Chapter 2

Quasi $P-$ waves at a corrugated interface between two dissimilar monoclinic elastic half-spaces\(^1\)

2.1 Introduction

Anisotropic materials are of continued interest since long because of their importance in the fields like engineering, geophysics and many others. Various problems concerning elastic wave propagation in anisotropic medium have been investigated by many researchers in the past. Singh and Khurana (2001) studied a problem of reflection and transmission of $P-$ and $SV-$ waves at a plane interface between two dissimilar monoclinic elastic half-spaces. They derived the expressions of reflection and transmission coefficients due to an incident plane quasi- longitudinal ($qP-$) and quasi- transverse ($qSV-$) waves. They only presented analytical results in closed form, but did not perform numerical computations to compute these coefficients for any particular material. Singh and Khurana (2002) also derived the expressions of phase speed of propagation of quasi- longitudinal and transverse waves in anisotropic elastic medium possessing monoclinic symmetry and showed that the phase speeds of these waves depend on their direction of propagation. They also derived the reflection coefficients, when a plane $qP-$/or $qSV-$ wave is made incident obliquely at a stress free plane boundary of a monoclinic half-space. In this chapter, we shall investigate the phenomena of reflection and transmission due to a plane $qP-$ wave incident obliquely at

\(^1\)International Journal of Solids and Structures, 44(1), 197-228(2007).
a corrugated interface between two monoclinic elastic half-spaces with different properties. Using Rayleigh’s method of approximation, the reflection and transmission coefficients corresponding to the plane interface and corresponding to the corrugated interface are obtained. These coefficients for the first order approximation of the corrugated interface of periodic nature are also obtained. The effect of corrugation and frequency parameters on these coefficients are analyzed for a particular model. The results obtained by Ben-Menahem and Singh (1981), and Singh and Khurana (2001) for the relevant problems are reduced as particular cases from the present formulation.

2.2 Formulation of the problem and solution

Let us consider a corrugated interface given by \( x_3 = \zeta(x_2) \) separating the two homogeneous monoclinic elastic half-spaces, namely \( H \) and \( H' \). The function \( \zeta \) is a periodic function of \( x_2 \), independent of \( x_1 \) and the mean value of which is zero. Let \( x_1 - x_2 \) plane be horizontal and the \( x_3 \)-axis be normal to this plane pointing downward. Let the half-space \( H \) be occupying the region: \( \zeta(x_2) \leq x_3 < \infty \), and the half-space \( H' \) be occupying the region: \(-\infty < x_3 \leq \zeta(x_2) \). We shall denote the elastic parameters, stresses and displacement components in the medium \( H \) without prime, and those in the medium \( H' \) with prime. We shall take the elastic plane of symmetry of the media \( H \) and \( H' \) as \( x_2x_3 \)- plane. The Fourier series representation of function \( \zeta(x_2) \) is given by

\[
\zeta(x_2) = \sum_{n=1}^{\infty} (\zeta_n e^{im\pi x_2} + \zeta_{-n} e^{-im\pi x_2}),
\]

(2.1)

where \( \zeta_n \) and \( \zeta_{-n} \) are the coefficients of series expansion of order \( n \), \( p \) is the wavenumber and \( i = \sqrt{-1} \). Introducing the constants \( d \), \( c_n \) and \( s_n \) such that

\[
\zeta_1 = \zeta_{-1} = \frac{d}{2}, \quad \zeta_n = \frac{c_n - is_n}{2}, \quad \zeta_{-n} = \frac{c_n + is_n}{2}, \quad (n = 2, 3, 4, 5...),
\]

(2.2)

we obtain

\[
\zeta(x_2) = d \cos(px_2) + \sum_{n=2}^{\infty} [c_n \cos(npx_2) + s_n \sin(npx_2)].
\]

(2.3)
If the interface shape is represented by only one cosine term, i.e., \( \zeta = d \cos px_2 \), then the wavelength of the corrugation will be \( \frac{2\pi}{P} \), where \( d \) is the amplitude of the corrugated interface. Our aim is to discuss the reflection and refraction phenomena of a plane \( qP- \) wave incident at the corrugated interface. We shall consider two dimensional problem in \( x_2 - x_3 \) plane.

Suppose a time harmonic plane \( qP- \) wave propagating through the medium \( H \) becomes incident at the corrugated interface, \( x_3 = \zeta(x_2) \). The incident \( qP- \) wave will give rise to reflected and transmitted \( qP- \) and \( qSV- \) waves at the interface. Since the interface is corrugated in nature, therefore the reflection and transmission phenomena of waves will be affected by the undulated nature of the interface. Thus, due to corrugation of the interface, there will be irregularly reflected and transmitted \( qP- \) and \( qSV- \) waves, in addition to the regularly reflected and transmitted \( qP- \) and \( qSV- \) waves (see Asano, 1961). The complete geometry of the problem is shown in Figure 2.1.

\[ \text{Figure 2.1: Geometry of the Problem.} \]
The total displacement in the medium $H$ will be the sum of the displacements caused by the incident wave, regularly reflected waves and irregularly reflected waves. Thus, the displacement components $u_2$ and $u_3$ along $x_2$ and $x_3$ directions respectively, in the medium $H$ (after dropping the common factor $e^{iut}$), are given by

\[ u_2 = A_0 \exp\left\{ -i\omega\left( \frac{x_2 \sin \theta - x_3 \cos \theta}{c_0} \right) \right\} + A \exp\left\{ -i\omega\left( \frac{x_2 \sin \theta + x_3 \cos \theta}{c_1} \right) \right\} + \sum_{n=1}^{\infty} \left[ A_n \exp\left\{ -i\omega\left( \frac{x_2 \sin \theta_n + x_3 \cos \theta_n}{c_1} \right) \right\} + A_n' \exp\left\{ -i\omega\left( \frac{x_2 \sin \theta_n' + x_3 \cos \theta_n'}{c_1} \right) \right\} \right] + B \exp\left\{ -i\omega\left( \frac{x_2 \sin \phi + x_3 \cos \phi}{c_2} \right) \right\} + \sum_{n=1}^{\infty} \left[ B_n \exp\left\{ -i\omega\left( \frac{x_2 \sin \phi_n + x_3 \cos \phi_n}{c_2} \right) \right\} \right] + B_n' \exp\left\{ -i\omega\left( \frac{x_2 \sin \phi_n' + x_3 \cos \phi_n'}{c_2} \right) \right\}], \tag{2.4} \]

\[ u_3 = A_{00} \exp\left\{ -i\omega\left( \frac{x_2 \sin \theta - x_3 \cos \theta}{c_0} \right) \right\} + A_0 \exp\left\{ -i\omega\left( \frac{x_2 \sin \theta + x_3 \cos \theta}{c_1} \right) \right\} + \sum_{n=1}^{\infty} \left[ A_{0n} \exp\left\{ -i\omega\left( \frac{x_2 \sin \theta_n + x_3 \cos \theta_n}{c_1} \right) \right\} + A_{0n}' \exp\left\{ -i\omega\left( \frac{x_2 \sin \theta_n' + x_3 \cos \theta_n'}{c_1} \right) \right\} \right] + B_0 \exp\left\{ -i\omega\left( \frac{x_2 \sin \phi + x_3 \cos \phi}{c_2} \right) \right\} + \sum_{n=1}^{\infty} \left[ B_{0n} \exp\left\{ -i\omega\left( \frac{x_2 \sin \phi_n + x_3 \cos \phi_n}{c_2} \right) \right\} \right] + B_{0n}' \exp\left\{ -i\omega\left( \frac{x_2 \sin \phi_n' + x_3 \cos \phi_n'}{c_2} \right) \right\}], \tag{2.5} \]

where the coefficients $A_0$, $A$, $A_n$, $A_n'$, $B$, $B_n$ and $B_n'$ are the amplitude factors of the $u_2$ component of the displacement, while the coefficients $A_{00}$, $A_0$, $A_{0n}$, $A_{0n}'$, $B_0$, $B_{0n}$ and $B_{0n}'$ are the amplitude factors of the $u_3$ component of the displacement. $\theta$ is the angle of incidence, $\theta$ is the angle of regularly reflected $qP-$ wave, $\theta_n$ is the angle of irregularly reflected $qP-$ wave on the right side of the regularly reflected $qP-$ wave, $\theta_n'$ is the angle of irregularly reflected $qP-$ wave on the left side of the regularly reflected $qP-$ wave, $\phi$ is the angle of regularly reflected $qSV-$ wave, $\phi_n$ is the angle of irregularly reflected $qSV-$ wave on the right side of the regularly reflected $qSV-$ wave and $\phi_n'$ is the angle of irregularly reflected $qSV-$ wave on the left side of the regularly reflected $qSV-$ wave. Each angle defined above is made with vertical $x_3-$ axis. $\omega$ is
the angular frequency, \( c_0 \) is the speed of propagation of incident wave, \( c_1 \) is the speed of propagation of reflected \( qP \)-waves, \( c_2 \) is the speed of propagation of reflected \( qSV \)-waves.

Since the incident \( qP \)-wave, regularly reflected \( qP \)-wave, irregularly reflected \( qP \)-waves, regularly reflected \( qSV \)-wave and irregularly reflected \( qSV \)-waves must satisfy the equations of motion in \( H \), i.e., equations (1.33) and (1.34), we can obtain the following relations among the coefficients as (see Singh and Khurana, 2001)

\[
A_0 = F_0 A_{00}, \quad A = F A^0, \quad A_n = F_n A_{0n}, \quad A'_n = F'_n A'_{0n},
\]

\[
B = F_{10} B_0, \quad B_n = F_{1n} B_{0n}, \quad B'_n = F'_{1n} B'_{0n}, \tag{2.6}
\]

where

\[
F_0 = \frac{V_0}{\rho c_0^2 - U_0} = \rho c_0^2 - Z_0, \quad F_1 = \frac{V_1}{\rho c_1^2 - U_{10}} = \rho c_1^2 - Z_{10},
\]

\[
F_n = \frac{V_{1n}}{\rho c_1^2 - U_{1n}} = \rho c_1^2 - Z_{1n}, \quad F'_n = \frac{V'_{1n}}{\rho c'_1^2 - U'_{1n}} = \rho c'_1^2 - Z'_{1n},
\]

\[
F_{10} = \frac{V_{20}}{\rho c_2^2 - U_{20}} = \rho c_2^2 - Z_{20}, \quad F_{1n} = \frac{V_{2n}}{\rho c_2^2 - U_{2n}} = \rho c_2^2 - Z_{2n},
\]

\[
F'_{1n} = \frac{V'_{2n}}{\rho c'_2^2 - U'_{2n}} = \rho c'_2^2 - Z'_{2n},
\]

and they define the coupling coefficients between various amplitude factors of the displacement components in medium \( H \). The expressions of symbols \( U \), \( V \) and \( Z \) with suffixes, e.g., \( U_0, V_0, U_{10}, \) etc. are obtained from the equation (1.38) on substituting the following values of \( p_2 \) and \( p_3 \) appropriately.

For the incident \( qP \)-wave at an angle \( \theta_0 \): \( p_2 = \sin \theta_0, \quad p_3 = -\cos \theta_0 \).

For regularly reflected \( qP \)-wave at an angle \( \theta \): \( p_2 = \sin \theta, \quad p_3 = \cos \theta \).

For irregularly reflected \( qP \)-wave at an angle \( \theta_n \): \( p_2 = \sin \theta_n, \quad p_3 = \cos \theta_n \).

For regularly reflected \( qSV \)-wave at an angle \( \phi' \): \( p_2 = \sin \phi', \quad p_3 = \cos \phi' \).

For irregularly reflected \( qSV \)-wave at an angle \( \phi \): \( p_2 = \sin \phi, \quad p_3 = \cos \phi \).

These expressions are given by

\[
V_0 = c_{24} \sin^2 \theta_0 + c_{34} \cos^2 \theta_0 - (c_{23} + c_{44}) \sin \theta_0 \cos \theta_0,
\]
The speed of propagation of incident and reflected waves are given by

\[ 2pc_0^2 = U_0 + Z_0 + \sqrt{(U_0 - Z_0)^2 + 4V_0^2}, \]
\[ 2pc_2^2 = U_{10} + Z_{10} + \sqrt{(U_{10} - Z_{10})^2 + 4V_{10}^2}, \]
\[ 2pc_2^2 = U_{20} + Z_{20} - \sqrt{(U_{20} - Z_{20})^2 + 4V_{20}^2}. \]

Similarly, the components of total displacement, \( u'_2 \) and \( u'_3 \) in the medium \( H' \) (after dropping the common factor \( e^{-\omega t} \)), are given by

\[
\begin{align*}
  u'_2 &= C \exp\left(-\frac{x_2 \sin \delta - x_3 \cos \delta}{c_3}\right) + \sum_{n=1}^{\infty} C_n \exp\left(-\frac{x_2 \sin \delta_n - x_3 \cos \delta_n}{c_3}\right) \]
\[ + C'_n \exp\left(-\frac{x_2 \sin \delta'_n - x_3 \cos \delta'_n}{c_3}\right) \]
\[ + D \exp\left(-\frac{x_2 \sin \gamma - x_3 \cos \gamma}{c_4}\right), \]
\end{align*}
\]
+ \sum_{n=1}^{\infty} [D_n \exp\left\{ -\omega \left( \frac{x_2 \sin \gamma_n - x_3 \cos \gamma_n}{c_4} \right) \right\} + D'_n \exp\left\{ -\omega \left( \frac{x_2 \sin \gamma'_n - x_3 \cos \gamma'_n}{c_4} \right) \right\}], (2.7)

u'_2 = C_0 \exp\left\{ -\omega \left( \frac{x_2 \sin \delta - x_3 \cos \delta}{c_3} \right) \right\} + \sum_{n=1}^{\infty} [C_{0n} \exp\left\{ -\omega \left( \frac{x_2 \sin \delta_n - x_3 \cos \delta_n}{c_3} \right) \right\} + 

C'_{0n} \exp\left\{ -\omega \left( \frac{x_2 \sin \delta'_n - x_3 \cos \delta'_n}{c_3} \right) \right\} + D_0 \exp\left\{ -\omega \left( \frac{x_2 \sin \gamma - x_3 \cos \gamma}{c_4} \right) \right\} + 

\sum_{n=1}^{\infty} [D_{0n} \exp\left\{ -\omega \left( \frac{x_2 \sin \gamma_n - x_3 \cos \gamma_n}{c_4} \right) \right\} + D'_{0n} \exp\left\{ -\omega \left( \frac{x_2 \sin \gamma'_n - x_3 \cos \gamma'_n}{c_4} \right) \right\}], (2.8)

where the coefficients $C$, $C_n$, $C'_n$, $D$, $D_n$, and $D'_n$ are the amplitude factors of the $u'_2$ component of the displacement, while the coefficients $C_0$, $C_{0n}$, $C'_{0n}$, $D_0$, $D_{0n}$, and $D'_{0n}$ are the amplitude factors of the $u'_3$ component of the displacement. $\delta$ is the angle of regularly refracted $qP-$ wave, $\delta'_n$ is the angle of irregularly refracted $qP-$ wave on the right side of the regularly refracted $qP-$ wave, $\gamma$ is the angle of regularly refracted $qSV-$ wave, $\gamma_n$ is the angle of irregularly refracted $qSV-$ wave on the right side of the regularly refracted $qSV-$ wave, and $\gamma'_n$ is the angle of irregularly refracted $qSV-$ wave on the left side of the regularly refracted $qSV-$ wave. Each angle defined above is made with vertical $x_3-$ axis. $c_3$ is the speed of propagation of refracted $qP-$ waves and $c_4$ is the speed of propagation of refracted $qSV-$ waves. Moreover, the regularly transmitted $qP-$ and $qSV-$ waves, irregularly transmitted $qP-$ and $qSV-$ waves must satisfy their equations of motion in $H'$, i.e., the equations similar to (1.33) and (1.34), we obtain (see Singh and Khurana, 2001)

\begin{align*}
C &= F_{20} C_0, \quad C_{2n} = F_{2n} C_{0n}, \quad C'_n = F'_{2n} C'_{0n}, \\
D &= F_{30} D_0, \quad D_n = F_{3n} D_{0n}, \quad D'_n = F'_{3n} D'_{0n}, \quad (2.9)
\end{align*}

where

\begin{align*}
F_{20} &= \frac{V_{30}}{\rho' c_3^2 - U_{30}} = \frac{\rho' c_3^2 - Z_{30}}{V_{30}}, \quad F_{2n} = \frac{V_{3n}}{\rho' c_3^2 - U_{3n}} = \frac{\rho' c_3^2 - Z_{3n}}{V_{3n}}.
\end{align*}
\[
F_{2n} = \frac{V''_{3n}}{\rho'c_2^2 - U''_{3n}} = \frac{\rho c_3^2 - Z'_{3n}}{V''_{3n}}, \quad F_{3n} = \frac{V_{4n}}{\rho'c_4^2 - U_{4n}} = \frac{\rho c_4^2 - Z_{4n}}{V_{4n}}.
\]
\[
F_{3n} = \frac{V_{4n}}{\rho'c_4^2 - U_{4n}} = \frac{\rho c_4^2 - Z_{4n}}{V_{4n}}, \quad F''_{3n} = \frac{V''_{4n}}{\rho'c_4^2 - U''_{4n}} = \frac{\rho c_4^2 - Z'_{4n}}{V''_{4n}}.
\]

define the coupling coefficients between various amplitude factors of the displacement components in medium \(H'\). The various symbols used can be obtained similar to as above by following values of \(p_2\) and \(p_3\) appropriately as:

For regularly transmitted \(qP\)– wave at an angle \(\delta\): \(p_2 = \sin \delta, \quad p_3 = \cos \delta\).

For irregularly transmitted \(qP\)– wave at an angle \(\delta_n\): \(p_2 = \sin \delta_n, \quad p_3 = \cos \delta_n\).

For regularly transmitted \(qSV\)– wave at an angle \(\gamma\): \(p_2 = \sin \gamma, \quad p_3 = \cos \gamma\).

For irregularly transmitted \(qSV\)– wave at an angle \(\gamma_n\): \(p_2 = \sin \gamma_n, \quad p_3 = \cos \gamma_n\).

They are given by

\[
V_{40} = c_{24}^2 \sin^2 \delta + c_{34}^2 \cos^2 \delta - (c_{23}^2 + c_{44}^2) \sin \delta \cos \delta,
\]

\[
U_{30} = c_{22}^2 \sin^2 \delta + c_{44}^2 \cos^2 \delta - 2c_{24}^2 \sin \delta \cos \delta,
\]

\[
Z_{30} = c_{24}^2 \sin^2 \delta + c_{33}^2 \cos^2 \delta - 2c_{24}^2 \sin \delta \cos \delta,
\]

\[
V_{4n} = c_{24}^2 \sin^2 \delta_n + c_{34}^2 \cos^2 \delta_n - (c_{23}^2 + c_{44}^2) \sin \delta_n \cos \delta_n,
\]

\[
U_{3n} = c_{22}^2 \sin^2 \delta_n + c_{44}^2 \cos^2 \delta_n - 2c_{24}^2 \sin \delta_n \cos \delta_n,
\]

\[
Z_{3n} = c_{44}^2 \sin^2 \delta_n + c_{33}^2 \cos^2 \delta_n - 2c_{24}^2 \sin \delta_n \cos \delta_n,
\]

\[
V_{40} = c_{24}^2 \sin^2 \gamma + c_{34}^2 \cos^2 \gamma - (c_{23}^2 + c_{44}^2) \sin \gamma \cos \gamma,
\]

\[
U_{30} = c_{22}^2 \sin^2 \gamma + c_{44}^2 \cos^2 \gamma - 2c_{24}^2 \sin \gamma \cos \gamma,
\]

\[
Z_{40} = c_{24}^2 \sin^2 \gamma + c_{33}^2 \cos^2 \gamma - 2c_{24}^2 \sin \gamma \cos \gamma,
\]

\[
V_{4n} = c_{24}^2 \sin^2 \gamma_n + c_{34}^2 \cos^2 \gamma_n - (c_{23}^2 + c_{44}^2) \sin \gamma_n \cos \gamma_n,
\]

\[
U_{3n} = c_{22}^2 \sin^2 \gamma_n + c_{44}^2 \cos^2 \gamma_n - 2c_{24}^2 \sin \gamma_n \cos \gamma_n,
\]

\[
Z_{3n} = c_{44}^2 \sin^2 \gamma_n + c_{33}^2 \cos^2 \gamma_n - 2c_{24}^2 \sin \gamma_n \cos \gamma_n,
\]

\[
V_{40} = c_{24}^2 \sin^2 \gamma' + c_{34}^2 \cos^2 \gamma' - (c_{23}^2 + c_{44}^2) \sin \gamma' \cos \gamma',
\]

\[
U_{30} = c_{22}^2 \sin^2 \gamma' + c_{44}^2 \cos^2 \gamma' - 2c_{24}^2 \sin \gamma' \cos \gamma',
\]

\[
Z_{4n} = c_{44}^2 \sin^2 \gamma' + c_{33}^2 \cos^2 \gamma' - 2c_{24}^2 \sin \gamma' \cos \gamma'.
\]
The speed of propagation of transmitted \( qP^- \) and \( qSV^- \) waves are given by

\[
2\rho'c_3^2 = U_{30} + Z_{30} + \sqrt{(U_{30} - Z_{30})^2 + 4V_{30}^2},
\]

\[
2\rho'c_4^2 = U_{40} + Z_{40} - \sqrt{(U_{40} - Z_{40})^2 + 4V_{40}^2}.
\]

In monoclinic medium, the angle of incidence is not equal to the angle of reflection (see Singh, 1999). The Snell’s law which is the relation between the angle of incidence and the angles of various reflected and transmitted waves in the monoclinic medium is given by (see Singh and Khurana, 2001)

\[
\frac{\sin \theta_0}{c_0(\theta_0)} = \frac{\sin \theta}{c_1(\theta)} = \frac{\sin \phi}{c_2(\phi)} = \frac{\sin \delta}{c_3(\delta)} = \frac{\sin \gamma}{c_4(\gamma)} = \frac{1}{c_a},
\]

where \( c_a \) is apparent velocity. The relations between the angles of regular and irregular waves are given by the following Spectrum theorem as

\[
\sin \theta_n - \sin \theta = \frac{npc_1}{\omega}, \quad \sin \theta_n' - \sin \theta = -\frac{npc_1}{\omega},
\]

\[
\sin \phi_n - \sin \phi = \frac{npc_2}{\omega}, \quad \sin \phi_n' - \sin \phi = -\frac{npc_2}{\omega},
\]

\[
\sin \delta_n - \sin \delta = \frac{npc_3}{\omega}, \quad \sin \delta_n' - \sin \delta = -\frac{npc_3}{\omega},
\]

\[
\sin \gamma_n - \sin \gamma = \frac{npc_4}{\omega}, \quad \sin \gamma_n' - \sin \gamma = -\frac{npc_4}{\omega}.
\]

Our interest lies in the computation of reflection and transmission coefficients. These coefficients will be derived by using the following boundary conditions at the corrugated interface.

### 2.3 Boundary conditions

The boundary conditions are the continuity of displacements and stresses (normal and shear) at the corrugated interface. The mathematical form of these boundary conditions can be expressed as: At \( x_3 = \zeta(x_2) \),
(i) continuity of displacements

\[ u_2 = u'_2, \quad u_3 = u'_3, \quad (2.12) \]

(ii) continuity of shear stress

\[ [\tau_{32} + (\tau_{33} - \tau_{22})\dot{\zeta}' - \tau_{23}\dot{\zeta}'] = [\tau'_{32} + (\tau'_{33} - \tau'_{22})\dot{\zeta}' - \tau'_{23}\dot{\zeta}'], \quad (2.13) \]

(iii) continuity of normal stress

\[ [\tau_{33} - 2\tau_{23}\dot{\zeta}' + \tau_{22}\dot{\zeta}'] = [\tau'_{33} - 2\tau'_{23}\dot{\zeta}' + \tau'_{22}\dot{\zeta}'], \quad (2.14) \]

where \( \dot{\zeta}' \) is the derivative of \( \zeta \) with respect to \( x_2 \) and is given by

\[ \dot{\zeta}' = \sum_{n=1}^{\infty} m_p(\zeta_n e^{\mu n x_2} - \zeta_n e^{-\mu n x_2}). \]

Using constitutive relations given by (1.31), the equations (2.13) and (2.14) can be written as

\[
\begin{align*}
&\left[(c_{23} - c_{22})\dot{\zeta}' + c_{24}(1 - \zeta'^2)\right]\frac{\partial u_2}{\partial x_2} + \left[(c_{33} - c_{23})\dot{\zeta}' + c_{34}(1 - \zeta'^2)\right]\frac{\partial u_3}{\partial x_3} \\
&\quad + \left[\left(c_{33} - c_{24}\right)\dot{\zeta}' + c_{44}(1 - \zeta'^2)\right]\left\{\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\right\} \\
&= \left[(c'_{23} - c'_{22})\dot{\zeta}' + c'_{24}(1 - \zeta'^2)\right]\frac{\partial u'_2}{\partial x_2} + \left[(c'_{33} - c'_{23})\dot{\zeta}' + c'_{34}(1 - \zeta'^2)\right]\frac{\partial u'_3}{\partial x_3} \\
&\quad + \left[\left(c'_{33} - c'_{24}\right)\dot{\zeta}' + c'_{44}(1 - \zeta'^2)\right]\left\{\frac{\partial u'_2}{\partial x_3} + \frac{\partial u'_3}{\partial x_2}\right\},
\end{align*}
\]

\[
\begin{align*}
&\left[c_{23} + c_{22}\zeta'^2 - 2c_{24}\zeta'\right]\frac{\partial u_2}{\partial x_2} + \left[c_{33} + c_{23}\zeta'^2 - 2c_{34}\zeta'\right]\frac{\partial u_3}{\partial x_3} + \left[c_{34} + c_{24}\zeta'^2 - 2c_{44}\zeta'\right]\left\{\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\right\} \\
&= \left[c'_{23} + c'_{22}\zeta'^2 - 2c'_{24}\zeta'\right]\frac{\partial u'_2}{\partial x_2} + \left[c'_{33} + c'_{23}\zeta'^2 - 2c'_{34}\zeta'\right]\frac{\partial u'_3}{\partial x_3} \\
&\quad + \left[c'_{34} + c'_{24}\zeta'^2 - 2c'_{44}\zeta'\right]\left\{\frac{\partial u'_2}{\partial x_3} + \frac{\partial u'_3}{\partial x_2}\right\}.
\end{align*}
\]
Chapter 2

Using equations (2.4), (2.5), (2.7), (2.8), (2.10) and (2.11) into (2.12), (2.15) and (2.16), we obtain

\[ A_0 \exp(\imath R_0) + A_0^\prime \exp(-\imath R) + \sum_{n=1}^{\infty} \left[ A_{0n} \exp(-\imath R_0) + A_{0n}^\prime \exp(-\imath R_0^\prime) \right] \]

\[ + B_0 \exp(-\imath Q) + \sum_{n=1}^{\infty} \left[ B_{0n} \exp(-\imath Q_0) + B_{0n}^\prime \exp(-\imath Q_0^\prime) \right] \]

\[ = C_0 \exp(\imath S) + \sum_{n=1}^{\infty} \left[ C_{0n} \exp(\imath S_0) + C_{0n}^\prime \exp(\imath S_0^\prime) \right] + D_0 \exp(\imath L) \]

\[ + \sum_{n=1}^{\infty} \left[ D_{0n} \exp(-\imath \alpha) + D_{0n}^\prime \exp(-\imath \alpha^\prime) \right], \quad (2.17) \]

\[ A_0 \exp(\imath R_0) + A \exp(-\imath R) + \sum_{n=1}^{\infty} \left[ A_n \exp(-\imath R_0) + A_n^\prime \exp(-\imath R_0^\prime) \right] \]

\[ + B \exp(-\imath Q) + \sum_{n=1}^{\infty} \left[ B_n \exp(-\imath Q_0) + B_n^\prime \exp(-\imath Q_0^\prime) \right] \]

\[ = C \exp(\imath S) + \sum_{n=1}^{\infty} \left[ C_n \exp(\imath S_0) + C_n^\prime \exp(\imath S_0^\prime) \right] + D \exp(\imath L) \]

\[ + \sum_{n=1}^{\infty} \left[ D_n \exp(-\imath \alpha) + D_n^\prime \exp(-\imath \alpha^\prime) \right], \quad (2.18) \]

\[ [(c_{23} - c_{22})\zeta' + c_{24}(1 - \zeta^2)] \left[ -P_0 \{ A_0 \exp(\imath R_0) + A \exp(-\imath R) \} - \sum_{n=1}^{\infty} [(P_0 + np) \right. \]

\[ \times A_n \exp(-\imath R_n) + (P_0 - np) A_n^\prime \exp(-\imath R_n^\prime)] - P_0 B \exp(-\imath Q) \]

\[ - \sum_{n=1}^{\infty} [(P_0 + np) B_n \exp(-\imath Q_n) + (P_0 - np) B_n^\prime \exp(-\imath Q_n^\prime)] \]

\[ + [(c_{33} - c_{23})\zeta' + c_{34}(1 - \zeta^2)] [R_0 A_{00} \exp(\imath R_0) - R A_0^\prime \exp(-\imath R) \]

\[ - \sum_{n=1}^{\infty} [A_{0n} R_n \exp(-\imath R_n) + A_{0n}^\prime R_n^\prime \exp(-\imath R_n^\prime)] - Q B_0 \exp(-\imath Q) \]

\[ - \sum_{n=1}^{\infty} [Q_n B_{0n} \exp(-\imath Q_n) + Q_n^\prime B_{0n}^\prime \exp(-\imath Q_n^\prime)] \]
\[
+ [(c_{34} - c_{24})\zeta' + c_{44}(1 - \zeta'^2)] [R_0 A_0 \exp(i\zeta R_0) - R A \exp(-i\zeta R)]
- \sum_{n=1}^{\infty} [R_n A_n e^{-mpz_2} \exp(-i\zeta R_n) + R'_n A'_n e^{mpz_2} \exp(-i\zeta R'_n)] - Q B \exp(-i\zeta Q)
- \sum_{n=1}^{\infty} [Q_n B_n e^{-mpz_2} \exp(-i\zeta Q_n) + Q'_n B'_n e^{mpz_2} \exp(-i\zeta Q'_n)] - P_0 A_{00} \exp(i\zeta R_0)
- P_0 A^0 \exp(-i\zeta R) - \sum_{n=1}^{\infty} [(P_0 + np) A_{0n} e^{-mpz_2} \exp(-i\zeta R_n) + (P_0 - np) A'_{0n} e^{mpz_2} \exp(-i\zeta R'_n)]
- P_0 B_0 \exp(-i\zeta Q) - \sum_{n=1}^{\infty} [(P_0 + np) B_{0n} e^{-mpz_2} \exp(-i\zeta Q_n) + (P_0 - np) B'_{0n} e^{mpz_2} \exp(-i\zeta Q'_n)]
= [(c_{23} - c_{22})\zeta' + c_{24}(1 - \zeta'^2)] [P_0 C \exp(i\zeta S) - \sum_{n=1}^{\infty} [(P_0 + np) C_n e^{-mpz_2} \exp(i\zeta S_n)]
+ (P_0 - np) C'_{zn} e^{mpz_2} \exp(i\zeta S'_n)] - P_0 D \exp(i\zeta L) - \sum_{n=1}^{\infty} [(P_0 + np) D_n e^{-mpz_2} \exp(i\zeta L_n)]
+ (P_0 - np) D'_{zn} e^{mpz_2} \exp(i\zeta L'_n)\] \] \[+ \sum_{n=1}^{\infty} [S_n C_{0n} e^{-mpz_2} \exp(i\zeta S_n) + S'_n C'_{0n} e^{mpz_2} \exp(i\zeta S'_n)] + L D_0 \exp(i\zeta L)
+ \sum_{n=1}^{\infty} [L_n D_{0n} e^{-mpz_2} \exp(i\zeta L_n) + L'_n D'_{0n} e^{mpz_2} \exp(i\zeta L'_n)] - P_0 C_0 \exp(i\zeta S)
- \sum_{n=1}^{\infty} [(P_0 + np) C_{0n} e^{-mpz_2} \exp(i\zeta S_n) + (P_0 - np) C'_{0n} e^{mpz_2} \exp(i\zeta S'_n)] - P_0 D_0 \exp(i\zeta L)
- \sum_{n=1}^{\infty} [(P_0 + np) D_{0n} e^{-mpz_2} \exp(i\zeta L_n) + (P_0 - np) D'_{0n} e^{mpz_2} \exp(i\zeta L'_n)], \] (2.19)
\[-P_0 B \exp(-i \zeta Q) - \sum_{n=1}^{\infty} [(P_0 + n \rho) B_n e^{-mpz^2} \exp(-i \zeta Q_n) + (P_0 - n \rho) B'_n e^{-mpz^2} \exp(-i \zeta Q'_n)] \]

\[+ c_{33} + c_{23} \zeta'^2 - 2c_{24} \zeta' |[R_0 A_{00} \exp(i \zeta R_0) - RA_0 \exp(-i \zeta R) - \sum_{n=1}^{\infty} |R_n A_{0n} e^{-mpz^2} \exp(-i \zeta R_n) \]

\[+ R'_n A'_{0n} e^{-mpz^2} \exp(-i \zeta R'_n)] - S B_0 \exp(-i \zeta S) - \sum_{n=1}^{\infty} |S_n B_{0n} e^{-mpz^2} \exp(-i \zeta S_n) \]

\[+ S'_n B'_{0n} e^{-mpz^2} \exp(-i \zeta S'_n)] + [c_{34} + c_{24} \zeta'^2 - 2c_{44} \zeta'] R_0 A_0 \exp(i \zeta R_0) - RA \exp(-i \zeta R) \]

\[- \sum_{n=1}^{\infty} |R_n A_{0n} e^{-mpz^2} \exp(-i \zeta R_n) + R'_n A'_{0n} e^{-mpz^2} \exp(-i \zeta R'_n)] - B Q \exp(-i \zeta Q) \]

\[- \sum_{n=1}^{\infty} |Q_n B_{0n} e^{-mpz^2} \exp(-i \zeta Q_n) + Q'_n B'_{0n} e^{-mpz^2} \exp(-i \zeta Q'_n)] - A_{00} B_0 \exp(i \zeta R_0) \]

\[-A'_0 P_0 \exp(-i \zeta R) - \sum_{n=1}^{\infty} |A_{0n} (P_0 + n \rho) e^{-mpz^2} \exp(-i \zeta R_n) + A'_0 (P_0 - n \rho) e^{-mpz^2} \exp(-i \zeta R'_n)] \]

\[-B_0 P_0 \exp(-i \zeta Q) - \sum_{n=1}^{\infty} |B_{0n} (P_0 + n \rho) e^{-mpz^2} \exp(-i \zeta Q_n) + B'_0 (P_0 - n \rho) e^{-mpz^2} \exp(-i \zeta Q'_n)] \]

\[= [\zeta'' + c_{22} \zeta'^2 - 2c_{24} \zeta'] [-C P_0 \exp(i \zeta S) - \sum_{n=1}^{\infty} (C_n (P_0 + n \rho) e^{-mpz^2} \exp(i \zeta S_n) \]

\[+ C'_n (P_0 - n \rho) e^{-mpz^2} \exp(i \zeta S'_n)] - D P_0 \exp(i \zeta L) - \sum_{n=1}^{\infty} |D_n (P_0 + n \rho) e^{-mpz^2} \exp(i \zeta L_n) \]

\[+ D'_n (P_0 - n \rho) e^{-mpz^2} \exp(i \zeta L'_n)] + [c_{33} + c_{23} \zeta'^2 - 2c_{24} \zeta'] |C_0 S \exp(i \zeta S) \]

\[+ \sum_{n=1}^{\infty} |C_{0n} S_n e^{-mpz^2} \exp(i \zeta S_n) + C'_0 S'_n e^{-mpz^2} \exp(i \zeta S'_n)] + D L_0 \exp(i \zeta L) \]

\[+ \sum_{n=1}^{\infty} |D_{0n} L_n e^{-mpz^2} \exp(i \zeta L_n) + D'_0 L'_n e^{-mpz^2} \exp(i \zeta L'_n)] + [c_{34} + c_{24} \zeta'^2 - 2c_{44} \zeta'] \]

\[\times C S \exp(i \zeta S) + \sum_{n=1}^{\infty} |C_n S_n e^{-mpz^2} \exp(i \zeta S_n) + C'_n S'_n e^{-mpz^2} \exp(i \zeta S'_n)] + D L \exp(i \zeta L) \]

\[+ \sum_{n=1}^{\infty} |D_n L_n e^{-mpz^2} \exp(i \zeta L_n) + D'_n L'_n e^{-mpz^2} \exp(i \zeta L'_n)] - C_0 P_0 \exp(i \zeta S) \]

\[= \sum_{n=1}^{\infty} |C_{0n} (P_0 + n \rho) e^{-mpz^2} \exp(i \zeta S_n) + C'_0 (P_0 - n \rho) e^{-mpz^2} \exp(i \zeta S'_n)] - D_0 P_0 \exp(i \zeta L) \]
\[-\sum_{n=1}^{\infty} [D_{0n}(P_0 + np)e^{-mpx^2} \exp(\kappa L_n) + D'_{0n}(P_0 - np)e^{mpx^2} \exp(\kappa L'_n)]], \quad (2.20)

where

\[ P_0 = \frac{\omega \sin \theta_0}{c_0}, \quad R_0 = \frac{\omega \cos \theta_0}{c_0}, \quad R = \frac{\omega \cos \theta}{c_1}, \quad R_n = \frac{\omega \cos \theta_n}{c_1}, \quad R'_n = \frac{\omega \cos \theta'_n}{c_1}, \]

\[ Q = \frac{\omega \cos \phi}{c_2}, \quad Q_n = \frac{\omega \cos \phi_n}{c_2}, \quad Q'_n = \frac{\omega \cos \phi'_n}{c_2}, \quad S = \frac{\omega \cos \delta}{c_3}, \quad S_n = \frac{\omega \cos \delta_n}{c_3}, \]

Equations (2.17)-(2.20) enable us to provide the formulae for the reflection and transmission coefficients corresponding to the corrugated interface. In case, the amplitude of the corrugated interface is neglected, then obviously, one would obtain the reflection and transmission coefficients due to incident \( qP \)-wave at a plane interface between two dissimilar anisotropic solid half-spaces in welded contact. Now, we proceed to find the solutions of the above equations by approximating the term involving corrugated function to a certain order.

### 2.4 Solution of the first order approximation

In order to apply Rayleigh’s method, we assume that the amplitude and slope of the corrugated interface are so small that

\[ \exp(\pm i\zeta R_0) \approx 1 \pm i\zeta R_0, \quad \exp(\pm i\zeta R) \approx 1 \pm i\zeta R, \quad \text{etc.}, \quad (2.21) \]

where we have retained only the linear term in the expansion. Putting equations (2.1), (2.6), (2.9) and (2.21) into (2.17) - (2.20) and collecting the terms independent of \( x \) and \( \zeta \), we obtain a system of four equations. These equations will provide us the reflection and transmission coefficients corresponding to plane interface for the relevant problem. These equations can be written in matrix form as

\[ MN = G, \quad (2.22) \]
where

\[
M = \begin{bmatrix}
1 & 1 & -1 & -1 \\
F & F_{10} & -F_{20} & -F_{30} \\
a & a_1 & -a_2 & -a_3 \\
b & b_1 & -b_2 & -b_3
\end{bmatrix}, \quad N = \begin{bmatrix}
\alpha^0 \\
\beta_0 \\
\gamma_0 \\
\delta_0
\end{bmatrix}, \quad G = \begin{bmatrix}
-1 \\
-F_0 \\
-1 \\
-1
\end{bmatrix},
\]

\[
a = \frac{FC_{24} + c_{44} + (Fc_{44} + c_{34}) \cot \theta}{F_0 c_{24} + c_{44} - (F_0 c_{44} + c_{34}) \cot \theta}, \quad a_1 = \frac{F_{10} c_{24} + c_{44} + (F_{10} c_{44} + c_{34}) \cot \phi}{F_{10} c_{24} + c_{44} - (F_0 c_{44} + c_{34}) \cot \theta},
\]

\[
a_2 = \frac{FC_{24} + c_{44} - (F_0 c_{44} + c_{34}) \cot \theta}{F_0 c_{24} + c_{44} - (F_0 c_{44} + c_{34}) \cot \theta}, \quad a_2 = \frac{F_{10} c_{24} + c_{44} - (F_{10} c_{44} + c_{34}) \cot \phi}{F_{10} c_{24} + c_{44} - (F_0 c_{44} + c_{34}) \cot \theta},
\]

\[
b = \frac{FC_{23} + c_{34} + (F_0 c_{34} + c_{33}) \cot \theta}{F_0 c_{23} + c_{34} - (F_0 c_{34} + c_{33}) \cot \theta}, \quad b_1 = \frac{F_{10} c_{23} + c_{34} + (F_{10} c_{34} + c_{33}) \cot \phi}{F_{10} c_{23} + c_{34} - (F_0 c_{34} + c_{33}) \cot \theta},
\]

\[
b_2 = \frac{FC_{23} + c_{34} - (F_0 c_{34} + c_{33}) \cot \theta}{F_0 c_{23} + c_{34} - (F_0 c_{34} + c_{33}) \cot \theta}, \quad b_2 = \frac{F_{10} c_{23} + c_{34} - (F_{10} c_{34} + c_{33}) \cot \phi}{F_{10} c_{23} + c_{34} - (F_0 c_{34} + c_{33}) \cot \theta},
\]

The elements of matrix \(N\) represent the ratios of the amplitude factors for the vertical components of the displacement, \(i.e., u_3\) and \(u_3'\) corresponding to the regularly reflected and transmitted waves. On solving equation (2.22), we obtain

\[
\frac{A^0}{A_{00}} = \frac{\Delta A^0}{\Delta}, \quad \frac{B_0}{A_{00}} = \frac{\Delta B_0}{\Delta}, \quad \frac{C_0}{A_{00}} = \frac{\Delta C_0}{\Delta}, \quad \frac{D_0}{A_{00}} = \frac{\Delta D_0}{\Delta}.
\]

Using equations (2.6), (2.9) and (2.23), one may obtain the ratios of the amplitude factors of the horizontal components of the displacement, \(i.e., u_2\) and \(u_2'\) corresponding to regularly reflected and transmitted waves, as

\[
\frac{A}{A_0} = \frac{\Delta A^0}{\Delta} \frac{F}{F_0}, \quad \frac{B}{A_0} = \frac{\Delta B_0}{\Delta} \frac{F_{10}}{F_0}, \quad \frac{C}{A_0} = \frac{\Delta C_0}{\Delta} \frac{F_{20}}{F_0}, \quad \frac{D}{A_0} = \frac{\Delta D_0}{\Delta} \frac{F_{30}}{F_0},
\]

where

\[
\Delta = \begin{vmatrix}
1 & 1 & -1 & -1 \\
F & F_{10} & -F_{20} & -F_{30} \\
a & a_1 & -a_2 & -a_3 \\
b & b_1 & -b_2 & -b_3
\end{vmatrix}
\]

and the values of \(\Delta A^0, \Delta B_0, \Delta C_0\) and \(\Delta D_0\) can be written by replacing first, second, third and fourth columns of the determinant in \(\Delta\) with the elements of column matrix \(G\) respectively.

Now, we are ready to compute the reflection and transmission coefficients corresponding
to the plane interface. The amplitude of the incident $qP-$ wave is given by

$$\sqrt{A_0^2 + A_{06}^2} = A_{00}\sqrt{1 + F_0^2}.$$  

Similarly, the amplitudes of the reflected $qP-$, reflected $qSV-$, transmitted $qP-$ and transmitted $qSV-$ waves are given by

$$\sqrt{A^2 + A^2_0} = A_0\sqrt{1 + F_0^2}, \quad \sqrt{B^2 + B^2_0} = B_0\sqrt{1 + F_0^2},$$

$$\sqrt{C^2 + C^2_0} = C_0\sqrt{1 + F^2_{20}}, \quad \sqrt{D^2 + D^2_0} = D_0\sqrt{1 + F^2_{30}},$$

respectively. Therefore, the reflection coefficients: $R_{pp}$ corresponding to the reflected $qP-$ wave, $R_{ps}$ corresponding to the reflected $qSV-$ wave and the transmission coefficients: $T_{pp}$ corresponding to the transmitted $qP-$ wave, $T_{ps}$ corresponding to the transmitted $qSV-$ wave, respectively, are given by

$$R_{pp} = \frac{A^0}{A_{00}} \sqrt{\frac{1 + F^2_0}{1 + F^2_{00}}}, \quad R_{ps} = \frac{B_0}{A_{00}} \sqrt{\frac{1 + F^2_{10}}{1 + F^2_{00}}},$$

$$T_{pp} = \frac{C_0}{A_{00}} \sqrt{\frac{1 + F^2_{20}}{1 + F^2_{00}}}, \quad T_{ps} = \frac{D_0}{A_{00}} \sqrt{\frac{1 + F^2_{30}}{1 + F^2_{00}}}. \quad (2.25)$$

These are the expressions of reflection and transmission coefficients due to incident $qP-$ wave corresponding to a plane interface between two monoclinic elastic half-spaces and exactly match with those obtained earlier by Singh and Khurana (2001) for the relevant problem.

Next, comparing the coefficients of $e^{-i\mu x^2}$ to both sides, we again obtain a system of four equations written in matrix form as

$$M_1 N_1 = G_1. \quad (2.26)$$
\[
M_1 = \begin{bmatrix}
1 & 1 & -1 & -1 \\
F_n & F_{1n} & -F_{2n} & -F_{3n} \\
d_3 & d_4 & -d_7 & -d_8 \\
e_3 & e_4 & -e_7 & -e_8
\end{bmatrix}, \quad N_1 = \begin{bmatrix}
\frac{A_{00}}{A_{00}} & \frac{A_{00}}{A_{00}} & \frac{B_{0}}{A_{00}} & \frac{C_{0}}{A_{00}} \\
\frac{B_{0}}{A_{00}} & \frac{B_{0}}{A_{00}} & \frac{C_{0}}{A_{00}} & \frac{D_{0}}{A_{00}} \\
\frac{C_{0}}{A_{00}} & \frac{D_{0}}{A_{00}} & \frac{D_{0}}{A_{00}} & \frac{D_{0}}{A_{00}} \\
\frac{D_{0}}{A_{00}} & \frac{D_{0}}{A_{00}} & \frac{D_{0}}{A_{00}} & \frac{D_{0}}{A_{00}}
\end{bmatrix}, \quad G_1 = \begin{bmatrix}
f_1 \\
f_2 \\
f_3 \\
f_4
\end{bmatrix},
\]

where

\[
f_1 = i \zeta_n \left[-R_0 + R \frac{A^0}{A_{00}} + Q \frac{B_0}{A_{00}} + S \frac{C_0}{A_{00}} + L \frac{D_0}{A_{00}} \right],
\]

\[
f_2 = i \zeta_n \left[-F_0 R_0 + FR \frac{A^0}{A_{00}} + F_0 \frac{Q}{A_{00}} + F_0 \frac{S C_0}{A_{00}} + F_0 \frac{L D_0}{A_{00}} \right],
\]

\[
f_3 = i(1 + d_1 \frac{A^0}{A_{00}} + d_2 \frac{B_0}{A_{00}} - d_3 \frac{C_0}{A_{00}} - d_4 \frac{D_0}{A_{00}}), \quad f_4 = i(1 + e_1 \frac{A^0}{A_{00}} + e_2 \frac{B_0}{A_{00}} - e_3 \frac{C_0}{A_{00}} - e_4 \frac{D_0}{A_{00}}),
\]

\[
d_k = \frac{d_{0k}}{d_0}, \quad e_k = \frac{e_{0k}}{e_0} \quad (k = 1, 2, \ldots, 8), \quad (2.27)
\]

\[
d_0 = \left\{ \left[ -c_{23} - c_{24} \right] n P_0 + \frac{P_0 R_0}{p} + \left[ c_{34} - c_{24} \right] n R_0 - c_{44} \frac{R_0^2}{p} \right\} F_0
\]

\[
+ \left( c_{33} - c_{23} \right) n R_0 - c_{34} \frac{R_0^2}{p} - \left( c_{34} - c_{24} \right) n P_0 + c_{44} \frac{P_0 R_0}{p} \zeta_n,
\]

\[
d_{01} = \left\{ \left[ (c_{23} - c_{22}) n P_0 - c_{24} \frac{P_0 R}{p} - (c_{34} - c_{24}) n R - c_{44} \frac{R^2}{p} \right] F
\]

\[
- \left( c_{33} - c_{23} \right) n R - c_{34} \frac{R^2}{p} - \left( c_{34} - c_{24} \right) n P_0 - c_{44} \frac{P_0 R}{p} \zeta_n,
\]

\[
d_{02} = \left\{ \left[ -c_{23} - c_{22} \right] n P_0 - c_{24} \frac{P_0 Q}{p} - (c_{34} - c_{24}) n Q - c_{44} \frac{Q^2}{p} \right\} F_{10}
\]

\[
- \left( c_{33} - c_{23} \right) n Q - c_{34} \frac{Q^2}{p} - \left( c_{34} - c_{24} \right) n P_0 - c_{44} \frac{P_0 Q}{p} \zeta_n,
\]

\[
d_{03} = \left\{ c_{44} \frac{R_n}{p} + c_{24} \frac{P_0}{p} + n \right\} F_n + c_{34} \frac{R_n}{p} + c_{44} \frac{R_0}{p} + n,
\]

\[
d_{04} = \left\{ -c_{44} \frac{Q_n}{p} + c_{24} \frac{P_0}{p} + n \right\} F_{1n} - c_{34} \frac{Q_n}{p} - c_{44} \frac{R_0}{p} + n,
\]

\[
d_{05} = \left\{ \left[ -c_{23} - c_{22} \right] n P_0 + c_{24} \frac{P_0 S}{p} + (c_{34} - c_{24}) n S - c_{44} \frac{S^2}{p} \right\} F_{20}
\]

\[
+ \left( c_{33} - c_{23} \right) n S - c_{34} \frac{S^2}{p} - \left( c_{34} - c_{24} \right) n P_0 + c_{44} \frac{P_0 S}{p} \zeta_n,
\]
The elements of the matrix $N_1$ represent the ratios of amplitude factors of the vertical components of displacement corresponding to the irregular waves at angles $\theta_n, \phi_n, \delta_n$ and $\gamma_n$. From equation (2.26), we get the following values of these ratios for the first order approximation of the corrugated interface as

\[
\begin{align*}
d_{06} &= \left\{ -(c'_{23} - c'_{22})nP_0 + c'_{24}\frac{P_0L}{p} + (c'_{34} - c'_{24})nL - c'_{44}\frac{L^2}{p} \right\}F_{00} \\
&\quad + (c'_{33} - c'_{23})nL - c'_{34}\frac{L^2}{p} - (c'_{34} - c'_{24})nP_0 + c'_{44}\frac{P_0L}{p} \right\} \zeta_{-n}, \\
d_{07} &= -\left\{ -c'_{44}\frac{S_n}{p} + c'_{24}\left(\frac{P_0}{p} + n\right) \right\}F_{2n} + c'_{34}\frac{S_n}{p} - \left(\frac{R_0}{p} + n\right)c'_{44}, \\
d_{08} &= -\left\{ -c'_{44}\frac{L_n}{p} + c'_{24}\left(\frac{P_0}{p} + n\right) \right\}F_{3n} + c'_{34}\frac{L_n}{p} - \left(\frac{R_0}{p} + n\right)c'_{44}, \\
e_0 &= \left\{ -c'_{23}\frac{P_0R_0}{p} + 2c'_{24}nP_0 - c'_{34}\frac{R_0^2}{p} - 2nc'_{44}R_0 \right\}F_0 - c'_{33}\frac{R_0^2}{p} - 2c'_{34}nR_0 + c'_{44}\frac{R_0^2}{p} + 2c'_{44}nP_0 \right\} \zeta_{-n}, \\
e_{01} &= \left\{ -c'_{23}\frac{P_0R}{p} + 2c'_{24}nP_0 - c'_{34}\frac{R^2}{p} - 2nc'_{44}R \right\}F_0 - c'_{33}\frac{R^2}{p} + 2c'_{34}nR - c'_{44}\frac{R^2}{p} + 2c'_{44}nR \right\} \zeta_{-n}, \\
e_{02} &= \left\{ -c'_{23}\frac{P_0Q}{p} + 2c'_{24}nP_0 - c'_{34}\frac{Q^2}{p} + 2nc'_{44}Q \right\}F_0 - c'_{33}\frac{Q^2}{p} + 2c'_{34}nQ - c'_{44}\frac{Q^2}{p} + 2nc'_{44}Q \right\} \zeta_{-n}, \\
e_{03} &= -\left\{ c'_{23}\left(\frac{P_0}{p} + n\right) + c'_{34}\frac{R_n}{p} \right\}F_n - c'_{33}\frac{R_n}{p} - c'_{44}\left(\frac{P_0}{p} + n\right), \\
e_{04} &= -\left\{ c'_{23}\left(\frac{P_0}{p} + n\right) + c'_{34}\frac{Q_n}{p} \right\}F_n - c'_{33}\frac{Q_n}{p} - c'_{44}\left(\frac{P_0}{p} + n\right), \\
e_{05} &= \left\{ -c'_{23}\frac{P_0S}{p} + 2c'_{24}nP_0 - c'_{34}\frac{S^2}{p} - 2nc'_{44}S \right\}F_0 - c'_{33}\frac{S^2}{p} - 2c'_{34}nS + c'_{44}\frac{P_0S}{p} - 2nc'_{44}Q \right\} \zeta_{-n}, \\
e_{06} &= \left\{ -c'_{23}\frac{P_0L}{p} + 2c'_{24}nP_0 - c'_{34}\frac{L^2}{p} - 2nc'_{44}L \right\}F_0 - c'_{33}\frac{L^2}{p} - 2c'_{34}nL + c'_{44}\frac{P_0L}{p} + 2nc'_{44}L \right\} \zeta_{-n}, \\
e_{07} &= -\left\{ c'_{23}\left(\frac{P_0}{p} + n\right) + c'_{34}\frac{S_n}{p} \right\}F_{2n} + c'_{34}\frac{S_n}{p} - c'_{44}\left(\frac{P_0}{p} + n\right), \\
e_{08} &= -\left\{ c'_{23}\left(\frac{P_0}{p} + n\right) + c'_{34}\frac{L_n}{p} \right\}F_{3n} + c'_{34}\frac{L_n}{p} - c'_{44}\left(\frac{P_0}{p} + n\right).
\end{align*}
\]

The elements of the matrix $N_1$ represent the ratios of amplitude factors of the vertical components of displacement corresponding to the irregular waves at angles $\theta_n, \phi_n, \delta_n$ and $\gamma_n$. From equation (2.26), we get the following values of these ratios for the first order approximation of the corrugated interface as

\[
\begin{align*}
A_{0n} &= \frac{\Delta A_{0n}}{\Delta_1}, \\
B_{0n} &= \frac{\Delta B_{0n}}{\Delta_1}, \\
C_{0n} &= \frac{\Delta C_{0n}}{\Delta_1}, \\
D_{0n} &= \frac{\Delta D_{0n}}{\Delta_1}.
\end{align*}
\]
Using equations (2.6), (2.9) and (2.28), the ratios of the amplitude factors of the horizontal components of displacement, are given by

\[
\frac{A_n}{A_0} = \frac{\Delta_{A_{0n}}}{\Delta_1} \frac{F_n}{F_0}, \quad \frac{B_n}{A_0} = \frac{\Delta_{B_{0n}}}{\Delta_1} \frac{F_{1n}}{F_0}, \quad \frac{C_n}{A_0} = \frac{\Delta_{C_{0n}}}{\Delta_1} \frac{F_{2n}}{F_0}, \quad \frac{D_n}{A_0} = \frac{\Delta_{D_{0n}}}{\Delta_1} \frac{F_{3n}}{F_0}, \quad (2.29)
\]

where

\[
\Delta_1 = \begin{vmatrix}
1 & 1 & -1 & -1 \\
F_n & F_{1n} & -F_{2n} & -F_{3n} \\
d_3 & d_4 & -d_7 & -d_8 \\
e_3 & e_4 & -e_7 & -e_8
\end{vmatrix}
\]

and the values of \(\Delta_{A_{0n}}, \Delta_{B_{0n}}, \Delta_{C_{0n}} \) and \(\Delta_{D_{0n}}\) can be written by replacing the corresponding elements in the first, second, third and fourth columns of the determinant in \(\Delta_1\) with the elements of column matrix \(G_1\) respectively.

Similarly, on comparing the coefficients of \(e^{\text{mpx}}\) to both sides, we obtain

\[
M_2 N_2 = G_2, \quad (2.30)
\]

where

\[
M_2 = \begin{bmatrix}
1 & 1 & -1 & -1 \\
F'_n & F'_{1n} & -F'_{2n} & -F'_{3n} \\
d'_3 & d'_4 & -d'_7 & -d'_8 \\
e'_3 & e'_4 & -e'_7 & -e'_8
\end{bmatrix}, \quad N_2 = \begin{bmatrix}
A_{0n}' \\
A_{0} \\
A_{0n}' \\
A_{0n}'
\end{bmatrix}, \quad \text{and} \quad G_2 = \begin{bmatrix}
f_1 \\
f_2 \\
f'_3 \\
f'_4
\end{bmatrix}.
\]

The quantities \(f_1\) and \(f_2\) are defined earlier, while the expressions of \(f'_3\) and \(f'_4\) are

\[
f'_3 = \iota(1 + d'_1 \frac{A^0}{A_{00}} + d'_2 \frac{B_0}{A_{00}} - d'_5 \frac{C_0}{A_{00}} - d'_6 \frac{D_0}{A_{00}}), \quad f'_4 = \iota(1 + e'_1 \frac{A^0}{A_{00}} + e'_2 \frac{B_0}{A_{00}} - e'_5 \frac{C_0}{A_{00}} - e'_6 \frac{D_0}{A_{00}}),
\]

\[
d'_k = \frac{d'_k}{d'_0}, \quad e'_k = \frac{e'_k}{e'_0}, \quad (k = 1, 2, ..., 8), \quad (2.31)
\]

\[
d'_0 = \frac{[\{(c_{23} - c_{22})n P_0 + c_{24} \frac{P_0 R_0}{p} - (c_{34} - c_{24})n R_0 - c_{44} \frac{R_0^2}{p}\}F_0}{p}.
\]

\[
- \frac{(c_{33} - c_{23})n R_0 - c_{34} \frac{R_0^2}{p} + (c_{24} - c_{24})n P_0 + c_{44} \frac{P_0 R_0}{p}]F_0}{p},
\]

\[
d'_01 = \frac{[\{(c_{23} - c_{22})n P_0 - c_{24} \frac{P_0 R}{p} + (c_{34} - c_{24})n R - c_{44} \frac{R^2}{p}\}F_0}{p}.
\]

63
\[ d_{02} = \left(\left(c_{23} - c_{24}\right)nR - c_{34} \frac{R^2}{p} + \left(c_{34} - c_{24}\right)nP_0 - c_{44} \frac{P_0R}{p}\right) \kappa_n, \]
\[ d_{03} = -\left(c_{44} \frac{R_n}{p} + c_{24} \left(\frac{P_0}{p} - n\right)\right)F_n - c_{34} \frac{R_n}{p} - c_{44} \left(\frac{P_0}{p} - n\right), \]
\[ d_{04} = -\left(c_{44} \frac{Q_n}{p} + c_{24} \left(\frac{P_0}{p} - n\right)\right)F_n - c_{34} \frac{Q_n}{p} - c_{44} \left(\frac{P_0}{p} - n\right), \]
\[ d_{05} = \left(\left(c_{23}' - c_{24}'\right)nP_0 + c_{24}' \frac{P_0S}{p} - \left(c_{34}' - c_{24}'\right)nS - c_{44}' \frac{S^2}{p}\right)F_{20}, \]
\[ -\left(c_{34}' - c_{34}\right)nS - c_{34}' \frac{S^2}{p} + \left(c_{34}' - c_{24}'\right)nP_0 + c_{44}' \frac{P_0S}{p} \kappa_n, \]
\[ d_{06} = \left(\left(c_{23}' - c_{24}'\right)nP_0 + c_{24}' \frac{P_0L}{p} - \left(c_{34}' - c_{24}'\right)nL - c_{44}' \frac{L^2}{p}\right)F_{30}, \]
\[ -\left(c_{34}' - c_{34}\right)nL - c_{34}' \frac{L^2}{p} + \left(c_{34}' - c_{24}'\right)nP_0 + c_{44}' \frac{P_0L}{p} \kappa_n, \]
\[ d_{07} = \left(c_{44}' \frac{S_n}{p} - c_{24} \left(\frac{P_0}{p} - n\right)\right)F_{2n} + c_{34}' \frac{S_n}{p} - c_{44} \left(\frac{P_0}{p} - n\right), \]
\[ d_{08} = \left(c_{44}' \frac{L_n}{p} - c_{24} \left(\frac{P_0}{p} - n\right)\right)F_{3n} + c_{34}' \frac{L_n}{p} - c_{44} \left(\frac{P_0}{p} - n\right), \]
\[ e_0 = \left(\left(c_{23} \frac{P_0R_0}{p} + 2c_{24}nP_0 - c_{34} \frac{R_0^2}{p} + 2nc_{44}R_0\right)F_0 - c_{33} \frac{R_0^2}{p} + 2c_{34}nR_0 + c_{34} \frac{R_0^2}{p} - 2c_{44}nP_0\right) \kappa_n, \]
\[ e_{01} = \left(\left(c_{23} \frac{P_0R}{p} - 2c_{24}nP_0 - c_{34} \frac{R^2}{p} - 2nc_{44}R\right)F - c_{33} \frac{R^2}{p} - 2c_{34}nR - c_{34} \frac{R^2}{p} - 2nc_{44}P_0\right) \kappa_n, \]
\[ e_{02} = \left(\left(c_{23} \frac{P_0Q}{p} - 2c_{24}nP_0 - c_{34} \frac{Q^2}{p} - 2nc_{44}Q\right)F_{10} - c_{33} \frac{Q^2}{p} - 2c_{34}nQ - c_{34} \frac{Q^2}{p} - 2nc_{44}P_0\right) \kappa_n, \]
\[ e_{03} = -\left(c_{23} \left(\frac{P_0}{p} - n\right) + c_{34} \frac{R_n}{p}\right)F_n - c_{33} \frac{R_n}{p} - c_{34} \left(\frac{P_0}{p} - n\right), \]
\[ e_{04} = -\left(c_{23} \left(\frac{P_0}{p} - n\right) + c_{34} \frac{Q_n}{p}\right)F_n - c_{33} \frac{Q_n}{p} - c_{34} \left(\frac{P_0}{p} - n\right), \]
\[ e_{05} = \left(\left(c_{23} \frac{P_0S}{p} - 2c_{24}nP_0 - c_{34} \frac{S^2}{p} + 2nc_{44}S\right)F_{20} - c_{33} \frac{S^2}{p} + 2c_{34}nS + c_{34} \frac{S^2}{p} - 2nc_{44}P_0\right) \kappa_n, \]
\[ e_{06} = \left(\left(c_{23} \frac{P_0L}{p} - 2c_{24}nP_0 - c_{34} \frac{L^2}{p} + 2nc_{44}L\right)F_{30} - c_{33} \frac{L^2}{p} + 2c_{34}nL + c_{34} \frac{L^2}{p} - 2nc_{44}P_0\right) \kappa_n, \]
The elements of matrix $N_2$ represent the ratios of the amplitude factors of the vertical components of displacement for the irregular waves at angles $\theta_n', \phi_n', \delta_n'$ and $\gamma_n'$ respectively. Solving equations in (2.30), we get that the ratios of amplitude factors of the vertical components of displacement corresponding to irregular waves are given by

$$
\frac{A_{0n}'}{A_0} = \frac{\Delta_{0n}'}{\Delta_1'}, \quad \frac{B_{0n}'}{A_0} = \frac{\Delta_{0n}'}{\Delta_1'}, \quad \frac{C_{0n}'}{A_0} = \frac{\Delta_{0n}'}{\Delta_1'}, \quad \frac{D_{0n}'}{A_0} = \frac{\Delta_{0n}'}{\Delta_1'}.
$$

(2.32)

Using equations (2.6), (2.9) and (2.32), we obtain the ratios of the amplitude factors of the horizontal components of displacement as

$$
\frac{A_n'}{A_0} = \frac{\Delta_{1n}'}{\Delta_1 F_0}, \quad \frac{B_n'}{A_0} = \frac{\Delta_{2n}'}{\Delta_1 F_0}, \quad \frac{C_n'}{A_0} = \frac{\Delta_{3n}'}{\Delta_1 F_0}, \quad \frac{D_n'}{A_0} = \frac{\Delta_{4n}'}{\Delta_1 F_0},
$$

(2.33)

where

$$
\Delta_1' = \begin{vmatrix}
1 & 1 & -1 & -1 \\
F_n' & F_{1n}' & -F_{2n}' & -F_{3n}' \\
E_3' & E_4' & -E_7' & -E_8'
\end{vmatrix}
$$

and the values of $\Delta_{1n}'$, $\Delta_{2n}'$, $\Delta_{3n}'$, and $\Delta_{4n}'$ can be written by replacing the corresponding elements of first, second, third and fourth columns of the determinant in $\Delta_1'$ with the elements of column matrix $G_2$ respectively.

The amplitudes of the various irregular waves can be obtained as

$$
\sqrt{A_n'^2 + A_{0n}^2} = A_0 \sqrt{1 + F_n'^2}, \quad \sqrt{A_n'^2 + A_{0n}^2} = A_0 \sqrt{1 + F_n'^2}, \quad \text{etc.}
$$

By dividing the amplitude of an irregular wave with the amplitude of incident wave, one can obtain the corresponding reflection/transmission coefficient. Let us denote the various coefficients for the first order approximation of the corrugation by the following notations.

**Reflection coefficients:**

- $R_{\theta_n}'$ - corresponding to the irregularly reflected $qP-$ wave at angle $\theta_n'$;

- $R_{\phi_n}'$ - corresponding to the irregularly reflected $qP-$ wave at angle $\phi_n'$. 

65
\( R_{\omega'}^{n} \) -corresponding to the irregularly reflected \( qSV - \) wave at angle \( \phi_n \),

\( R_{\omega''}^{n} \) -corresponding to the irregularly reflected \( qSV - \) wave at angle \( \phi'_n \),

**Transmission coefficients:**

\( T_p^{n} \) -corresponding to the irregularly transmitted \( qP - \) wave at angle \( \delta_n \),

\( T_{p'}^{n} \) -corresponding to the irregularly transmitted \( qP - \) wave at angle \( \delta'_n \),

\( T_{\omega}^{n} \) -corresponding to the irregularly transmitted \( qSV - \) wave at angle \( \gamma_n \),

\( T_{\omega'}^{n} \) -corresponding to the irregularly transmitted \( qSV - \) wave at angle \( \gamma'_n \),

The expressions of these coefficients are given by

\[
R_{p}^{n} = \frac{A_{\omega n}}{A_{00}} \sqrt{\frac{1 + F_{p}^{2}}{1 + F_{0}^{2}}}, \quad R_{p'}^{n} = \frac{A'_{\omega n}}{A_{00}} \sqrt{\frac{1 + F_{p'}^{2}}{1 + F_{0}^{2}}}, \quad R_{\omega}^{n} = \frac{B_{\omega n}}{A_{00}} \sqrt{\frac{1 + F_{\omega}^{2}}{1 + F_{0}^{2}}}, \quad R_{\omega'}^{n} = \frac{B'_{\omega n}}{A_{00}} \sqrt{\frac{1 + F_{\omega'}^{2}}{1 + F_{0}^{2}}},
\]

\[
R_{\omega'}^{n} = \frac{B'_{\omega n}}{A_{00}} \sqrt{\frac{1 + F_{\omega'}^{2}}{1 + F_{0}^{2}}}, \quad T_{p}^{n} = \frac{C_{\omega n}}{A_{00}} \sqrt{\frac{1 + F_{p}^{2}}{1 + F_{0}^{2}}}, \quad T_{p'}^{n} = \frac{C'_{\omega n}}{A_{00}} \sqrt{\frac{1 + F_{p'}^{2}}{1 + F_{0}^{2}}}, \quad T_{\omega}^{n} = \frac{D_{\omega n}}{A_{00}} \sqrt{\frac{1 + F_{\omega}^{2}}{1 + F_{0}^{2}}}, \quad T_{\omega'}^{n} = \frac{D'_{\omega n}}{A_{00}} \sqrt{\frac{1 + F_{\omega'}^{2}}{1 + F_{0}^{2}}},
\]

(2.34)

It can be seen that these coefficients depend on the coefficients, \( \zeta_{n} \) and \( \zeta_{-n} \) and hence on the corrugation of the interface. In the subsequent section, we shall consider a special type of interface following the cosine law. We shall deduce directly the reflection and transmission coefficients for the first order of approximation of the corrugation for the cosine law interface.

**2.5 Special case: A periodic interface**

When the interface is represented by only one cosine term, i.e., \( \zeta = d \cos(px) \), \( d \) being the amplitude of the corrugation, then

\[
\zeta_{-n} = \zeta_{n} = \begin{cases} 
0 & \text{if } n \neq 1, \\
\frac{d}{2} & \text{if } n = 1.
\end{cases}
\]

In this case, the reflection and transmission coefficients of irregularly reflected and transmitted waves for the first order approximation of the corrugation can be obtained
directly by putting \( n = 1 \) into (2.34). The relevant coefficients can be written as

\[
R'_p = \frac{A_{01}}{A_{00}} \sqrt{1 + \frac{F_{11}^2}{1 + F_{00}^2}}, \quad R'_{p'} = \frac{A'_{01}}{A_{00}} \sqrt{1 + \frac{F_{11}^2}{1 + F_{00}^2}}, \quad R_{p'} = \frac{B_{01}}{A_{00}} \sqrt{1 + \frac{F_{11}^2}{1 + F_{00}^2}},
\]

\[
R'_{p''} = \frac{B_{01}}{A_{00}} \sqrt{1 + \frac{F_{11}^2}{1 + F_{00}^2}}, \quad T'_p = \frac{C_{01}}{A_{00}} \sqrt{1 + \frac{F_{21}^2}{1 + F_{00}^2}}, \quad T'_{p'} = \frac{C'_{01}}{A_{00}} \sqrt{1 + \frac{F_{21}^2}{1 + F_{00}^2}},
\]

\[
T'_{p''} = \frac{D_{01}}{A_{00}} \sqrt{1 + \frac{F_{21}^2}{1 + F_{00}^2}}, \quad T_{p''} = \frac{D'_{01}}{A_{00}} \sqrt{1 + \frac{F_{21}^2}{1 + F_{00}^2}}, \tag{2.35}
\]

where

\[
\begin{align*}
\Delta_1 &= \begin{vmatrix}
1 & 1 & -1 & -1 \\
F_1 & F_{11} & -F_{21} & -F_{31} \\
d_{13} & d_{14} & -d_{17} & -d_{18} \\
e_{13} & e_{14} & -e_{17} & -e_{18}
\end{vmatrix}, \\
\Delta_2' &= \begin{vmatrix}
1 & 1 & -1 & -1 \\
F'_{1} & F'_{11} & -F'_{21} & -F'_{31} \\
d'_{13} & d'_{14} & -d'_{17} & -d'_{18} \\
e'_{13} & e'_{14} & -e'_{17} & -e'_{18}
\end{vmatrix}
\end{align*}
\]

and the values of \( \Delta_{A_{01}}, \Delta_{B_{01}}, \Delta_{C_{01}} \) and \( \Delta_{D_{01}} \) can be written by replacing the elements of first, second, third and fourth columns of the determinant in \( \Delta_2 \) with the corresponding elements of column vector \([f_{11} f_{12} f_{13} f_{14}]^T \) respectively. Similarly, the values of \( \Delta'_{A_{01}}, \Delta'_{B_{01}}, \Delta'_{C_{01}} \) and \( \Delta'_{D_{01}} \) can be set up by replacing with the corresponding elements of column vector \([f_{11} f_{12} f'_{13} f'_{14}]^T \) in \( \Delta'_2 \). Their expressions are given below as

\[
\begin{align*}
\Delta_{A_{01}} &= \frac{A_{01}}{A_{00}}, \\
\Delta_{B_{01}} &= \frac{B_{01}}{A_{00}}, \\
\Delta_{C_{01}} &= \frac{C_{01}}{A_{00}}, \\
\Delta_{D_{01}} &= \frac{D_{01}}{A_{00}},
\end{align*}
\]

\[
\begin{align*}
\Delta'_{A_{01}} &= \frac{A'_{01}}{A_{00}}, \\
\Delta'_{B_{01}} &= \frac{B'_{01}}{A_{00}}, \\
\Delta'_{C_{01}} &= \frac{C'_{01}}{A_{00}}, \\
\Delta'_{D_{01}} &= \frac{D'_{01}}{A_{00}},
\end{align*}
\]

\[
\begin{align*}
\Delta_1 &= \frac{1 + F_{00}^2}{1 + F_{00}^2}, \\
\Delta_2' &= \frac{1 + F_{00}^2}{1 + F_{00}^2}, \tag{2.35}
\end{align*}
\]

and the values of \( \Delta_{A_{01}}, \Delta_{B_{01}}, \Delta_{C_{01}} \) and \( \Delta_{D_{01}} \) can be written by replacing the elements of first, second, third and fourth columns of the determinant in \( \Delta_2 \) with the corresponding elements of column vector \([f_{11} f_{12} f_{13} f_{14}]^T \) respectively. Similarly, the values of \( \Delta'_{A_{01}}, \Delta'_{B_{01}}, \Delta'_{C_{01}} \) and \( \Delta'_{D_{01}} \) can be set up by replacing with the corresponding elements of column vector \([f_{11} f_{12} f'_{13} f'_{14}]^T \) in \( \Delta'_2 \). Their expressions are given below as

\[
\begin{align*}
J_{11} &= \frac{1}{2} \left[ \frac{d_{11}}{c_0} - \frac{\cos \theta_0}{c_1} + \frac{\cos \theta_0 \Delta_{A_0}}{c_1 \Delta} + \frac{\cos \theta_0 \Delta_{B_0}}{c_2 \Delta} + \frac{\cos \theta_0 \Delta_{C_0}}{c_3 \Delta} + \frac{\cos \theta_0 \Delta_{D_0}}{c_4 \Delta} \right], \\
J_{12} &= \frac{1}{2} \left[ \frac{d_{12}}{c_0} \cos \theta_0 + \frac{F_{12} \cos \theta_0 \Delta_{A_0}}{c_1 \Delta} + \frac{F_{12} \cos \theta_0 \Delta_{B_0}}{c_2 \Delta} + \frac{F_{12} \cos \theta_0 \Delta_{C_0}}{c_3 \Delta} + \frac{F_{12} \cos \theta_0 \Delta_{D_0}}{c_4 \Delta} \right], \\
J_{13} &= \frac{1}{2} \left[ d_{13} \cos \theta_0 \Delta_{A_0} + d_{13} \cos \theta_0 \Delta_{B_0} - d_{13} \cos \theta_0 \Delta_{C_0} - d_{13} \cos \theta_0 \Delta_{D_0} \right], \\
J_{14} &= \frac{1}{2} \left[ e_{14} \cos \theta_0 \Delta_{A_0} + e_{14} \cos \theta_0 \Delta_{B_0} - e_{14} \cos \theta_0 \Delta_{C_0} - e_{14} \cos \theta_0 \Delta_{D_0} \right], \\
J_{15} &= \frac{1}{2} \left[ d_{15} \cos \theta_0 \Delta_{A_0} + d_{15} \cos \theta_0 \Delta_{B_0} - d_{15} \cos \theta_0 \Delta_{C_0} - d_{15} \cos \theta_0 \Delta_{D_0} \right], \\
J_{16} &= \frac{1}{2} \left[ e_{16} \cos \theta_0 \Delta_{A_0} + e_{16} \cos \theta_0 \Delta_{B_0} - e_{16} \cos \theta_0 \Delta_{C_0} - e_{16} \cos \theta_0 \Delta_{D_0} \right],
\end{align*}
\]
\[ f'_{14} = e(1 + e'_{11} \Delta \rho + e'_{12} \Delta \theta + e'_{13} \Delta \gamma, - e'_{10} \Delta \rho_0), \]
\[ d_{1k} = \frac{d_{2k}}{d_{00}}, \quad e_{1k} = \frac{e_{2k}}{e_{00}}, \quad d'_{1k} = \frac{d'_{2k}}{d'_{00}}, \quad e'_{1k} = \frac{e'_{2k}}{e'_{00}}, \quad (k = 1, 2, \ldots, 8), \]
\[ d_{00} = \left\{ -(c_{23} - c_{22}) \frac{\omega \sin \theta_0}{c_0} + c_{24} \frac{\omega^2 \sin \theta_0 \cos \theta_0}{pc_0^2} + (c_{34} - c_{24}) \frac{\omega \cos \theta_0}{c_0} - c_{44} \frac{\omega^2 \cos^2 \theta_0}{pc_0^2} \right\} F_0 \]
\[ + (c_{33} - c_{23}) \frac{\omega \cos \theta_0}{c_0} - c_{34} \frac{\omega^2 \cos \theta_0}{pc_0^2} - (c_{34} - c_{24}) \frac{\omega \sin \theta_0}{c_0} + c_{44} \frac{\omega^2 \sin \theta_0 \cos \theta_0}{pc_0^2} \frac{d}{2}, \]
\[ d_{21} = \left\{ -(c_{23} - c_{22}) \frac{\omega \sin \theta_0}{c_0} - c_{24} \frac{\omega^2 \sin \theta_0 \cos \theta_0}{pc_0^2 c_1} - (c_{34} - c_{24}) \frac{\omega \cos \theta_0}{c_1} - c_{44} \frac{\omega^2 \cos \theta_0}{pc_0^2 c_1} \right\} F \]
\[ - (c_{33} - c_{23}) \frac{\omega \cos \theta_0}{c_1} - c_{34} \frac{\omega^2 \cos \theta_0}{pc_1^2 c_1} - (c_{34} - c_{24}) \frac{\omega \sin \theta_0}{c_0} - c_{44} \frac{\omega^2 \sin \theta_0 \cos \theta_0}{pc_0^2 c_1} \frac{d}{2}, \]
\[ d_{22} = \left\{ -(c_{23} - c_{22}) \frac{\omega \sin \theta_0}{c_0} - c_{24} \frac{\omega^2 \sin \theta_0 \cos \phi}{pc_0^2 c_2} - (c_{34} - c_{24}) \frac{\omega \cos \phi}{c_2} - c_{44} \frac{\omega^2 \cos \phi}{pc_0^2 c_2} \right\} F_1 \]
\[ - (c_{33} - c_{23}) \frac{\omega \cos \phi}{c_2} - c_{34} \frac{\omega^2 \cos \phi}{pc_2^2 c_2} - (c_{34} - c_{24}) \frac{\omega \sin \phi}{c_0} - c_{44} \frac{\omega^2 \sin \phi}{pc_0^2 c_2} \frac{d}{2}, \]
\[ d_{23} = \left\{ c_{44} \frac{\omega \cos \theta_1}{pc_1} + c_{24} \frac{\omega \sin \theta_0}{c_0 c_1} + 1 \right\} F_1 + c_{34} \frac{\omega \cos \phi}{pc_1} + c_{44} \frac{\omega \cos \phi}{pc_0^2 c_1} + 1, \]
\[ d_{24} = - \left\{ c_{44} \frac{\omega \cos \theta_1}{pc_2} + c_{24} \frac{\omega \sin \theta_0}{c_0 c_2} + 1 \right\} F_1 + c_{34} \frac{\omega \cos \phi}{pc_2} + c_{44} \frac{\omega \cos \theta_0}{pc_0^2 c_2} + 1, \]
\[ d_{25} = \left\{ -(c'_{23} - c'_{22}) \frac{\sin \theta_0}{c_0} + c'_{24} \frac{\omega^2 \sin \theta_0 \cos \delta}{pc_0^2 c_3} + (c'_{34} - c'_{24}) \frac{\omega \cos \delta}{c_3} - c_{44} \frac{\omega^2 \cos^2 \delta}{pc_0^2 c_3} \right\} F_{20} \]
\[ + (c'_{33} - c'_{23}) \frac{\omega \cos \delta}{c_3} - c_{34} \frac{\omega^2 \cos \delta}{pc_0^2 c_3} - (c'_{34} - c'_{24}) \frac{\omega \sin \theta_0}{c_0} + c_{44} \frac{\omega^2 \sin \theta_0 \cos \delta}{pc_0^2 c_3} \frac{d}{2}, \]
\[ d_{26} = \left\{ -(c'_{23} - c'_{22}) \frac{\sin \theta_0}{c_0} + c'_{24} \frac{\omega^2 \sin \theta_0 \cos \gamma}{pc_0^2 c_4} + (c'_{34} - c'_{24}) \frac{\omega \cos \gamma}{c_4} - c_{44} \frac{\omega^2 \cos^2 \gamma}{pc_0^2 c_4} \right\} F_{30} \]
\[ + (c'_{33} - c'_{23}) \frac{\omega \cos \gamma}{c_4} - c_{34} \frac{\omega^2 \cos \gamma}{pc_0^2 c_4} - (c'_{34} - c'_{24}) \frac{\omega \sin \theta_0}{c_0} + c_{44} \frac{\omega^2 \sin \theta_0 \cos \gamma}{pc_0^2 c_4} \frac{d}{2}, \]
\[ d_{27} = - \left\{ c_{44} \frac{\omega \cos \delta}{pc_3} + c_{24} \frac{\omega \sin \theta_0}{c_0 c_3} + 1 \right\} F_{21} + c'_{34} \frac{\omega \cos \delta}{pc_3} - (\omega \cos \theta_0 + 1)c_{44}, \]
\[ d_{28} = - \left\{ c_{44} \frac{\omega \cos \gamma_1}{pc_4} + c_{24} \frac{\omega \sin \theta_0}{c_0 c_4} + 1 \right\} F_{31} + c_{34} \frac{\omega \cos \gamma_1}{pc_4} - (\omega \cos \theta_0 + 1)c_{44}, \]
\[ e_{00} = \left\{ (c_{23} - c_{22}) \frac{\omega^2 \sin \theta_0 \cos \theta_0}{pc_0^2 c_0} + 2c_{24} \frac{\omega \sin \theta_0}{c_0} - c_{34} \frac{\omega^2 \cos^2 \theta_0}{pc_0^2 c_0} + 2c_{44} \frac{\omega \cos \theta_0}{c_0} \right\} F_0 \]
\[ - c_{33} \frac{\omega^2 \cos^2 \theta_0}{c_0^2} - 2c_{34} \frac{\omega \cos \theta_0}{c_0} + c_{34} \frac{\omega^2 \cos^2 \theta_0}{pc_0^2 c_0} + 2c_{44} \frac{\omega \cos \theta_0}{c_0} \frac{d}{2}. \]
\[ e_{21} = \left\{ -c_{23} \frac{\omega^2 \sin \theta_0 \cos \theta}{c_0 c_1} + 2c_{24} \omega \frac{\sin \theta_0}{c_0} - c_{34} \frac{\omega^2 \cos^2 \theta}{p c_1^2} + 2c_{44} \frac{\omega \cos \theta}{c_1} \right\} F - c_{33} \frac{\omega^2 \cos^2 \theta}{p c_1^2} \]

\[ + 2c_{34} \frac{\omega \cos \theta}{c_1} - c_{34} \frac{\omega^2 \sin \theta_0 \cos \theta}{p c_0 c_1} + 2c_{44} \frac{\omega \sin \theta_0}{c_0} \frac{d}{2}, \]

\[ e_{22} = \left\{ -c_{23} \frac{\omega^2 \sin \theta_0 \phi}{p c_0 c_2} + 2c_{24} \frac{\sin \theta_0}{c_0} - c_{34} \frac{\omega^2 \cos^2 \phi}{p c_2^2} + 2c_{44} \frac{\omega \cos \phi}{c_2} \right\} F_{10} - c_{33} \frac{\omega^2 \cos^2 \phi}{p c_2^2} \]

\[ + 2c_{34} \frac{\omega \cos \phi}{c_2} - c_{34} \frac{\omega^2 \sin \theta_0 \cos \phi}{p c_0 c_2} + 2c_{44} \frac{\omega \sin \theta_0}{c_0} \frac{d}{2}, \]

\[ e_{23} = -\left\{ c_{23} \left( \frac{\omega \sin \theta_0}{p c_0} + 1 \right) + c_{34} \frac{\omega \cos \theta_1}{p c_1} \right\} F_1 - c_{33} \frac{\omega \cos \theta_1}{p c_1} - c_{34} \left( \frac{\omega \sin \theta_0}{p c_0} + 1 \right), \]

\[ e_{24} = -\left\{ c_{23} \left( \frac{\omega \sin \theta_0}{p c_0} + 1 \right) + c_{34} \frac{\omega \cos \phi_1}{p c_2} \right\} F_{11} - c_{33} \frac{\omega \cos \phi_1}{p c_2} - c_{34} \left( \frac{\omega \sin \theta_0}{p c_0} + 1 \right), \]

\[ e_{25} = \left\{ -c_{23} \frac{\omega^2 \sin \theta_0 \cos \delta}{p c_0 c_3} + 2c_{24} \frac{\omega \sin \theta_0}{c_0} - c_{34} \frac{\omega^2 \cos^2 \delta}{p c_3^2} - 2c_{34} \frac{\omega \cos \delta}{c_3} \right\} F_{20} - c_{33} \frac{\omega^2 \cos^2 \delta}{p c_3^2} \]

\[ - 2c_{34} \frac{\omega \cos \delta}{c_3} + c_{34} \frac{\omega^2 \sin \theta_0 \cos \delta}{p c_0 c_3} + 2c_{44} \frac{\omega \sin \theta_0}{c_0} \frac{d}{2}, \]

\[ e_{26} = \left\{ -c_{23} \frac{\omega^2 \sin \theta_0 \cos \gamma}{p c_0 c_4} + 2c_{24} \frac{\omega \sin \theta_0}{c_0} - c_{34} \frac{\omega^2 \cos^2 \gamma}{p c_4^2} - 2c_{34} \frac{\omega \cos \gamma}{c_4} \right\} F_{30} - c_{33} \frac{\omega^2 \cos^2 \gamma}{p c_4^2} \]

\[ - 2c_{34} \frac{\omega \cos \gamma}{c_4} + c_{34} \frac{\omega^2 \sin \theta_0 \cos \gamma}{p c_0 c_4} + 2c_{44} \frac{\omega \sin \theta_0}{c_0} \frac{d}{2}, \]

\[ e_{27} = \left\{ -c_{23} \left( \frac{\omega \sin \theta_0}{p c_0} + 1 \right) + c_{34} \frac{\omega \cos \delta_1}{p c_3} \right\} F_{21} + c_{33} \frac{\omega \cos \delta_1}{p c_3} - c_{34} \left( \frac{\omega \sin \theta_0}{p c_0} + 1 \right), \]

\[ e_{28} = \left\{ -c_{23} \left( \frac{\omega \sin \theta_0}{p c_0} + 1 \right) + c_{34} \frac{\omega \cos \gamma_1}{p c_4} \right\} F_{31} + c_{33} \frac{\omega \cos \gamma_1}{p c_4} - c_{34} \left( \frac{\omega \sin \theta_0}{p c_0} + 1 \right), \]

\[ d'_{00} = \left\{ \left( c_{33} - c_{23} \right) \frac{\omega \cos \theta_0}{c_0} - c_{34} \frac{\omega \cos \theta_0}{p c_0^2} + \left( c_{34} - c_{24} \right) \frac{\omega \cos \theta_0}{c_0} - c_{44} \frac{\omega \sin \theta_0 \cos \theta_0}{p c_0^2} \right\} F_0 \]

\[ - \left( c_{33} - c_{23} \right) \frac{\omega \cos \theta_0}{c_0} - c_{34} \frac{\omega \cos \theta_0}{p c_0^2} + \left( c_{34} - c_{24} \right) \frac{\omega \cos \theta_0}{c_0} - c_{44} \frac{\omega \sin \theta_0 \cos \theta_0}{p c_0^2} \frac{d}{2}, \]

\[ d'_{21} = \left\{ \left( c_{33} - c_{23} \right) \frac{\omega \sin \theta_0}{c_0} - c_{34} \frac{\omega \sin \theta_0 \cos \theta_0}{p c_0 c_1} + \left( c_{34} - c_{24} \right) \frac{\omega \cos \theta_0}{c_1} - c_{44} \frac{\omega \sin \theta_0 \cos \theta_0}{p c_0 c_1} \right\} F \]

\[ + \left( c_{33} - c_{23} \right) \frac{\omega \cos \theta_0}{c_1} - c_{34} \frac{\omega \cos \theta_0}{p c_0^2} + \left( c_{34} - c_{24} \right) \frac{\omega \sin \theta_0 \cos \theta_0}{p c_0 c_1} \frac{d}{2}, \]

\[ d'_{22} = \left\{ \left( c_{33} - c_{23} \right) \frac{\omega \sin \theta_0}{c_0} - c_{34} \frac{\omega \sin \theta_0 \cos \phi}{p c_0 c_2} + \left( c_{34} - c_{24} \right) \frac{\omega \cos \phi}{c_2} - c_{44} \frac{\omega \sin \theta_0 \cos \phi}{p c_0 c_2} \right\} F_{10} \]

\[ + \left( c_{33} - c_{23} \right) \frac{\omega \cos \phi}{c_2} - c_{34} \frac{\omega \cos \phi}{p c_0^2} + \left( c_{34} - c_{24} \right) \frac{\omega \sin \theta_0 \cos \phi}{p c_0 c_2} \frac{d}{2}, \]
\[ \begin{align*}
  d'_{23} &= \left\{ c_{44} \frac{\omega \cos \theta_1'}{p_{c1}} + c_{24} \left( \frac{\omega \sin \theta_0}{p_{c0}} - 1 \right) \right\} F'_{1} - c_{34} \frac{\omega \cos \theta_1'}{p_{c1}} - c_{44} \left( \frac{\omega \cos \theta_0}{p_{c0}} - 1 \right), \\
  d'_{24} &= \left\{ c_{44} \frac{\omega \cos \delta_1'}{p_{c2}} + c_{24} \left( \frac{\omega \sin \theta_0}{p_{c0}} - 1 \right) \right\} F'_{11} - c_{34} \frac{\omega \cos \delta_1'}{p_{c2}} - c_{44} \left( \frac{\omega \cos \theta_0}{p_{c0}} - 1 \right), \\
  d'_{25} &= \left\{ (c'_{23} - c'_{22}) \frac{\omega \sin \theta_0}{c_0} + c'_{24} \frac{\omega^2 \sin \theta_0 \cos \delta}{p_{c0} c_3} - (c'_{34} - c'_{24}) \frac{\omega \cos \delta}{c_3} - c'_{44} \left( \frac{\omega^2 \cos^2 \delta}{p_{c3}^2} \right) \right\} F_{20} \\
  d'_{26} &= \left\{ (c'_{23} - c'_{22}) \frac{\omega \sin \theta_0}{c_0} + c'_{24} \frac{\omega^2 \sin \theta_0 \cos \gamma}{p_{c0} c_4} - (c'_{34} - c'_{24}) \frac{\omega \cos \gamma}{c_4} - c'_{44} \left( \frac{\omega^2 \cos^2 \gamma}{p_{c4}^2} \right) \right\} F_{30} \\
  d'_{27} &= \left\{ c_{44} \frac{\omega \cos \delta_1'}{p_{c3}} - c_{24} \left( \frac{\omega \sin \theta_0}{p_{c0}} - 1 \right) \right\} F_{21} + c'_{34} \frac{\omega \cos \delta_1'}{p_{c3}} - c_{44} \left( \frac{\omega \cos \theta_0}{p_{c0}} - 1 \right), \\
  d'_{28} &= \left\{ c_{44} \frac{\omega \cos \gamma_1'}{p_{c4}} - c_{24} \left( \frac{\omega \sin \theta_0}{p_{c0}} - 1 \right) \right\} F_{31} + c'_{34} \frac{\omega \cos \gamma_1'}{p_{c4}} - (\frac{\omega \cos \theta_0}{p_{c0}} - 1) c'_{44}, \\
  e'_{00} &= \left\{ (c_{23} \frac{\omega^2 \sin \theta_0 \cos \theta}{p_{c0}^2} + 2 c_{24} \omega \sin \theta_0 \frac{\omega^2 \cos^2 \theta}{p_{c0}^2} + 2 c_{44} \omega \cos \theta_0 \frac{\omega^2 \sin \theta_0}{p_{c0}^2} \right\} F_{0} \\
  c_{33} \frac{\omega^2 \cos^2 \theta_0}{p_{c0}^2} + 2 c_{34} \omega \cos \theta_0 \frac{\omega^2 \cos^2 \theta_0}{p_{c0}^2} - 2 c_{44} \omega \sin \theta_0 \frac{\omega^2 \sin \theta_0}{p_{c0}^2}, \\
  e'_{21} &= \left\{ (c_{23} \frac{\omega \sin \theta_0 \cos \theta}{c_0} - 2 c_{24} \frac{\omega \sin \theta_0}{c_0} - c_{34} \frac{\omega^2 \cos \theta}{p_{c1}^2} - 2 c_{44} \frac{\omega \cos \theta_0}{c_1} \right\} F \\
  -2 c_{33} \frac{\omega^2 \cos \theta_0}{p_{c1}^2} + 2 c_{34} \omega \cos \theta_0 \frac{\omega^2 \cos \theta_0}{p_{c1}^2} - c_{44} \omega \sin \theta_0 \frac{\omega^2 \sin \theta_0}{c_1}, \\
  e'_{22} &= \left\{ (c_{23} \frac{\omega \sin \theta_0 \cos \phi}{p_{c0} c_2} - 2 c_{24} \frac{\omega \sin \theta_0}{c_0} - c_{34} \frac{\omega^2 \cos \phi}{p_{c0}^2} - 2 c_{44} \frac{\omega \cos \phi}{c_2} \right\} F_{10} \\
  -2 c_{33} \frac{\omega^2 \cos \phi}{c_2^2} - 2 c_{34} \frac{\omega \cos \phi}{c_0} - c_{44} \frac{\omega \sin \theta_0 \cos \phi}{p_{c0} c_2} - 2 c_{44} \frac{\omega \sin \theta_0 \cos \phi}{c_0}, \\
  e'_{23} &= \left\{ (c_{23} \frac{\omega \sin \theta_0}{p_{c0}} - 1) + c_{24} \frac{\omega \cos \theta_1'}{p_{c1}} \right\} F'_{1} - c_{33} \frac{\omega \cos \theta_1'}{p_{c1}} - c_{44} \left( \frac{\omega \cos \theta_0}{p_{c0}} - 1 \right), \\
  e'_{24} &= \left\{ (c_{23} \frac{\omega \sin \theta_0}{p_{c0}} - 1) - c_{34} \frac{\omega \cos \theta_1'}{p_{c2}} \right\} F_{11} - c_{33} \frac{\omega \cos \theta_1'}{p_{c2}} - c_{44} \left( \frac{\omega \sin \theta_0}{p_{c0}} - 1 \right), \\
  e'_{25} &= \left\{ (c_{23} \frac{\omega \sin \theta_0 \cos \delta}{p_{c0} c_3} - 2 c_{24} \frac{\omega \sin \theta_0}{c_0} - c_{34} \frac{\omega^2 \cos \delta}{p_{c3}^2} + 2 c_{44} \frac{\omega \cos \delta}{c_3} \right\} F_{20} \\
  -2 c_{33} \frac{\omega^2 \cos^2 \delta}{p_{c3}^2} + 2 c_{34} \frac{\omega \cos \delta}{c_3} + c_{34} \frac{\omega \sin \theta_0 \cos \delta}{p_{c0} c_3} - 2 c_{44} \frac{\omega \sin \theta_0 \cos \delta}{c_0}, \\
  70 \end{align*} \]
Chapter 2

\[ e''_{26} = \left\{ \frac{c'_3}{c'_4} \omega^2 \sin \theta_0 \cos \gamma - 2c'_{24} \omega \sin \theta_0 \frac{c'_3}{c'_4} + \frac{c'_{34}}{c'_4} \frac{\omega^2 \cos^2 \gamma}{c'_0} + 2c'_4 \omega \cos \gamma \right\} F_{30} \]

\[ -c'_{33} \omega^2 \cos^2 \gamma \frac{c'_4}{c'_4} + 2c'_{34} \omega \cos \gamma \frac{c'_4}{c'_0 c'_4} + c'_{34} \frac{\omega^2 \sin \theta_0 \cos \gamma}{c'_0 c'_4} - 2c'_4 \omega \sin \theta_0 \frac{d}{c'_0} \frac{d}{2} \]

\[ e''_{27} = \left\{ -c'_{23} \left( \frac{\omega \sin \theta_0}{c'_0} - 1 \right) + c'_{34} \frac{\omega \cos \delta_1}{c'_3} \right\} F'_{21} + c'_{33} \omega \cos \delta'_1 \frac{c'_3}{c'_3} - c'_{34} \left( \frac{\omega \sin \theta_0}{c'_0} - 1 \right) \]

\[ e''_{28} = \left\{ -c'_{23} \left( \frac{\omega \sin \theta_0}{c'_0} - 1 \right) + c'_{34} \frac{\omega \cos \gamma'_1}{c'_4} \right\} F''_{31} + c'_{33} \omega \cos \gamma'_1 \frac{c'_3}{c'_4} - c'_{34} \left( \frac{\omega \sin \theta_0}{c'_0} - 1 \right) \]

The expressions of \( F_1, F'_1, F_{11}, \) etc. can be written by putting \( n = 1 \) in the expressions of \( F_n, F'_n, F_{1n}, \) etc. defined earlier.

It can be easily verified that when the corrugation of the interface is zero, i.e., when \( d = 0 \), then the determinants \( \Delta_A_{01}, \Delta_{02}, \Delta_{C01}, \Delta_{011}, \) etc. vanish and hence all the coefficients given by \( R_1^{1p}, R_1^{1q}, R_1^{1r}, \) etc. vanish. In this case, we are left with formulae given in (2.25), which are the reflection and transmission coefficients corresponding to the plane interface between two dissimilar monoclinic elastic half-spaces.

### 2.6 Particular cases

(a) In an orthotropic medium, the plane of symmetry coincides with the coordinate plane. Therefore, for orthotropic media, we shall have \( c'_{14} = c'_{24} = c'_{34} = c'_{56} = 0 \) in medium \( H \) and \( c'_{14} = c'_{24} = c'_{34} = c'_{56} = 0 \) in medium \( H' \). In this case, the two half-spaces reduce to orthotropic-orthotropic half-spaces and the values of \( V_0, U_0, Z_0, \) etc. in equations (2.6) and (2.9) with \( n = 1 \) become

\[ V_0 = -(c_{23} + c_{44}) \sin \theta_0 \cos \theta_0, \quad U_0 = c_{22} \sin^2 \theta_0 + c_{44} \cos^2 \theta_0, \quad Z_0 = c_{44} \sin^2 \theta_0 + c_{33} \cos^2 \theta_0, \]

\[ V_{10} = (c_{23} + c_{44}) \sin \theta \cos \theta, \quad U_{10} = c_{22} \sin^2 \theta + c_{44} \cos^2 \theta, \quad Z_{10} = c_{44} \sin^2 \theta + c_{33} \cos^2 \theta, \]

\[ V_{11} = (c_{23} + c_{44}) \sin \theta_1 \cos \theta_1, \quad U_{11} = c_{22} \sin^2 \theta_1 + c_{44} \cos^2 \theta_1, \quad Z_{11} = c_{44} \sin^2 \theta_1 + c_{33} \cos^2 \theta_1, \]

\[ V_{20} = (c_{23} + c_{44}) \sin \phi \cos \phi, \quad U_{20} = c_{22} \sin^2 \phi + c_{44} \cos^2 \phi, \quad Z_{20} = c_{44} \sin^2 \phi + c_{33} \cos^2 \phi, \]

\[ V_{21} = (c_{23} + c_{44}) \sin \phi_1 \cos \phi_1, \quad U_{21} = c_{22} \sin^2 \phi_1 + c_{44} \cos^2 \phi_1, \quad Z_{21} = c_{44} \sin^2 \phi_1 + c_{33} \cos^2 \phi_1, \]

\[ V_{30} = -(c_{23} + c_{44}) \sin \delta \cos \delta, \quad U_{30} = c_{22} \sin^2 \delta + c_{44} \cos^2 \delta, \quad Z_{30} = c_{44} \sin^2 \delta + c_{33} \cos^2 \delta, \]

71
\[ V_{31} = -(c'_{23} + c'_{44}) \sin \gamma \cos \delta_1, \quad U_{31} = c'_{22} \sin^2 \delta_1 + c'_{44} \cos^2 \delta_1, \quad Z_{31} = c_{44} \sin^2 \delta_1 + c_{33} \cos^2 \delta_1, \]
\[ V_{40} = -(c'_{23} + c'_{44}) \sin \gamma \cos \gamma, \quad U_{40} = c'_{22} \sin^2 \gamma + c'_{44} \cos^2 \gamma, \quad Z_{40} = c_{44} \sin^2 \gamma + c_{33} \cos^2 \gamma, \]
\[ V_{41} = -(c'_{23} + c'_{44}) \sin \gamma_1 \cos \gamma_1, \quad U_{41} = c'_{22} \sin^2 \gamma_1 + c'_{44} \cos^2 \gamma_1, \quad Z_{41} = c_{44} \sin^2 \gamma_1 + c_{33} \cos^2 \gamma_1 \]
and the expressions for \( V_{11}, U_{11} \) and \( Z_{11} \) are obtained respectively from \( V_{11}, U_{11} \) and \( Z_{11} \) by replacing \( \theta_1 \) with \( \theta'_1 \); for \( V_{21}, U_{21} \) and \( Z_{21} \) from \( V_{21}, U_{21} \) and \( Z_{21} \) by replacing \( \phi_1 \) with \( \phi'_1 \); for \( V'_{31}, U'_{31} \) and \( Z'_{31} \) from \( V_{31}, U_{31} \) and \( Z_{31} \) by replacing \( \delta_1 \) by \( \delta'_1 \); and for \( V''_{41}, U''_{41} \) and \( Z''_{41} \) from \( V_{41}, U_{41} \) and \( Z_{41} \) by replacing \( \gamma_1 \) with \( \gamma'_1 \).

\[
\begin{align*}
 a &= \frac{1 + F \cot \theta}{1 - F_0 \cot \theta_0}, \quad a_1 = \frac{1 + F_{10} \cot \phi}{1 - F_0 \cot \theta_0}, \quad a_2 = \frac{c_{44}(1 - F_20 \cot \delta)}{c_{44}(1 - F_0 \cot \theta_0)}, \\
 a_3 &= \frac{c'_{44}(1 - F_{30} \cot \gamma)}{c_{44}(1 - F_0 \cot \theta_0)}, \quad b = \frac{F_{23} + c_{33} \cot \theta}{c_{23} F_0 - c_{33} \cot \theta_0}, \quad b_1 = \frac{F_{10} c_{23} + c_{33} \cot \phi}{c_{23} F_0 - c_{33} \cot \theta_0}, \\
 b_2 &= \frac{F_{30} c_{23} + c'_{33} \cot \delta}{c_{23} F_0 - c_{33} \cot \theta_0}, \quad b_3 = \frac{F_{30} c_{23} + c_{33} \cot \gamma}{c_{23} F_0 - c_{33} \cot \theta_0}.
\end{align*}
\]

\( F, F_0, F_{10}, \) etc. can be modified from the expressions of these already given. With these modified values, the reflection and transmission coefficients of the \( qP^- \) waves at the plane interface between two dissimilar orthotropic half-spaces are given by equation (2.25). The coefficients corresponding to irregular waves for the first order approximation of the corrugated interface given by \( \zeta = d \cos(p x_2) \) between two dissimilar orthotropic half-spaces are given by equation (2.35) with the the following modified values

\[
\begin{align*}
 d_{00} &= \left\{ \left( c_{23} - c_{22} \right) \cos \theta_0 \right\} F_0 + \left( c_{33} - c_{23} \right) \cos \theta_0 F_{10} + c_{44} \frac{\omega^2 \sin \theta_0 \cos \theta_0}{pc_0^2} \frac{d}{2}, \\
 d_{21} &= \left\{ \left( c_{23} - c_{22} \right) \cos \theta_0 \right\} F + \left( c_{33} - c_{23} \right) \cos \theta F_{10} - c_{44} \frac{\omega^2 \sin \theta_0 \cos \theta}{pc_0^2} \frac{d}{2}, \\
 d_{22} &= \left\{ \left( c_{23} - c_{22} \right) \cos \theta_0 \right\} F_{10} - \left( c_{33} - c_{23} \right) \cos \theta_{10} + c_{44} \frac{\omega^2 \sin \theta_0 \cos \theta}{pc_0^2} \frac{d}{2}, \\
 d_{23} &= \left( \frac{\omega \cos \theta_1}{pc_1} F_1 + \frac{\omega \cos \theta_0}{pc_0} (1) \right) c_{44}, \quad d_{24} = -c_{44} \frac{\omega \cos \phi_1}{pc_2} F_{11} + \frac{\omega \cos \theta_0}{pc_0} (1), \\
 d_{25} &= \left\{ \left( c'_{23} - c'_{22} \right) \cos \theta_0 \right\} F_{20} + \left( c'_{33} - c'_{23} \right) \cos \delta c_3 + c_{44} \frac{\omega^2 \sin \theta_0 \cos \delta}{pc_0^2} \frac{d}{2}, \\
 d_{26} &= \left\{ \left( c'_{23} - c'_{22} \right) \cos \theta_0 \right\} F_{30} + \left( c'_{33} - c'_{23} \right) \cos \gamma \frac{d}{pc_0^2} + c_{44} \frac{\omega^2 \sin \theta_0 \cos \gamma}{pc_0^2} \frac{d}{2},
\end{align*}
\]

72
\[ d_{27} = c_4^4 \left( -\frac{\cos \delta_0}{pc_3} F_{21} + \frac{\cos \theta_0}{pc_0} + 1 \right), \quad d_{28} = c_4^4 \left( -\frac{\cos \gamma_1}{pc_4} F_{31} + \frac{\cos \theta_0}{pc_0} + 1 \right), \]

\[ c_{00} = \left[ \left\{ c_{23} \right\} \frac{\sin \theta_0 \cos \theta_0}{pc_0^2} - 2c_{44} \left( \frac{\cos \theta_0}{c_0} \right) \frac{F_0}{c_0} - c_{33} \frac{\cos^2 \theta_0}{c_0^2} + 2c_{44} \frac{\sin \theta_0}{c_0} \right] \frac{d}{2}, \]

\[ c_{21} = \left[ \left\{ -c_{23} \right\} \frac{\sin \theta_0 \cos \theta_0}{c_0 c_1} + 2c_{44} \frac{\cos \theta_0}{c_1} \right] F - c_{33} \frac{\cos^2 \theta_0}{c_1^2} + 2c_{44} \frac{\sin \theta_0}{c_1} \frac{d}{2}, \]

\[ c_{22} = \left[ \left\{ -c_{23} \right\} \frac{\sin \theta_0 \cos \phi}{pc_0 c_1} + 2c_{44} \frac{\cos \phi}{c_2} \right] F_{10} - c_{33} \frac{\cos^2 \phi}{c_2^2} + 2c_{44} \frac{\sin \theta_0}{c_2} \frac{d}{2}, \]

\[ c_{23} = -c_{23} \left( \frac{\sin \theta_0}{pc_0} + 1 \right) F_1 + c_{33} \frac{\cos \theta_1}{pc_1}, \quad c_{24} = -c_{23} \left( \frac{\sin \theta_0}{pc_0} + 1 \right) F_{11} - c_{33} \frac{\cos \theta_1}{pc_2}, \]

\[ c_{25} = \left[ \left\{ -c_{23} \right\} \frac{\sin \theta_0 \cos \delta}{pc_0 c_3} - 2c_{44} \frac{\cos \delta}{c_3} \right] \frac{F_20}{c_3} - c_{33} \frac{\cos^2 \delta}{pc_3^2} + 2c_{44} \frac{\sin \theta_0}{pc_3} \frac{d}{2}, \]

\[ c_{26} = \left[ \left\{ -c_{23} \right\} \frac{\sin \theta_0 \cos \gamma}{pc_0 c_4} - 2c_{44} \frac{\cos \gamma}{c_4} \right] \frac{F_30}{pc_4^2} - c_{33} \frac{\cos^2 \gamma}{pc_4^2} + 2c_{44} \frac{\sin \theta_0}{pc_4} \frac{d}{2}, \]

\[ c_{27} = -c_{23} \left( \frac{\sin \theta_0}{pc_0} + 1 \right) F_{21} + c_{33} \frac{\cos \delta_1}{pc_3}, \quad c_{28} = -c_{23} \left( \frac{\sin \theta_0}{pc_0} + 1 \right) F_{31} + c_{33} \frac{\cos \gamma_1}{pc_4}, \]

\[ d'_{00} = \left[ \left\{ (c_{23} - c_{22}) \frac{\sin \theta_0}{c_0} - c_{44} \frac{\cos^2 \theta_0}{pc_0^2} \right\} F_0 - (c_{33} - c_{23}) \frac{\cos \theta_0}{c_0} + c_{44} \frac{\sin \theta_0 \cos \theta_0}{pc_0} \right] \frac{d}{2}, \]

\[ d'_{21} = \left[ \left\{ (c_{23} - c_{22}) \frac{\sin \theta_0}{c_0} - c_{44} \frac{\cos^2 \theta_0}{pc_0^2} \right\} F_0 + (c_{33} - c_{23}) \frac{\cos \theta_0}{c_0} - c_{44} \frac{\sin \theta_0 \cos \theta_0}{pc_0} \right] \frac{d}{2}, \]

\[ d'_{22} = \left[ \left\{ (c_{23} - c_{22}) \frac{\sin \theta_0}{c_0} - c_{44} \frac{\cos^2 \phi}{pc_2^2} \right\} F_{10} + (c_{33} - c_{23}) \frac{\cos \phi}{c_2} - c_{44} \frac{\sin \theta_0 \cos \phi}{pc_2} \right] \frac{d}{2}, \]

\[ d'_{23} = -c_{44} \frac{\cos \theta_1}{pc_1} F_{11} - c_{44} \left( \frac{\cos \theta_0}{pc_0} - 1 \right), \quad d'_{24} = -c_{44} \frac{\cos \phi}{pc_2} F_{11} + \frac{\cos \theta_1}{pc_2} - 1, \]

\[ d'_{25} = \left[ \left\{ (c'_{23} - c_{22}) \frac{\sin \theta_0}{c_0} - c_{44} \frac{\cos^2 \delta}{pc_3^2} \right\} F_{20} - (c'_{33} - c'_{23}) \frac{\cos \delta}{c_3} + c_{44} \frac{\sin \theta_0 \cos \delta}{pc_3} \right] \frac{d}{2}, \]

\[ d'_{26} = \left[ \left\{ (c'_{23} - c_{22}) \frac{\sin \theta_0}{c_0} - c_{44} \frac{\cos^2 \gamma}{pc_4^2} \right\} F_{30} - (c'_{33} - c'_{23}) \frac{\cos \gamma}{c_4} + c_{44} \frac{\sin \theta_0 \cos \gamma}{pc_4} \right] \frac{d}{2}, \]

\[ d'_{27} = c_{44} \left( \frac{\cos \delta_1}{pc_3} \frac{F_{21}}{pc_0} - \frac{\cos \theta_0}{pc_0} + 1 \right), \quad d'_{28} = c_{44} \left( \frac{\cos \gamma_1}{pc_4} \frac{F_31}{pc_0} - \frac{\cos \theta_0}{pc_0} + 1 \right), \]

\[ e'_{00} = \left[ \left\{ c_{23} \frac{\sin \theta_0 \cos \theta_0}{pc_0^2} + 2c_{44} \frac{\cos \theta_0}{c_0} \right\} F_0 - c_{33} \frac{\cos^2 \theta_0}{c_0^2} - 2c_{44} \frac{\sin \theta_0}{c_0} \right] \frac{d}{2}, \]

\[ e'_{21} = \left[ \left\{ -c_{23} \frac{\sin \theta_0 \cos \theta_0}{c_0 c_1} - 2c_{44} \frac{\cos \theta_0}{c_1} \right\} F_3 - c_{33} \frac{\cos^2 \theta_0}{pc_1^2} - c_{44} \frac{\sin \theta_0}{c_1} \right] \frac{d}{2}, \]

\[ e'_{22} = \left[ \left\{ -c_{23} \frac{\sin \theta_0 \cos \phi}{pc_0 c_2} - 2c_{44} \frac{\cos \phi}{c_2} \right\} F_{10} - c_{33} \frac{\cos^2 \phi}{c_2^2} - 2c_{44} \frac{\sin \theta_0}{c_2} \right] \frac{d}{2}. \]
\[ e'_{23} = -c_{23} \left( \frac{\omega \sin \theta_0}{p c_0} - 1 \right) F'_1 - c_{33} \frac{\omega \cos \theta'_1}{p c_1} , \quad e'_{24} = -c_{23} \left( \frac{\omega \sin \theta_0}{p c_0} - 1 \right) F'_{11} - c_{33} \frac{\omega \cos \phi'_1}{p c_2} , \]
\[ e'_{27} = -c_{23} \left( \frac{\omega \sin \theta_0}{p c_0} - 1 \right) F'_1 + c_{33} \frac{\omega \cos \delta'_1}{p c_3} , \quad e'_{28} = -c_{23} \left( \frac{\omega \sin \theta_0}{p c_0} - 1 \right) F'_{31} + c_{33} \frac{\omega \cos \gamma'_1}{p c_4} , \]
\[ e'_{25} = \left( \frac{c'_{23} \omega^2 \sin \theta_0 \cos \delta}{p c_0 c_3} + 2 c'_{44} \frac{\omega \cos \phi}{c_4} \right) F_{20} - c'_{33} \frac{\omega^2 \cos^2 \gamma}{p c_4^2} - 2 c'_{44} \frac{\omega \sin \theta_0}{c_0} \frac{d}{c_0^2} , \]
\[ e'_{26} = \left( \frac{c'_{23} \omega^2 \sin \theta_0 \cos \gamma}{p c_0 c_4} + 2 c'_{44} \frac{\omega \cos \phi}{c_4} \right) F_{30} - c'_{33} \frac{\omega^2 \cos^2 \gamma}{p c_4^2} - 2 c'_{44} \frac{\omega \sin \theta_0}{c_0} \frac{d}{c_0^2} , \]
\[ e'_{27} = -c'_{23} \left( \frac{\omega \sin \theta_0}{p c_0} - 1 \right) F_{21} + c'_{33} \frac{\omega \cos \delta'_1}{p c_3} , \quad e'_{28} = -c'_{23} \left( \frac{\omega \sin \theta_0}{p c_0} - 1 \right) F'_{41} + c'_{23} \frac{\omega \cos \gamma'_1}{p c_4} . \]

(b) To reduce the problem for transversely isotropic media, we shall use the following values of elastic constants: In medium \( H \), \( c_{12} = c_{13} \), \( c_{22} = c_{33} \), \( c_{35} = c_{66} \), \( c_{23} = c_{22} - 2c_{44} \), \( c_{14} = c_{24} = c_{34} = c_{56} = 0 \) and same values of the corresponding constants in medium \( H' \). Inserting these values of elastic constants, one can obtain the relevant reflection and transmission coefficients of various reflected and transmitted waves from the formulae given in (2.25) and (2.35).

(c) For the homogeneous isotropic media, we shall put the values of elastic parameters in the medium \( H \) and \( H' \) as follows
\[ c'_{14} = c'_{24} = c'_{34} = c'_{56} = 0 \text{ in } H \text{ and } c'_{14} = c'_{24} = c_{44} = c_{56} = 0 \text{ in } H' \], so that \( c_{11} = c_{22} = c_{33} = \lambda + 2\mu \), \( c'_{11} = c'_{22} = c'_{33} = \lambda' + 2\mu' \), \( c_{12} = c_{23} = c_{34} = \lambda \), \( c_{12} = c'_{13} = c'_{23} = \lambda' \), \( c_{44} = c_{56} = \mu \) and \( c'_{44} = c'_{56} = \mu' \). For these values, the expressions of various other relevant quantities shall reduce to
\[ c_1 = c_0 = \sqrt{\lambda + 2\mu} = \alpha , \quad c_2 = \sqrt{\frac{\mu}{\rho}} = \beta , \quad c_3 = \sqrt{\frac{\lambda' + 2\mu'}{\rho'}} = \alpha' , \quad c_4 = \sqrt{\frac{\mu'}{\rho'}} = \beta' , \]
\[ F_0 = -F = -\tan \theta_0 = -\tan \theta , \quad F_{10} = -\cot \phi , \quad F_{20} = -\tan \delta , \quad F_{30} = \cot \gamma , \quad a = 1 , \]
\[ a_1 = -\frac{\cos \phi}{2 \sin^2 \phi}, \quad a_2 = \frac{\phi'}{\rho \beta'^2} , \quad a_3 = -\frac{\rho' \beta'^2 \cos 2 \gamma}{2 \rho \beta^2 \sin^2 \gamma} , \quad b = -1 , \quad b_1 = -\frac{\sin 2 \theta_0 \cot \phi}{\cos 2 \phi} \]
\[ b_2 = -\frac{\rho' \beta'^2 \sin 2 \gamma \sin 2 \theta_0}{\rho \alpha^2 \cos 2 \phi}, \quad b_3 = -\frac{\rho' \beta'^2 \sin 2 \theta_0 \cot \gamma}{\rho a^2 \cos 2 \phi}. \]

With these modified values, the reflection and transmission coefficients due to incident \( qP \) wave, corresponding to the plane interface between two dissimilar isotropic elastic half-spaces are given by the equation (2.25) which are exactly the same as obtained by Ben-menahen and Singh (1981) for the concerned case.
2.7 Numerical results and discussion

In this section, we shall compute the reflection and transmission coefficients at different angles of incidence \( \theta_0 \) for the special type of interface considered in the Section - 2.5. First, we shall compute the angles of propagation of various reflected and transmitted waves for a given angle of incidence. To find out the angle \( \phi \) for reflected \( qP - \) wave, the angle \( \delta \) for transmitted \( qP - \) wave and angle \( \gamma \) for transmitted \( qSV - \) wave, for a given value of angle of incidence, we shall make use of Snell’s law given by equation (2.10), in which we shall take the apparent velocity \( c_a = c/p_2 \), \( c \) being the unknown phase speed. We define the dimensionless apparent velocity \( \bar{c} \) through \( \bar{c} = \frac{c_a}{\beta} = \frac{c}{p_2\beta} \). With these, equation (1.40) becomes

\[
\bar{c}^4 - (\bar{U} + \bar{Z})\bar{c}^2 + (\bar{U}\bar{Z} - \bar{V}^2) = 0, \tag{2.36}
\]

where

\[
\bar{U} = \frac{U}{c_{44}p_2^2} = p_0^2 + 2\bar{c}_{24}p_0 + \bar{c}_{22}, \quad \bar{V} = \frac{V}{c_{44}p_2^2} = \bar{c}_{34}p_0^2 + (1 + \bar{c}_{23})p_0 + \bar{c}_{24},
\]

\[
\bar{Z} = \frac{Z}{c_{44}p_2^2} = \bar{c}_{33}p_0^2 + 2\bar{c}_{34}p_0 + 1, \quad p_0 = \frac{p_3}{p_2}, \quad \bar{c}_{ij} = \frac{C_{ij}}{c_{44}}, \quad \beta = \sqrt{\frac{c_{44}}{\rho}}.
\]

From equation (2.36), for a given value of \( p_0 \) there are two roots of \( \bar{c}^2 \) corresponding to speed of propagation of \( qP - \) and \( qSV - \) waves and for a given value of \( \bar{c} \), there are two roots of \( p_0 \) corresponding to angle of propagation of \( qP - \) and \( qSV - \) waves. Substituting the values of \( \bar{U} \), \( \bar{V} \) and \( \bar{Z} \) into equation (2.36), we obtain

\[
g_0p_0^4 + g_1p_0^3 + g_2p_0^2 + g_3p_0 + g_4 = 0, \tag{2.37}
\]

where \( g_0 = \bar{c}_{33} - \bar{c}_{34}^2 \), \( g_1 = 2(\bar{c}_{24}\bar{c}_{33} - \bar{c}_{23}\bar{c}_{34}) \), \( g_2 = 1 + \bar{c}_{22}\bar{c}_{33} + 2\bar{c}_{24}\bar{c}_{34} - (1 + \bar{c}_{23})^2 - (1 + \bar{c}_{33})^2 \), \( g_3 = 2[\bar{c}_{22}\bar{c}_{34} - \bar{c}_{23}\bar{c}_{24} - (\bar{c}_{24} + \bar{c}_{34})\bar{c}_0^2] \), \( g_4 = \bar{c}^4 - (1 + \bar{c}_{22})\bar{c}_0^2 + \bar{c}_{22} - \bar{c}_{24}^2 \).

Transforming the above equation by using the transformation \( q = \frac{1}{p_0} = \frac{p_3}{p_2} \), we obtain

\[
g_4q^4 + g_3q^3 + g_2q^2 + g_1q + g_0 = 0. \tag{2.38}
\]

For a given angle of incidence of a plane \( qP - \) wave (\( p_2 = \sin \theta_0, p_3 = -\cos \theta_0 \)), one can calculate from equation (2.36) two roots as shown in (1.40), a root \( \bar{c}_P^2 \) (corresponding to
$qP$-wave and other root $c_2^2$ (corresponding to $qSV$-wave). Now, using this value of $c_1^2$ into equation (2.37), one would obtain two roots of $q^2$. One of the roots of $q^2$ would give the direction of propagation of reflected $qP$-wave and the other root will give that of $qSV$-wave in the medium $H$ (the medium in which the plane $qP$-wave is made incident at the interface). Obviously, we shall consider only positive roots of $q$. In this way, the two positive roots of equation (2.38) will give the directions of the reflected $qP$- and $qSV$-waves. Let $q_3$ and $q_4$ be the two positive roots such that $q_3 > q_4$. Then, the directions of propagation of the regularly reflected $qP$- and $qSV$-waves are respectively given by $\tan^{-1}q_3$ and $\tan^{-1}q_4$ (see Singh and Khurana, 2001). Similarly, to find the directions of propagation of the transmitted $qP$- and $qSV$-waves, an equation similar to (2.38) is set up for the medium $H'$ by making corresponding changes to $g_0$, $g_1$, $g_2$, $g_3$ and $g_4$ appropriately. The directions of propagation of the regularly transmitted $qP$- and $qSV$-waves are respectively obtained through $\tan^{-1}q_3'$ and $\tan^{-1}q_4'$, where $q_3'$ and $q_4'$ are the positive roots of the relevant equation concerning the medium $H'$ such that $q_3' > q_4'$. Once these directions of regularly reflected and transmitted waves are determined, one can find out the directions of irregularly reflected and transmitted waves through Spectrum theorem given by relations in (2.11).

In order to calculate these directions of all reflected and transmitted waves and hence the reflection and transmission coefficients of all reflected and transmitted waves, we take the following values of relevant elastic parameters.

For monoclinic half-space $H$ (Lithium Tantalate):

\[
c_{24} = 0.11 \times 10^{11} N/m^2, \quad c_{23} = 0.80 \times 10^{11} N/m^2, \quad c_{34} = 0, \quad c_{44} = 0.94 \times 10^{11} N/m^2, \\
c_{33} = 2.75 \times 10^{11} N/m^2, \quad c_{22} = 2.33 \times 10^{11} N/m^2, \quad \rho = 7400 \ kg/m^3.
\]

For monoclinic half-space $H'$ (Lithium Neobate like material):

\[
c_{24}' = -0.09 \times 10^{11} N/m^2, \quad c_{23}' = 0.75 \times 10^{11} N/m^2, \quad c_{34}' = 0, \quad c_{44}' = 1.06 \times 10^{11} N/m^2, \\
c_{33}' = 2.45 \times 10^{11} N/m^2, \quad c_{22}' = 2.03 \times 10^{11} N/m^2, \quad \rho' = 4700 \ kg/m^3 \text{ and corrugation parameter } (pd) \text{ and frequency parameter } (\omega^2) \text{ are taken as } pd = 0.000125 \text{ and } \omega^2 = 0.01, \text{ whenever not mentioned.}
\]

The variation of directions of propagation of the regularly reflected and regularly transmitted waves with the angle of incidence are shown in Figure 2.2. The directions of propagation of the regularly reflected and regularly transmitted $qP$- and $qSV$-waves increase with the increase of the angle of incidence. It is found that the directions of propagation of reflected $qP$- and $qSV$-waves exist only for the angle of incidence
in the range $0^\circ < \theta_0 \leq 81^\circ$, while that of the transmitted $qP-$ and $qSV-$ waves exist for

![Figure 2.2: Variation of the angle of propagation of regularly reflected and transmitted waves with angle of incidence, $\theta_0$.](image)

the range $0^\circ < \theta_0 < 90^\circ$. Figures 2.3 - 2.6 show the variation of the modulus of reflection and transmission coefficients of the regular and irregular waves with the angle of incidence. In Figure 2.3, Curve - I shows the variation of the reflection coefficient $R_{pp}$ of regular $qP-$ wave, which is nearly zero near normal incidence and then increases monotonically with increase of the angle of incidence $\theta_0$ attaining maximum value near grazing incidence. Curves - II and III show the variation of the coefficients $R_p$ and $R_p^l$ with the angle $\theta_0$. The values of coefficient $R_p^l$ increase with $\theta_0$ upto $\theta_0 = 38^\circ$, then go on decreasing till vanishes at $\theta_0 = 52^\circ$ and then again increases upto the maximum values at $\theta_0 = 62^\circ$, thereafter decreases till $\theta_0 = 70^\circ$. The coefficient $R_p^l$ attains local maxima at $\theta_0 = 38^\circ$ & $52^\circ$, while outside the neighborhood of these angles of incidence, its values are very small. In Figure 2.4, Curve - I shows the variation of reflection coefficient $R_{ps}$ with $\theta_0$. We see that it starts from certain value and increases very slowly.
Figure 2.3: Variation of the reflection coefficients of reflected $qP$-waves with angle of incidence, $\theta_0$.

Figure 2.4: Variation of the reflection coefficients of reflected $qSV$-waves with angle of incidence, $\theta_0$.

Figure 2.5: Variation of the transmission coefficients of transmitted $qP$-waves with angle of incidence, $\theta_0$.

Figure 2.6: Variation of the transmission coefficients of transmitted $qSV$-waves with angle of incidence, $\theta_0$. 
with $\theta_0$ till $\theta_0 = 28^0$, thereafter it decreases with further increase of the angle of incidence and approaches to the value zero at $\theta_0 = 56^0$. Afterwards, it increases with increase of the angle of incidence attaining its maximum value near grazing incidence. Behaviors of the coefficients $R^1_{pp}$ and $R^1_{ps}$ with $\theta_0$ are similar as the behavior of the coefficients $R^1_p$ and $R^1_{p'}$ with $\theta_0$. In Figure 2.5, Curve - I shows that the transmission coefficient $T_{pp}$ starts from certain value at the normal incidence, thereby increasing a little bit up to certain value of the angle of incidence and then it decreases with the further increase of the angle of incidence. Curve - I in Figure 2.6 shows that the variation of the transmission coefficient $T^1_{av}$, is parabolic in the range $0^0 < \theta_0 < 46^0$ and thereby increasing with the increase of the angle of incidence. It can be noticed from Figures 2.3 and 2.5 that the behaviors of the coefficients $R_{pp}$ and $T_{pp}$ (depicted by the Curve-I) with angle of incidence, is opposite, while from Figure 2.4 and 2.6, one can note that the behaviors of the coefficients $R_{ps}$ and $T_{ps}$ with angle of incidence, are similar. Moreover, we observe from Figures 2.3 - 2.6 that the behaviors of the coefficients corresponding to the Curve -II and corresponding to the Curve - III are similar. These coefficients in Curves -II and III are depicted after magnifying 10 times their original values. It can be observed from these figures that the amplitude ratios of the regular waves are greater than the amplitude ratios of irregular waves. The peaks appearing in Curves - II and III, in these figures are due to two reasons (i) there occur change in sign in the amplitude ratios at these particular angles of incidence and (ii) they have been depicted after magnifying 10 times their original values for visibility.

The variation of the modulus of reflection and transmission coefficients of the irregularly reflected and irregularly transmitted $qP-$ and $qSV-$ waves with the corrugation parameter $pd$ are shown through Figures 2.7 - 2.10 when the angle of incidence, $\theta_0 = 25^0$. It is clear from these figures that the coefficients corresponding to all irregular waves increase linearly with the increase of corrugation parameter, but with different rates. However, the coefficients corresponding to regular waves are independent of corrugation parameter as was expected beforehand. Figures 2.11 - 2.14 show the variation of the modulus of reflection and transmission coefficients corresponding to the reflected and transmitted $qP-$ and $qSV-$ waves with the frequency parameter, $\frac{\omega}{\omega}$ when the angle of incidence, $\theta_0 = 25^0$. We see that the amplitude ratios corresponding to the
Figure 2.7: Variation of the reflection coefficients of reflected $qP$ waves with corrugation parameter ($pd$), when $\theta_0 = 25^\circ$.

Figure 2.8: Variation of the reflection coefficients of reflected $qSV$ waves with corrugation parameter ($pd$), when $\theta_0 = 25^\circ$.

Figure 2.9: Variation of the transmission coefficients of transmitted $qP$ waves with corrugation parameter ($pd$), when $\theta_0 = 25^\circ$.

Figure 2.10: Variation of the transmission coefficients of transmitted $qSV$ waves with corrugation parameter ($pd$), when $\theta_0 = 25^\circ$. 

80
Figure 2.11: Variation of the reflection coefficients of reflected \( qP \)-waves with frequency parameter \( \frac{E}{\omega} \), when \( \theta_0 = 25^\circ \).

Figure 2.12: Variation of the reflection coefficients of reflected \( qSV \)-waves with frequency parameter \( \frac{E}{\omega} \), when \( \theta_0 = 25^\circ \).

Figure 2.13: Variation of the transmission coefficients of transmitted \( qP \)-waves with frequency parameter \( \frac{E}{\omega} \), when \( \theta_0 = 25^\circ \).

Figure 2.14: Variation of the transmission coefficients of transmitted \( qSV \)-waves with frequency parameter \( \frac{E}{\omega} \), when \( \theta_0 = 25^\circ \).
Figure 2.15: Variation of the reflection coefficient $R_{\phi_1}^1(\times 10)$ at an angle $\theta_1$ with the angle of incidence for different values of $pd$.  

Figure 2.16: Variation of the reflection coefficient $R_{\phi_1}^1(\times 10)$ at an angle $\phi_1$ with the angle of incidence for different values of $pd$.  

Figure 2.17: Variation of the transmission coefficient $T_{\phi_1}^1(\times 10)$ at an angle $\delta_1$ with the angle of incidence for different values of $pd$.  

Figure 2.18: Variation of the transmission coefficient $T_{\phi_1}^1(\times 10)$ at an angle $\gamma_1$ with the angle of incidence for different values of $pd$.  

82
Chapter 2

Figure 2.19: Variation of $R^p_\theta(\times 10)$ at an angle $\theta_1$ with the angle of incidence for different values of $FR(= \frac{E_2}{\omega})$.

Figure 2.20: Variation of $R^\|_\gamma(\times 10)$ at an angle $\phi_1$ with the angle of incidence for different values of $FR(= \frac{E_2}{\omega})$.

Figure 2.21: Variation of $T^p_\phi(\times 10)$ at an angle $\delta_1$ with the angle of incidence for different values of $FR(= \frac{E_2}{\omega})$.

Figure 2.22: Variation of $T^\perp_\gamma(\times 10)$ at an angle $\gamma_1$ with the angle of incidence for at different values of $FR(= \frac{E_2}{\omega})$. 
regular waves are independent of frequency parameter, while the amplitude ratios corresponding to the irregular waves are significantly influenced with the frequency parameter. One can see from these figures that the coefficients $R_p^1$, $R_{sv}^1$, $T_p^1$ and $T_{sv}^1$ decrease with the increase of the frequency parameter, while the coefficients $R_p^1$, $R_{sv}^1$, $T_p^1$ and $T_{sv}^1$ decrease with the increase of $\frac{pd}{\omega}$ up to certain value of $\frac{pd}{\omega}$, then go on increasing thereby making a maximum value at $\frac{pd}{\omega} = 0.12$ and thereafter, they decrease with $\frac{pd}{\omega}$. The variation of the reflection coefficients ($R_p^1$, $R_{sv}^1$) and transmission coefficients ($T_p^1$, $T_{sv}^1$) corresponding to the irregular waves with the angle of incidence at the different values of the corrugation and frequency parameters are depicted in Figures 2.15 - 2.22. It is noted that the reflection and transmission coefficients of the irregularly reflected and irregularly transmitted $qP-$ and $qSV-$ waves increase with the increase of the corrugation parameter, while they decrease with the increase of the frequency parameter almost at every angle of incidence. All these coefficients have similar behavior with the angle of incidence for different values of the $pd$ and $\frac{pd}{\omega}$. Thus, the reflection and transmission coefficients corresponding to the irregular waves are functions of $pd$,
Figure 2.25: Variation of $T_{\gamma}^1(\times 10)$ at an angle $\theta'_1$ with the angle of incidence for different values of $pd$.

Figure 2.26: Variation of $T_{\gamma}^1(\times 10)$ at an angle $\gamma'_1$ with the angle of incidence for different values of $pd$.

Figure 2.27: Variation of $R_{\gamma}^1(\times 10)$ at an angle $\theta'_1$ with the angle of incidence for different values of $FR(=\frac{pd}{\omega})$.

Figure 2.28: Variation of $R_{\gamma}^1(\times 10)$ at an angle $\phi'_1$ with the angle of incidence for different values of $FR(=\frac{pd}{\omega})$. 
Similarly, from Figures 2.23 - 2.30, we have found that the coefficients $R_{\nu'}$, $R_{\nu''}$, $T_{\nu'}$ and $T_{\nu''}$ corresponding to the irregular waves have similar behavior with the angle of incidence for different values of corrugation and frequency parameters. Moreover, the values of these coefficients are found to increase with the increase of $pd$, while their values are found to decrease with the increase of $\frac{p^2}{\omega^2}$. 