CHAPTER 5

THEORITICAL COMPARISION
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5.1 MONTE CARLO TECHNIQUE

Monte Carlo method is a versatile computing technique for determining the macroscopic properties of a system whose microscopic behaviour consists of a series of random choices from probability distributions. Monte Carlo method is a statistical method, the amount of computing required for a precise estimate of the answer must be large with the rapid speed of computer, it is possible to obtain convenient solutions for many values of parameters. The selection of resonance parameters for total cross section data has been accomplished by Monte Carlo technique, especially for case in which more than one energy level contributes to the cross section. Flow chart for Monte Carlo calculation of the neutron transmission of an infinite slab is shown in the Fig.(5.01).

In the transmission measurements, it is assumed that elastic scattering distribution is isotropic, inelastic scattering cross section is zero and the scattering nuclei must have infinite mass so that there is no energy loss in the collision. The slab of material with mean free path $\lambda$ is perpendicular to the z-axis, the front face of the slab
lying in the xy-plane and the incident neutrons travelling in the positive z-direction.

A flow chart is a convenient method of indicating the computing procedure. The expressions in the rectangular boxes describe the computation done at that particular section of the code.

In the transmission experiment, the initial xy coordinates of the neutron do not affect its future behaviour, the incident neutrons are taken to be directed along the z-axis. The value of N is initially set to zero and is advanced one unit as each new particle is introduced. When N becomes equal to N*, the selected number of histories to be calculated, the computer is instructed to print the number P which succeeded in passing through the slab. The transmission will be equal to P/N*. The distance to the first collision is taken to be \( d = -\lambda \ln R \), where R is the random number between 0 and 1. Using this scheme the distribution of distances to collision will be exponential with mean free path \( \lambda \).

When the position of the collision is known, the z-coordinate is tested to observe, if the particle is inside the slab. If it is passed through the material, z will be greater than the thickness T of the slab and an additional unit is added to the register containing P. If the \( Z < 0 \), the particle has been reflected at the front face and is lost.
If the point of collision is within the slab, the test: \( R : \mu \lambda \) is made to decide whether the particle was absorbed or not. If \( R \leq \mu \lambda \), a new source particle is selected.

The Monte Carlo method becomes increasingly useful, particularly, if complicated shielding geometries and loss of neutron energy in collisions are introduced. To obtain the results with required precision, some modifications are needed. The following are the two important modifications to Monte Carlo calculation.

**Weighing:**

It is one of the simplest modifications of Monte Carlo calculation to obtain reasonable accuracy. In this method the particle is forced to act in a preferred manner rather than in the manner required by the physical probabilities. The deviation of the behaviour from reality is corrected to assigning to each particle a weight such that at the end of the problem, the sum of the weights is taken rather than the sum of the particles to obtain correct answer.
In the transmission experiment, if the particle is absorbed as a result of the random number being less than \( \mu \lambda \), the information recorded during previous collisions is wasted. This difficulty is eliminated by weighting. The weight is initially set to 1 and at each collision the weight is reduced by a factor \( 1 - \mu \lambda \). At the end of the procedure a tally is made for the weights of escaping particles rather than their numbers.

**Use of expectation values:**

The modification use of expectation values is applicable when the intensity of the neutron beam passing through the slab is regarded as a sum contributed by neutrons having many collisions within the slab. The Flow chart to compute the position of each collision in the slab is shown in the Fig. (5.02). In this modification instead of making tallies of success at the time, a particle reaches \( Z=T \), the probability of reaching \( Z=T \) is recorded at each collision point with this change the program computes for each collision the probability that the neutron will emerge before the next collision and these probabilities are added to form expectation value \( v_e \). In this way observation is tallied each time a particle points in a direction which may lead, in a single flight through the slab.
Fig. 5.02
Monte Carlo technique simulates experiment by neutron generation events such as scattering or absorption with a frequency proportional to their probability of occurrence. However unlike real experiment, the neutron histories can be followed individually so that neutrons which are multiply scattered can be separated from those which are singly scattered. The fraction of multiple scattering in the detected intensity can then be calculated.

5.2 STATISTICS OF NEUTRON TRANSMISSION

Statistical methodology depends essentially on mathematical tools for its development. In order to make problems analytically tractable, number of assumptions had to be made on the nature of the data, models formulated had to be fairly simple even if unrealistic and criteria for inference had to be reasonably manageable. The Monte Carlo technique is used to analyze huge data sets, formulate highly complex models when called for formulate criteria’s not necessarily amenable to analytically tractable and easy computable solutions. The increasing use of randomization techniques and the emergence of resampling method such as boot strap and other cross validation method increase the application of Monte Carlo techniques\(^{(5.01, 5.02)}\).
Let $\mu (E)$ be the absorption coefficient as a function of energy $E$.

Consider a slab of thickness 't' split it up into number of smaller slabs of thickness $dt$ such that

$$Ndt = t \quad \text{(5.01)}$$

$$N \rightarrow \infty$$

$$dt \rightarrow 0$$

Made the assumption that what happens in a given slab of thickness $dt$ is independent of what happens in other smaller slabs. The probability of a neutron being absorbed in a thickness $dt$ is given by

$$P_a(dt) = \mu \ dt \quad \text{(5.02)}$$

The survival probability of the neutron is

$$P_s(dt) = [1 - P_a(dt)] = 1 - \mu \ dt \quad \text{(5.03)}$$

Over a thickness $t$, the total survival probability is given by

$$P_s(t) = N \rightarrow \infty (1 - \mu \ dt) (1 - \mu \ dt) \ldots (1 - \mu \ dt) = e^{-\mu t} \quad \text{(5.04)}$$

$$dt \rightarrow 0$$

If there are $\phi_0$ neutrons incident, the number of neutron expected to survive is
\[ \phi = \phi_0 e^{-\mu t} \]  \hspace{1cm} (5.05)

Now consider the case \( \mu = \mu_1 + \mu_2 + \mu_3 \)

Where \( \mu_1, \mu_2 \) and \( \mu_3 \) stand for different places. The probability of a given neutron to be absorbed at place 1 over a length \( \tau \). Let a neutron survive over the length \( \tau \), the probability of survive is

\[ P_s (\tau) = e^{\mu \tau} \]  \hspace{1cm} (5.06)

But let the neutron get absorbed in an interval \( \tau \) and \( \tau + d\tau \). The probability of the neutron being absorbed is

\[ P_1 (\tau) = P_s (\tau) P_a (d\tau) \]

\[ = e^{\mu \tau} \cdot \mu_1 d\tau \]  \hspace{1cm} (5.07)

The total probability of the neutron to be absorbed anywhere between \([0, t]\) is

\[ P_a^{(1)} (t) = \int_0^t e^{\mu \tau} \cdot \mu_1 d\tau \]

\[ = \frac{\mu_1}{\mu} (1 - e^{-\mu t}) \]  \hspace{1cm} (5.08)
Out of $\phi_0$ neutrons, $\phi$ neutrons survive, the number of neutrons getting absorbed is

$$\phi_0 - \phi = \phi_0 \left(1 - e^{-\mu t}\right) \quad (5.09)$$

Out of $\phi_0$ neutrons, the number of neutrons getting absorbed at place 1 is

$$\phi_a^{(1)} = \phi_0 \left(1 - e^{-\mu_1 t}\right) \quad (5.10)$$

at place 2

$$\phi_a^{(2)} = \phi_0 \left(1 - e^{-\mu_2 t}\right) \quad (5.11)$$

at place 3

$$\phi_a^{(3)} = \phi_0 \left(1 - e^{-\mu_3 t}\right) \quad (5.12)$$

From equations (5.10), (5.11) and (5.12)

$$\phi_a^{(1)} + \phi_a^{(2)} + \phi_a^{(3)} = \phi_0 \left(1 - e^{-\mu t}\right) = \phi_0 - \phi \quad (5.13)$$

Therefore the number of neutrons survived or transmitted is
\[ \phi = \phi_0 e^{-\mu t} \quad (5.14) \]

One of the most easy methods used for making shielding calculations is based on the concept of removal cross-section. It is the effective probability for the removal of neutrons from the incident neutron beam \(^{[5.03],[5.04]}\).

Fig. (5.03) shows the point source geometry used for the neutron transmission measurements.

Table (5.01) shows the comparison between the values of macroscopic removal cross section measured using threshold foil detectors and the values of macroscopic removal cross section measured using Monte Carlo technique for different shield materials. The experimental and calculated values are in agreement with the calculated values within the experimental error.
$S_d$ - Disc source (Disc shaped)
$D_p$ - Point detector, $C$ - Concrete

Fig. 5.03 Schematic Diagram of the Geometry used in Computations
<table>
<thead>
<tr>
<th>Material</th>
<th>Aluminium $^{27}$Al(n,p) $^{27}$Mg</th>
<th>Iron $^{56}$Fe(n,p) $^{56}$Mn</th>
<th>Aluminium $^{27}$Al(n,$\alpha$) $^{24}$Na</th>
<th>Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron $(\rho = 7.87 \text{ gm/cm}^3)$</td>
<td>0.1723</td>
<td>0.1686</td>
<td>0.1658</td>
<td>0.1764</td>
</tr>
<tr>
<td>Lead $(\rho = 11.35 \text{ gm/cm}^3)$</td>
<td>0.1117</td>
<td>0.1126</td>
<td>0.1134</td>
<td>0.1158</td>
</tr>
<tr>
<td>Polypropylene $(\rho = 0.91 \text{ gm/cm}^3)$</td>
<td>0.1024</td>
<td>0.0986</td>
<td>0.0964</td>
<td>0.1017</td>
</tr>
<tr>
<td>Polyacrylic acid $(\rho = 1.34 \text{ gm/cm}^3)$</td>
<td>0.0794</td>
<td>0.0778</td>
<td>0.0767</td>
<td>0.0783</td>
</tr>
<tr>
<td>Concrete $(\rho = 2.40 \text{ gm/cm}^3)$</td>
<td>0.0783</td>
<td>0.0762</td>
<td>0.0734</td>
<td>0.0775</td>
</tr>
</tbody>
</table>

Table (5.01)

Experimental & Calculated Values of Macroscopic Removal Cross-Section For Different Materials