \gamma_2 \gamma'_2 + \frac{\mu'_T}{k \mu'_L} (\epsilon'_H + 1) ] \right],
$$

$$
\Delta'_{B_1} = \frac{ikd}{2} \left[ \left( 1 + B_0 \right) [ - \gamma'_1^2 - \frac{\mu'T}{k \mu' L} (\epsilon'H + 1) - (1 - B_0) \frac{\mu'_L}{\mu_L} \gamma_1 \gamma'_2 + D_0 \frac{\mu'_L}{\mu_L} \gamma_2 \gamma'_2 + \frac{\mu'_T}{k \mu'_L} (\epsilon'_H + 1) ] \right],
$$

(b) If the direction of magnetic field is perpendicular to the direction of wave propagation and the direction of self-reinforcement is along $z-$ axis in both the media, then $\phi = \phi' = \pi/2$, $a_1 = a_2 = a'_1 = a'_2 = 0$; $a_3 = a'_3 = 1$. In this case, the quantities $P_m$, $Q_m$ and $S_m$ defined in equation (5.1) reduce to

$$
P_1 = \mu_e H_0^2 + \mu_L, \quad P_2 = \mu'_e H'_0^2 + \mu'_L, \quad Q_1 = \mu_T, \quad Q_2 = \mu'_T, \quad S_m = 0
$$

and the quantities $\gamma_m$ and $\gamma'_m$ reduce to

$$
\gamma_1 = \frac{\cot \theta}{\epsilon}, \quad \gamma'_1 = \frac{\cot \theta'_1}{\epsilon}, \quad \gamma_2 = \frac{\cot \delta}{\epsilon'}, \quad \gamma'_2 = \frac{\cot \delta'_1}{\epsilon'},
$$

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where
\[ \epsilon = \sqrt{\epsilon_H + \frac{\mu_L}{\mu_T}} \quad \text{and} \quad \epsilon' = \sqrt{\epsilon_H' + \frac{\mu_L'}{\mu_T'}} \]
then the expressions of the coefficients \( B_0 \) and \( D_0 \) are given by (5.21) with following modified value of \( M \) as
\[ M = \frac{\epsilon^2 \mu_T \gamma_2}{\epsilon' \mu_T \gamma_1} \]
The expressions of the coefficients \( B_1, D_1, B'_1 \) and \( D'_1 \) for the first order approximation of the corrugation are given by equation (5.30) with modifications in the expressions of numerators and denominators as
\[ \Delta_1 = \gamma_1' + \frac{\mu_T \epsilon'^2}{\mu_T \epsilon'^2} \gamma_2', \quad \Delta'_1 = \gamma_1' + \frac{\mu_T \epsilon'^2}{\mu_T \epsilon'^2} \gamma_2' \]
\[ \Delta_{B_1} = \frac{i k d}{2} [ (1 + B_0) (\gamma_2' + \frac{\rho}{k \epsilon'^2}) + (1 - B_0) \gamma_1' \epsilon'^2 \frac{\mu_T}{\epsilon' \mu_T} + \frac{\mu_T}{\mu_T} D_0 \{ \epsilon^2 \gamma_2 (\gamma_2 - \gamma_2') - \frac{\rho}{k} \}] \]
\[ \Delta_{D_1} = \frac{i k d}{2} [ (1 + B_0) (\gamma_2' - \frac{\rho}{k \epsilon'^2}) - (1 - B_0) \gamma_1' \epsilon'^2 \frac{\mu_T}{\epsilon' \mu_T} + \frac{\mu_T}{\mu_T} D_0 \{ \epsilon^2 \gamma_2 (\gamma_2 - \gamma_2') + \frac{\rho}{k} \}] \]
\[ \Delta'_{B_1} = \frac{i k d}{2} [ (1 + B_0) (\gamma_2' - \frac{\rho}{k \epsilon'^2}) + (1 - B_0) \gamma_1' \epsilon'^2 \frac{\mu_T}{\epsilon' \mu_T} + \frac{\mu_T}{\mu_T} D_0 \{ \epsilon^2 \gamma_2 (\gamma_2 - \gamma_2') + \frac{\rho}{k} \}] \]
\[ \Delta'_{D_1} = \frac{i k d}{2} [ (1 + B_0) (\gamma_2' + \frac{\rho}{k \epsilon'^2}) - (1 - B_0) \gamma_1' \epsilon'^2 \frac{\mu_T}{\epsilon' \mu_T} + \frac{\mu_T}{\mu_T} D_0 \{ \epsilon^2 \gamma_2 (\gamma_2 - \gamma_2') - \frac{\rho}{k} \}] \]
(c) If the reinforcement of both the half-spaces are neglected, we shall be left with the problem of reflection and refraction of \( SH \) - waves at a corrugated interface between two dissimilar uniform elastic half-spaces. In this case, we put
\[ \mu_L = \mu_T = \mu, \quad \mu'_L = \mu'_T = \mu', \quad a_1 = a'_1 = a_2 = a'_2 = 0, \quad a_3 = a'_3 = 1, \]
\[ H_0 = H'_0 = 0, \quad Q_1 = P_1 = \mu, \quad S_1 = S_2 = 0, \quad Q_2 = P_2 = \mu' \]
and obtain \( \gamma_1 = \cot \theta, \gamma_1' = \cot \theta', \gamma_2 = \cot \delta, \gamma_2' = \cot \delta' \), \( \gamma_3' = \cot \delta' \).
The expressions of the coefficients \( B_0 \) and \( D_0 \) are now given by equation (5.21) with modified value of \( M \) as
\[ M = \frac{\mu' \gamma_2}{\mu \gamma_1} \]
The expressions of the coefficients $B_i$, $D_i$, $B'_i$ and $D'_i$ for the first order approximation are given by equation (5.30), where now

$$
\Delta_i = \gamma_1^i + \frac{\mu'}{\mu} \gamma_2^i, \quad \Delta'_i = \gamma'_1^i + \frac{\mu'}{\mu} \gamma'_2^i,
$$

$$
\Delta_{B_i} = \frac{ikd}{2} [(1 + B_0)(-\gamma_1^2 + \frac{p}{k}) + (1 - B_0)\frac{\mu'}{\mu} \gamma_1^2 + \frac{\mu'}{\mu} D_0(\gamma_2^2 - \gamma_2^1 - \frac{p}{k})],
$$

$$
\Delta_{D_i} = \frac{ikd}{2} [(1 + B_0)(-\gamma_1^2 + \frac{p}{k}) - (1 - B_0)\gamma_1^1 + D_0\{\frac{\mu'}{\mu} (\gamma_2^2 - \frac{p}{k}) + \gamma_2^1\}],
$$

$$
\Delta'_{B_i} = \frac{ikd}{2} [(1 + B_0)(-\gamma_1^2 - \frac{p}{k}) + (1 - B_0)\frac{\mu'}{\mu} \gamma_1^2 + \frac{\mu'}{\mu} D_0(\gamma_2^2 - \gamma_2^1 + \frac{p}{k})],
$$

$$
\Delta'_{D_i} = \frac{ikd}{2} [(1 + B_0)(-\gamma_1^2 - \frac{p}{k}) - (1 - B_0)\gamma_1^1 - D_0\{\frac{\mu'}{\mu} (\gamma_2^2 + \frac{p}{k}) + \gamma_2^1\}].
$$

It is easy to verify that the expressions of $B_0$ and $D_0$ obtained for the present case are in full agreement with those given in Bullen and Bolt (1985) for the relevant problem corresponding to the plane interface. Also, it can be verified that the expressions of the coefficients $B_i$, $B'_i$, $D_i$ and $D'_i$ match with those presented by Asano (1960) for the relevant problem corresponding to the corrugated interface.

### 5.8 Numerical results and discussion

To study the effect of amplitude of corrugation of the interface, frequency and reinforcement parameters with different values of the angle of incidence on the reflection and refraction coefficients, we have computed them numerically for a specific model.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_L$</td>
<td>$2.68 \times 10^9$ N/m²,</td>
</tr>
<tr>
<td>$\mu_T$</td>
<td>$5.68 \times 10^9$ N/m²,</td>
</tr>
<tr>
<td>$p_1$</td>
<td>$2.2 \times 10^9$ Kg/m³,</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.5,</td>
</tr>
<tr>
<td>$\phi$, $\phi'$</td>
<td>$20^0$,</td>
</tr>
<tr>
<td>$\mu'_L$</td>
<td>$2.95 \times 10^9$ N/m²,</td>
</tr>
<tr>
<td>$\mu'_T$</td>
<td>$4.88 \times 10^9$ N/m²,</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>$2.9 \times 10^9$ Kg/m³,</td>
</tr>
<tr>
<td>$a'_1$</td>
<td>0.3.</td>
</tr>
</tbody>
</table>
For this purpose, we have taken the above following values of relevant parameters. The frequency parameter and angle of incidence are respectively taken as $\frac{\omega d}{\beta_1} = 0.1$ and $\theta = 30^\circ$, wherever not mentioned. The numerical computations are made for the interface considered in the Section 5.6 - Special case. We have taken different values of reinforcement parameters, namely, $(\epsilon_H, \epsilon'_H) = (0, 0), (0.1, 0.2), (0.2, 0.3), (0.3, 0.4)$ to study the effect of reinforcement parameters on the reflection and transmission coefficients both corresponding to the plane interface and corresponding to the corrugated interface.

Figures 5.2 and 5.3 show the variation of modulus of reflection and transmission coefficients corresponding to the regular waves with the angle of incidence $\theta$. We observe from Figure 5.2 that the reflection coefficient $B_0$ increases with increase of parameters $\epsilon_H$ and $\epsilon'_H$ as $\theta$ increases. There occurs a critical angle at $\theta = 70^\circ$ when $(\epsilon_H, \epsilon'_H) = (0.3, 0.4)$. The effect of parameters $\epsilon_H$ and $\epsilon'_H$ on the coefficient $B_0$ is found to be minimum and almost negligible in the range $0^\circ < \theta \leq 30^\circ$, while it is found to be maximum and noticeable in rest of the range of the angle of incidence. We also note that the value of the amplitude ratio $B_0$ is minimum at $\theta = 1^\circ$ and it attains

![Figure 5.2: Variation of amplitude ratio $B_0$ with angle of incidence, $\theta$ for different values of $(\epsilon_H, \epsilon'_H)$.](image)

![Figure 5.3: Variation of amplitude ratio $B_0$ with angle of incidence, $\theta$ for different values of $(\epsilon_H, \epsilon'_H)$.](image)
maximum value at grazing incidence, for all values of the parameters $\epsilon_H$ and $\epsilon'_H$ considered. The behavior of the modulus value of the refraction coefficient $D_0$ with angle of incidence is depicted in Figure 5.3, at different set of values of $(\epsilon_H, \epsilon'_H)$. Its nature of dependence on $\theta$ is found to be exactly opposite to that of the coefficient $B_0$ as it was expected in view of equation (5.21). Thus both the coefficients $B_0$ and $D_0$ are found to be influenced by the parameters $\epsilon_H$ and $\epsilon'_H$, however their behavior with increasing angle of incidence is reverse.

Figures 5.4 - 5.7 show the variation of the modulus of reflection and refraction coefficients $B_1$, $B'_1$, $D_1$ and $D'_1$ with the angle of incidence. From Figure 5.4, we note that the reflection coefficient $B_1$ has certain value near normal incidence, its value decreases with $\theta$ attaining a minimum value at certain angle of incidence and thereafter increases with $\theta$. In Figure 5.5, we note that the reflection coefficient $B'_1$ has maximum value at $\theta = 1^\circ$ and thereafter, it goes on decreasing with $\theta$ for all set of values of $(\epsilon_H, \epsilon'_H)$. From Figure 5.6, we also note that the refraction coefficient $D_1$ has maximum value at $\theta = 1^\circ$ and its value first decreases with $\theta$ up to a certain angle of incidence and thereafter as $\theta$ increases, its value increases very slowly. Critical angles are found...
to occur for this coefficient at $\theta = 82^\circ$ and at $\theta = 68^\circ$ for $(\epsilon_H, \epsilon'_H) = (0.2, 0.3)$ and $(\epsilon_H, \epsilon'_H) = (0.3, 0.4)$ respectively. In Figure 5.7, we see that for the values of parameters $(\epsilon_H, \epsilon'_H) = (0, 0)$ and $(0.1, 0.2)$, the refraction coefficient $D'_1$ has maximum value in the vicinity of normal incidence, thereafter its value goes on decreasing with $\theta$ and takes minimum value at certain angle of incidence and then there is a small increase in its value in the vicinity of grazing incidence. For the set of values $(\epsilon_H, \epsilon'_H) = (0.2, 0.3)$, this coefficient has different behavior in the vicinity of grazing incidence. A critical angle at $\theta = 72^\circ$ is found to exist for $D'_1$ when $(\epsilon_H, \epsilon'_H) = (0.3, 0.4)$. In these figures,

**Figure 5.6**: Variation of amplitude ratio $B'_1 \times 10$ with angle of incidence, $\theta$ for different values of $(\epsilon_H, \epsilon'_H)$.

**Figure 5.7**: Variation of amplitude ratio $D'_1(\times 10)$ with angle of incidence, $\theta$ for different values of $(\epsilon_H, \epsilon'_H)$.

the effect is found to be minimum and almost negligible near the normal incidence, while all the coefficients corresponding to the first order approximation of the corrugation are found to be strongly influenced near grazing incidence. In view of critical angles occurring in Figures 5.2 - 5.7, it is worth to mention that for the incident $SH$-wave at an interface, the critical angles occur when somewhere a quantity under the square root sign becomes negative. In that case, the reflection and transmission coefficients become complex. For example, the angles $\sin^{-1} \left( \sqrt{\mu_T/(Q_1 - a_1^2 P_1)} \right)$ and

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\( \sin^{-1}\left(\frac{\mu_T}{\sqrt{Q_2 - \alpha_2^2 P_2}}\right) \) are the possible critical angles for the reflected and refracted \( SH^- \) waves. Hence, for the incidence at an angle greater than these critical angles, the quantities \( \gamma_1 \) or \( \gamma_2 \) become purely imaginary. Consequently, the reflection or refraction coefficients become complex.

Figures 5.8 - 5.13 show the variation of amplitude ratios \( B_0, D_0, B_1, B'_1, D_1 \) and \( D'_1 \) with parameter \( \omega d/\beta_1 \) when \( \theta = 30^\circ \) and at different values of the set of parameters \((\epsilon_H, \epsilon'_H)\). From Figures 5.8 and 5.9, it is found that the reflection and refraction coefficients corresponding to the plane interface, namely, \( B_0 \) and \( D_0 \) do not depend on the frequency parameter \( \omega d/\beta_1 \), however, the amplitude ratio \( B_0 \) increases and the amplitude ratio \( D_0 \) decreases with increase in the values of the set of parameters \((\epsilon_H, \epsilon'_H)\).

From Figures 5.10, 5.12 and 5.13, it is seen that the amplitude ratios \( B_1, D_1, D'_1 \) increase linearly at different rates for different values of the set of parameters \((\epsilon_H, \epsilon'_H)\), with the frequency parameter \( \omega d/\beta_1 \). Figure 5.11 shows that the amplitude ratio \( B'_1 \) exhibits parabolic behavior in the range \( 0 < \omega d/\beta_1 \leq 3 \) of the frequency parameter.

**Figure 5.8:** Variation of amplitude ratio \( D_1 \) with angle of incidence, \( \theta \) for different values of \((\epsilon_H, \epsilon'_H)\).

**Figure 5.9:** Variation of amplitude ratio \( D'_1(\times 10) \) with angle of incidence, \( \theta \) for different values of \((\epsilon_H, \epsilon'_H)\).
Figure 5.10: Variation of amplitude ratio $B_1$ with frequency parameter, $\frac{\omega_1}{\omega_0}$ for different values of $(\epsilon_H, \epsilon_H')$.

Figure 5.11: Variation of amplitude ratio $D_1'$ with frequency parameter, $\frac{\omega_1}{\omega_0}$ for different values of $(\epsilon_H, \epsilon_H')$.

Figure 5.12: Variation of amplitude ratio $D_1$ with frequency parameter, $\frac{\omega_1}{\omega_0}$ for different values of $(\epsilon_H, \epsilon_H')$.

Figure 5.13: Variation of amplitude ratio $D_1'$ with frequency parameter, $\frac{\omega_1}{\omega_0}$ for different values of $(\epsilon_H, \epsilon_H')$. 
and then there is monotonic increase in its value for all values of set of parameters \((\epsilon_H, \epsilon_H')\). It is noticed from this figure that the effect of the set of parameters \((\epsilon_H, \epsilon_H')\) is almost nil in the range \(0 < \omega d/\beta_1 \leq 3\), while there is significant effect in rest of the range and this effect increases with increase of frequency parameter \(\omega d/\beta_1\). Figures 5.14 - 5.17 depict the variation of the amplitude ratios of the coefficients corresponding to the first order approximation of the corrugation, with angle of incidence at the different values of the corrugation parameter, namely, \(pd = 0.0001, 0.0005\) and \(0.001\) when \(\omega d/\beta_1 = 0.1\). Here, it is to be noted that since the amplitude ratios \(B_0\) and \(D_0\) do not depend on corrugation parameter, therefore there is no effect of change of corrugation parameter on these coefficients. From Figure 5.14, we see that the reflection coefficient \(B_1\) is strongly influenced by the corrugation parameter \(pd\), in the vicinity of grazing and normal incidences. The effect of parameter \(pd\) on the coefficient \(B_1\) is almost negligible in the range \(40^\circ < \theta < 60^\circ\). Figure 5.15 shows that the coefficient \(B'_1\) is almost unaltered by the corrugation parameter \(pd\), in the entire range of angle of
Figure 5.16: Variation of amplitude ratio $D_1(\times 10)$ with angle of incidence, $\theta$ for different values of $pd$ and $FOD = 0.1$

Figure 5.17: Variation of amplitude ratio $D_1'(\times 10)$ with angle of incidence, $\theta$ for different values of $pd$ and $FOD = 0.1$

Figure 5.18: Variation of amplitude ratio $B_1(\times 10^3)$ with $pd$ when $\phi = \phi' = 0^\circ, \theta = 30^\circ$ for different values of $(\epsilon_H, \epsilon_H')$.

Figure 5.19: Variation of amplitude ratio $B_1'(\times 10^3)$ with $pd$ when $\phi = \phi' = 0^\circ, \theta = 30^\circ$ for different values of $(\epsilon_H, \epsilon_H')$. 

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incidence except near the normal incidence, where its value increases with the increase of parameter $pd$. The coefficients $D_1$ and $D'_1$ are found to be influenced by the corrugation parameter only in the vicinity of the normal and grazing incidences. In Figures 5.16 and 5.17, we see that the effect of parameter $pd$ on the coefficients $D_1$ and $D'_1$ is almost nil in the range $25^0 < \theta < 60^0$ approximately.

Figures 5.18 - 5.21 show the variation of the modulus of amplitude ratios $B_1$, $B'_1$, $D_1$ and $D'_1$ with the corrugation parameter $pd$ at different set of values of $(\epsilon_H, \epsilon'_H)$ when $\phi = \phi' = 0^0$ and $\theta = 30^0$, while Figures 5.22 - 5.25 show the variation of the amplitude ratios of the irregular waves when $\phi = \phi' = 90^0$. We note from these figures that all the reflection and refraction coefficients are linear functions of the corrugation parameter. It is also clear from these figures that amplitude ratios $B_1$, $B'_1$ and $D_1$ increase linearly with the increase of parameter $pd$, while the amplitude ratio $D'_1$ decreases linearly with the increase of parameter $pd$. It is found that the coefficient $B'_1$ decreases with the

**Figure 5.20:** Variation of amplitude ratio $D_1(\times 10^3)$ with $pd$ when $\phi = \phi' = 0^0, \theta = 30^0$ for different values of $(\epsilon_H, \epsilon'_H)$.

**Figure 5.21:** Variation of amplitude ratio $D'_1(\times 10^3)$ with $pd$ when $\phi = \phi' = 0^0, \theta = 30^0$ for different values of $(\epsilon_H, \epsilon'_H)$. 

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Figure 5.22: Variation of amplitude ratio $B_1(\times 10^3)$ with $pd$ when $\phi = \phi' = 90^0, \theta = 30^0$ for different values of $(\epsilon_H, \epsilon'_H)$.

Figure 5.23: Variation of amplitude ratio $B_1'(\times 10^3)$ with $pd$ when $\phi = \phi' = 90^0, \theta = 30^0$ for different values of $(\epsilon_H, \epsilon'_H)$.

Figure 5.24: Variation of amplitude ratio $D_1(\times 10^3)$ with $pd$ when $\phi = \phi' = 90^0, \theta = 30^0$ for different values of $(\epsilon_H, \epsilon'_H)$.

Figure 5.25: Variation of amplitude ratio $D_1'(\times 10^3)$ with $pd$ when $\phi = \phi' = 90^0, \theta = 30^0$ for different values of $(\epsilon_H, \epsilon'_H)$.
increase in the set of values of \((\epsilon_H, \epsilon_H')\), while the behavior of other amplitude ratios with \((\epsilon_H, \epsilon_H')\) is not similar to \(B'_1\). Thus, we conclude that, when \(\phi = \phi' = 0^\circ\) or \(90^\circ\), the corrugation parameter \(pd\) has strong effect on the coefficients corresponding to the first order approximation of the corrugation.